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PHYSICS-INFORMED NEURAL NETWORK MODELING OF NONLOCAL CROWD DYNAMICS FOR EVACUATION SCENARIOS

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We investigate a nonlocal continuum model of crowd dynamics using a physics-informed neural network approach. The crowd is described by a system of nonlinear conservation laws in which the flux incorporates advection, diffusion, and nonlocal interaction terms accounting for density-dependent motion and limited perception of surrounding agents. Nonlocal effects are modeled through spatial convolutions with smooth kernels, enabling agents to respond to averaged density gradients rather than purely local information. The governing system of partial differential equations is solved using a physics-informed neural network known as PINN, which approximates the solution over the entire space–time domain while enforcing the physical constraints through automatic differentiation. The nonlocal interaction terms are implemented in a stable discrete convolution form, ensuring numerical robustness during training. The approach is demonstrated on the interaction of two pedestrian groups moving in opposite directions in a one-dimensional corridor. The results exhibit the formation and propagation of density fronts, the gradual merging of flows, and the emergence of stable mixed zones. A characteristic feature of the solution is the partial interpenetration of the groups without rigid collisions, reflecting realistic collective motion. To validate the method, the PINN solution is compared with a reference finite-difference scheme based on a Rusanov flux. Qualitative agreement is observed in front structure and mixing dynamics, while quantitative deviations in key characteristics remain within a few percent. A systematic parameter study shows that the PINN-based solution remains stable under variations of advection velocity, diffusion coefficient, and nonlocal interaction radius, in contrast to the finite-difference scheme, which exhibits strong stability limitations. These results demonstrate that PINN provides a robust and physically consistent tool for modeling nonlinear nonlocal crowd dynamics.

Keywords: *Physics-informed neural networks; Nonlocal crowd dynamics; Evacuation modeling; Advection-diffusion; Convolution kernels; Robustness; Complex geometries*

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1. INTRODUCTION

With the increasing density of urban development, the growing number of mass events, and the risks associated with emergencies, there is an increasing need for effective methods to analyze collective motion and evacuation processes. Such problems require mathematical models that account for interactions among agents, the influence of environmental geometry, and the temporal evolution of density. From a mathematical perspective, collective motion can be described within a continuum framework using conservation laws for density. Such models have been developed since the 1970s [1] and have been successfully applied to describe pedestrian and traffic flows [2, 3, 4]. In more advanced formulations, nonlocal interactions are taken into account, where the motion is determined not only by the local density but also by its distribution in a surrounding neighborhood. This leads to the appearance of terms represented as integral convolutions with smooth kernels [5]. These models allow the incorporation of effects such as limited perception and the tendency to avoid regions of high density. Despite their physical consistency, nonlocal models pose significant challenges for numerical solution. The presence of integral terms increases computational complexity and complicates the stability of classical numerical schemes. Moreover, traditional discretization-based methods scale poorly when extended to higher-dimensional problems [6, 7].

In recent years, PINN has emerged as an alternative approach for solving differential equations [8], in which physical laws are directly embedded into the loss function of the neural network. This approach enables the computation of solutions without explicit domain discretization and naturally incorporates boundary and initial conditions. Further developments of the method focus on improving training stability and solution quality [9, 10]. Nevertheless, the application of PINN to nonlinear equations with nonlocal interactions remains insufficiently explored. In particular, the presence of convolution terms requires a dedicated implementation within the PINN framework and may significantly affect the stability and accuracy of the resulting solutions. In the present work, a crowd dynamics model is used as a representative example to investigate the applicability of PINNs to nonlocal equations of this type. One of the objectives is to assess the stability of the computational method and the reliability of the obtained solutions for such problems.

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2. MODEL

We model the crowd as a continuum with density $\rho(t,x)$. When many agents of the same type move in similar directions, their motion can be treated statistically, and the overall crowd behaves like a compressible fluid. [1, 2, 3, 4] Accordingly, we adopt a conservation law (continuity equation)

$$\partial_t \rho + \nabla \cdot \mathbf{j} = 0, \tag{1}$$

where $\mathbf{j} = \rho \mathbf{u}$, and $\mathbf{u}$ is the velocity vector field. In contrast to an exact description involving a number of equations equal to the number of agents, this hydrodynamic approach allows for describing collective motion without tracking each person individually and significantly simplifies the mathematics of the problem, reducing the number of equations of motion to one.

To correctly represent the velocity of agents in a crowd, the direction and the magnitude of motion should be treated independently, in accordance with [5], was demonstrated that the dependence $v(\rho) = v_{\text{free}}(1 - \rho/\rho_{\max})$ reflects real behavior in a traffic jam. Here, $\rho_{\max}$ is associated with the finite size of the agents, and v_{free} is the maximum speed of free movement of the agents in the absence of crowding. In the considered scenario (in the absence of a crowd, agents, optimizing their direction of movement), move in the preferred direction $\mathbf{e}(x)$, which is unitary vector. The boundary conditions of movement in the corridor are $\mathbf{e} = \pm 1$, depending on whether the movement is left or right.

An attempt to avoid high-density areas is described by a vector $\mathbf{i}[\rho(x)]$. To model this behavior, it was proposed [5] to move along $-\nabla\rho$. In addition, in order to account for the radius of the agent's field of view, where it estimates the density, the density is averaged over a certain neighborhood with a smooth kernel η, which leads us to the following expression for $\mathbf{i}[\rho(x)]$

$$\mathbf{i}[\rho(x)] \; = \; -\,\varepsilon \, \frac{\nabla(\rho * \eta)(x)}{\sqrt{1 + \left\|\nabla(\rho * \eta)(x)\right\|^2}}\,, \qquad (\rho * \eta)(x) = \int_{-L}^{L} \rho(y)\, \eta(x-y)\, \mathrm{d}y, \tag{2}$$

where $*$ denotes spatial convolution with a kernel in the vicinity L. The coefficient $\varepsilon > 0$ controls the reaction strength: larger values correspond to stronger avoidance of crowded regions, while smaller values produce less sensitive trajectories. The denominator in (2) limits the magnitude of the deviation, preventing the reaction from becoming infinitely large. The natural presence of velocity dispersion among agents in a crowd is described by a diffusive flux term $\mathbf{q} = -D\,\nabla\rho$ Fick's law. Thus, total flux takes the form:

$$\mathbf{j} = \rho\, v(\rho)\, \big(\mathbf{e}(x) + \mathbf{i}[\rho(x)]\big) + \mathbf{q}, \tag{3}$$

being the non-linear expression in ρ. Totally, equation (1) for one type of agents takes the following form:

$$\partial_t \rho + \nabla\cdot\Big(\rho\, v(\rho)\big(\mathbf{e} + \mathbf{i}[\rho]\big)\Big) = D\,\Delta\rho, \tag{4}$$

here and further we omitt the notation of the dependence on (x).

The process of mixing flows of agents of two types is described by a system of equations of form (4). The mutual influence between agents of different types is accounted by the additional term in (3) of the form:

$$\mathbf{i}_k[\rho_l] \; = \; -\,\varepsilon_{k,l}\, \frac{\nabla(\rho_l * \eta_k)}{\sqrt{1 + \left\|\nabla(\rho_l * \eta_k)\right\|^2}}\,, \tag{5}$$

where $\varepsilon_{k,l}$ controls how strongly each group reacts to the local density of another group, and η_k defines the perceptual radius of the agents of type k where k and any other sub-indexes take values 1 and 2. Thus, the complete system describing the motion of the interacting groups is a nonlinear, nonlocal system of partial differential equations (PDEs) and has the following form:

$$\begin{cases} \partial_t \rho_1 + \nabla\cdot\Big(\rho_1\, v_1(\rho)\big(\mathbf{e}_1 + \mathbf{i_1}[\rho_1] + \mathbf{i_1}[\rho_2]\big)\Big) = D\,\Delta\rho_1, \\ \partial_t \rho_2 + \nabla\cdot\Big(\rho_2\, v_2(\rho)\big(\mathbf{e}_2 + \mathbf{i_2}[\rho_1] + \mathbf{i_2}[\rho_2]\big)\Big) = D\,\Delta\rho_2, \end{cases} \tag{6}$$

with $\rho = \rho_1 + \rho_2$ with Neumann boundary conditions corresponding to free outflow of agents. Developing PINNs for such systems is a central motivation of this work.

3. PINN COMPUTATION

The system equations (6) is a strongly nonlinear system of PDEs with nonlocal integral terms. We employ a PINN approach, originally introduced in [5] to solve it. The analysis is carried out on a space-time domain $0 \le x \le L$ with $L = 10$, and for times $0 \le t \le 9$. All variables and parameters in the model are expressed in dimensionless form. Before being passed into the neural network, the input coordinates are transformed using a Fourier feature embedding, which maps them to a higher-dimensional space using sinusoidal functions in order to improve the representation of sharp spatial and temporal variations. This transformation affects only the neural network input representation and does not alter the governing equations. The maximum frequency of the Fourier features is set to $N_{\text{freq}} = 8$, which provides sufficient resolution of high-frequency solution components while maintaining training stability and avoiding unnecessary high-frequency modes that do not improve accuracy.

Here we present the implementation of the network consisting of 6 hidden layers with 256 neurons each, using the `tanh` activation function. The output layer contains two neurons corresponding to the two densities, and applies the `softplus` activation to ensure non-negativity of the predicted densities. All derivatives entering the equations are computed using TensorFlow's automatic differentiation. Direct integration inside the PINN, unlike differentiation, leads to instability during training. Therefore, the integral operator is rewritten in discrete form as a precomputed convolution matrix. This allows the integral term to be evaluated as a matrix multiplication, which remains differentiable and stable within the TensorFlow framework.

At the initial time, the crowd is assumed to form a configuration with well-defined boundaries. Therefore, the initial density distributions of the two agent groups are prescribed as localized super-Gaussian profiles.

$$\rho_i(x,0) = A_i \exp\left(-\left|\frac{x-x_i}{\sigma_i}\right|^{2m_i}\right) \tag{7}$$

where ρ_1 corresponds to the faster group and ρ_2 to the slower one. The slower group is initially located ahead of the faster group, such that interaction occurs only after a finite time interval. The boundary conditions correspond to an open-boundary problem with zero inflow at the left boundary and a free outflow at the right boundary. Training is performed in stages. During the first stage, the network is fitted to the initial and boundary conditions and to the correct conserved mass, establishing a stable baseline. Subsequently, the network progressively enforces the governing equations. By adjusting the weights in the loss function, we control the relative importance of the physical terms and suppress non-physical deviations, which improves the stability of the training process.

The PINN approach preserves the correctness of the solution across a wide range of physical parameters, since it does not rely on step-by-step temporal advancement and is not constrained by conditions such as the Courant–Friedrichs–Lewy restriction [11]. The neural network approximates the solution over the entire domain simultaneously, which makes the method robust to changes in velocity, the radius of nonlocal interaction, and diffusion coefficients.

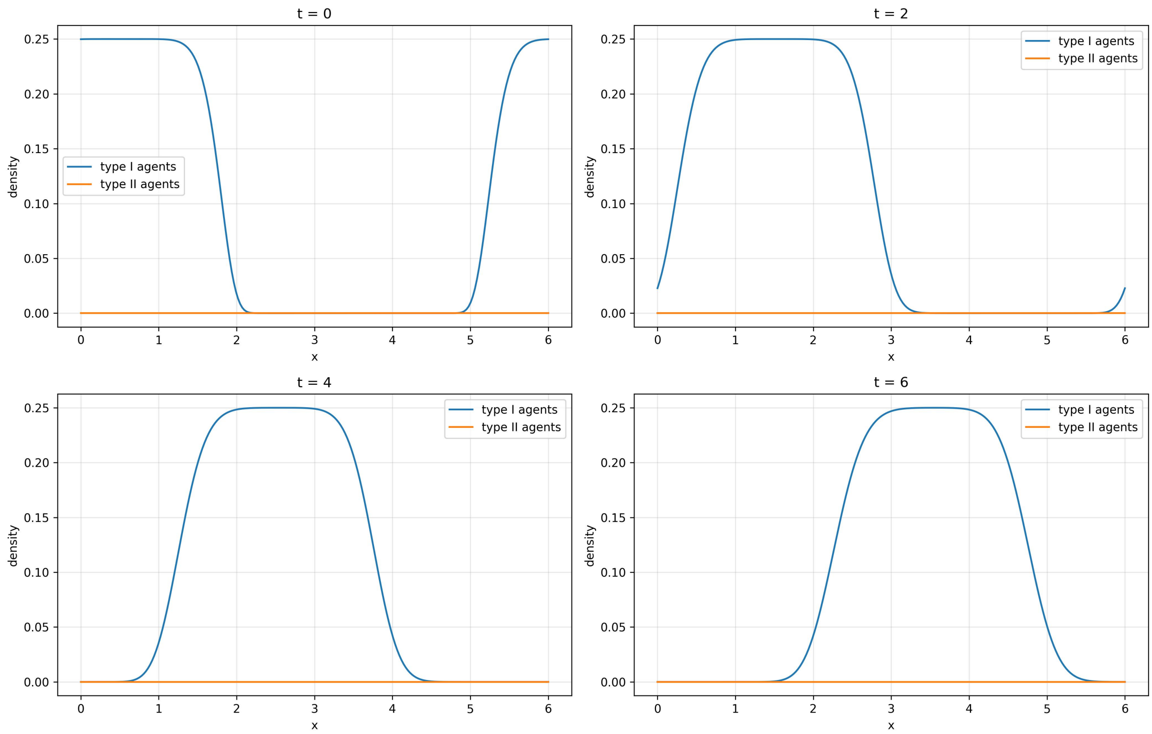

Figure 1. Motion of a single-type crowd

For clarity, and to illustrate the interaction effects between two groups, we first demonstrate that the numerical model correctly reproduces the dynamic of a single crowd moving along the corridor. The network is trained for 7000 epochs without further improvement in accuracy. Fig. 1 shows the spatial density profiles at time instants $t = 0.0$, 2.0, 4.0 and 6.0. The simulation was performed with parameters $A = 0.25$, $x = 0.6$, $\sigma = 1.2$, $m = 4$, and $v = 0.5$. As expected, the group moves as a whole.

Fig. 2 shows a qualitative change in the collective dynamics two groups of agents, that arises when a second is introduced. The second simulation was performed with parameters $A_{1,2} = 0.25$, $x_1 = 1.59$, $x_2 = 0.52$, $\sigma_{1,2} = 1.2$, $m_{1,2} = 4$, and $v_1 = 0.5$, $v_2 = 0.38$.

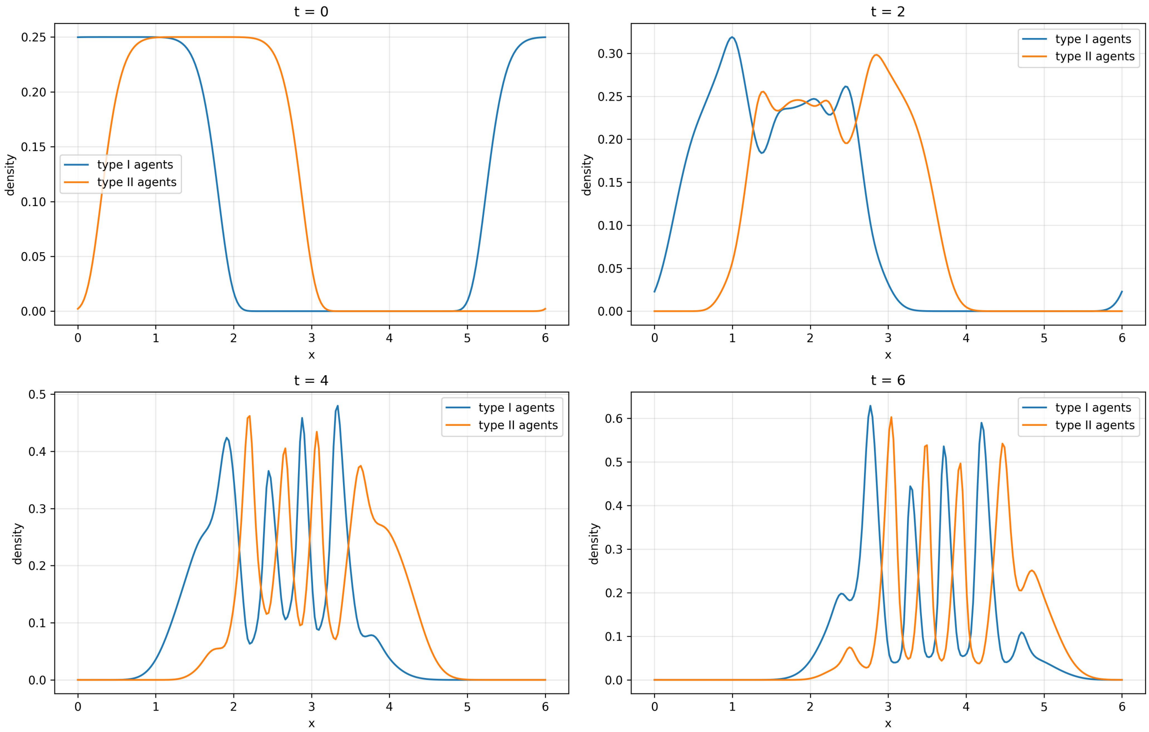

Figure 2. Spatial density profiles at selected time instants, illustrating the formation of quasiperiodic structures and interacting fronts. Both groups move in the positive direction of the x-axis, with the group of type I agents having a higher velocity.

The flow becomes fragmented into subgroups, and diffusive transport through low-density areas enables faster agents to overtake slower ones. As dispersion progresses, the overall density decreases, allowing unobstructed bypassing of slower agents and leading to the eventual disappearance of the periodic ordering. At the initial moment ($t = 0.0$), the faster group begins to catch up with the slower one. At the next stage ($t = 2.0$), the two groups already overlap significantly, and the difference in their preferred velocities leads to a deformation of the initial profiles and to the emergence of several local density maxima in the interaction zone. As the process develops further ($t = 4.0$), the model exhibits a characteristic phenomenon: the faster blue group gradually penetrates through the slower one, and quasiperiodic structures form. By $t = 6.0$ and beyond, this zone stabilizes into a broad segment that gradually shifts to the left, capturing mutual interpenetration and "pressing through" of the flows. The visualization demonstrates that the model reproduces key effects: gradual merging of flows, formation of compacted regions, and their subsequent displacement. The faster group overtakes the slower one and, after passing through it, forms a gap, a region of reduced total density. This behavior corresponds to the real behavior; people do not form a continuous medium; they can bypass, flow around, and partially pass through local clusters while maintaining their own speed, proving that social pressure and density allow for it.

The density fronts that emerge in the model have direct analogues in the physics of continuous media. Similar structures are formed, for example, in the exhaust plume of a jet engine: the hot jet, interacting with the surrounding air, creates zones of sharp density variation that move, stretch, flow around obstacles, and can partially "penetrate" through slower layers of the medium. These fronts are not rigid surfaces; they are dynamic transition zones where material moving at one speed enters the region of material moving at another speed. Similarly, in the crowd-motion model, the front represents a transition zone between two groups of people with different velocities and densities. It shifts, stretches, and can seep into the interior of the other group without an instantaneous equalization of speeds, reflecting the real kinematics of human movement.

Figure 3 presents the space-time diagrams of the densities $\rho_1(x, t)$ and $\rho_2(x, t)$. They demonstrate an evolutionary transition from an almost uniform distribution within each group to a quasiperiodic structure arising as a consequence of the interaction between the groups. As the process develops, alternating bands of higher and lower density are formed, with the density maxima of one group largely corresponding to the density minima of the other. This reflects a coordinated spatial modulation of the two interacting flows.

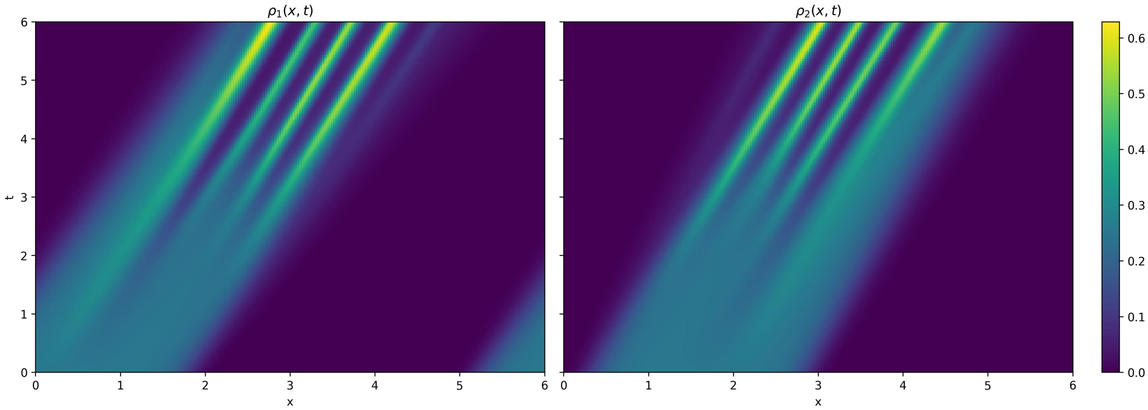

Figure 3. The space-time diagrams of the agent distributions in both groups

4. FDM VERIFICATION

1D, 2D problems without experimental data, with known parameters, are suited for solution by numerical methods such as the FDM. The model is discretized on a uniform one-dimensional grid, where the node values store the densities of the two groups. Thus, we deliberately considered simplefied problem in 1D to verify the PINN approach.

Both models, FDM and PINN, are run with identical initial conditions. The resulting density profiles, front positions, and mixing dynamics of the two groups are then compared. PINN reproduces the key features obtained by the FDM model: the shapes of the fronts and the interaction zones match visually, and the quantitative deviations remain small. To provide a qualitative assessment, we conducted simultaneous comparative runs for the FDM and PINN methods with initial conditions varying from $0.8 \cdot A_1$ to $1.2 \cdot A_1$. The relative discrepancies in the peak positions and front widths did not exceed 6% across all simulations considered. In addition, both models exhibited convergence under spatial grid refinement, which confirms the robustness of the obtained results and the consistency of the numerical implementation. Thus, the FDM framework serves as a reliable reference model against which the PINN demonstrates physically consistent behavior.

5. RESULTS ANALYSIS

The results demonstrate that PINN provides a stable and accurate reconstruction of solutions to nonlinear differential equations with nonlocal terms and remains operational over an extended parameter range. The obtained solution consistently reproduces the key dynamical features of two interacting pedestrian groups. At the initial moment the densities are fully separated, while during the evolution the faster group overtakes the slower one and forms a stable transition zone. This behavior realistic reflects pedestrian flow, where individuals partially interpenetrate rather than collide as rigid bodies.

The density fronts evolve without numerical artifacts: within the interaction region the gradients increase and then decrease smoothly. No nonphysical oscillations or discontinuities are observed. An analysis of temporal interaction areas shows that the deformation of the distribution remains regular and physically meaningful throughout the entire simulation interval. An examination of global characteristics shows that the front position, the width of the interaction areas, and the maximum density values change monotonically and without abrupt jumps, indicating a correct and stable reproduction of the nonlocal interaction mechanism.

Within the considered assumptions, the proposed PINN formulation demonstrates consistent performance. The structure of the method suggests applicability to more general problem classes beyond the present setting, including higher dimensions.

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ФІЗИЧНО-ІНФОРМОВАНЕ НЕЙРОМЕРЕЖЕВЕ МОДЕЛЮВАННЯ НЕЛОКАЛЬНОЇ ДИНАМІКИ НАТОПУ ДЛЯ СЦЕНАРІЇВ ЕВАКУАЦІЇ

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Ми досліджуємо нелокальну континуальну модель динаміки натовпу, використовуючи фізико-інформований нейронний мережевий підхід. Натовп описується системою нелінійних законів збереження, в яких потік включає адвекцію, дифузію та нелокальні члени взаємодії, що враховують рух, залежний від щільності, та обмежене сприйняття навколишніх агентів. Нелокальні ефекти моделюються за допомогою просторових згорток з гладкими ядрами, що дозволяє агентам реагувати на усереднені градієнти щільності, а не на суто локальну інформацію. Система диференціальних рівнянь з частинними похідними, що керує, розв'язується за допомогою фізико-орієнтованої нейронної мережі, відомої як PINN, яка апроксимує розв'язок по всьому просторово-часовому домену, одночасно дотримуючись фізичних обмежень за допомогою автоматичного диференціювання. Нелокальні члени взаємодії реалізовані у стабільній дискретній згортковій формі, що забезпечує числову стійкість під час навчання. Підхід демонструється на взаємодії двох груп пішоходів, що рухаються в протилежних напрямках в одновимірному коридорі. Результати демонструють формування та поширення фронтів щільності, поступове злиття потоків та виникнення стабільних змішаних зон. Характерною особливістю розв'язку є часткове взаємопроникнення груп без жорстких зіткнень, що відображає реалістичний колективний рух. Для валідації методу розв'язок PINN порівнюється з еталонною схемою скінченних різниць, заснованою на потоці Русанова. Якісна відповідність спостерігається у структурі фронту та динаміці змішування, тоді як кількісні відхилення в ключових характеристиках залишаються в межах кількох відсотків. Систематичне дослідження параметрів показує, що розв'язок на основі PINN залишається стабільним при змінах швидкості адвекції, коефіцієнта дифузії та радіуса нелокальної взаємодії, на відміну від схеми скінченних різниць, яка демонструє сильні обмеження стійкості. Ці результати демонструють, що PINN забезпечує надійний та фізично узгоджений інструмент для моделювання нелінійної нелокальної динаміки натовпу.

Ключові слова: *фізично-орієнтовані нейронні мережі; нелокальна динаміка натовпу; моделювання евакуації; адвекція-дифузія; ядра згортки; робастність; складні геометрії*