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ANISOTROPIC COSMIC EVOLUTION AND STATEFINDER DIAGNOSTICS OF A BIANCHI TYPE VI$_0$ UNIVERSE WITH HYBRID SCALE FACTOR IN $f(R, L_m)$ GRAVITY

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We investigate the dynamics of an anisotropic Bianchi type VI$_0$ cosmological model within the framework of $f(R, L_m)$ gravity by adopting a curvature–matter coupled form $f(R, L_m) = \frac{R}{2} + L_m^{\alpha}$. A hybrid expansion law $a(t) = e^{\beta t} t^{\gamma}$ is employed to describe the cosmic evolution, allowing a smooth transition from an early decelerated phase to a late-time accelerated expansion. The parameter $\alpha = 0.4$ is chosen to ensure stable and physically viable behavior of the model. The analysis shows that the scale factor and spatial volume increase monotonically with cosmic time, indicating a continuously expanding universe. The deceleration parameter exhibits a transition from positive to negative values and approaches the de Sitter limit at late times. The equation of state parameter evolves from a matter-dominated regime toward the quintessence region and asymptotically approaches $\omega = -1$. The energy density remains positive and decreases with time, while the pressure stays negative throughout the evolution, supporting accelerated expansion. Furthermore, the statefinder diagnostics in the $\{r, s\}$ and $\{r, q\}$ planes demonstrate that the model deviates from standard cosmology at intermediate epochs and converges to the ΛCDM fixed points at late times. These results indicate that the proposed model provides a consistent and viable description of dark energy-driven cosmic evolution.

Keywords: *$f(R, L_m)$ gravity; Bianchi type VI$_0$ universe; Hybrid expansion law; Anisotropic cosmology; Statefinder Diagnostics; Cosmic acceleration*

1. INTRODUCTION

Modern cosmology has undergone a significant transformation following compelling observational evidence indicating that the universe is currently undergoing a phase of late-time accelerated expansion. This phenomenon was first established through observations of Type Ia supernovae (SNe Ia) [1, 2], and was subsequently supported by measurements of the cosmic microwave background (CMB) radiation [3], large-scale structure (LSS) surveys [4], and high-precision observations from the Planck mission [5]. Within the framework of standard cosmology, the ΛCDM model provides the simplest explanation for this acceleration through the inclusion of the cosmological constant Λ. Despite its remarkable observational success, this model suffers from well-known theoretical issues, particularly the fine-tuning and cosmic coincidence problems [6–8]. These limitations have motivated the development of alternative theoretical approaches beyond general relativity. Among these, modified theories of gravity have emerged as promising alternatives. In particular, extensions such as $f(R)$ gravity and related frameworks have been extensively studied [9–11]. A further generalization is provided by the $f(R, L_m)$ theory of gravity, in which the gravitational action depends on both the Ricci scalar R and the matter Lagrangian density L_m [12]. This formulation introduces a non-minimal coupling between matter and geometry, leading to modified field equations and non-conservation of the energy-momentum tensor [13, 14]. Recent investigations have explored the cosmological implications of this theory in various contexts, including anisotropic models and hybrid expansion behavior [15–17]. Although the Friedmann–Lemaître–Robertson–Walker (FLRW) metric successfully describes a homogeneous and isotropic universe, it does not account for possible anisotropies that may have existed during the early stages of cosmic evolution. In this context, Bianchi-type cosmological models provide a natural generalization by allowing directional dependence in expansion rates. The pioneering works of Ellis and MacCallum [18], along with Collins [19], established the importance of anisotropic cosmologies. several studies have demonstrated that such models can effectively describe more realistic cosmic scenarios and their dynamical evolution [20, 21]. To describe a realistic cosmic history that includes both early-time deceleration and late-time acceleration, the hybrid expansion law (HEL), given by $a(t) = e^{\beta t} t^{\gamma}$, has been widely employed due to its ability to unify both phases of cosmic evolution [22]. In addition, the statefinder diagnostic pair $\{r, s\}$ introduced by Sahni et al. [23] provides an effective tool for distinguishing different dark energy models and for comparing them with the standard ΛCDM scenario. In particular, Bianchi type VI$_0$ cosmological models within modified gravity frameworks have been investigated in several studies, including those addressing anisotropic dynamics and exact solutions [24]. The formulation and field equations adopted in the present work are closely related to recent developments in this direction [25]. More recently, considerable attention has been devoted to examining the cosmological viability and broader implications of $f(R, L_m)$ gravity. Singh et al. [26] investigated a constrained cosmological scenario within this framework and demonstrated its potential for describing the observed accelerated expansion. The applicability of matter–geometry coupling to early-universe phenomena was further explored by Jaybhaye et al. [27] in the context of cosmological baryogenesis. Shukla et al. [28] analyzed FLRW dynamics by incorporating an equation-of-state parametrization, while Solanki et al. [29] extended the scope of $f(R, L_m)$ gravity to wormhole configurations. A more comprehensive examination of the cosmological background and perturbative evolution was presented by Sahlu et al. [30], providing useful constraints on the model parameters and its observational behavior. Beyond conventional late-time cosmology, bouncing scenarios

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and their dynamical stability have also been investigated within this theory [31]. Recent studies have further applied matter–geometry coupling to string cosmological models [32], illustrating the versatility of the $f(R, L_m)$ framework in describing different matter sources and cosmic geometries. The growing emphasis on observational consistency is evident in recent analyses of late-time cosmic dynamics using contemporary datasets [33], as well as Bayesian investigations aimed at assessing the ability of $f(R, L_m)$ models to alleviate the Hubble tension [34]. Navarro-Coyd'an et al. [35] showed that matter–geometry coupling can support an accelerating late-time Universe and de Sitter-like evolution without adding a separate vacuum contribution. Goswami and Pradhan [36] assessed the consistency of this gravity framework with current cosmological observations and obtained bounds on the model parameters. Recent studies have further explored the cosmological relevance of $f(R, L_m)$ gravity by investigating its implications for anisotropic cosmological models, matter–geometry coupling, and the evolution of different matter sources under modified gravitational dynamics [37]. Collectively, these developments indicate that $f(R, L_m)$ gravity provides a flexible theoretical setting for studying early- and late-time cosmic evolution, non-standard matter configurations, anisotropic effects, and observational departures from the standard ΛCDM paradigm.

Motivated by these developments, the present work investigates a Bianchi type VI_0 cosmological model within the framework of $f(R, L_m)$ gravity by employing a hybrid expansion law. The physical behavior of the model is analyzed through key cosmological parameters, including the deceleration parameter, equation of state parameter, and statefinder diagnostics. The results are examined to assess the consistency of the model with current observational data and the standard cosmological paradigm.

1.1. $f(R, L_m)$ Field Equations

The gravitational dynamics in $f(R, L_m)$ gravity are described by a generalized action in which the Ricci scalar R and the matter Lagrangian density L_m are treated as independent variables. The action is given by

$$S = \int f(R, L_m)\sqrt{-g}\, d^4x. \tag{1}$$

where g denotes the determinant of the metric tensor.

In the present work, we consider the specific functional form

$$f(R, L_m) = \frac{R}{2} + L_m^{\alpha}, \tag{2}$$

where α is a constant parameter that characterizes the strength of the matter–geometry coupling. The corresponding partial derivatives of the function are

$$f_R = \frac{\partial f}{\partial R} = \frac{1}{2}, \quad f_{L_m} = \frac{\partial f}{\partial L_m} = \alpha L_m^{\alpha-1}. \tag{3}$$

By varying the action with respect to the metric tensor, the field equations in $f(R, L_m)$ gravity are obtained as [12, 14]

$$f_R R_{ij} - \frac{1}{2}(f - f_{L_m} L_m) g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j) f_R = \frac{1}{2} f_{L_m} T_{ij}, \tag{4}$$

where $\Box = \nabla^k\nabla_k$ is the d'Alembert operator and ∇_i denotes the covariant derivative.

In $f(R, L_m)$ gravity, the choice of matter Lagrangian density for a perfect fluid is not unique. Common choices such as $L_m = -p$ and $L_m = \rho$ have been widely discussed in the literature [12–14]. In this work, we adopt the physically motivated choice $L_m = \rho$, which yields

$$f_{L_m} = \alpha\rho^{\alpha-1}. \tag{5}$$

Since $f_R = \frac{1}{2}$ is constant, the derivative term vanishes identically, i.e.,

$$(g_{ij}\Box - \nabla_i\nabla_j) f_R = 0.$$

Substituting the above results into the field equations, we obtain

$$\frac{1}{2}R_{ij} - \frac{1}{2}\left(\frac{R}{2} + (1-\alpha)\rho^{\alpha}\right) g_{ij} = \frac{1}{2}\alpha\rho^{\alpha-1} T_{ij}. \tag{6}$$

Multiplying throughout by 2 and simplifying, the modified field equations reduce to

$$R_{ij} - \frac{1}{2} R g_{ij} = \alpha\rho^{\alpha-1} T_{ij} - (\alpha-1)\rho^{\alpha} g_{ij}. \tag{7}$$

This equation represents a modified form of Einstein's field equations in which the effective gravitational coupling becomes matter-dependent through the factor $\alpha\rho^{\alpha-1}$, and an additional term proportional to ρ^{α} emerges, which can be interpreted as an effective cosmological contribution arising from matter–geometry coupling.

2. GEOMETRICAL SETUP AND FIELD EQUATIONS

We consider a spatially homogeneous but anisotropic Bianchi type VI_0 space-time, represented by the metric

$$ds^2 = -dt^2 + A^2(t)\, dx^2 + B^2(t) e^{-2x}\, dy^2 + C^2(t) e^{2x}\, dz^2, \tag{8}$$

where $A(t)$, $B(t)$, and $C(t)$ denote the directional scale factors corresponding to the three spatial directions.

In order to analyze the geometrical structure of the model and derive the corresponding gravitational field equations, it is necessary to evaluate the curvature quantities associated with the metric. In particular, the Ricci tensor R_{ij}, which characterizes the curvature of space-time, is obtained through contraction of the Riemann curvature tensor. It can be expressed in terms of the Christoffel symbols Γ^k_{ij} as follows:

$$R_{ij} = \partial_j \Gamma^k_{ik} - \partial_k \Gamma^k_{ij} + \Gamma^k_{il}\Gamma^l_{jk} - \Gamma^k_{ij}\Gamma^l_{kl}. \tag{9}$$

The Ricci scalar, which provides a measure of the curvature of space-time, is defined as the contraction of the Ricci tensor with the metric tensor, and is given by

$$R = g^{ij} R_{ij}. \tag{10}$$

For the considered Bianchi type VI_0 metric, this expression reduces to

$$R = -2\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right) - 2\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right) + \frac{2}{A^2}. \tag{11}$$

We assume that the matter content of the universe is described by a perfect fluid in a space-time characterized by the metric signature $(-,+,+,+)$. The corresponding energy–momentum tensor is given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}, \tag{12}$$

where ρ and p denote the energy density and isotropic pressure of the fluid, respectively, and u^i represents the four-velocity vector. The four-velocity satisfies the normalization condition

$$u^i u_i = -1. \tag{13}$$

For a comoving observer, the fluid remains at rest in the chosen coordinate system, and hence the four-velocity is expressed as

$$u^i = (1, 0, 0, 0). \tag{14}$$

Using this form of the four-velocity along with the metric components, the non-zero components of the energy–momentum tensor are obtained as

$$T_{00} = \rho, \quad T_{11} = pA^2, \quad T_{22} = pB^2 e^{-2x}, \quad T_{33} = pC^2 e^{2x}. \tag{15}$$

3. FIELD EQUATIONS FOR BIANCHI TYPE VI_0 SPACE-TIME

By substituting the metric functions corresponding to the Bianchi type VI_0 space-time into the modified field equations of $f(R, L_m)$ gravity, we obtain a set of coupled non-linear differential equations that govern the dynamical evolution of the model. The independent field equations can be written as follows:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = \alpha\rho^{\alpha-1}p - (\alpha - 1)\rho^\alpha, \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \alpha\rho^{\alpha-1}p - (\alpha - 1)\rho^\alpha, \tag{17}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \alpha\rho^{\alpha-1}p - (\alpha - 1)\rho^\alpha, \tag{18}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = (1 - 2\alpha)\rho^\alpha, \tag{19}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \tag{20}$$

Equation (20) establishes a first-order differential relation between the metric functions B and C. This relation can be expressed in the form

$$\frac{d}{dt}(\ln B - \ln C) = 0, \tag{21}$$

which indicates that the difference $\ln B - \ln C$ remains invariant with respect to cosmic time.

Integrating Eq. (21), we obtain

$$\ln B - \ln C = \ln k, \tag{22}$$

where k is a constant of integration. This relation can be rewritten as

$$\frac{B}{C} = k. \tag{23}$$

Since the constant k can be absorbed through an appropriate rescaling of the spatial coordinates, it is convenient to choose $k = 1$ without affecting the generality of the solution. Consequently, Eq. (23) reduces to

$$B = C. \tag{24}$$

This result imposes a constraint on the metric potentials, effectively reducing the number of independent scale factors. As a consequence, the system of field equations becomes simpler and more manageable for further analysis. By applying the condition $B = C$ obtained in Eq. (24), the system of field equations (16)–(19) simplifies considerably. The resulting set of independent differential equations can be expressed as

$$2\frac{\ddot{B}}{B}+\left(\frac{\dot{B}}{B}\right)^2+\frac{1}{A^2}=\alpha\rho^{\alpha-1}p-(\alpha-1)\rho^{\alpha}, \tag{25}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=\alpha\rho^{\alpha-1}p-(\alpha-1)\rho^{\alpha}, \tag{26}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\left(\frac{\dot{B}}{B}\right)^2-\frac{1}{A^2}=(1-2\alpha)\rho^{\alpha}. \tag{27}$$

4. HYBRID SCALE FACTOR AND PHYSICAL PARAMETERS

To close the system of field equations, we assume that the ratio of the shear scalar σ to the expansion scalar θ remains constant, i.e.,

$$\frac{\sigma}{\theta}=\text{constant}. \tag{28}$$

This condition leads to a linear relation among the directional Hubble parameters and, upon integration, yields

$$A=B^m, \tag{29}$$

where m is a constant characterizing the anisotropy of the model. Using the relation $B=C$ (Eq. (24)), we obtain

$$A=B^m=C^m. \tag{30}$$

The average scale factor a is defined through the relation

$$a^3=ABC, \tag{31}$$

which corresponds to the spatial volume. Substituting Eq. (30), we get

$$a^3=B^{m+2}, \tag{32}$$

leading to

$$B=C=a^{\frac{3}{m+2}},\quad A=a^{\frac{3m}{m+2}}. \tag{33}$$

To describe the cosmic evolution, we employ a hybrid expansion law characterized by the scale factor

$$a(t)=e^{\beta t}t^{\gamma}, \tag{34}$$

where β and γ are positive constants. This form allows a smooth transition from an early decelerating phase to a late-time accelerated expansion. Differentiating Eq. (34) with respect to cosmic time t, the first derivative of the scale factor is obtained as

$$\dot{a}=e^{\beta t}t^{\gamma}\left(\beta+\frac{\gamma}{t}\right), \tag{35}$$

The second derivative takes the form

$$\ddot{a}=e^{\beta t}t^{\gamma}\left[\left(\beta+\frac{\gamma}{t}\right)^2-\frac{\gamma}{t^2}\right], \tag{36}$$

and the third derivative is given by

$$\dddot{a}=e^{\beta t}t^{\gamma}\left[\left(\beta+\frac{\gamma}{t}\right)^3-\frac{3\gamma}{t^2}\left(\beta+\frac{\gamma}{t}\right)+\frac{2\gamma}{t^3}\right]. \tag{37}$$

The Hubble parameter, which characterizes the rate of expansion of the universe, is defined as

$$H=\frac{\dot{a}}{a}. \tag{38}$$

Substituting the expression for $\dot{a}$ from Eq. (35) into Eq. (38), we obtain

$$H=\beta+\frac{\gamma}{t}. \tag{39}$$

The spatial volume of the anisotropic universe is given by

$$V=a^3, \tag{40}$$

which represents the total volume expansion of the cosmological model. Using the hybrid scale factor defined in Eq. (34), the volume can be expressed as

$$V=e^{3\beta t}t^{3\gamma}. \tag{41}$$

The expansion scalar θ describes the rate of volume expansion of the universe and is related to the Hubble parameter by

$$\theta=3H. \tag{42}$$

Substituting the expression for H from Eq. (39), we obtain

$$\theta = 3\left(\beta + \frac{\gamma}{t}\right). \tag{43}$$

The deceleration parameter q provides information about the acceleration or deceleration behavior of the universe and is defined as

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{44}$$

Differentiating Eq. (39) with respect to cosmic time t, we obtain

$$\dot{H} = -\frac{\gamma}{t^2}. \tag{45}$$

Substituting Eqs. (39) and (45) into Eq. (44), the deceleration parameter is expressed as

$$q = -1 + \frac{\gamma}{(\beta t + \gamma)^2}. \tag{46}$$

From Eq. (46), it follows that at early cosmic times (small t), the second term dominates, leading to $q > 0$, which corresponds to a decelerating phase. In contrast, for sufficiently large values of t, the parameter approaches $q \to -1$, indicating a transition to an accelerated expansion phase consistent with late-time cosmological observations. By utilizing the reduced field equations (25)–(27) together with the hybrid scale factor defined in Eq. (34), the expression for the energy density of the cosmic fluid is obtained as

$$\rho(t) = \frac{1}{1-2\alpha}\left[\frac{9(2m+1)}{(m+2)^2}\left(\beta + \frac{\gamma}{t}\right)^2 - \frac{1}{\left(e^{\beta t}t^{\gamma}\right)^{\frac{6m}{m+2}}}\right]^{\frac{1}{\alpha}} \tag{47}$$

with the restriction $\alpha \neq \frac{1}{2}$ to avoid singular behavior.

Substituting the above expression for the energy density into the modified field equations, we obtain the corresponding pressure of the cosmic fluid as

$$p = \frac{1}{\alpha\rho^{\alpha-1}}\left[\frac{(15-6m)}{(m+2)^2}\left(\beta + \frac{\gamma}{t}\right)^2 + \frac{6}{m+2}\left(\frac{\gamma(\gamma-1)}{t^2} + \frac{2\beta\gamma}{t} + \beta^2\right) + \frac{1}{\left(e^{\beta t}t^{\gamma}\right)^{\frac{6m}{m+2}}} - (1-\alpha)\rho^{\alpha}\right]. \tag{48}$$

5. STATEFINDER DIAGNOSTIC

To analyze the evolutionary behavior of the proposed cosmological model, we make use of the statefinder diagnostic, which provides a geometrical characterization of dark energy models through higher-order derivatives of the scale factor. The statefinder parameters are defined as

$$r = \frac{\dddot{a}}{aH^3}, \tag{49}$$

$$s = \frac{r-1}{3\left(q - \frac{1}{2}\right)}, \tag{50}$$

where q denotes the deceleration parameter.

Using the expression of the deceleration parameter derived earlier, we write

$$q = -1 + \frac{\gamma}{t^2\left(\beta + \frac{\gamma}{t}\right)^2}. \tag{51}$$

Substituting the expressions for a, $\dot{a}$, $\ddot{a}$, and $\dddot{a}$ from Eqs. (34), (35), and (37), along with the Hubble parameter given in Eq. (39), into Eq. (49), we obtain the statefinder parameter r in the form

$$r = 1 - \frac{3\gamma}{t^2\left(\beta + \frac{\gamma}{t}\right)^2} + \frac{2\gamma}{t^3\left(\beta + \frac{\gamma}{t}\right)^3}. \tag{52}$$

Further, substituting Eqs. (51) and (52) into Eq. (50), the parameter s can be expressed as

$$s = \frac{-3\gamma}{3t^2\left(\beta + \frac{\gamma}{t}\right)^2\left(q - \frac{1}{2}\right)} + \frac{2\gamma}{3t^3\left(\beta + \frac{\gamma}{t}\right)^3\left(q - \frac{1}{2}\right)}. \tag{53}$$

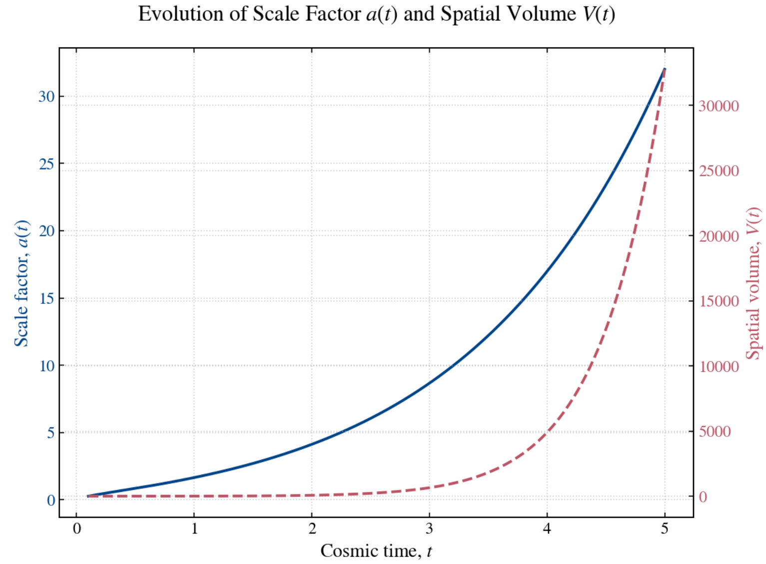

Figure 1. Evolution of the scale factor $a(t)$ and spatial volume $V(t)$ with cosmic time t for $\beta = 0.5$ and $\gamma = 0.6$.

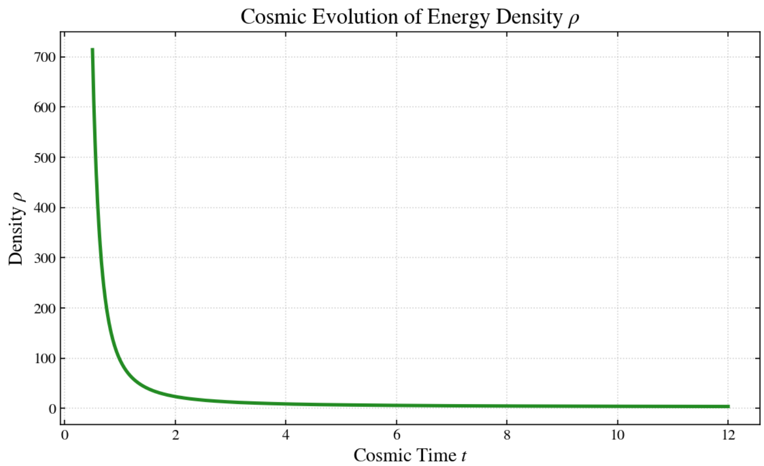

Figure 2. Energy density $\rho(t)$ versus cosmic time t for $\beta = 0.5$, $\gamma = 0.6$, and $\alpha = 0.4$.

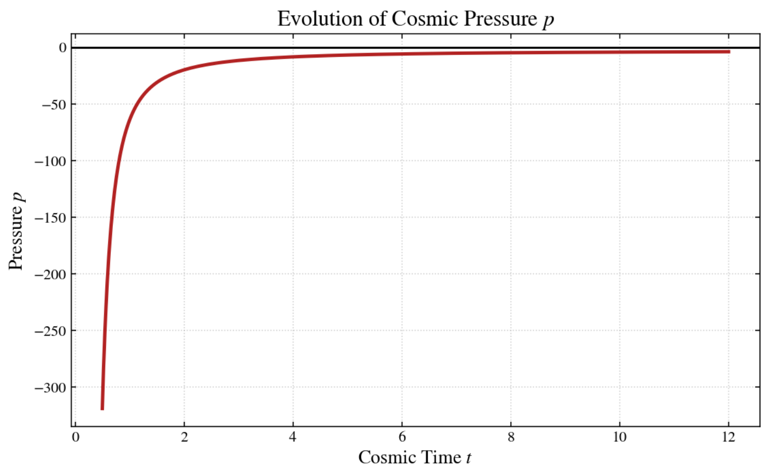

Figure 3. Thermodynamic pressure $p(t)$ as a function of cosmic time t for $\beta = 0.5$ and $\gamma = 0.6$.

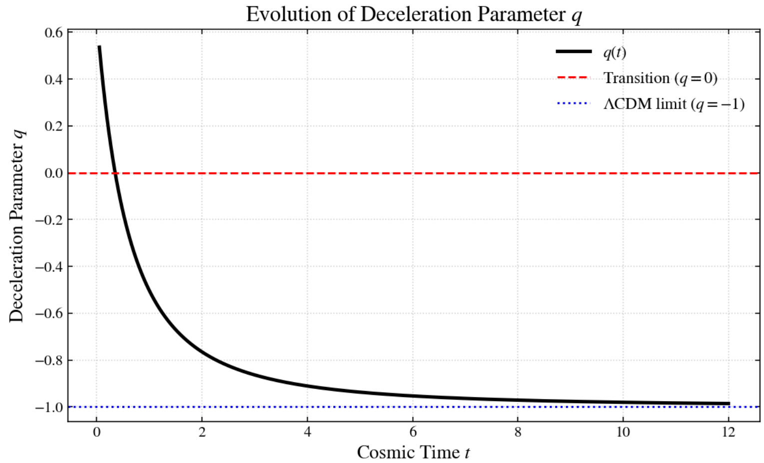

Figure 4. Deceleration parameter $q(t)$ versus cosmic time t for $\beta = 0.5$ and $\gamma = 0.6$.

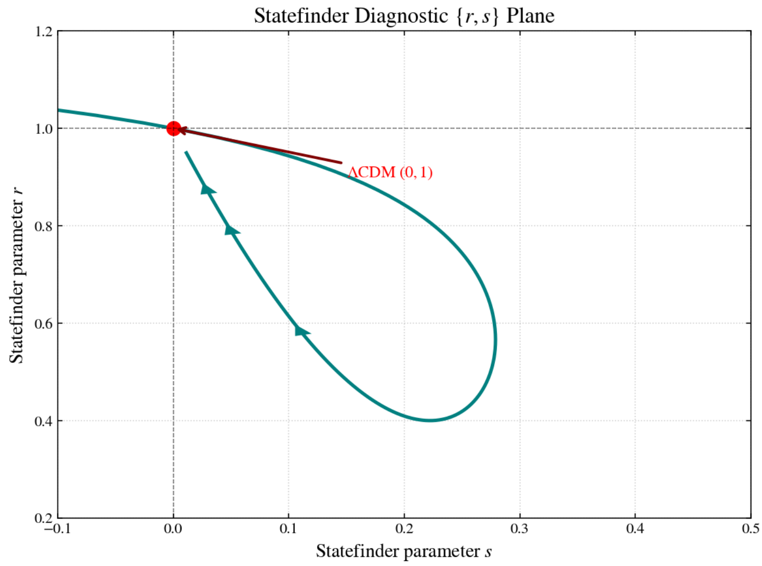

Figure 5. Statefinder trajectory in the (s, r) plane for $\beta = 0.5$ and $\gamma = 0.6$.

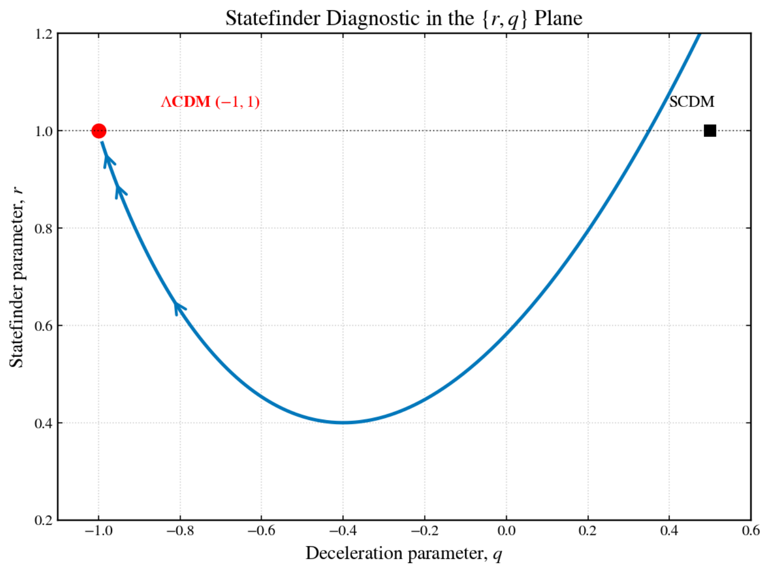

Figure 6. Statefinder trajectory in the (q, r) plane for $\beta = 0.5$ and $\gamma = 0.6$.

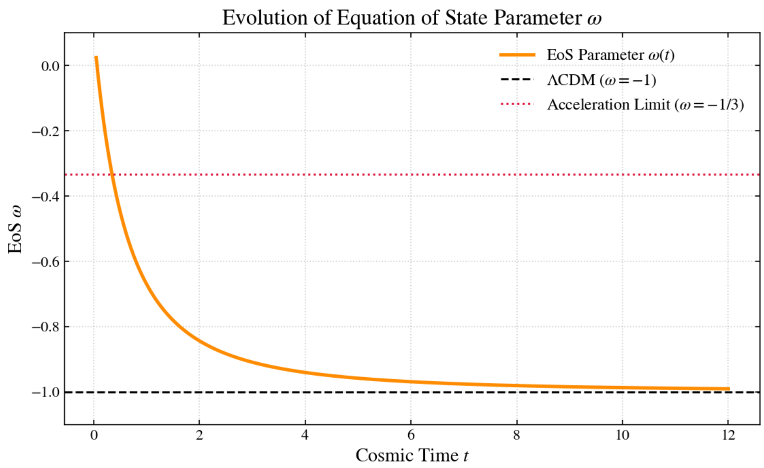

Figure 7. Equation of state parameter $\omega(t)$ versus cosmic time t for $\beta = 0.5$ and $\gamma = 0.6$.

6. DISCUSSION AND CONCLUDING REMARKS

The present study investigates a dynamical dark energy model governed by a hybrid expansion law within the framework of $f(R, L_m)$ gravity. Based on the analytical expressions and graphical analysis of the cosmological parameters, the following conclusions are drawn:

- The evolution of the scale factor $a(t)$ and spatial volume $V(t)$, shown in Fig. 1, demonstrates a continuous and monotonic increase with cosmic time. This confirms a persistently expanding universe without any indication of recollapse. The rapid growth of the spatial volume at later times reflects the accelerated expansion governed by the hybrid expansion law.

- The energy density $\rho(t)$, depicted in Fig. 2, remains strictly positive throughout the evolution and decreases monotonically with time. The initially large value of ρ at early times followed by gradual dilution is consistent with an expanding universe and satisfies the Weak Energy Condition.

- The thermodynamic pressure $p(t)$, shown in Fig. 3, remains negative for all cosmic time. The large negative pressure at early epochs gradually approaches a small negative value at late times, providing the repulsive effect responsible for the observed accelerated expansion.

- The deceleration parameter $q(t)$, illustrated in Fig. 4, exhibits a smooth transition from a decelerated phase ($q > 0$) to an accelerated phase ($q < 0$). The transition occurs at early cosmic time, and at late times q asymptotically approaches -1, indicating convergence toward a de Sitter phase.

- The statefinder diagnostics in the $\{r, s\}$ and $\{r, q\}$ planes, shown in Fig. 5 and Fig. 6, respectively, reveal that the model deviates from the ΛCDM behavior during intermediate stages and eventually converges to the fixed points $(0, 1)$ and $(-1, 1)$ respectively.

- The equation of state parameter $\omega(t)$, illustrated in Fig. 7, evolves from values close to zero at early times into the quintessence region ($-1 < \omega < -1/3$), and asymptotically approaches $\omega \to -1$. The condition $\omega < -1/3$ is consistent with the negative values of the deceleration parameter shown in Fig. 4, confirming accelerated expansion.

Overall, the proposed model successfully describes a transition from an early decelerated phase to a late-time accelerated universe. The consistent behavior of q, ω, ρ, and p, along with the convergence of statefinder parameters toward the ΛCDM limits, confirms the physical viability of the model as a realistic description of dark energy-dominated cosmic evolution.

6.1. Statefinder Diagnostic Analysis

The $\{r, s\}$ statefinder diagnostic, introduced by Sahni et al. [23], is employed to characterize the evolutionary trajectory of the proposed dark energy model and to distinguish it from standard cosmological scenarios. As shown in Fig. 5, the trajectory originates in the vicinity of the ΛCDM fixed point $(s, r) = (0, 1)$ and then evolves into the region $s > 0$ with $r < 1$, which is typically associated with quintessence-like behavior.

A notable feature of the evolution is the non-monotonic, hook-like trajectory observed in the s–r plane. The model departs from the ΛCDM point during the intermediate stages of cosmic evolution, reaching a minimum value of the statefinder parameter r before turning back toward the fixed point. This behavior reflects the dynamical nature of the dark energy component.

The directional arrows in the figure indicate the progression of cosmic time, showing that the trajectory gradually returns and asymptotically approaches the de Sitter attractor at $(s, r) = (0, 1)$. This convergence suggests that the model becomes effectively indistinguishable from the ΛCDM scenario at late times.

Overall, the characteristic loop-like evolution in the $\{r, s\}$ plane highlights the transient deviation from the cosmological constant behavior and provides a useful diagnostic feature for comparing the present model with other dark energy scenarios.

6.2. Geometrical Diagnostic in the $\{r, q\}$ Plane

To further investigate the expansion dynamics, we analyze the trajectory in the $\{r, q\}$ statefinder–deceleration plane, as shown in Fig. 6. This diagnostic provides a geometrical interpretation of cosmic evolution by relating the deceleration parameter q and the statefinder parameter r [23].

The trajectory begins in the decelerated regime ($q > 0$) with $r \approx 1$, and then evolves toward lower values of r as the universe transitions into the accelerated phase ($q < 0$). A distinct minimum in r is observed during the intermediate stage of evolution, indicating a departure from the standard ΛCDM behavior.

As cosmic time progresses, the trajectory turns upward and asymptotically approaches the de Sitter fixed point $(q, r) = (-1, 1)$. This convergence confirms that the model ultimately evolves toward a late-time accelerated phase consistent with the ΛCDM scenario.

The smooth and continuous nature of the trajectory across the $q = 0$ transition demonstrates a stable evolution from deceleration to acceleration, without any abrupt changes in the expansion dynamics. The characteristic curved path in the $\{r, q\}$ plane reflects the dynamical nature of the model and distinguishes it from the constant behavior of the standard cosmological constant model.

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АНІЗОТРОПНА КОСМІЧНА ЕВОЛЮЦІЯ ТА ДІАГНОСТИКА СТАНУ ВСЕСВІТУ ТИПУ Б'ЯНКІ VI_0 З ГІБРИДНИМ МАСШТАБНИМ ФАКТОРОМ У $f(R,L_m)$ ГРАВІТАЦІЇ

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Ми досліджуємо динаміку анізотропної космологічної моделі типу Б'янкі VI_0 в рамках гравітації $f(R,L_m)$, приймаючи форму зв'язку кривизни та матерії $f(R,L_m) = \frac{R}{2} + L_m^{\alpha}$. Для опису космічної еволюції використовується гібридний закон розширення $a(t) = e^{\beta t} t^{\gamma}$, що дозволяє плавний перехід від ранньої уповільненої фази до прискореного розширення наприкінці часу. Параметр $\alpha = 0.4$ вибрано для забезпечення стабільної та фізично життєздатної поведінки моделі. Аналіз показує, що масштабний коефіцієнт та просторовий об'єм монотонно зростають з космічним часом, що вказує на безперервно розширюваний Всесвіт. Параметр уповільнення демонструє перехід від позитивних до негативних значень та наближається до межі де Сіттера на пізніх етапах часу. Параметр рівняння стану еволюціонує від режиму домінування матерії до області квінтесенції та асимптотично наближається до $\omega = -1$. Густина енергії залишається позитивною та зменшується з часом, тоді як тиск залишається негативним протягом усієї еволюції, що підтверджує прискорене розширення. Крім того, діагностика стану в площинах $\{r,s\}$ та $\{r,q\}$ демонструє, що модель відхиляється від стандартної космології в проміжні епохи та сходиться до фіксованих точок ΛCDM на пізніх етапах часу. Ці результати вказують на те, що запропонована модель забезпечує послідовний та життєздатний опис космічної еволюції, керованої темною енергією.

Keywords: $f(R,L_m)$ *гравітація; Всесвіт типу Б'янкі* VI_0*; гібридний закон розширення; анізотропна космологія; діагностика statefinder; космічне прискорення*