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COSMOLOGICAL MODEL WITH POLYTROPIC BULK VISCOSITY AND VARYING GRAVITATIONAL AND COSMOLOGICAL CONSTANTS

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This paper investigates Einstein's field equations within a cosmological model that incorporates a polytropic bulk viscous fluid, with the gravitational and cosmological constants varying in a Bianchi type-I universe. It is observed that different time-dependent scenarios yield variations in physical parameters, which hold significant implications for cosmic evolution. Key physical parameters, including energy density, the gravitational constant, expansion scalar, the cosmological constant, the bulk viscosity coefficient and shear scalar are examined for their physical significance. The geometrical and observational interpretations of the cosmological model are also investigated.

Keywords: *Bianchi Type-I; Shear Scalar; Bulk Viscosity; Expansion Scalar*

1. INTRODUCTION

Cosmology investigates the origin and evolution of the universe, with a focus on its largest-scale structures and dynamics, including the motion of celestial bodies. Advances in the twentieth century established the Big Bang as the prevailing cosmological model, now widely accepted by the scientific community. These developments have also stimulated speculation regarding the very beginning of the universe. Bianchi type-I spacetimes represent simple generalizations of the flat Friedmann-Robertson-Walker (FRW) models. Observations, such as the late-time cosmic acceleration (Perlmutter et al. [1–3]; Riess et al. [4]; Allen et al. [5]; Peebles et al. [6]; Padmanabhan [7]; Lima [8]; Khadekar [9]; Dhankar [10–18]; Munyeshyaka [19–21]; Gadbail [22]; Panda [23]; Kumar [24]; Thakran [25]; Samanta [26]; Sanyal [27]), reveal that the universe is undergoing accelerated expansion.

The cosmological constant (Λ) is a subject of considerable interest, with recent observations suggesting a value of approximately $10^{-55}cm^{-2}$, which is dramatically smaller than the predictions from particle physics by about 120 orders of magnitude. Λ remains a cornerstone of modern cosmological theory. Bulk viscosity, arising from phase transitions in grand unified theories, may contribute to an inflationary phase in the early universe. There is substantial evidence supporting a non-zero cosmological constant Riess et al. [28]. For instance, Zhang et al. [29] investigated Friedmann cosmology on a codimension-2 brane with time-varying tension, demonstrating that Λ can remain independent of the brane tension. Tiwari and Sharma [30] studied Bianchi type-III models incorporating strings and bulk viscosity within general relativity, showing that such models can be shear-free. Wang [31–33], examined Bianchi type I–IX models in the presence of string clouds. Numerous studies have explored viscous fluid cosmologies in the early universe. Tiwari et al. [34] analyzed Bianchi type-I string cosmologies with bulk viscosity and a time-dependent Λ. Zeyauddin and Saha [35] studied Bianchi type-V models incorporating bulk viscosity and particle production. Beesham et al. [36] proposed a Bianchi type-I anisotropic model with a bulk viscous fluid, variable gravitational constant G, and Λ, where the bulk viscosity η is related to the energy density ρ via $\eta = \eta_0\rho^n$, with $\eta_0 \leq 0$. They derived the modified Friedmann equation $3H^2 = G\rho + \sigma^2 + \Lambda$, where σ denotes shear and H is the Hubble parameter.

Further investigations by Tiwari [37], Tiwari and jha [38], Tiwari and Dwivedi [39] (2008–2010) explored isotropic and anisotropic cosmologies, examining relationships among the cosmological constant Λ, the inverse power of the scale factor (R^{-m}), and the Hubble parameter H, with m as a constant. Tiwari and Sharma [40] analyzed LRS Bianchi type-II models featuring a decaying Λ, while Tiwari et al. [41] studied Bianchi type-I models with a perfect fluid in general relativity. Numerous other researchers [42–62] have also investigated the behavior and implications of the cosmological constant. Another intriguing topic is the possible variability of the gravitational constant G. While the conventional view, based on Newtonian gravity and Einstein's general relativity, treats G as constant, some theories allow for its variation over cosmic time. A time-varying G could revolutionize fundamental physics and have profound astrophysical consequences.

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Dirac [63] proposed that G decreases inversely with time when measured in atomic units, noting its approximate value as 10^{-40} at the present epoch.

Cosmological models that include bulk viscosity, together with time-dependent gravitational and cosmological constants, have received significant attention in contemporary theoretical physics [64, 65]. Bulk viscosity arises naturally in cosmic fluids due to particle interactions and phase transitions, and it can have a substantial impact on the dynamics of the early universe [66]. In anisotropic spacetimes, such as Bianchi type-I models, viscous effects provide a useful framework for exploring deviations from perfect fluid behavior while remaining compatible with observational constraints on cosmic expansion. Models with variable gravitational G and cosmological Λ constants offer an alternative to the standard ΛCDM framework, potentially addressing unresolved issues related to dark energy and late-time cosmic acceleration [67]. Polytropic equations of state have been applied to describe bulk viscous fluids, extending the barotropic case and capturing more intricate thermodynamic properties [68, 69]. These models often admit exact solutions to Einstein's field equations, allowing detailed investigations of the universe's expansion history and the dissipation of anisotropy [64]. Moreover, the combined influence of viscous effects and evolving G and Λ can produce scenarios consistent with current observations of cosmic acceleration without invoking exotic fields [70]. In this study, we focus on a Bianchi type-I cosmological model with polytropic bulk viscosity, permitting both G and Λ to vary over cosmic time, and we examine the evolution of key cosmological parameters.

The structure of this paper is as follows: Section 2 presents the Bianchi type-I bulk viscous polytropic cosmological model with variable G and Λ. In Section 3, we derive solutions to Einstein's field equations under specific equations of state and compute various physical quantities, including scale factors, the Hubble parameter, the expansion scalar, and the deceleration parameter. Section 4 summarizes our conclusions.

2. THE METRIC AND IT'S FIELD EQUATIONS

The spatially homogeneous and anisotropic Bianchi type-I spacetime is described by,

$$ds^2 = -dt^2 + X_1^2(t)dx^2 + X_2^2(t)dy^2 + X_3^2(t)dz^2, \tag{1}$$

where, $X_1(t)$, $X_2(t)$ and $X_3(t)$ are functions dependent solely on cosmic time t.

The energy–momentum tensor of a bulk viscous fluid is given by

$$T_\mu^\nu = (\rho + \bar{p})u_\mu u^\nu - \bar{p}\, g_\mu^\nu, \tag{2}$$

where the effective pressure is defined as

$$\bar{p} = p - 3\xi H. \tag{3}$$

Here, ρ and p denote the energy density and equilibrium pressure, respectively, ξ is the coefficient of bulk viscosity, H is the Hubble parameter, and the four-velocity u^μ satisfies the normalization condition $g_{\mu\nu}u^\mu u^\nu = -1$.

The Einstein field equations are expressed as

$$R_\mu^\nu - \frac{1}{2}Rg_\mu^\nu = -8\pi G T_\mu^\nu + \Lambda g_\mu^\nu, \tag{4}$$

where G and Λ are the time-dependent gravitational and cosmological constants, respectively.
The expressions for the shear scalar ρ and the expansion scalar Θ are provided as follows:

$$\Theta = u^\nu_{;\mu} = \frac{1}{3}\left(\frac{\dot{X}_1}{X_1} + \frac{\dot{X}_2}{X_2} + \frac{\dot{X}_3}{X_3}\right) = 3H,$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}$$

$$= \frac{1}{3}\left(\frac{\dot{X}_1^2}{X_1^2} + \frac{\dot{X}_2^2}{X_2^2} + \frac{\dot{X}_3^2}{X_3^2} - \frac{\dot{X}_1\dot{X}_2}{X_1X_2} - \frac{\dot{X}_2\dot{X}_3}{X_2X_3} - \frac{\dot{X}_1\dot{X}_3}{X_1X_3}\right). \tag{5}$$

The Einstein's field equation (4) supporting the metric (1) now become

$$\frac{\ddot{X}_2}{X_2} + \frac{\ddot{X}_3}{X_3} + \frac{\dot{X}_2\dot{X}_3}{X_2X_3} = -8\pi G\bar{p} + \Lambda, \tag{6}$$

$$\frac{\ddot{X}_1}{X_1} + \frac{\ddot{X}_3}{X_3} + \frac{\dot{X}_1\dot{X}_3}{X_1X_3} = -8\pi G\bar{p} + \Lambda, \tag{7}$$

$$\frac{\ddot{X}_1}{X_1} + \frac{\ddot{X}_2}{X_2} + \frac{\dot{X}_1\dot{X}_2}{X_1X_2} = -8\pi G\bar{p} + \Lambda, \tag{8}$$

$$\frac{\dot{X}_1\dot{X}_2}{X_1X_2}+\frac{\dot{X}_2\dot{X}_3}{X_2X_3}+\frac{\dot{X}_1\dot{X}_3}{X_1X_3}=8\pi G\rho+\Lambda, \tag{9}$$

The divergence of the Einstein tensor produces an additional equation for the time variations of Λ and G, namely:

$$\left(R^{\nu}_{\mu}-\frac{1}{2}Rg^{\nu}_{\mu}\right)_{;\nu}=0$$

which leads to

$$\left(8\pi GT^{\nu}_{\mu}-\Lambda g^{\nu}_{\mu}\right)_{;\nu}=0$$

yielding

$$8\pi\dot{G}\rho+\dot{\Lambda}+8\pi G\left\{\dot{\rho}+\left(\frac{\dot{X}_1}{X_1}+\frac{\dot{X}_2}{X_2}+\frac{\dot{X}_3}{X_3}\right)(\rho+\bar{p})\right\}=0. \tag{10}$$

The energy conservation Eq. (10) is divided into two equations after using Eq. (4), we get

$$\dot{\rho}+\left(\frac{\dot{X}_1}{X_1}+\frac{\dot{X}_2}{X_2}+\frac{\dot{X}_3}{X_3}\right)(\rho+\bar{p})=0, \tag{11}$$

$$\dot{\Lambda}+8\pi\dot{G}\rho=8\pi G\xi\left(\frac{\dot{X}_1}{X_1}+\frac{\dot{X}_2}{X_2}+\frac{\dot{X}_3}{X_3}\right)^2=0. \tag{12}$$

3. SOLUTIONS OF THE FIELD EQUATIONS

We consider the relation as

$$\xi=\xi_0\rho^{\beta} \tag{13}$$

where, $\xi_0>0$, and β is taken as a constant.
To derive the complete solution, we further consider the polytropic relation as follows:

$$\bar{p}=\alpha\rho^{\zeta} \tag{14}$$

where α and ζ are constants characterizing the polytropic fluid, and ρ denotes the energy density. For simplicity, we set the constant ζ to unity.

We assume that the solution to the system of equations takes the following form:

$$\frac{\dot{X}_1}{X_1}=\frac{\dot{X}_2}{X_2}=\frac{\alpha_1}{t^n},\frac{\dot{X}_3}{X_3}=\frac{\alpha_2}{t^n} \tag{15}$$

where, α_1, α_2 both are constants [4].

Upon integrating equation (15), we obtain the solution as follows:

$$X_1=X_2=a.exp\left[\frac{\alpha_1t^{1-n}}{(1-n)}\right],X_3=b.exp\left[\frac{\alpha_2t^{1-n}}{(1-n)}\right]. \tag{16}$$

where a and b are the constant of integration.
After using Eqns. (14) & (15) into Eq. (11), we have the values as given

$$\frac{\dot{\rho}}{\rho}=-\frac{(1+\alpha)(2\alpha_1+\alpha_2)}{t^n} \tag{17}$$

After Integrating on both sides, we get

$$\rho=\alpha_3exp\left[-(1+\alpha)(2\alpha_1+\alpha_2)\frac{t^{1-n}}{(1-n)}\right], \tag{18}$$

Differentiating Eq. (18) on both sides, we have

$$\dot{\rho}=-\alpha_3\frac{(\alpha+1)(2\alpha_1+\alpha_2)}{t^n}exp\left[-\frac{(1+\alpha)(2\alpha_1+\alpha_2)}{(1-n)}t^{1-n}\right], \tag{19}$$

By using Eq. (9), (15), (16) and differentiating on both sides, we have

$$-\frac{2n(\alpha_1^2+2\alpha_1\alpha_2)}{t^{(1+2n)}} = 8\pi G\dot{\rho} + 8\pi G\xi\left[\frac{(2\alpha_1+\alpha_2)}{t^n}\right]^2, \tag{20}$$

Again, substituting Eqns. (13) & (19) into Eq. (20), we get the values as

$$G = -\frac{n(\alpha_1^2+2\alpha_1\alpha_2)}{4\pi\alpha_3 t^{2n+1}}.exp\left\{\frac{(\alpha+1)(2\alpha_1+\alpha_2)}{(1-n)}t^{1-n}\right\}\left[-\frac{(\alpha+1)(2\alpha_1+\alpha_2)}{t^n}+\xi_0\alpha_3^{\beta-1}.\right.$$

$$\left. exp\left\{-\frac{(\beta-1)(1+\alpha)(2\alpha_1+\alpha_2)}{(1-n)}t^{1-n}\right\}\right]^{-1} \tag{21}$$

$$\Lambda = \frac{(\alpha_1^2+2\alpha_1\alpha_2)}{t^{2n}} + 2n\frac{(\alpha_1^2+2\alpha_1\alpha_2)}{t^{2n+1}}\left[-\frac{(\alpha+1)(2\alpha_1+\alpha_2)}{t^n}+\xi_0\alpha_3^{\beta-1}\right.$$

$$\left. .exp\left\{-\frac{(\beta-1)(1+\alpha)(2\alpha_1+\alpha_2)}{(1-n)}t^{1-n}\right\}\right]^{-1} \tag{22}$$

It should be noted that the parameters α_1, α_2, and α_3 are dimensional constants with dimensions T^{n-1}, as dictated by the ansatz $\dot{X}_i/X_i = \alpha_i t^{-n}$, the dimension of t being T. Consequently, the expression for Λ remains dimensionally consistent with dimension L^{-2} (or T^{-2}) for arbitrary values of n. From the Eq. (13) and Eq. (18), we have

$$\xi = \xi_0\alpha_3^{\beta}.exp\left\{-\frac{\beta(1+\alpha)(2\alpha_1+\alpha_2)}{(1-n)}t^{1-n}\right\} \tag{23}$$

Here $n \neq 1$.
Hence the metric in Eq. (1) simplifies to the following:

$$ds^2 = -dt^2 + a^2.exp\left(\frac{2\alpha_1 t^{1-n}}{(1-n)}\right)(dx^2+dy^2)$$

$$+b^2.exp\left(\frac{2\alpha_2 t^{1-n}}{(1-n)}\right)dz^2. \tag{24}$$

For the model in Eq. (24), the energy density ρ, Hubble constant H_i, and coefficient of bulk viscosity ξ are given by:

$$\rho = \alpha_3.exp\left\{\frac{(1+\alpha)(2\alpha_1+\alpha_2)}{(n-1)t^{n-1}}\right\} \tag{25}$$

$$H_1 = H_2 = \frac{\alpha_1}{t^n}, H_3 = \frac{\alpha_2}{t^n} \tag{26}$$

where $n > 1$.

$$\xi = \xi_0\alpha_3^{\beta}.exp\left\{-\frac{\beta(1+\alpha)(2\alpha_1+\alpha_2)}{(1-n)}t^{1-n}\right\} \tag{27}$$

$$\bar{p} = \alpha.\alpha_3.exp\left\{\frac{(1+\alpha)(2\alpha_1+\alpha_2)}{(n-1)}t^{n-1}\right\} \tag{28}$$

4. GRAPHICAL REPRESENTATION OF PHYSICAL QUANTITIES

Figures 1 and 2 illustrate the temporal evolution of the energy density, ρ, which exhibits a mild increase at early times followed by a more pronounced growth at later cosmic epochs. This behavior is indicative of a cosmological model featuring dynamical phase transitions. The initial slow rise in ρ is consistent with radiation- or matter-dominated expansion, whereas the subsequent rapid enhancement signals a transition toward dark energy domination, driving the onset of late-time cosmic acceleration. In this context, the accelerated growth of ρ at late times reflects the increasing dynamical influence of dark energy on the expansion history of the universe.

Alternatively, the sharp spike observed in the inset may be interpreted as a brief inflationary phase in the primordial universe, characterized by a rapid amplification of the energy density over a short interval of cosmic time. The density profile shown in Fig. 3 further corroborates this interpretation, displaying a sequence of quasi-stationary plateaus punctuated by abrupt transitions. The presence of distinct vertical features indicates that the energy density remains approximately

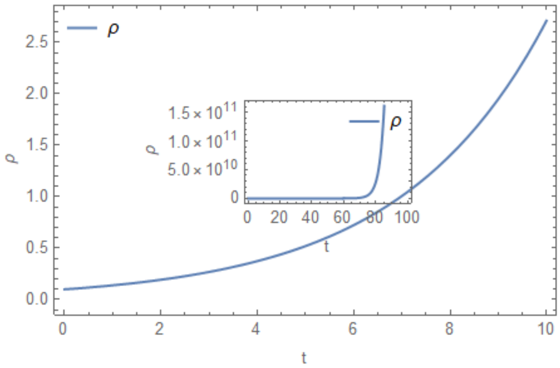

Figure 1. The relationship between energy density ρ and cosmic time t is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

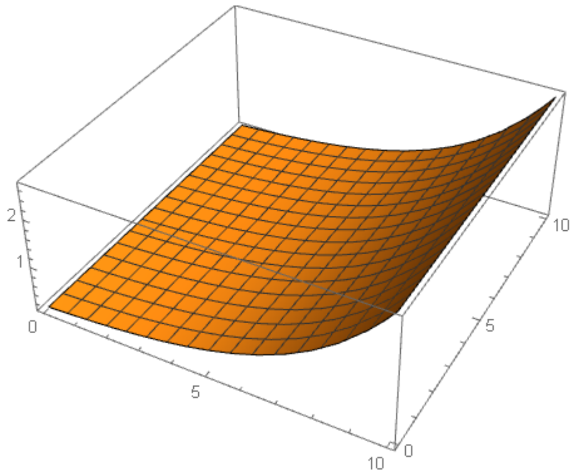

Figure 2. The relationship between energy density ρ and cosmic time t in three-Dimensional is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

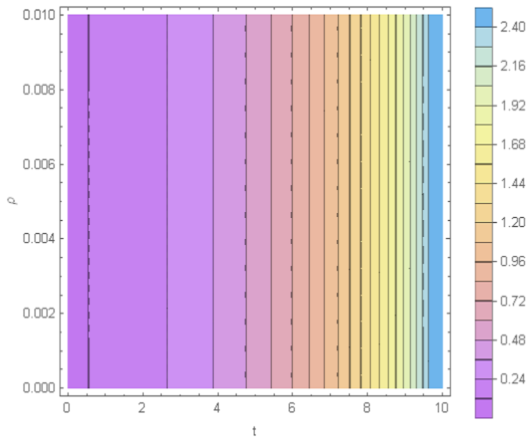

Figure 3. The relationship between energy density ρ and cosmic time t in Density plot view is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

constant over extended epochs before undergoing rapid changes at specific cosmic times, consistent with discrete phase transitions in the underlying cosmological dynamics.

Figures (4–6) illustrate the temporal evolution of the pressure, p, which grows approximately exponentially with cosmic time. During the early epochs ($t \ll t_0$), this growth is modest, while the pronounced steepening at later times signals a major dynamical transition, potentially corresponding to primordial inflation or a cosmological phase change, such as the onset of late-time acceleration driven by dark energy.

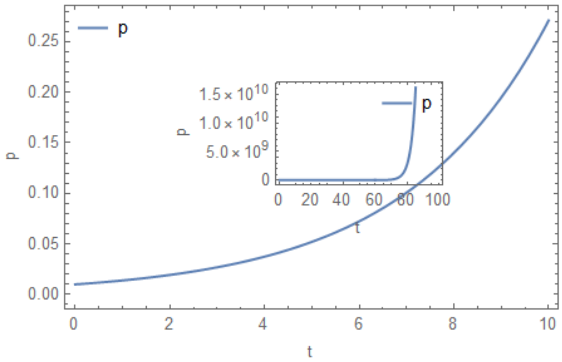

Figure 4. The relationship between pressure p and cosmic time t is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

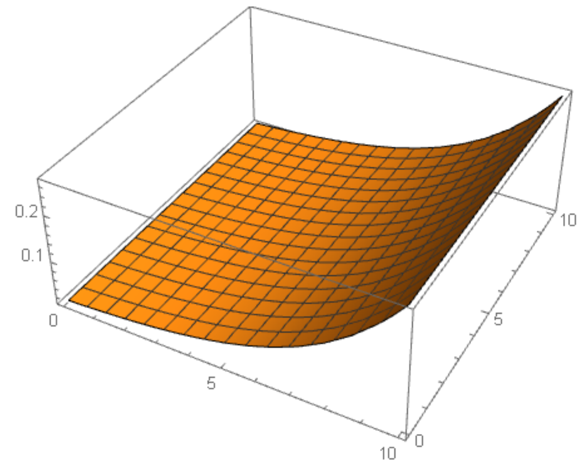

Figure 5. The relationship between pressure p and cosmic time t in three-Dimensional is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

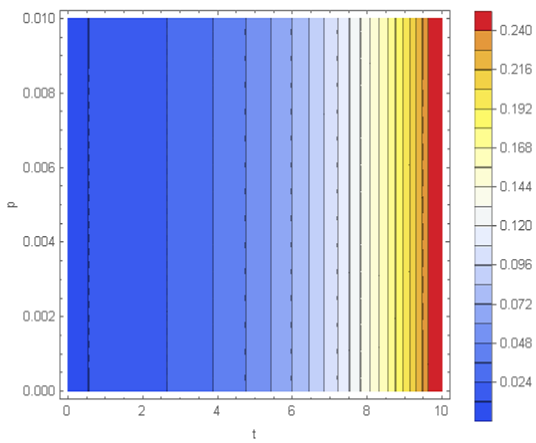

Figure 6. The relationship between pressure p and cosmic time t in Density plot view is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

Figures (7–9) show a monotonic decrease of the gravitational constant, G, over cosmic time, consistent with a broad spectrum of observational and experimental bounds, as detailed in the concluding section. This trend supports the plausibility of a slowly varying gravitational coupling within the current cosmological paradigm.

The evolution of the cosmological constant, Λ, depicted in Figs. (10–12), exhibits a three-phase pattern: rapid initial growth during the early universe, a peak in an intermediate epoch, and a subsequent decline toward near-zero values in the late universe. These dynamics align with theoretical expectations, with Λ increasing exponentially or as a power law

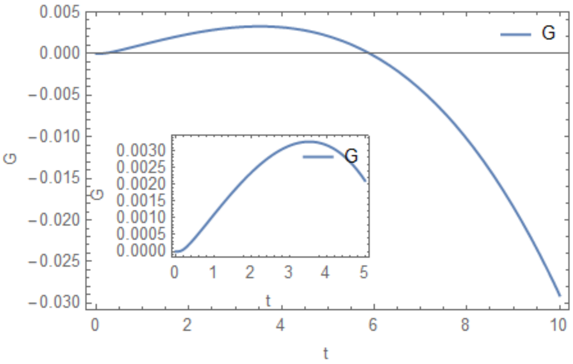

Figure 7. The relationship between Gravitational Constant G and cosmic time t is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

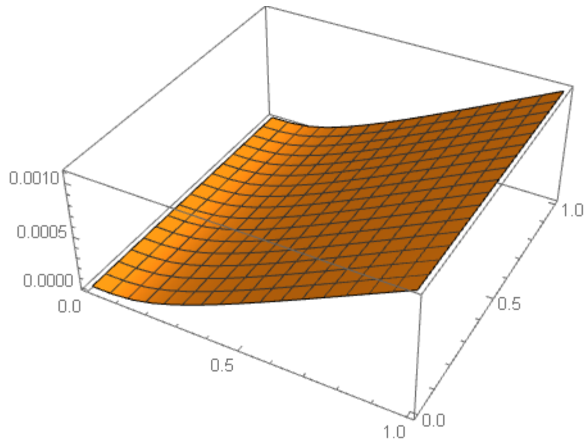

Figure 8. The relationship between Gravitational Constant G and cosmic time t in three-Dimensional is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

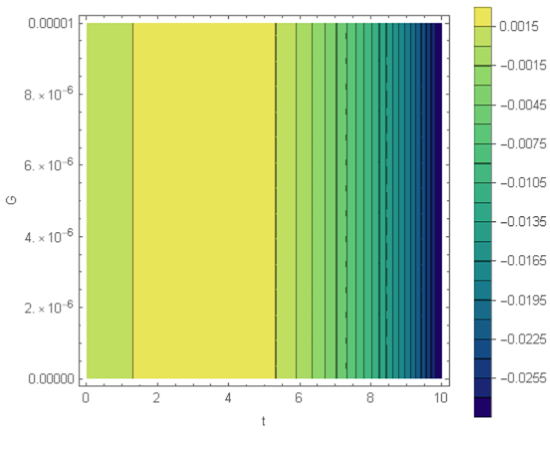

Figure 9. The relationship between Gravitational Constant G and cosmic time t in Density plot view is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

in the primordial phase, reaching a maximum in the intermediate era, and asymptotically approaching the small observed value at late times.

Finally, the behavior of the bulk viscosity coefficient, ξ, as a function of cosmic time is presented in Figs. (13–15) via density plots and three-dimensional visualizations. In the early universe, ξ rises rapidly, significantly impacting expansion dynamics; its growth continues at a slower rate during the intermediate epoch, and in the late universe, ξ stabilizes to an approximately constant value, reflecting the damping of dissipative effects as the universe expands.

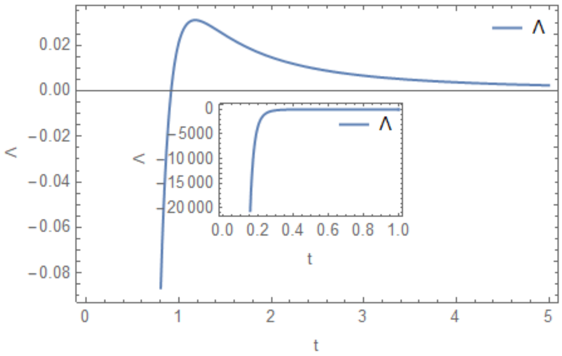

Figure 10. The relationship between Cosmological Constant Λ and cosmic time t is defined with the parameters: $\alpha = 0.1, n = 2, \alpha_1 = 0.1, \alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

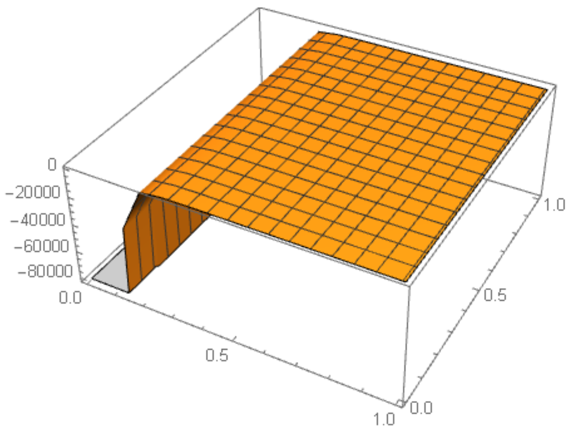

Figure 11. The relationship between Cosmological Constant Λ and cosmic time t in three-Dimensional is defined with the parameters: $\alpha = 0.1, n = 2, \alpha_1 = 0.1, \alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

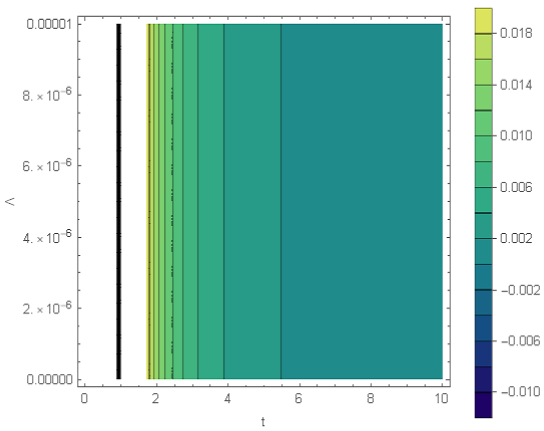

Figure 12. The relationship between Cosmological Constant Λ and cosmic time t in Density plot view is defined with the parameters: $\alpha = 0.1, n = 2, \alpha_1 = 0.1, \alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

5. SPECIAL CASES:

When $\frac{\alpha_1}{2} = \alpha_2$ and $\beta = 1$, the Cosmological constant Λ, the shear scalar σ, Gravitational constant G, the expansion scalar θ and Hubble constant H_i for the Eq. (24) is expressed as,

$$G = -\frac{2n\alpha_2^2}{\pi\alpha_3 t^{2n+1}}.exp\left\{\frac{5\alpha_2(\alpha+1)}{1-n}t^{1-n}\right\}.\left\{\xi_0 - \frac{5\alpha_2(\alpha+1)}{t^n}\right\}^{-1}$$

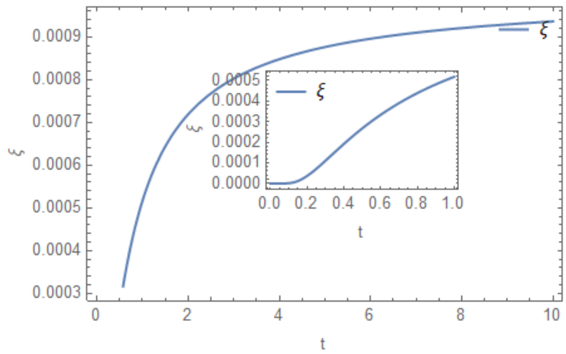

Figure 13. The relationship between coefficient of bulk viscosity ξ and cosmic time t is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

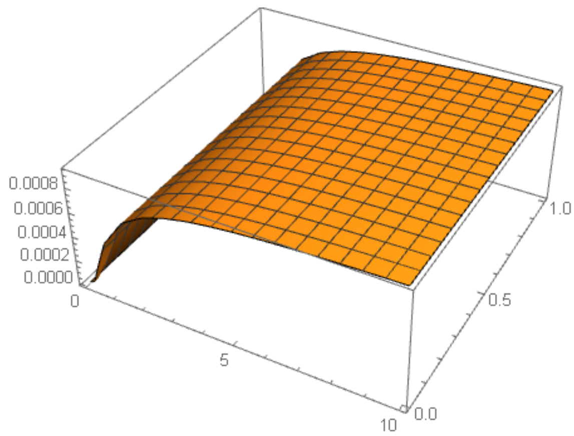

Figure 14. The relationship between coefficient of bulk viscosity ξ and cosmic time t in three-Dimensional is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

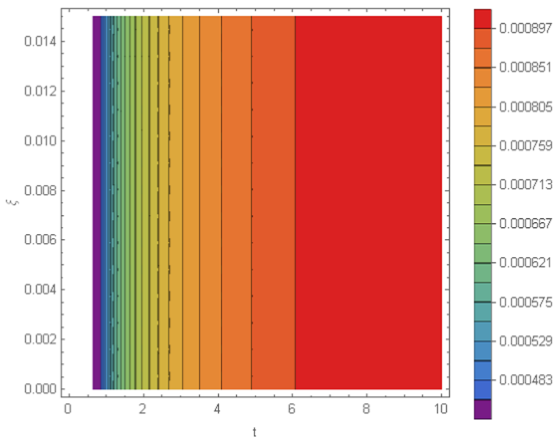

Figure 15. The relationship between coefficient of bulk viscosity ξ and cosmic time t in Density plot view is defined with the parameters: $\alpha = 0.1$, $n = 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.1$.

$$\Lambda = \frac{8\alpha_2^2}{t^{2n}} + \frac{16n\alpha_2^2}{t^{2n+1}} \left\{ -\frac{5\alpha_2(\alpha+1)}{t^n} + \xi_0 \right\}^{-1}$$

It should be noted that α_2 has the dimension T^{n-1}, as implied by the ansatz $\dot{X}_3/X_3 = \alpha_2 t^{-n}$, while ξ_0 has the dimension T^{-1}. Consequently, the above expression for Λ possesses the correct physical dimension T^{-2} (or L^{-2}) for arbitrary values of n, where the dimension of t is T. Further,

$$H_1 = H_2 = \frac{2\alpha_2}{t^n}, H_3 = \frac{\alpha_2}{t^n}$$

$$\Theta = \left(\frac{\dot{X}_1}{X_1} + \frac{\dot{X}_2}{X_2} + \frac{\dot{X}_3}{X_3}\right) = \frac{2\alpha_1 + \alpha_2}{t^n}$$

$$\frac{\dot{G}}{G} \propto H, \Lambda \propto \frac{1}{t^2}, H \propto \frac{1}{t}, G > 0, \rho > 0$$

.

When $n > 0$, the model in Eq. (24) initiates with a Big-Bang at $t = 0$, while the expansion scalar Θ decreases as time t increases. On the other hand, when $n < 0$, the expansion in the model increases over time. When $t \to 0$, both the Hubble parameter H and the shear scalar σ approach infinity; however, as $t \to \infty$, the Hubble parameter H and the shear scalar σ approach zero. Since, $\lim_{t\to\infty} \frac{\sigma}{\Theta} \neq 0$, the model however does not achieve isotropy for large values of t. As time t increases, the spatial volume V decreases, and the rate of expansion also slows down. Since, $\xi = \xi_0 \rho^\beta$ and $\beta > 1$, the model indicates the presence of inflationary phases.
When $n = 1$, Eq. (15) is expressed as,

$$\frac{\dot{X}_1}{X_1} = \frac{\dot{X}_2}{X_2} = \frac{\alpha_1}{t}, \frac{\dot{X}_3}{X_3} = \frac{\alpha_2}{t}$$

After integrating on both sides, we get

$$X_1 = X_2 = \alpha_4.t^{\alpha_1}, X_3 = \alpha_5.t^{\alpha_2}$$

Therefore, metric Eq. (1) reduces to

$$ds^2 = -dt^2 + \alpha_4^2.t^{2\alpha_1}(dx^2 + dy^2) + \alpha_5^2.t^{2\alpha_2}dz^2$$

Thus, the Gravitational constant G, the energy density ρ, Cosmological constant Λ, coefficient of bulk viscosity ξ, Hubble constant H_i, the shear scalar σ, spacial volume V and expansion scalar Θ are expressed as follows:

$$\rho = N.exp\left\{\frac{(1+\alpha)(2\alpha_1+\alpha_2)^2}{t}\right\}$$

$$\bar{p} = \alpha.N.exp\left\{\frac{(1+\alpha)(2\alpha_1+\alpha_2)^2}{t}\right\}$$

where N is the integration constant.

$$G = \frac{2\alpha_1(\alpha_1 + 2\alpha_2)t}{8\pi(1+\alpha)(2\alpha_1+\alpha_2)}\left[exp\left\{\frac{(1+\alpha)(2\alpha_1+\alpha_2)^2}{t}\right\}\right]^{-1}$$

$$\Lambda = \alpha_1(2\alpha_1+\alpha_2)t.\left\{\frac{1}{t^3} - \frac{2N}{(1+\alpha)(2\alpha_1+\alpha_2)}\right\}$$

$$\xi = \xi_0 N^\beta.exp\left\{\frac{\beta(1+\alpha)(2\alpha_1+\alpha_2)^2}{t}\right\}$$

$$H_1 = H_2 = \frac{\alpha_1}{t}, H_3 = \frac{\alpha_2}{t}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\alpha_1 - \alpha_2}{t}\right) \tag{29}$$

$$\Theta = \frac{\dot{X}_1}{X_1} + \frac{\dot{X}_2}{X_2} + \frac{\dot{X}_3}{X_3} = \frac{2\alpha_1+\alpha_2}{t}$$

$$V = \alpha_4\alpha_5 t^{(2\alpha_1+\alpha_2)}$$

$$\frac{\dot{G}}{G} \propto H, \Lambda \propto \frac{1}{t^2}, H \propto \frac{1}{t}, G > 0, \rho > 0.$$

It may be noted that the above expression for Λ in eqn. (29) is dimensionally consistent provided the parameter N carries the dimension T^{-3} (or L^{-3} in geometrized units), thereby ensuring that Λ has the correct dimension T^{-2} (or L^{-2}), where T or L is the dimension of t. The expansion scalar Θ, shear scalar σ, Hubble parameter H, and cosmological constant Λ all exhibit a decreasing trend with time. As cosmic time t increases, σ, H, Θ, and Λ gradually decline, indicating that the universe experienced a rapid expansion in the past, which is slowing down over time. This behavior is consistent with observations reported by numerous astronomers (Riess et al. [4, 28], Perlmutter et al. [1], etc.). In contrast, the gravitational constant G and the volume of the universe V show a positive correlation with time. Since the ratio $\frac{\sigma}{\Theta}$ remains constant, the model does not approach isotropy at late times. Moreover, if $\alpha_1 = \alpha_2$, it implies that the polytropic Bianchi type-I space-time is shear-free.

6. CONCLUSIONS

The decreasing trend of the gravitational constant, G, with cosmic time inferred from Figs. (7–9) is consistent with a broad range of independent constraints spanning cosmological, astrophysical, and laboratory scales. CMB observations from *Planck* (2018) limit fractional variations to $\Delta G/G < 0.1\%$ over ~ 13.8 Gyr, while BAO measurements from SDSS, BOSS, and DES constrain $\Delta G/G < 0.2\%$ over ~ 10 Gyr. Type Ia supernovae from the Union2.1 compilation impose $\Delta G/G < 0.3\%$ over ~ 9 Gyr. At shorter timescales, gravitational redshift tests of compact objects bound $\dot{G}/G < 10^{-13}\,\mathrm{yr}^{-1}$, binary pulsar timing yields $\dot{G}/G < 10^{-12}\,\mathrm{yr}^{-1}$, advanced LIGO O1–O2 observations constrain $\dot{G}/G < 10^{-15}\,\mathrm{yr}^{-1}$, and high-precision torsion balance and Cavendish-type experiments further tighten laboratory limits to $\dot{G}/G < 10^{-14}$–$10^{-15}\,\mathrm{yr}^{-1}$.

Astrophysical observations further support a mild temporal decrease of the gravitational constant, with analyses of the M67 star cluster indicating a $\sim 5\%$ reduction in G over ~ 4.5 Gyr, while galaxy rotation curve studies based on the SPARC dataset suggest a $\sim 3\%$ decrease over ~ 10 Gyr. Ongoing and forthcoming experimental and observational programs will continue to investigate the compelling possibility of a time–varying gravitational constant, G, with unprecedented accuracy. The Square Kilometre Array (SKA) is expected to place stringent constraints on deviations in G through high–precision measurements of baryon acoustic oscillations and cosmic shear, while the Laser Interferometer Space Antenna (LISA) will probe such variations using gravitational–wave observations of compact binary systems. In parallel, next–generation torsion balance experiments will further tighten laboratory bounds on both temporal and spatial variations of G, and the James Webb Space Telescope (JWST) will provide complementary astrophysical constraints by examining stellar evolution and age determinations across diverse cosmic environments.

In this paper, we derived solutions to Einstein's general relativity equations for a Bianchi type-I spacetime with a polytropic bulk viscous fluid. We observed that as time approaches $t \longrightarrow 0$, the spatial volume $V \longrightarrow \infty$, indicating that the universe begins with zero volume and expands infinitely in both the distant past and future. The influence of bulk viscosity on cosmic evolution, particularly in its early stages, appears to be significant. Additionally, when $t \longrightarrow 0$, the energy density ρ, cosmological constant Λ, expansion scalar Θ, shear scalar σ, coefficient of bulk viscosity ξ all tend to infinity. Conversely, as $t \longrightarrow \infty$, these quantities approach zero.

Since $\frac{\sigma}{\Theta}$ remains constant, the model does not approach isotropy for large values of t. The spatial volume V increases with time if $\alpha_2 > 0$, and since $\xi = \xi_0\rho^{\beta}, \beta > 0$, the model leads to an inflationary solution. The gravitational constant G increases with the universe's age, suggesting that gravity was weaker in the past and has been strengthening over time, consistent with previous results from Abdussattar and others (Vishwakarma [52], chow [45], Levitt [47]). The cosmological constant follows $\Lambda \propto \frac{1}{t^2}$, aligning with models from Kalligas et al. [46], Berman et al. [42], Berman et al. [42], Bertolami [43, 44] Singh [49], Tiwari [50, 51], is physically consistent with current observations indicating a small Λ in the present universe. Ultimately, the solutions developed in this work offer a coherent theoretical framework for analyzing the evolution of a Bianchi type-I universe with a polytropic bulk viscous fluid and time-varying G and Λ. They clarify the combined roles of bulk viscosity, anisotropy, and dynamical fundamental constants, while remaining consistent with observational constraints and providing meaningful insights into cosmic evolution. Further comprehensive analysis demonstrate that different time-dependent scenarios produce significant variations in key physical parameters, impacting the universe's dynamical evolution. The geometrical structure and observational implications of the model are further explored, offering insights into its consistency with cosmological data and theoretical predictions.

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КОСМОЛОГІЧНА МОДЕЛЬ З ПОЛІТРОПІЧНОЮ ОБ'ЄМНОЮ В'ЯЗКІСТЮ ТА ЗМІННИМИ ГРАВІТАЦІЙНИМИ ТА КОСМОЛОГІЧНИМИ СТАЛИМИ

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У цій статті досліджуються рівняння поля Ейнштейна в рамках космологічної моделі, яка включає політропну об'ємну в'язку рідину, причому гравітаційні та космологічні константи змінюються у Всесвіті типу Біанкі I. Спостерігається, що різні сценарії, що залежать від часу, призводять до змін фізичних параметрів, що має значні наслідки для космічної еволюції. Ключові фізичні параметри, включаючи густину енергії, гравітаційну сталу, скаляр розширення, космологічну сталу, коефіцієнт об'ємної в'язкості та скаляр зсуву, досліджуються на предмет їхнього фізичного значення. Також досліджуються геометричні та спостережувальні інтерпретації космологічної моделі.

Ключові слова: *Б'янкі I типу; скаляр зсуву; об'ємна в'язкість; скаляр розширення*