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MAGNETIZED ANISOTROPIC DARK ENERGY COSMOLOGICAL MODEL IN $f(T)$ GRAVITY WITH A SPECIAL LAW OF THE HUBBLE PARAMETER

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In this paper, we investigate a magnetized anisotropic dark energy cosmological model in the framework of $f(T)$ gravity for a locally rotationally symmetric Bianchi type-I spacetime. Choosing $f(T) = 0$ reduces the theory to teleparallel gravity, dynamically equivalent to general relativity, enabling direct comparison with standard cosmology. Exact solutions of the field equations are obtained by assuming a special law of the Hubble parameter with a nonzero constant S, yielding power-law behavior of the directional scale factors. A uniform magnetic field aligned along one spatial direction produces an early-time anisotropy whose energy density decays with cosmic expansion. The energy density, pressure, the equation-of-state parameter, and the energy conditions are analyzed. The pressure remains negative, while the equation-of-state parameter evolves dynamically and approaches the range $-1 \le \omega < -\frac{1}{3}$ at late times, consistent with SN Ia and CMB constraints. The null and weak energy conditions are violated at late times, whereas the strong energy condition is violated throughout, implying acceleration.

Keywords: *Magnetized dark energy; $f(T)$ gravity; LRS Bianchi type-I model; Anisotropic universe; Cosmic time*

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1. INTRODUCTION

Research on magnetized anisotropic dark energy with constant deceleration parameter in $f(T)$ gravity has emerged as a critical area of inquiry due to its potential to explain the accelerated expansion of the universe while addressing anisotropies observed in cosmic structures. The evolution of modified teleparallel gravity theories,particularly $f(T)$ gravity, has been extensively studied since the early 2010s, offering an alternative to curvature-based modifications such as $f(R)$ gravity by utilizing torsion scalar T as the fundamental geometric quantity [1–3]. Initial models demonstrated the capability of $f(T)$ gravity to reproduce late-time acceleration consistent with observational data [4, 5], while subsequent works incorporated anisotropic cosmologies, such as Bianchi type-I and VI models, to capture large-scale anisotropies hinted by supernova and cosmic microwave background observations [6–8].

The inclusion of magnetic fields alongside anisotropic dark energy components further enriches the modeling of cosmic evolution, reflecting realistic astrophysical conditions [9–11]. These developments underscore the theoretical and observational significance of exploring anisotropic dark energy within $f(T)$frameworks, especially under constraints like constant deceleration parameters that simplify and clarify cosmic dynamics [12].

Despite these advances, the specific problem of characterizing magnetized anisotropic dark energy models with a constant deceleration parameter in the context of $f(T_0)$ gravity remains insufficiently addressed.Although several studies have examined anisotropic models with varying deceleration parameters or without magnetic fields [13, 14], and others have explored magnetized fluids in different gravity theories,a comprehensive treatment combining magnetization, anisotropy, and constant deceleration within $f(T_0)$ gravity is lacking [15, 16].

Studies on modified teleparallel gravity have significantly expanded over the last decade, particularly in the context of anisotropic cosmology and dark energy dynamics.The early foundational work [17–20] explored perturbations, large-scale structure, phantom divide crossing, and the reconstruction of $f(T)$ models capable of describing inflation, ΛCDM behavior, and future singularities. These studies generally did not include magnetic fields but provided key insights into stability, perturbation growth, thermodynamics, and late-time acceleration within torsion-based gravity. Extensions such as $f(T, B)$,$f(T, T_G)$, and $f(T, \mathcal{T})$ [21–23] further broadened the theoretical landscape, addressing unified inflation–acceleration phases, cosmographic constraints, and ghost-free stability features. A large set of works has examined anisotropic dark energy models within f(T) gravity and related theories. [24–26] investigated Bianchi-type cosmologies, incorporating constant or variable deceleration parameters through Hubble parameter variation laws, power-law expansions, or reconstruction schemes. These studies revealed transitions from deceleration to acceleration, isotropization trends, phantom behavior, observational compatibility with SNe Ia, CMB, BAO, and alleviation of tensions such as H0.However, in most cases magnetic fields were either omitted or treated only implicitly through anisotropic pressure components, thus limiting the analysis of realistic anisotropic environments.

Recent observations of cosmic magnetic fields have motivated magnetized anisotropic models, mostly in GR, $f(R)$, or $f(Q, T)$ gravity. Some works such as [27–29] explicitly modeled magnetic fields in Bianchi types I, II, V, VI_0, and

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Kantowski–Sachs universes. These studies demonstrated that magnetic fields significantly influence shear evolution, skewness parameters, isotropization, and the expansion behavior of the universe. Many models exhibited anisotropy persistence, delayed isotropization, or strong early-time magnetic dominance. [30, 31] incorporated magnetic effects indirectly via cosmic string fluids within $f(T)$ or $f(Q)$ gravity, showing that torsion–magnetization interactions modify late-time evolution and observational characteristics. Although these studies highlighted the cosmological importance of magnetic fields, they did not integrate them into the teleparallel framework with a constant deceleration parameter.

Meanwhile, several recent works [32–35] focused on observational constraints, phantom crossings, holographic reconstructions, Kaniadakis entropy, emergent scenarios, and hybrid scale factor dynamics. These contributions further emphasize the viability of modified gravity, but still lack explicit treatment of magnetized anisotropic fluids within $f(T)$ cosmology under CDP assumptions.

Despite significant progress in magnetized anisotropic dark energy models within modified gravity, several important challenges remain unresolved. Most existing studies idealize magnetic fields as uniform or one-directional, neglecting realistic magnetohydrodynamic effects and backreaction on anisotropy, shear evolution, and dark energy dynamics [36,37]. Additionally, the widespread use of constant deceleration parameters oversimplifies cosmic evolution and limits the ability to model realistic transitions between decelerated and accelerated phases, particularly in magnetized anisotropic settings [38]. Observational constraints on such models remain weak, as current datasets primarily focus on isotropic scenarios and insufficiently probe anisotropic and magnetized signatures in CMB, large-scale structure, and polarization data [39].

Moreover, the coupling between magnetic fields and torsion scalar modifications in $f(T)$ gravity, perturbation theory in anisotropic magnetized backgrounds, thermodynamic consistency, and systematic exploration of viable $f(T)$ functional forms remain insufficiently explored. Addressing these issues through numerical simulations, realistic magnetic field modeling, and comprehensive observational analyses is essential for developing physically viable magnetized anisotropic cosmological models.

2. $f(T)$ GRAVITY FORMALISM AND MAGNETIZED DARK ENERGY

Teleparallel gravity provides an equivalent formulation of General Relativity (GR) in which gravitation is described by spacetime torsion rather than curvature. The dynamical variables are the tetrad fields $e^a{}_\mu$, which relate the spacetime metric $g_{\mu\nu}$ to the Minkowski metric η_{ab} through

$$g_{\mu\nu} = \eta_{ab}\, e^a{}_\mu e^b{}_\nu. \tag{1}$$

In teleparallel gravity, the torsion tensor is defined as

$$T^\lambda{}_{\mu\nu} = e_a{}^\lambda \left(\partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu\right), \tag{2}$$

and the torsion scalar T is constructed from suitable contractions of the torsion tensor,

$$T = S_\lambda{}^{\mu\nu} T^\lambda{}_{\mu\nu}, \tag{3}$$

where $S_\lambda{}^{\mu\nu}$ denotes the superpotential.

In $f(T)$ gravity, the gravitational action is generalized by replacing the torsion scalar T with an arbitrary function $f(T)$,

$$S = \int [T + f(T) + L_m] e d^4x \tag{4}$$

where $e = \det(e^a{}_\mu) = \sqrt{-g}$ and $\mathcal{L}_m$ represents the matter Lagrangian. Variation of this action with respect to the tetrad fields yields second-order field equations, which constitutes an important advantage of $f(T)$ gravity over curvature-based modifications such as $f(R)$ gravity.

We obtain the following equation of motion by functionally varying the action in equation (3) with regard to the tetrads:

$$S_\mu{}^{\rho\nu}\, \partial_\rho T\, f_{TT} + \left[e^{-1} e^a{}_\mu\, \partial_\rho (e\, e_a{}^\alpha S_\alpha{}^{\rho\nu}) + T^\alpha{}_{\mu\beta}\, S_\alpha{}^{\nu\beta}\right](1 + f_T) + \frac{1}{4}\, \delta^\nu_\mu\, (T + f) = T^\nu_\mu. \tag{5}$$

where $T^\nu{}_\mu$ is the energy–momentum tensor and

$$f_T(T) = \frac{df(T)}{dT}.$$

The field equation (5) is written in terms of tetrads and their partial derivatives and therefore appears formally different from Einstein's field equations. However, by choosing $f(T) = 0$, this is dynamically equivalent to the GR. The energy momentum tensor for magnetized dark energy is given by

$$T^\nu_\mu = \text{diag}\left[-p_x, -p_y, -p_z, \rho\right]. \tag{6}$$

Then, one may parametrize it as follows:

$$T^{\nu}_{\mu} = \text{diag}\left[-p_x + \rho_B,\ -p_y - \rho_B,\ -p_z - \rho_B,\ \rho + \rho_B\right] \tag{7}$$

where ρ is the energy density of the fluid,ρ_B is the energy density of the magnetic field, p_x, p_y, p_z are pressures. Zeldovich et al [40] underlined the importance of magnetic field for a variety of astrophysical phenomena.Magnetic field could have cosmological origin [41].

3. MODEL AND FIELD EQUATIONS

In this work, we consider the spatially homogeneous and anisotropic Bianchi type-I space-time as

$$ds^2 = dt^2 - A^2(t)\,dx^2 - B^2(t)\left(dy^2 + dz^2\right), \tag{8}$$

where the metric potentials A and B are functions of cosmic time t only. The corresponding torsion scalar T is given by

$$T = -2\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right). \tag{9}$$

Using the equation of motion (4),the stress–energy tensor (6), and the Bianchi type-I space-time (8) , the field equations can be written as

$$(T + f) + 4(1 + f_T)\left(\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right) + 4\frac{\dot{B}}{B}\dot{T}f_{TT} = -(p - \rho_B) \tag{10}$$

$$(T + f) + 2(1 + f_T)\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3\frac{\dot{A}\dot{B}}{AB}\right) + 2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{T}f_{TT} = -(p + \rho_B) \tag{11}$$

$$(T + f) + 4(1 + f_T)\left(\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB}\right) = \rho + \rho_B \tag{12}$$

The system of equations from equations (10)-(12) may reduce to

$$\left(4\frac{\ddot{B}}{B} + 2\frac{\dot{B}^2}{B^2}\right) = -(p - \rho_B) \tag{13}$$

$$\left(2\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB}\right) = -(p + \rho_B) \tag{14}$$

$$\left(2\frac{\dot{B}^2}{B^2} + 4\frac{\dot{A}\dot{B}}{AB}\right) = \rho + \rho_B \tag{15}$$

4. SOLUTION OF THE FIELD EQUATION

The field equations (13)-(15), we get three differential equations containing five unknowns — namely A, B, p, ρ and ρ_B.To determine the values of these unknowns, we use the energy–momentum conservation equation $T^{\nu}_{\mu;\nu} = 0$, which leads to two separate conservation equations corresponding to the anisotropic fluid and the magnetic field [42].

$$\dot{\rho} + (1 + \omega)\rho\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0 \tag{16}$$

$$\rho_B = \frac{\beta}{B^4} \tag{17}$$

In what follows, we define certain kinematical quantities of the space-time, specifically the mean scale factor and the volume.

$$a^3 = V = AB^2 \tag{18}$$

The directional Hubble parameters along the x-, y-, and z-axes, respectively, for the LRS Bianchi type-I metric are defined as

$$H_x = \frac{\dot{A}}{A}, \quad H_y = \frac{\dot{B}}{B}, \quad H_z = \frac{\dot{B}}{B}. \tag{19}$$

To express volumetric expansion rate of Universe, the mean Hubble parameter is defined as

$$H = \frac{1}{3}(H_x + H_y + H_z) \tag{20}$$

From equations (18)-(20), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{21}$$

For the purpose of solving the field equations, we consider a physical condition according to which the expansion scalar is proportional to the shear scalar, i.e.,

$$A = B^n \tag{22}$$

where n is the proportionality constant. The motivation for considering Eq. (22) which is from the work of [9,43,44]. The line element (6) is fully characterized by the Hubble parameter H. Hence, we assume that the mean Hubble parameter H is related to the average scale factor a through the relation

$$H = k_1 a^{-s} \tag{23}$$

where $k_1 > 0$ and $s \geq 0$ are constants. This particular law of variation of the Hubble parameter leads to a constant value of the deceleration parameter. Such a relation was originally introduced by Bermann [48] in the study of FRW cosmological models. Subsequently, several researchers [45–48] have extensively employed this special law of the Hubble parameter to investigate flat FRW and Bianchi-type cosmological models.

By adopting this special law of variation of the Hubble parameter, we obtain exact solutions of the field equations corresponding to an anisotropic cosmological model exhibiting a negative constant deceleration parameter. The deceleration parameter q is define as,

$$q = -\frac{a\,\ddot{a}}{\dot{a}^2} \tag{24}$$

From eqs (21) and (23), we get

$$\dot{a} = k_1 a^{-s+1}, \tag{25}$$

$$\ddot{a} = -k_1^2 (s-1) a^{-2s+1} \tag{26}$$

By using Eqs. (24)–(25), we obtain a constant deceleration parameter for the mean scale factor as

$$q = s - 1, \quad s \neq 0 \tag{27}$$

For $0 < s < 1$, the deceleration parameter satisfies $-1 < q < 0$, indicating accelerated expansion. The case $s = 1$ corresponds to a coasting universe ($q = 0$), while $s > 1$ leads to a decelerating phase ($q > 0$). The special case $s = 0$ yields $q = -1$, representing exponential expansion. Using eq.(25), we get the law of average scale factor as,

$$a = (Dt + c_1)^{\frac{1}{s}}, \quad s \neq 0. \tag{28}$$

where c_1 is the constant of integration. Using Eqs. (20), (21), (22), and (28), the exact expression for the scale function is given by

$$A(t) = l_2\,(Dt + c_1)^{\frac{n}{r}} \tag{29}$$

$$B(t) = l_1\,(Dt + c_1)^{\frac{1}{r}} \tag{30}$$

where

$l_1 = c_1^{-\frac{1}{n+2}}$, $l_2 = l_1^n$ and $r = \frac{n+2}{3}s$

Therefore, model (8) becomes

$$ds^2 = dt^2 - l_2^2\,(Dt + c_1)^{\frac{2n}{r}}\,dx^2 - l_1^2\,(Dt + c_1)^{\frac{2}{r}}\left(dy^2 + dz^2\right) \tag{31}$$

The space–time model (31) describes an LRS Bianchi Type-I anisotropic cosmological model filled with magnetized dark energy, where the magnetic field is aligned along one spatial direction, leading to anisotropic expansion.

For model (31),the expression for kinematical parameters, i.e. the Hubble parameter H, the scalar expansion θ, mean anisotropic parameter A_m and shear scalar σ are given by

$$H = \frac{(2n+1)D}{3r(Dt + c_1)} \tag{32}$$

$$\theta = 3H = \frac{(2n+1)D}{r(Dt + c_1)}, \tag{33}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2(n-1)^2}{(2n+1)^2} \tag{34}$$

$$\sigma^2 = \frac{3}{2}A_m H^2 = \frac{(n-1)^2 D^2}{3r^2(Dt+c_1)^2} \tag{35}$$

The energy density for magnetic field is obtained by substituting eq.(30) in eq.(17)

$$\rho_B = \frac{\beta}{l_1^4\,(Dt+c_1)^{\frac{4}{r}}} \tag{36}$$

Figure 1 depicts the variation of the magnetic field energy density ρ_B with cosmic time. It is observed that ρ_B diverges at the initial epoch, indicating a Big Bang type singularity, and decreases monotonically as the universe expands. At late times, the magnetic field contribution becomes negligible, favoring an isotropic dark-energy–dominated universe. Using Eqs. (15), (22), (29), (30) and (36), the energy density of the fluid is obtained as follows:

$$\rho = \frac{2(2n+1)D^2}{r^2\,(Dt+c_1)^2} - \frac{\beta}{l_1^4\,(Dt+c_1)^{\frac{4}{r}}} \tag{37}$$

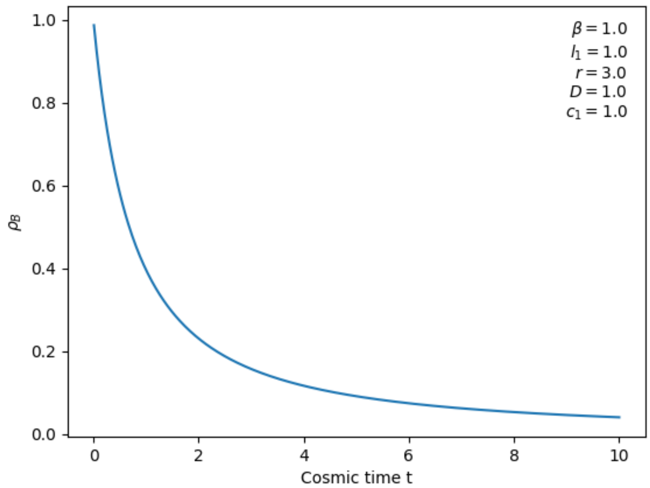

Figure 1. Energy density of the magnetic field versus cosmic time.

Figure 2 illustrates the variation of the fluid energy density ρ with cosmic time. It is observed that ρ diverges at the initial epoch, indicating a Big Bang type singularity. The energy density decreases sharply during the early evolution and attains a minimum before gradually approaching zero at late times. This behavior suggests that magnetic effects dominate the early universe, while the fluid evolves toward a dark-energy–dominated phase at late times. Using Eqs. (14), (22), (29), (30) and (36), the pressure of the fluid is obtained as follows:

$$p = -\left[\frac{\beta}{l_1^4\,(Dt+c_1)^{\frac{4}{r}}} + \frac{2D^2(n+1)(n+1-r)}{r^2\,(Dt+c_1)^2}\right] \tag{38}$$

Figure 3 illustrates the variation of pressure p with cosmic time t for the magnetized dark energy model. The pressure remains negative throughout the cosmic evolution, diverges at the initial epoch, and asymptotically approaches zero at late times, indicating a transition toward a dark-energy-dominated universe.

$$\omega = -\frac{1}{\rho}\left[\frac{\beta}{l_1^4\,(Dt+c_1)^{\frac{4}{r}}} + \frac{2D^2(n+1)(n+1-r)}{r^2\,(Dt+c_1)^2}\right] \tag{39}$$

The equation-of-state parameter ω evolves dynamically and attains negative values at late times. For suitable choices of the model parameters, ω lies in the range $-1 \le \omega < -\frac{1}{3}$, which is consistent with the constraints from SN Ia and CMB observations. Hence, the present model is observationally viable and capable of describing the late-time accelerated expansion of the universe.

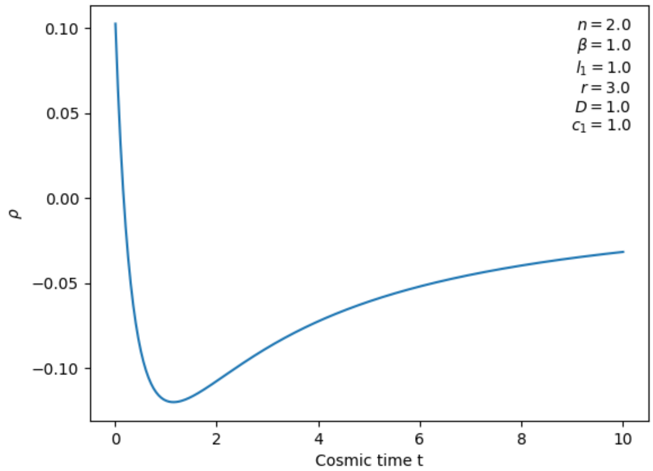

Figure 2. Energy density versus cosmic time.

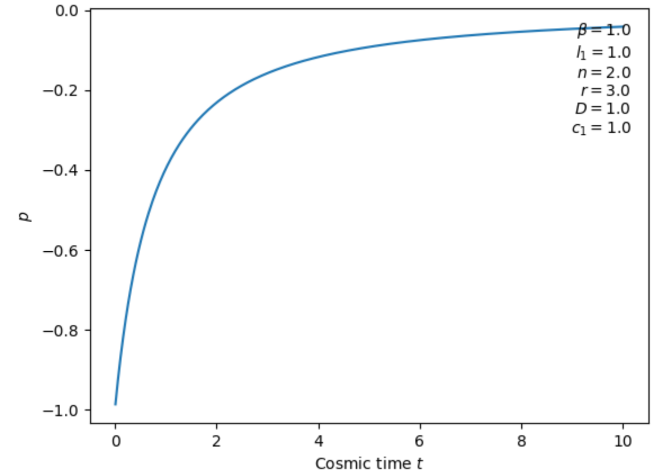

Figure 3. Pressure versus cosmic time.

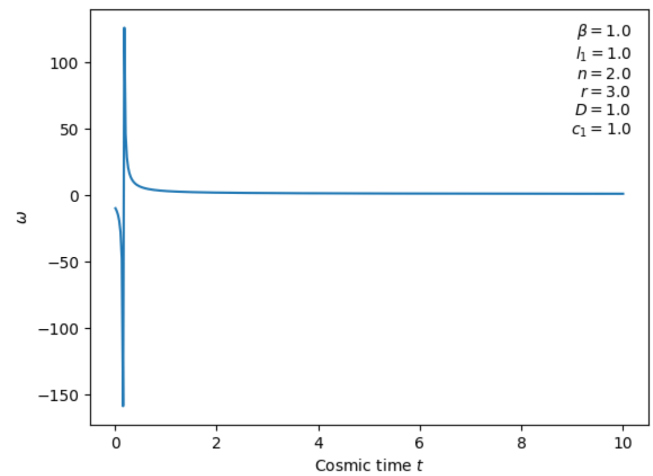

Figure 4. EoS parameter cosmic time.

5. ENERGY CONDITION

The Energy Conditions for the Present Model are given by

Null Energy Condition (NEC) The null energy condition is defined as

$$\rho + p \geq 0. \tag{40}$$

$$\rho + p = \frac{2D^2}{r^2\,(Dt + c_1)^2}\left[(2n+1) - (n+1)(n+1-r)\right] - \frac{2\beta}{l_1^4\,(Dt + c_1)^{\frac{4}{r}}} \tag{41}$$

For the present model, $\rho + p$ becomes negative at late times, indicating that the NEC is violated in the dark-energy–dominated phase of the universe.

Weak Energy Condition (WEC) The weak energy condition requires

$$\rho \geq 0, \qquad \rho + p \geq 0. \tag{42}$$

Although the energy density ρ remains positive for suitable choices of the model parameters, the violation of $\rho + p$ at late times implies a partial violation of the WEC.

Strong Energy Condition (SEC) The strong energy condition is given by

$$\rho + 3p \geq 0. \tag{43}$$

$$\rho + 3p = \frac{2D^2}{r^2\,(Dt + c_1)^2}\left[(2n+1) - 3(n+1)(n+1-r)\right] - \frac{4\beta}{l_1^4\,(Dt + c_1)^{\frac{4}{r}}} \tag{44}$$

For the present model, $\rho+3p$ is found to be negative throughout the cosmic evolution, which signifies a clear violation of the SEC. This violation is a necessary condition for the accelerated expansion of the universe.

Dominant Energy Condition (DEC) The dominant energy condition is expressed as

$$\rho \geq |p|. \tag{45}$$

In the present scenario, the dominance of negative pressure at late times leads to the violation of the DEC, indicating the presence of exotic dark-energy behavior. Figure 5 displays the combined evolution of the energy conditions for the

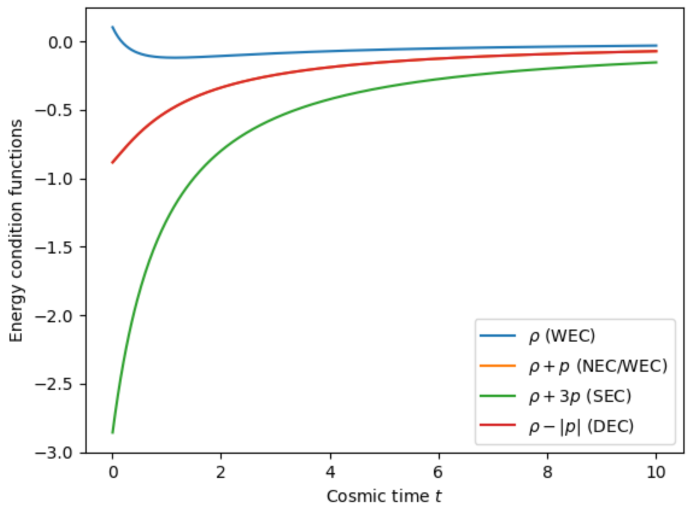

Figure 5. Evolution of the energy conditions with cosmic time t.

present magnetized dark energy model.It is observed that the energy density ρ remains positive, satisfying the weak energy condition at early times. However, the quantity $\rho + p$ becomes negative at late times, indicating the violation of the null and weak energy conditions.Moreover, $\rho + 3p$ remains negative throughout the cosmic evolution, implying a persistent violation of the strong energy condition, which is necessary for late-time accelerated expansion. The violation of $\rho - |p|$ at late times shows that the dominant energy condition is also not satisfied, confirming the presence of exotic dark-energy behavior in the model.

6. CONCLUSIONS

In this work, we have investigated a magnetized anisotropic dark energy cosmological model in the framework of $f(T)$ gravity for an LRS Bianchi type-I space–time. By choosing $f(T) = 0$, the theory reduces to teleparallel gravity, which is dynamically equivalent to general relativity, allowing a consistent comparison with standard cosmological scenarios.

A special law of the Hubble parameter with $S \neq 0$ has been employed to obtain exact solutions of the field equations. The resulting model describes an anisotropic universe with directional scale factors evolving as power-law functions of cosmic time. The presence of a uniform magnetic field aligned along one spatial direction introduces anisotropy at early times, while its contribution decays rapidly as the universe expands.

The physical behavior of the model has been examined through the evolution of the energy density, pressure, equation-of-state parameter, and energy conditions. The pressure remains negative throughout the cosmic evolution, while the energy density is positive for suitable choices of model parameters. The equation-of-state parameter ω evolves dynamically and attains negative values at late times, lying in the range $-1 \leq \omega < -\frac{1}{3}$, which is consistent with the observational constraints from Type Ia supernovae and cosmic microwave background observations.

Furthermore, the null and weak energy conditions are found to be violated at late times, whereas the strong energy condition is violated throughout the cosmic evolution. The violation of the strong energy condition confirms the presence of accelerated expansion in the universe. The dominant energy condition is also violated in the late-time regime, indicating the dominance of dark energy with strong negative pressure.

Overall, the present magnetized anisotropic dark energy model in $f(T)$ gravity with a special law of the Hubble parameter provides a viable description of the late-time accelerated expansion of the universe. The model exhibits a transition from a magnetically dominated anisotropic early universe to an effectively isotropic dark-energy-dominated phase at late times, in agreement with current cosmological observations.

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КОСМОЛОГІЧНА МОДЕЛЬ НАМАГНІЧЕНОЇ АНІЗОТРОПНОЇ ТЕМНОЇ ЕНЕРГІЇ В $f(T)$ ГРАВІТАЦІЇ ЗІ СПЕЦІАЛЬНИМ ЗАКОНОМ ПАРАМЕТРА ХАББЛА

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У цій статті ми досліджуємо космологічну модель намагніченої анізотропної темної енергії в рамках $f(T)$ гравітації для локально обертально-симетричного простору-часу типу Біанкі I. Вибір $f(T) = 0$ зводить теорію до телепаралельної гравітації, динамічно еквівалентної загальній теорії відносності, що дозволяє пряме порівняння зі стандартною космологією. Точні розв'язки рівнянь поля отримуються, припускаючи спеціальний закон параметра Хаббла з ненульовою константою S, що призводить до степеневої поведінки напрямлених масштабних коефіцієнтів. Однорідне магнітне поле, орієнтоване вздовж одного просторового напрямку, створює анізотропію на ранніх етапах часу, густина енергії якої зменшується з розширенням космосу. Аналізуються густина енергії, тиск, параметр рівняння стану та енергетичні умови. Тиск залишається негативним, тоді як параметр рівняння стану динамічно розвивається та наближається до діапазону $-1 \leq \omega < -\frac{1}{3}$ на пізніх етапах часу, що узгоджується з обмеженнями SN Ia та реліктового випромінювання. Умови нульової та слабкої енергії порушуються на пізніх етапах часу, тоді як умова сильної енергії порушується протягом усього часу, що передбачає прискорення.

Ключові слова: *намагнічена темна енергія; $f(T)$ гравітація; LRS модель Біанкі типу I; анізотропний Всесвіт; космічний час*