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# BIANCHI TYPE–VII COSMOLOGICAL MODEL WITH TSALLIS–BARROW HOLOGRAPHIC DARK ENERGY IN LYRA GEOMETRY

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In this work, we investigate an anisotropic Bianchi type–VII cosmological model in the framework of Lyra geometry filled with perfect fluid matter and Tsallis–Barrow holographic dark energy. The modified Einstein field equations are derived, and exact solutions are obtained by assuming a power-law average scale factor for a decelerating universe. Expressions for various cosmological parameters, such as the Hubble parameter, expansion scalar, shear scalar, matter density, dark energy density, and density parameters, are derived and analyzed. The behaviour of these parameters indicates that the universe is expanding continuously, with the expansion rate decreasing with cosmic time. The anisotropy parameter decreases gradually, indicating that the universe evolves towards isotropy at late times. Energy conditions, stability analysis, and cosmological diagnostics, including the statefinder and Om parameters, are also examined to evaluate the model's physical viability. The results suggest that Tsallis–Barrow entropy corrections in Lyra geometry provide a consistent framework for studying anisotropic cosmological evolution and dark energy dynamics.
**Keywords:** *Bianchi type–VII cosmology; Lyra geometry; Tsallis–Barrow holographic dark energy; Anisotropic Universe, Modified gravity; Cosmological diagnostics*
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## 1. INTRODUCTION

Modern observational cosmology strongly indicates that the universe is currently undergoing accelerated expansion. The first observational evidence for this phenomenon came from high-redshift Type Ia supernova observations, which revealed that distant supernovae appear dimmer than expected in a decelerating universe [1]. Subsequent observations of the cosmic microwave background radiation and baryon acoustic oscillations further confirmed this result [2,3]. These observations suggest that nearly 70% of the universe's total energy density is composed of a mysterious component known as dark energy.

The simplest candidate for dark energy is the cosmological constant introduced by Einstein in the framework of general relativity. Although the cosmological constant model successfully explains many observations, it faces theoretical problems such as the fine-tuning and coincidence problems [4]. These issues have motivated the development of alternative dark energy models, including scalar-field models, modified gravity theories, and holographic dark energy models.

The holographic principle, originally proposed in the context of black hole thermodynamics, provides a theoretical foundation for holographic dark energy models [5]. In this approach, the dark energy density is related to the infrared cutoff of the universe. Recently, entropy-corrected holographic dark energy models have been proposed to incorporate quantum gravitational effects. Among these models, the Tsallis–Barrow holographic dark energy framework has attracted considerable attention because it combines two different entropy modifications arising from non-extensive statistical mechanics and quantum gravitational corrections [6–8].

In addition to modified dark energy models, modified gravitational theories have also been proposed to explain cosmic acceleration. Lyra geometry, introduced as an alternative to Riemannian geometry, modifies Einstein's field equations by introducing a displacement vector field [9-13]. This modification has been used to study various cosmological models.

Furthermore, anisotropic cosmological models such as Bianchi spacetimes provide a more general description of the early universe. Among them, the Bianchi type VII model represents an anisotropic generalization of the Friedmann universe.

Inspired by the above developments, the present study explores a Bianchi type–VII anisotropic cosmological model in the framework of Lyra geometry incorporating Tsallis–Barrow holographic dark energy. The paper is organized as follows. Section 2 introduces the space–time metric and derives the model's field equations. In Section 3, exact solutions of the cosmological model are obtained. Section 4 examines the behaviour of the key physical and cosmological parameters. Section 5 presents various cosmological diagnostic tools used to analyse the model. Section 6 compares the obtained results with standard cosmological models. Section 7 discusses the physical implications of the results, and finally, Section 8 summarizes the main findings and conclusions of the work.

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## 2. METRIC AND FIELD EQUATIONS

We consider the anisotropic Bianchi type–VII metric given by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{2mx}dy^2 + C^2(t)e^{2mx}dz^2. \quad (1)$$

The average scale factor is defined as

$$a = (ABC)^{1/3}. \quad (2)$$

The average spatial volume is

$$V = a^3. \quad (3)$$

The modified Einstein field equations in Lyra geometry are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -T_{ij}. \quad (4)$$

The displacement vector is chosen as

$$\phi_i = (\beta(t),0,0,0). \quad (5)$$

The energy-momentum tensor for a perfect fluid is

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij}. \quad (6)$$

The total density and pressure are

$$\rho = \rho_m + \rho_{TB}, p = p_m + p_{TB}. \quad (7)$$

The energy density of Tsallis–Barrow holographic dark energy is defined as

$$\rho_{TB} = 3c^2H^{2(2-\delta)}. \quad (8)$$

Where $c$ is a model constant, $\delta$ is the entropy deformation parameter.

## 3. SOLUTION OF THE MODEL

To obtain exact solutions, we assume the relation

$$A = B^r, \quad (9)$$

which yields

$$A = a^{\frac{3r}{r+2}}, B = C = a^{\frac{3}{r+2}}. \quad (10)$$

We assume a power-law scale factor

$$a(t) = a_0(t + t_0)^s. \quad (11)$$

The Hubble parameter becomes

$$H = \frac{s}{t+t_0}. \quad (12)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1-s}{s}. \quad (13)$$

For $0 < s < 1$, The universe exhibits decelerating expansion.

## 4. PHYSICAL PROPERTIES OF THE MODEL

The expansion scalar is

$$\theta = 3H = \frac{3s}{t+t_0}. \quad (14)$$

The shear scalar becomes

$$\sigma^2 = \frac{3(r-1)^2s^2}{(r+2)^2(t+t_0)^2}. \quad (15)$$

The anisotropy parameter is

$$\Delta = \frac{2(r-1)^2}{(r+2)^2}. \quad (16)$$

The matter density evolves as

$$\rho_m = \rho_{m0}(t + t_0)^{-3s(1+\omega_m)}. \tag{17}$$

The Matter pressure is
Since $p_m = \omega_m \rho_m$, we obtain

$$p_m = \omega_m \rho_{m0}(t + t_0)^{-3s(1+\omega_m)} \tag{18}$$

The dark energy density becomes

$$\rho_{TB} = 3c^2 \left(\frac{s}{t+t_0}\right)^{2(2-\delta)} \tag{19}$$

Where $c$= model constant, $\delta$= Tsallis–Barrow entropy parameter.
These expressions indicate that the expansion rate decreases with cosmic time and the universe gradually approaches isotropy.

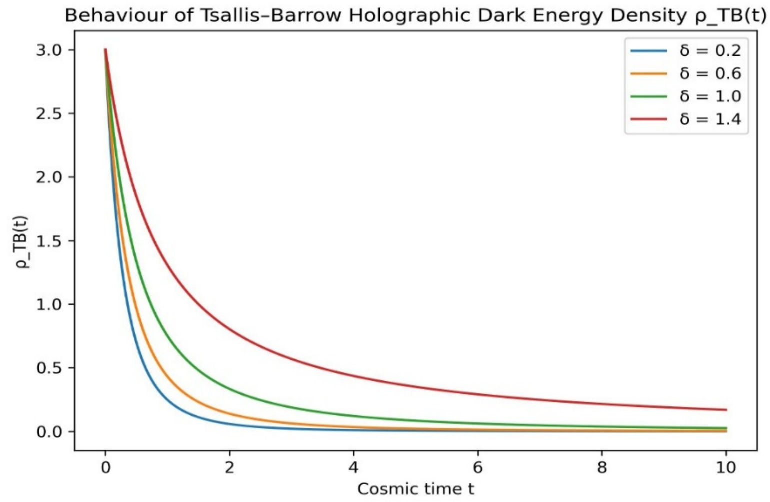


**Figure 1.** $\rho_{TB}$ vs. Cosmic time t

The graph was plotted using the following parameter values: c =1, s= 1, $t_0$=0.5, $\delta = 0.2, 0.6, 1.0, 1.4$

We observed from Figure 1, (i) $\rho_{TB}(t)$ decreases with cosmic time $t$ (ii) Larger values of $\delta$produce a slower decay of dark energy density. (iii) For small $\delta$, The density falls rapidly at early times. (iv) At late times, all curves approach zero, indicating dilution of dark energy density during cosmic evolution.

The Tsallis–Barrow equation of state parameter is

$$\omega_{TB} = -1 + \frac{2(2-\delta)}{3s} \tag{20}$$

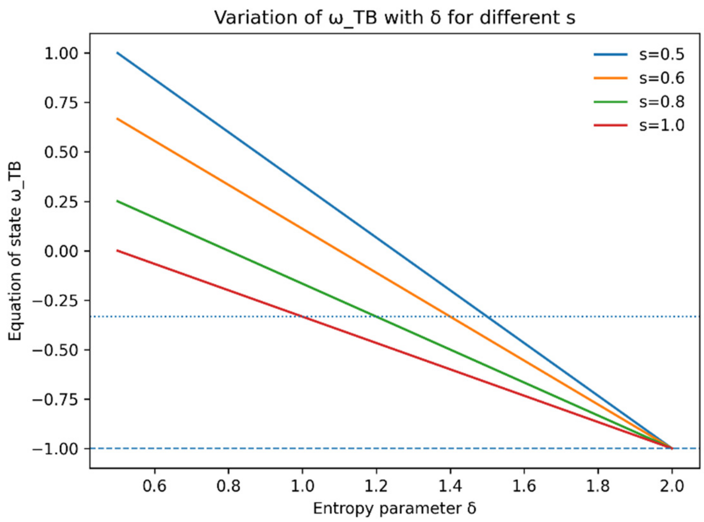


**Figure 2.** $\omega_{TB}$ vs. Cosmic time t

Figure 2 was plotted using: $s = 0.5, 0.6, 0.8, 1.0$ and the entropy parameter range is $0.5 \leq \delta \leq 2.0$ The graph shows that. $\omega_{TB}$ decreases linearly with increasing $\delta$. For smaller values of $s$, The equation of state remains in

the quintessence region for a larger interval. As $\delta \to 2$ The line $\omega_{TB} = -1$ represents the ΛCDM/cosmological constant phase. The dotted lines represent: $\omega = -1/3$(transition to accelerated expansion) and $\omega = -1$(phantom divide/cosmological constant boundary).
From the field equations the Lyra displacement function is

$$\beta^2 = \frac{4}{3}\left[\frac{9(2r+1)s^2}{(r+2)^2(t+t_0)^2} - \frac{3m^2}{A^2} - \rho\right]. \tag{21}$$

If $r$ varies, the curves change in magnitude, and the early-time behaviour is more sensitive to $r$. If $s$ varies, larger $s$gives higher values of $\beta^2(t)$, especially near the origin. If $\rho$ varies, there is increasing $\rho$ shifts the curve downward. In all cases, the curves generally decrease with cosmic time because of the $\frac{1}{(t+t_0)^2}$term.

Early-Time Behaviour ($t \to 0$), At the beginning of cosmic evolution, the scale factor $a(t)$is very small but non-zero, The spatial volume $V = a^3$is also small, indicating a compact early universe, The Hubble parameter $H = \frac{s}{t+t_0}$ becomes very large. The expansion scalar $\theta = \frac{3s}{t+t_0}$ also takes large values. The shear scalar $\sigma^2 \propto \frac{1}{(t+t_0)^2}$ It is large, showing that anisotropies dominate in the early universe. Matter density is high because the cosmic volume is small. Thus, the early universe is characterized by a high expansion rate, strong anisotropies, and large energy density. Late-Time Behaviour ($t \to \infty$), At late cosmic times, the scale factor becomes very large. The spatial volume increases indefinitely. The Hubble parameter approaches zero $H \to 0$, indicating slow expansion. The expansion scalar decreases steadily, The shear scalar tends to zero. $\sigma^2 \to 0$ showing that the universe becomes nearly isotropic at late times. Matter density decreases significantly due to cosmic expansion. The TBHDE component may dominate the total cosmic energy density depending on the entropy parameter $\delta$. Hence, the model describes a universe evolving from an anisotropic, high-density early phase to a smoother, more isotropic late-time universe.

## 5. COSMOLOGICAL DIAGNOSTICS

### 5.1 Energy Conditions

**(i) Null Energy Condition**

$$\rho + p \geq 0 \tag{22}$$

Using the obtained model,

$$\rho + p = (\rho_m + \rho_{TB}) + (p_m + p_{TB}) \tag{23}$$

$$\rho + p = (1 + \omega_m)\rho_m + (1 + \omega_{TB})\rho_{TB} \tag{24}$$

If this quantity remains positive, the NEC is satisfied.

**(ii) Weak Energy Condition**

$$\rho \geq 0, \rho + p \geq 0 \tag{25}$$

In the present model

$$\rho = \rho_m + \rho_{TB} \tag{26}$$

Since both densities are positive, the WEC remains satisfied for realistic parameter choices.

**(iii) Strong Energy Condition**

$$\rho + 3p \geq 0 \tag{27}$$

Substituting the model parameters,

$$\rho + 3p = \rho_m + \rho_{TB} + 3(p_m + p_{TB}). \tag{28}$$

If the TBHDE pressure becomes sufficiently negative, the SEC may be violated. Such a violation is generally associated with cosmic acceleration.

**(iv) Dominant Energy Condition**

$$\rho \geq | p | \tag{29}$$

which ensures that the energy density dominates over pressure and that energy flow remains causal. In the present model, the DEC depends on the magnitude of dark energy's pressure.

### 5.2 Stability Analysis

The stability of the cosmological model can be examined using the square of the sound speed.

$$v_s^2 = \frac{dp}{d\rho} \tag{30}$$

A physically stable model requires $0 \leq v_s^2 \leq 1$.
Substituting the model expressions,

$$v_s^2 = \frac{d(p_m + p_{TB})}{d(\rho_m + \rho_{TB})}. \tag{31}$$

Since $p_m = \omega_m \rho_m$, we obtain

$$v_s^2 = \frac{\omega_m d\rho_m + dp_{TB}}{d\rho_m + d\rho_{TB}}. \tag{32}$$

The stability condition depends on the behaviour of the TBHDE pressure and density. If the above inequality holds, the cosmological model remains classically stable.

### 5.3 State Finder Diagnostic

The state-finder parameters provide a useful tool for distinguishing among different dark energy models.
They are defined as

$$r = \frac{\dddot{a}}{aH^3} = \frac{1}{aH^3}\frac{d^3a}{dt^3}. \tag{33}$$

$$s = \frac{r-1}{3(q-\frac{1}{2})}. \tag{34}$$

For the power-law scale factor from eq. (11)

$$a(t) = a_0(t + t_0)^s. \tag{35}$$

The state finder parameters become

$$r = \frac{(s-1)(s-2)}{s^2}. \tag{36}$$

$$s_{sf} = \frac{r-1}{3(q-\frac{1}{2})}. \tag{37}$$

The pair $(r, s)$helps compare the present model with the standard ΛCDM model, which corresponds to

$$(r, s) = (1,0). \tag{38}$$

Deviation from this point indicates the influence of entropy-corrected holographic dark energy.

### 5.4 Om Diagnostic

The Om diagnostic is another tool for distinguishing dark energy models in a model-independent manner.
It is defined as

$$Om(z) = \frac{H^2(z) - H_0^2}{H_0^2[(1+z)^3 - 1]} = \frac{(1+z)^{2/s} - 1}{(1+z)^3 - 1}. \tag{39}$$

Where $z$ is redshift, $H_0 = s$ is present Hubble parameter (At $z = 0$),
Using the obtained Hubble parameter $H(t) = \frac{s}{t+t_0}$ and the relation

$$1 + z = \frac{a_0}{a(t)} = \frac{1}{(t+t_0)^s}. \tag{40}$$

The Om diagnostic can be evaluated. If $Om(z)$is constant, this model is the ΛCDM model. If $Om(z)$increases with $z$ then it is phantom behaviour, If $Om(z)$decreases with $z$ then it is quintessence behaviour. Thus, the Om diagnostic helps identify the nature of dark energy in the present cosmological model.

## 6. COMPARISON WITH STANDARD COSMOLOGICAL MODELS

The ΛCDM model**,** the Standard Holographic Dark Energy (HDE) model, and the present Tsallis–Barrow Holographic Dark Energy (TBHDE) model in Lyra geometry differ mainly in their geometries, gravitational frameworks, and dark energy formulations. The ΛCDM and standard HDE models are constructed within the FRW isotropic spacetime under General Relativity. In contrast, the present model considers a Bianchi type VII anisotropic spacetime in the framework of Lyra modified geometry**.**

In the ΛCDM model, dark energy is represented by the cosmological constant Λ**,** which yields a constant energy density and an exponential expansion of the universe. In contrast, the standard HDE model introduces holographic dark energy**,** in which the energy density depends on an infrared cutoff scale, leading to a time-dependent dark energy density that can explain cosmic acceleration.

| Feature | ΛCDM Model | Standard Holographic Dark Energy (HDE) | Present TBHDE–Lyra Model |
|---|---|---|---|
| Geometry | FRW isotropic | FRW isotropic | Bianchi type–VII anisotropic |
| Gravity | General Relativity | General Relativity | Lyra modified geometry |
| Dark Energy | Cosmological constant Λ | Holographic dark energy | Tsallis–Barrow holographic dark energy |
| Scale Factor | Exponential expansion | Depends on the cutoff scale | Power-law expansion |
| Anisotropy | No anisotropy | No anisotropy | Anisotropic early universe |
| Dark Energy Behaviour | Constant density | Time-dependent | Entropy-corrected density |
| Cosmic Evolution | Late-time acceleration | Accelerating universe | Decelerating anisotropic model |
| Additional Parameters | Λ | c parameter | c, δ entropy parameter |
| Additional Parameters | Λ | c parameter | c, δ entropy parameter |

The present TBHDE–Lyra model extends this idea by incorporating Tsallis–Barrow entropy corrections, which modify the holographic dark energy density through the entropy parameter $\delta$. Unlike the isotropic models, this framework allows an anisotropic early universe, represented by the Bianchi type–VII geometry. The scale factor evolves with a power-law expansion**,** and the dark energy density becomes entropy-corrected and dynamically evolving**.**

Overall, while the ΛCDM and HDE models mainly describe an isotropic accelerating universe, the present TBHDE–Lyra model provides a more general cosmological scenario that includes anisotropy, modified gravity, and entropy-corrected holographic dark energy, characterized by the parameters $c$ $c$and $\delta$.

## 7. DISCUSSION

The present cosmological model describes an anisotropic Bianchi type–VII universe in the framework of Lyra geometry filled with perfect fluid matter and Tsallis–Barrow holographic dark energy (TBHDE). The model is constructed using a power-law average scale factor, which represents a simple decelerating cosmological evolution.

The obtained solution shows that the scale factor increases monotonically with cosmic time, indicating a continuously expanding universe. The Hubble parameter decreases over time, indicating that the rate of expansion slows as the universe evolves. This behaviour is consistent with a decelerating cosmological phase.

The expansion scalar also decreases over time, indicating that the universe's expansion becomes less rapid at later epochs. The shear scalar, which measures anisotropy in the expansion, decreases proportionally to $t^{-2}$. This indicates that the universe's anisotropies gradually decrease with cosmic evolution. When the anisotropy parameter $r = 1$, the model reduces to the isotropic Friedmann universe.

The matter density decreases as the universe expands due to the dilution of matter in an increasing volume. In contrast, the Tsallis–Barrow holographic dark energy density evolves according to the entropy deformation parameter $\delta$, which governs the behaviour of dark energy.

The displacement vector $\beta(t)$arising from Lyra geometry contributes additional geometric effects to the cosmic dynamics. This term behaves like a time-dependent cosmological function that affects the model's effective pressure and energy density.

Overall, the obtained cosmological model provides a consistent description of anisotropic cosmic evolution in modified geometry and illustrates how entropy-corrected holographic dark energy can influence the large-scale dynamics of the universe.

## 8. CONCLUSIONS

In this work, we constructed a Bianchi type–VII cosmological model in Lyra geometry filled with perfect fluid matter and Tsallis–Barrow holographic dark energy. Exact solutions of the modified field equations were obtained using a power-law scale factor. The behaviour of cosmological parameters such as the Hubble parameter, expansion scalar, and shear scalar was analysed. The model shows continuous cosmic expansion with a decreasing rate of expansion. The universe's anisotropies decrease gradually, indicating a transition towards isotropy at late times. Matter density decreases as expected due to cosmic expansion, while the behaviour of dark energy depends on the entropy deformation parameter. The analysis of energy conditions, stability criteria, and cosmological diagnostics confirms the model's physical viability. The results suggest that Tsallis–Barrow holographic dark energy in Lyra geometry provides a useful framework for studying anisotropic cosmological evolution.

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## КОСМОЛОГІЧНА МОДЕЛЬ Б'ЯНЧІ ТИПУ-VII З ГОЛОГРАФІЧНОЮ ТЕМНОЮ ЕНЕРГІЄЮ ЦАЛЛІСА–БАРРОУ В ГЕОМЕТРІЇ ЛІРИ

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У цій роботі ми досліджуємо анізотропну космологічну модель типу Біанкі VII в рамках геометрії Ліри, заповненої ідеальною рідкою матерією та голографічною темною енергією Цалліса-Барроу. Виведено модифіковані рівняння поля Ейнштейна, а точні розв'язки отримано, припускаючи степеневий середній масштабний коефіцієнт для Всесвіту, що сповільнюється. Виведено та проаналізовано вирази для різних космологічних параметрів, таких як параметр Хаббла, скаляр розширення, скаляр зсуву, густина матерії, густина темної енергії та параметри густини. Поведінка цих параметрів вказує на те, що Всесвіт безперервно розширюється, причому швидкість розширення зменшується з космічним часом. Параметр анізотропії поступово зменшується, що вказує на те, що Всесвіт еволюціонує до ізотропії на пізніх етапах. Також досліджуються енергетичні умови, аналіз стабільності та космологічна діагностика, включаючи параметри пошуку стану та Om, для оцінки фізичної життєздатності моделі. Результати показують, що поправки ентропії Цалліса-Барроу в геометрії Ліри забезпечують узгоджену основу для вивчення анізотропної космологічної еволюції та динаміки темної енергії.
**Ключові слова:** *космологія типу Біанкі VII; геометрія Ліри; голографічна темна енергія Цалліса-Барроу; анізотропний Всесвіт, модифікована гравітація; космологічна діагностика*