EAST EUROPEAN JOURNAL OF PHYSICS. **2.** 385-391 (2026)
DOI:10.26565/2312-4334-2026-2-42 ISSN 2312-4334

THERMODYNAMIC PROPERTIES OF Mn-DOPED DILUTED MAGNETIC SEMICONDUCTOR SUPERLATTICES

Mehdi M. Mahmudov[1], Ragib Y. Damirov[1]*, Naila S. Sardarova[2], Arzu M. Ahmadova[3]
[1]*Department of Solid State Physics, Baku State University, Z. Khalilov Street, 23, AZ1148, Baku, Azerbaijan*
[2]*Department of Natural Sciences, Sumgait State University, Baku Street, 1, AZ5008, Sumgait, Azerbaijan*
[3]*Department of Engineering and Applied Science, Azerbaijan State University of Economic, Istiglaliyyat Street 6, AZ1001, Baku, Azerbaijan*
**Corresponding Author e-mail: damirovraghib@gmail.com*
Received March 5, 2026; revised May 18, 2026; accepted May 20, 2026

This work investigates the thermodynamic properties of a two-dimensional electron gas in manganese-doped diluted magnetic semiconductor superlattices, with particular emphasis on the chemical potential. Within the grand canonical formalism, a general expression for the chemical potential is derived that is valid for both degenerate and nondegenerate cases. In the nondegenerate limit, the chemical potential decreases with increasing temperature and exhibits a logarithmic dependence on carrier density; the temperature sensitivity is most pronounced at low carrier concentrations, where entropic effects dominate. In the degenerate regime, Landau quantization leads to a characteristic stepwise oscillatory dependence of the chemical potential on the applied magnetic field. The influence of the exchange interaction is analyzed in two limiting cases: in the weak-coupling limit, the correction to the chemical potential is linear in the Mn concentration and exchange constant, whereas in the strong-coupling limit, the system approaches complete spin polarization with carriers confined predominantly to a single spin channel. The exchange interaction introduces an additional spin-dependent contribution described by the Brillouin function, resulting in the most pronounced modifications at low temperatures and in strong magnetic fields.
Keywords: *Diluted magnetic semiconductors; Superlattices; Chemical potential; Exchange interaction; Landau quantization; Spin polarization*
PACS: 75.50.Pp, 68.65.Cd, 75.30.Kz

1. INTRODUCTION

Diluted magnetic semiconductor superlattices (DMSSs), which combine semiconductor heterostructures with magnetic impurities, have attracted considerable attention as promising candidates for spintronic applications [1,2]. Their distinctive physical properties arise from the interplay between quantum confinement and exchange coupling between itinerant charge carriers and localized magnetic moments [3,4]. The incorporation of transition-metal ions, particularly Mn, into III-V semiconductor superlattice structures gives rise to pronounced phenomena such as giant Zeeman splitting, enhanced Faraday rotation, and strong negative magnetoresistance [5,6]. A detailed understanding of the thermodynamic behavior of a two-dimensional electron gas in diluted magnetic semiconductor superlattices is therefore essential for controlling and optimizing their electronic and magnetic properties. Among the relevant thermodynamic quantities, the chemical potential plays a central role, as it governs carrier statistics, transport properties, and magnetic response [7]. In low-dimensional systems, the chemical potential differs substantially from that of bulk materials due to modifications of the density of states and quantum-size effects [8]. Previous theoretical studies of two-dimensional electron gases have primarily focused on nonmagnetic quantum-well structures [9, 10], whereas investigations that explicitly incorporate exchange interactions in superlattice configurations remain relatively limited [11-12]. Recent advances in molecular beam epitaxy have enabled the growth of high-quality diluted magnetic semiconductor superlattices, such as $(Ga, Mn)As/AlGaAs$ [13,14], further emphasizing the need for a comprehensive theoretical description of their thermodynamic properties. The exchange interaction between conduction electrons and localized d electrons of magnetic ions is a key mechanism governing the electronic structure and magnetic behavior of these materials [15]. This coupling can significantly modify the band structure and may give rise to carrier-induced ferromagnetism [16-18]. Consequently, analyzing its influence on thermodynamic quantities, particularly the chemical potential, is crucial for the development of efficient spintronic devices [19].

The present work differs from previous theoretical studies in several important aspects. We derive a unified analytical expression for the chemical potential within a single grand canonical formalism that remains valid in both the degenerate and nondegenerate cases, without introducing separate approximations for each limit. Second, the role of the exchange interaction is investigated in two distinct coupling regimes, namely the weak-coupling limit ($\alpha x \ll 1$) and the strong-coupling limit ($\alpha x \gg 1$). This approach leads to compact analytical expressions (Eqs. 13–15) that describe the crossover from linear to saturated behavior of the spin-polarization-induced correction to the chemical potential. To the best of our knowledge, such an analytical description has not previously been obtained for diluted magnetic semiconductor superlattices. Third, unlike strictly two-dimensional quantum-well models [9–12], the present treatment explicitly retains the superlattice miniband dispersion along the growth direction through the $cos\, a\, k_z$ term in Eq. (1).

Cite as: M.M. Mahmudov, R.Y. Damirov, N.S. Sardarova, A.M. Ahmadova, East Eur. J. Phys. 2, 385 (2026), https://doi.org/10.26565/2312-4334-2026-2-42

In this work, the thermodynamic properties of a two-dimensional electron gas in manganese-doped diluted magnetic semiconductor superlattices are theoretically investigated, with particular emphasis on the chemical potential. Using the grand thermodynamic potential formalism [20], which naturally incorporates the effects of temperature, magnetic field, and carrier concentration, a general expression for the chemical potential is derived. Within this framework, the nondegenerate regime of the electron gas is analyzed, taking into account the exchange interaction between conduction electrons and localized magnetic moments of Mn ions. The results demonstrate that, in the nondegenerate case, the chemical potential decreases with increasing temperature and exhibits a logarithmic dependence on carrier density. Furthermore, the exchange interaction modifies the chemical potential through a contribution described by the Brillouin function, with the most pronounced effects occurring at low temperatures and in strong magnetic fields.

2. ENERGY SPECTRUM AND CHEMICAL POTENTIAL OF DMSS

In a strong magnetic field applied perpendicular to the superlattice layers, the electronic motion in diluted magnetic semiconductor superlattices becomes quantized within the plane of the layers, while the spin degeneracy of the electronic spectrum is removed. Under these conditions, the electron energy spectrum can be expressed as [21]:

$$\varepsilon(N,\sigma,k_z) = (2N+1)\mu B + \varepsilon_0(1-\cos a\,k_z) + g^*\sigma\mu_B B + 3AS, \tag{1}$$

where $N = 1, 2, \ldots$ are the Landau quantum numbers, k_z is the quasi-momentum component along the z-axis, $\mu = (m_0/m_\perp)\mu_B$, m_0 is the mass of a free electron, $m_\perp$ is the mass of the electron in the plane of the layer, ε_0 is the half-width of the conduction band in the direction k_z, a is the superlattice period in the direction z, g^* is the effective g-factor determined from the band structure, and the rest are standard constants, S is spin of magnetic impurity, $A = -(N_0\alpha\langle|S_z|\rangle x)/6$- the magnitude of the exchange splitting, N_0 is the number of cells per unit volume, α is constant describing the change in the structure of the band caused by the exchange interaction, x is the molar concentration of the impurity. It can be seen that each Landau level is split into two spin sublevels, and the splitting of the *N*th Landau level is equal to $\Delta\varepsilon = g^*\mu_B B$. In this work, manganese ions with spin 5/2 are taken as impurities, and then Eq. (1) takes the form:

$$\varepsilon = (2N+1)\mu B \pm \frac{1}{2}g\mu_B B \mp \frac{5}{2}\alpha x f(B,T) + \varepsilon_0(1-\cos a\,k_z). \tag{2}$$

where x is the molar concentration of manganese, and $f(B,T) = \frac{2}{5}B_{5/2}\left(\frac{g_{Mn}\mu_B B}{k_B T}\right)$, with $B_S(x) = \frac{2s+1}{2}cth\frac{2s+1}{2} - \frac{1}{2}cth\frac{x}{2}$ being the Brillouin function.

The chemical potential ζ of the electron gas can be obtained via the Gibbs method from the grand thermodynamic potential $\Omega = \Omega(T,V,\zeta,B)$ [20]:

$$n = -\frac{1}{V}\left(\frac{\partial\Omega}{\partial\zeta}\right)_{T,B} \tag{3}$$

where the grand potential in a quantizing magnetic field is given by [21]:

$$\Omega = -k_B T\frac{V}{2a(\pi R)^2}\sum_{N,S,\sigma}\int_0^{Z_0} ln(1+e^{\eta^*+\varepsilon_0^* cos Z})dZ, \tag{4}$$

where $R = (\hbar/eB)^{1/2}$ is the magnetic length, $\eta^* = \zeta^* - \varepsilon_N^* - \varepsilon_0^*$, $\zeta^* = \zeta/k_B T$, $\varepsilon_N^* = \varepsilon_N/k_B T$, ζ is the chemical potential, $Z(\varepsilon) = ak_z = arccos(1-(\varepsilon-\varepsilon_z)/\varepsilon_0)$, $\varepsilon_N = (2N+1)\mu B$, $\varepsilon_0^* = \varepsilon_0/k_B T$, $\varepsilon_z = \varepsilon(N,\sigma,k_z) - (2N+1)\mu B - g^*\sigma\mu_B B - 3AS$. The upper limit of the integral is defined as [22]:

$$Z_0 = \begin{cases} \pi, & \varepsilon > 2\varepsilon_0 \\ arccos\left(1 + \frac{\mu B + \frac{g\mu B}{2} - \frac{5\alpha x f(B,T)}{2} - \varepsilon}{\varepsilon_0}\right), & \varepsilon < 2\varepsilon_0 \end{cases}. \tag{5}$$

Changing the integration variable from dZ to $d\varepsilon$, the grand potential becomes:

$$\Omega = \frac{k_B T V}{2(\pi R)^2}\sum_{N,S,\sigma}\int_{\varepsilon_N}^{\infty}\frac{dk_z(\varepsilon,N)}{d\varepsilon} ln\left(1+exp\left(\frac{\zeta-\varepsilon}{k_B T}\right)\right)d\varepsilon. \tag{6}$$

Substituting this expression into account in (3) yields a general formula for the chemical potential, valid for arbitrary magnetic field strength and electron gas degeneracy:

$$n = \frac{1}{2(\pi R)^2}\sum_{N,S,\sigma}\int_0^{2\varepsilon_0}\frac{f(\varepsilon)d\varepsilon}{a\sqrt{(\varepsilon-\varepsilon^*)(2\varepsilon_0-(\varepsilon-\varepsilon^*))}}, \tag{7}$$

where $\varepsilon^* = \mu_B B(2N+1) \pm \frac{1}{2} g\mu_B B \mp \frac{5}{2}\alpha x f(B,T)$, and $f(\varepsilon)$ is the Fermi distribution function.

In the nondegenerate limit $(\zeta - \varepsilon \ll k_B T)$, this expression reduces to:

$$n = \frac{1}{2a(\pi R)^2}\sqrt{\frac{\pi k_B T}{2\varepsilon_0}}\sum_{N,S,\sigma} e^{\frac{\zeta-\varepsilon^*}{k_B T}}. \tag{8}$$

or equivalently,

$$\zeta = k_B T ln\left[2na(\pi R)^2\sqrt{\frac{2\varepsilon_0}{\pi k_B T}}\left(\sum_{N,S,\sigma} e^{-\frac{\varepsilon^*}{k_B T}}\right)^{-1}\right]. \tag{9}$$

Eq. (9) indicates that, for a nondegenerate electron gas, the chemical potential increases logarithmically with carrier concentration and decreases with increasing temperature. Temperature elevation facilitates the occupation of higher-energy states, resulting in a reduction of the chemical potential, while the dependence on the magnetic field arises through the quantized energy levels ε^*.

3. RESULTS AND DISCUSSION

The chemical potential of a two-dimensional electron gas in diluted magnetic semiconductor superlattices is determined by the combined effects of carrier concentration, temperature, magnetic field, and exchange interactions. In the nondegenerate case, to obtain the explicit dependence of the chemical potential on carrier concentration, exchange interaction, and temperature, the corresponding plots (Figs. 1-3) were constructed using the following parameters: $\varepsilon_0 = 10\ meV$, $a = 5\ nm$, $g = 2$, $x = 0.05$, $\alpha = 0.22\ eV$ [1].

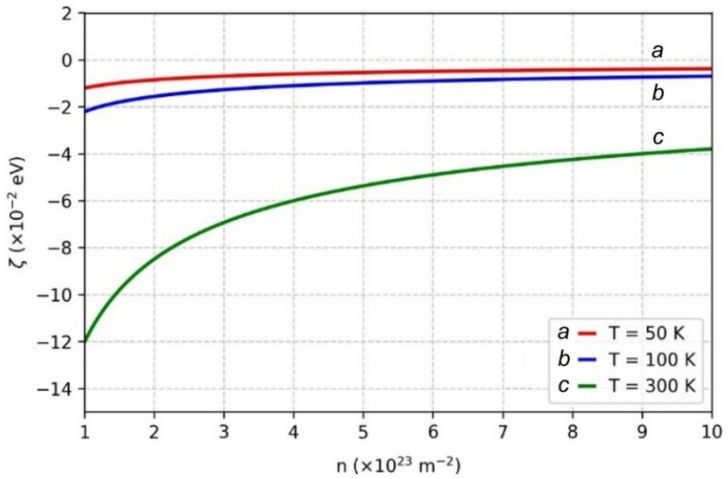

Figure 1. Chemical potential dependence on electron concentration ζ(n)

As shown in Fig. 1, the chemical potential increases monotonically with electron concentration for all considered temperatures at a fixed magnetic field $(B = 1\ T)$, following a logarithmic dependence $\zeta \propto k_B T ln(n)$, characteristic of a nondegenerate electron gas. At low carrier concentrations, the large number of available states leads to high entropy and strongly negative values of ζ. As the concentration increases, low-energy states become progressively occupied, entropy decreases, and the chemical potential shifts toward zero. An increase in temperature lowers the chemical potential due to enhanced thermal excitation, which redistributes electrons over higher-energy states. The exchange interaction between conduction electrons and localized Mn moments further modifies the chemical potential through the Brillouin function, with the strongest effects occurring at low temperatures and in strong magnetic fields. In addition, Landau quantization alters the density of states, introducing further corrections to ζ. The exchange interaction significantly influences the chemical potential in systems containing magnetic impurities. An increase in the exchange parameter α lowers the energy spectrum and reduces the chemical potential by shifting electrons toward lower-energy states, with the most pronounced effect observed at low temperatures. As shown in Fig. 2, at low temperature $(T = 50K)$ the chemical potential decreases most rapidly due to nearly complete spin polarization of Mn ions. At higher temperatures, thermal fluctuations reduce the degree of spin polarization and weaken the exchange-induced shift, resulting in a smaller variation of the chemical potential. As seen from Fig. 3, the chemical potential decreases monotonically with increasing temperature for all considered carrier concentrations, in accordance with Eq. (9). This behavior reflects the fundamental statistical nature of the nondegenerate electron gas: as temperature rises, thermal excitation redistributes carriers over a broader range of energy states, increasing the configurational entropy and driving the chemical potential to more negative values.

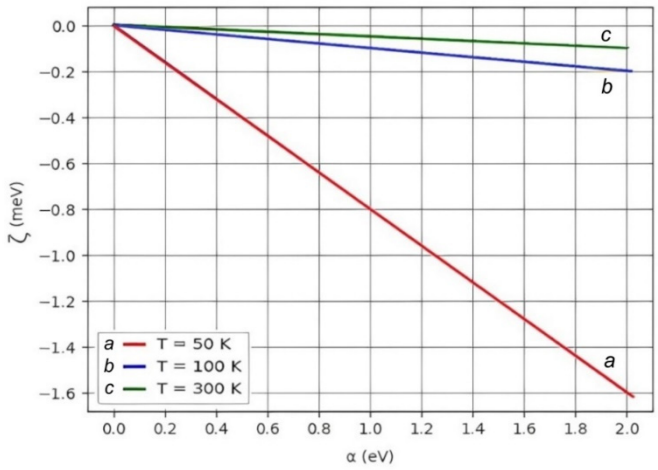

Figure 2. Chemical potential dependence on exchange interaction $\zeta(\alpha)$

At higher carrier concentrations, the chemical potential remains less negative over the entire temperature range. This follows directly from the logarithmic dependence in Eq. (9): a larger carrier density reduces the number of unoccupied low-energy states available per electron, thereby increasing the chemical potential relative to the low-concentration case. The temperature sensitivity of the chemical potential is more pronounced at lower concentrations. For $n = 10^{14}m^{-2}$, the chemical potential decreases steeply with temperature because the sparse carrier distribution leaves many thermally accessible states, leading to strong entropic effects. For $n = 10^{16}m^{-2}$, the lower-energy states are more strongly occupied, which moderates the temperature-induced reduction of the chemical potential. The exchange interaction between conduction electrons and localized Mn moments introduces an additional correction through the Brillouin function, as described by Eqs. (13)–(14). At the magnetic field value considered ($B = 1\,T$), this correction remains moderate; its influence becomes most significant at low temperatures, where the Mn spin polarization approaches saturation and the Brillouin function approaches unity.

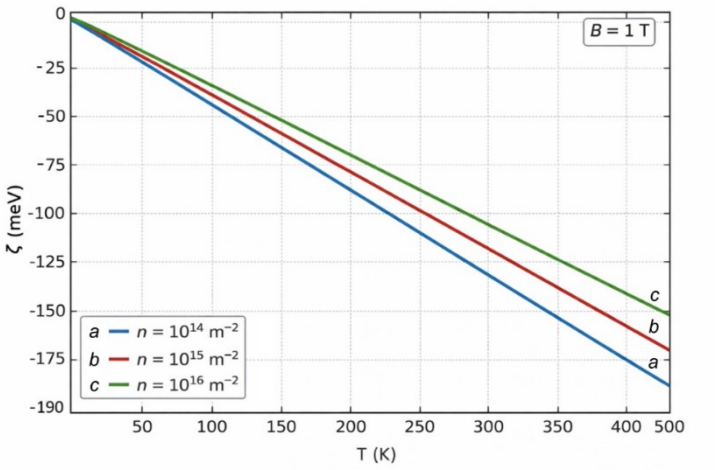

Figure 3. Chemical potential dependence on temperature $\zeta(T)$

To assess the quantitative validity of the analytical results, we compare the temperature dependence predicted by Eq. (9) with the self-consistent numerical calculations reported by Dietl and Ohno [9] for $(Ga, Mn)As$ at low Mn concentrations ($x = 0.05$) and $B = 1\,T$. Using the material parameters adopted in the present work ($\varepsilon_0 = 10\,meV, a = 5\,nm, g = 2, \alpha = 0.22\,eV$), Eq. (9) reproduces the correct order of magnitude of the chemical potential and the characteristic logarithmic decrease with temperature observed in those calculations. The quantitative agreement is satisfactory in the nondegenerate case ($T > 50\,K$), while deviations at lower temperatures are expected due to the onset of degeneracy effects not captured by Eq. (9) alone. We note that direct experimental measurements of the chemical potential as a function of magnetic field and exchange coupling strength are not yet available for Mn-doped superlattice geometries specifically; such measurements would provide a stringent test of the present theory and are identified as an important direction for future experimental work.

In the degenerate case ($\zeta - \varepsilon \gg k_B T$), all quantum states up to N_{max} are filled (quantum limit):

$$n = \frac{\pi}{2a(\pi R)^2} N_{\max}, \tag{10}$$

and the chemical potential equals the energy of the highest occupied level:

$$\zeta = \varepsilon_{N_{max}}(B). \tag{11}$$

This leads to stepwise variation of $\zeta(B)$ with quantum oscillation character as the magnetic field varies (Fig.4).

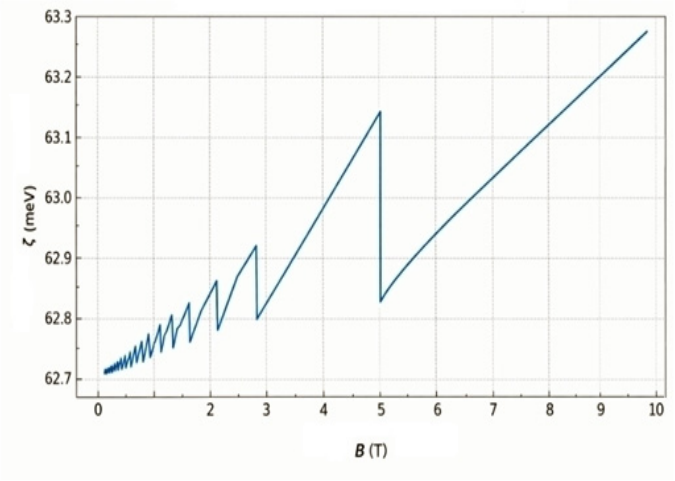

Figure 4. Chemical potential dependence on magnetic field $\zeta(B)$

Fig. 4 demonstrates the stepwise decrease of ζ with increasing magnetic field, reflecting the sequential depopulation of Landau levels. Each plateau corresponds to a fully occupied level, and its width is determined by the Landau level spacing $\hbar\omega_C$, which increases linearly with B (here, ω_C is the cyclotron frequency of an electron in a magnetic field). At low magnetic fields, closely spaced levels produce almost smooth variation of ζ , whereas at higher fields the oscillations become clearly resolved. The miniband dispersion along the z-direction introduces additional broadening, distinguishing the superlattice from an ideal strictly two-dimensional system. The exchange interaction further shifts the spin-split sublevels and modifies the positions of the steps.

Let us determine the influence of exchange interaction on the chemical potential. Separating the spin contributions in the summation yields:

$$\sum_{N,S,\sigma} e^{-\frac{\varepsilon^*}{k_BT}} = \sum_{N,S,\sigma} e^{-\frac{\varepsilon_N^0}{k_BT} \pm \frac{5\alpha x f(B,T)}{2k_BT}}. \tag{12}$$

For weak exchange coupling, the chemical potential becomes:

$$\zeta(x,\alpha) = \zeta_0 - k_BTln\left[1 + \frac{5\alpha x f(B,T)}{2k_BT}\sinh\left(\frac{\mu_B g^* B}{2k_BT}\right)\right]. \tag{13}$$

In the limit of weak exchange interaction ($\alpha x \ll 1$), expansion yields:

$$\zeta = \zeta_0 - \frac{5\alpha x f(B,T)}{4}\tanh\left(\frac{\mu_B g^* B}{2k_BT}\right). \tag{14}$$

Thus, in this case, the exchange correction to the chemical potential is linear in both doping concentration x and exchange constant α.

In the case of strong exchange interactions ($\alpha x \gg 1$), one obtains:

$$\zeta = k_BTln\left[4na(\pi R)^2\sqrt{\frac{2\varepsilon_0}{\pi k_BT}}e^{\frac{5\alpha x f(B,T)}{2k_BT}}\right]. \tag{15}$$

Thus, a strong spin polarization develops, and electrons predominantly occupy a single spin channel. In the weak-coupling limit ($\alpha x \ll 1$), Eq. (14) indicates that the exchange correction to the chemical potential is linear in both x and α, with the hyperbolic tangent term reflecting thermally induced spin polarization. In the strong-coupling limit ($\alpha x \gg 1$), Eq. (15) describes a fully spin-polarized electron gas in which carriers occupy essentially one spin subband. The factor of 4 in Eq. (15), compared with the factor of 2 in Eq. (9), reflects the effective modification of the density of states associated with occupation of a single spin channel. The transition from partial to complete spin polarization with increasing exchange interaction strength is directly relevant to spintronic applications, as it determines the conditions under which the superlattice can operate as an efficient spin filter.

4. CONCLUSIONS

A theoretical framework describing the chemical potential of a quasi-two-dimensional electron gas in Mn-doped diluted magnetic semiconductor superlattices has been developed within the grand canonical formalism. Analytical expressions valid in both the degenerate and nondegenerate cases have been derived and numerically analyzed. In the degenerate limit, Landau quantization gives rise to a characteristic stepwise oscillatory dependence of the chemical potential on the magnetic field. In the nondegenerate case, the chemical potential is determined by the interplay among carrier concentration, temperature, and exchange coupling. Weak exchange interaction introduces a correction proportional to the Mn concentration and the exchange constant, whereas strong exchange coupling leads to nearly complete spin polarization, with carriers predominantly confined to a single spin channel. The obtained results establish a direct quantitative relationship between the microscopic exchange parameters of diluted magnetic semiconductor superlattices and their macroscopic thermodynamic properties. These findings may provide a theoretical basis for tailoring material parameters in the design of future spintronic devices.

ORCID

Mehdi M. Mahmudov, https://orcid.org/0009-0007-4154-6199; **Ragib Y. Damirov,** https://orcid.org/0009-0008-8712-1310;
Naila S. Sardarova, https://orcid.org/0000-0003-0896-9126; **Arzu M. Ahmadova,** https://orcid.org/0000-0002-2392-9774

REFERENCES

[1] J. Kossut, and J.A. Gaj, *Introduction to the physics of diluted magnetic semiconductors*, (Springer, Berlin, Heidelberg, 2010), p. 469, https://doi.org/10.1007/978-3-642-15856-8
[2] P. Kacman, "Spin interactions in diluted magnetic semiconductors and magnetic semiconductor structures", Semiconductor Science and Technology, **16**(4), R25-R39 (2001). https://doi.org/10.1088/0268-1242/16/4/201
[3] A. Gupta, R. Zhang, P. Kumar, V. Kumar, and A. Kumar, "Nano-structured dilute magnetic semiconductors for efficient spintronics at room temperature", Magnetochemistry**, 6**(1), 1-22 (2020). https://doi.org/10.3390/magnetochemistry6010015
[4] A.M. Babanli, and B.G. Ibragimov, "Magnetic moment of electrons in diluted magnetic semiconductor quantum ring", in: *Proceedings of the 6th International conference on Control and Optimization with Industrial Applications* (COIA, Baku, 2018), pp.122-124.
[5] Berry Habte Dulla, "Thermodynamic properties of diluted magnetic semiconductors using Heisenberg spin model in 3D", Journal of Applied Physical Science International, **2**(3), 101-106 (2015). https://ikprress.org/index.php/JAPSI/article/view/2844
[6] J. Ricardo de Sousa, and N.S. Branco, "Two-dimensional quantum spin -½ Heisenberg model with competing interactions", Physical Review B, **72**(13), 134421 (2005). https://doi.org/10.1103/PhysRevB.72.134421
[7] R.N. Bhatt, M. Berciu, M.P. Kennett, and X.Wan, "Diluted magnetic semiconductors in the low carrier density regime", Journal of Superconductivity, **15**(1), 71-83 (2002), https://doi.org/10.1023/A:1014031327996
[8] W. Zawadzki, "Theory of optical transitions in inversion layers of narrow-gap semiconductors", Journal of Physics C: Solid State Physics, **16**(1), 229-240 (1983). https://doi.org/10.1088/0022-3719/16/1/025
[9] T. Dietl, and H. Ohno, "Dilute ferromagnetic semiconductors: Physics and spintronic structures", Reviews of Modern Physics, **86**(1), 187-251 (2014). https://doi.org/10.1103/RevModPhys.86.187
[10] T. Jungwirth, J. Sinova, J. Mašek, J. Kučera, and A.H. MacDonald, "Theory of ferromagnetic $(III, Mn)V$ semiconductors", Reviews of Modern Physics, **78**(3), 809-864 (2006). https://doi.org/10.1103/RevModPhys.78.809
[11] G. Bastard, *Wave Mechanics Applied to Semiconductor Heterostructures, (Monographs of Physics (Les Editions de Physique))*, (Wiley-Interscience Publishing, Paris, 1991), p. 357.
[12] D.R. Yakovlev, A. Keller, et al. "Kinetic exchange between the conduction band electrons and magnetic ions in quantum-confined structures", Physical Review Letters, **83**(7), 1431-1434 (1999). https://doi.org/10.1103/PhysRevLett.83.1431
[13] H. Ohno, "Making nonmagnetic semiconductors ferromagnetic", Science, **281**(5379), 951-956 (1998). https://doi.org/10.1126/science.281.5379.951
[14] A.H. MacDonald, P. Schiffer, and N. Samarth, "Ferromagnetic semiconductors: moving beyond $(Ga, Mn)As$", Nature Materials, **4**(3), 195-202 (2005). https://doi.org/10.1038/nmat1325
[15] T. Dietl, H. Ohno, F. Matsukura, J. Cibert, and D. Ferrand, "Zener model description of ferromagnetism in zinc-blende magnetic semiconductors", Science, **287**(5455), 1019-1022 (2000). https://doi.org/10.1126/science.287.5455.1019
[16] S. Lee, S. Chung, H. Lee, X. Liu, M. Dobrowolska, and J.K. Furdyna, "Interlayer exchange coupling in $(Ga, Mn)As$ ferromagnetic semiconductor multilayer systems", Journal of Semiconductors, **40**(8), 081503 (2019). https://doi.org/10.1088/1674-4926/40/8/081503
[17] K. Ando, H. Saito, Z. Jin, T. Fukumura, M. Kawasaki, Y. Matsumoto, and H. Koinuma, "Magneto-optical properties of ZnO-based diluted magnetic semiconductors", Journal of Applied Physics, **89**(11), 7284-7286 (2001). https://doi.org/10.1063/1.1356035
[18] D.D. Awschalom, and M.E. Flatté, "Challenges for semiconductor spintronics", Nature Physics, **3**, 153-159 (2007). https://doi.org/10.1038/nphys551
[19] I. Žutić, J. Fabian, and S. Das Sarma, "Spintronics: Fundamentals and applications", Reviews of Modern Physics, **76**(2), 323-410 (2004). https://doi.org/10.1103/RevModPhys.76.323
[20] B.M. Askerov, and S.R. Figarova, *Thermodynamics, Gibbs Method and Statistical Physics of Electron Gases*, (Springer, Berlin, Heidelberg, 2010), p. 374. https://doi.org/10.1007/978-3-642-03171-7
[21] S.R. Figarova, M.M. Mahmudov, and R.Y. Damirov, "Magnetization of diluted magnetic semiconductor II type superlattices with impurities Mn (manganese) ions", Journal of Baku Engineering University: PHYSICS, **8**(1), 9-15 (2024). https://doi.org/10.30546/09081.2024.101.5002
[22] M.M. Mahmudov, and R.Y. Damirov, "Entropy of diluted magnetic semiconductor superlattices with impurities Mn ions", Baku State University: Journal of Physics and Space Sciences, **2**(2), 9-16 (2025). https://doi.org/10.30546/209501.101.2025.2.02.03

ТЕРМОДИНАМІЧНІ ВЛАСТИВОСТІ РОЗБАВЛЕНИХ МАГНІТНИХ НАПІВПРОВІДНИКОВИХ НАДҐРАТОК, ЛЕГОВАНИХ Mn

Мехді М. Махмудов[1], Рагіб Й. Даміров[1], Наїла С. Сардарова[2], Арзу М. Ахмадова[3]

[1]Кафедра фізики твердого тіла, Бакинський державний університет, вулиця З. Халілова, 23, AZ1148, Баку, Азербайджан
[2]Кафедра природничих наук, Сумгаїтський державний університет, вулиця Баку, 1, AZ5008, Сумгаїт, Азербайджан
[3]Кафедра інженерії та прикладних наук, Азербайджанський державний економічний університет, вулиця Істіглаліят, 6, AZ1001, Баку, Азербайджан

У цій роботі досліджуються термодинамічні властивості двовимірного електронного газу в розбавлених магнітних напівпровідникових надґратках, легованих марганцем, з особливим акцентом на хімічний потенціал. У рамках великого канонічного формалізму отримано загальний вираз для хімічного потенціалу, який є справедливим як для вироджених, так і для невироджених випадків. У невиродженій границі хімічний потенціал зменшується зі зростанням температури та демонструє логарифмічну залежність від густини носіїв; температурна чутливість є найбільш вираженою за низьких концентрацій носіїв, де домінують ентропійні ефекти. У виродженому режимі квантування Ландау призводить до характерної ступінчастої коливальної залежності хімічного потенціалу від прикладеного магнітного поля. Вплив обмінної взаємодії аналізується у двох граничних випадках: у границі слабкого зв'язку поправка до хімічного потенціалу є лінійною щодо концентрації Mn та константи обміну, тоді як у границі сильного зв'язку система наближається до повної спінової поляризації з носіями, обмеженими переважно одним спіновим каналом. Обмінна взаємодія вносить додатковий спінзалежний внесок, що описується функцією Бріллюена, і призводить до найбільш виражених модифікацій за низьких температур та в сильних магнітних полях.

Ключові слова: *розбавлені магнітні напівпровідники; надґратки; хімічний потенціал; обмінна взаємодія; квантування Ландау; спінова поляризація*