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## UNIFIED DESCRIPTION OF PHOTO AND ELECTRO PROCESSES ON LIGHT NUCLEI IN COVARIANT APPROACH WITH EXACTLY CONSERVED EM CURRENT

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Generalized gauge invariant electro-break up process amplitude is considered. One is a sum of traditional pole series and the regular part. The deposits of regular part of amplitude, and its physical sense, are explored. A transition from virtual to real photon is considered in photon point limit  $q^2 \rightarrow 0$ . The general analysis for electro-break up process of component scalar system is given. Precisely conserved nuclear electromagnetic currents at arbitrary  $q^2$  are received.

**KEY WORDS:** regular part of amplitude, electro-break up process, photo-break up process, limit of the amplitude, light nuclei

### ЄДИНИЙ ОПИС ФОТО ТА ЕЛЕКТРО ПРОЦЕСІВ НА ЛЕГКИХ ЯДРАХ У КОВАРІАНТНОМУ ПІДХОДІ З ЕМ СТРУМОМ ЩО ТОЧНО ЗБЕРІГАЄТЬСЯ

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Розглядається узагальнена калібрувально-замкнута амплітуда електропроцесу, яка складається з традиційного полюсного ряду та регулярної частини. Вивчено вклад регулярної частини амплітуди та її фізичний зміст. Розглянуто границю амплітуди електропроцесу у переході в фотонну точку ( $q^2 \rightarrow 0$ ), тобто при переході від віртуальних фотонів до реальних. Здійснено загальний аналіз процесу електророзщеплювання складеної скалярної системи. Отримано точне зберігання ядерних електромагнітних токів для довільних  $q^2$ .

**КЛЮЧОВІ СЛОВА:** регулярна частина амплітуди, електророзщеплення, фоторозщеплення, границя амплітуди, легкі ядра

### ОБЪЕДИНЕННОЕ ОПИСАНИЕ ФОТО И ЭЛЕКТРО ПРОЦЕССОВ НА ЛЕГКИХ ЯДРАХ В КОВАРИАНТНОМ ПОДХОДЕ С ТОЧНО СОХРАНЯЮЩИМСЯ ЭМ ТОКОМ

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Рассмотрена обобщенная калибровочно-замкнутая амплитуда электропроцесса, которая состоит из традиционного полюсного ряда и регулярной составляющей. Изучен вклад регулярной части амплитуды и ее физический смысл. Рассмотрен предел амплитуды электропроцесса при переходе в фотонную точку ( $q^2 \rightarrow 0$ ), т.е. при переходе от виртуальных фотонов к реальным. Проведен общий анализ процесса электрорасщепления составной скалярной системы. Получено точное сохранение ядерных электромагнитных токов при произвольных  $q^2$ .

**КЛЮЧЕВЫЕ СЛОВА:** регулярная часть амплитуды, электрорасщепление, фоторасщепление, предел амплитуды, легкие ядра

The main goal of present paper is to explore the properties of generalized gauge-invariant amplitude. The main aspect in exploration of electro process on light nuclei is detection of the amplitudes regular part role, as compactness measure of compound strongly connected system. Electromagnetic aspect of amplitudes regular part is that one can reconstruct the deposit of many-particle electric intersections in addition to one-particle mechanisms. In addition, this part determines the requirement of total hadron current conservation. Moreover, amplitudes regular part expands the upper bound of vertex function classes, which does not lead to the cross-section increment with the respect to photon energy enlargement.

Universality of electromagnetic interaction in form of minimal connection consists in the following statement. Photons ( $q^2 = 0$ ) with polarization vectors  $\varepsilon_\mu$  and  $\varepsilon_\mu + \lambda q_\mu$  and arbitrary parameter  $\lambda \in (-\infty; +\infty)$  are indistinguishable during the interaction  $e\varepsilon_\mu J^\mu$  only with conserved current  $q_\nu J^\nu = 0$ . There is opinion in literature

[1, 2], that in processes with virtual photons ( $q^2 \neq 0$ ) gauge arbitrariness of virtual photon propagator (longitudinal

part  $\tilde{\varepsilon}_\mu = e \frac{j^\nu(\text{lepton})}{q^2} (-g_{\nu\mu} + \frac{q_\nu q_\mu}{q^2})$ ,  $\tilde{\varepsilon}_\mu q^\mu = 0$  in matrix element  $M = e\tilde{\varepsilon}_\mu J^\mu(\text{hadrons})$ ,

$J^\mu(\text{hadrons}) = \sum_{i=\{s,t,u\}} J_i^\mu$ ) can be removed by conserved lepton current. This only ensures gauge independence for deposits of distinct mechanisms of hadron current  $q_\mu J^\mu(\text{hadrons}) = 0$  even when lepton current is not conserved. In literature, the efforts to outflank such situation lead to suggestion to make additional heuristic substitution  $J_\mu(\text{hadrons}) \rightarrow J_\mu(\text{hadrons}) - \frac{qJ(\text{hadrons})q_\mu}{q^2}$  - gauge zero shifting. Therefore, the matrix element  $M = e\tilde{\varepsilon}_\mu(J_\mu(\text{hadrons}) - \frac{qJ(\text{hadrons})q_\mu}{q^2})$  is zero with the respect to substitution  $\tilde{\varepsilon}_\mu \rightarrow q_\mu$  even if  $q_\mu J^\mu(\text{hadrons}) \neq 0$ . However, if hadron current is not conserved simultaneously with the lepton one then electro-break up amplitude does not have a photon point limit ( $q^2 \rightarrow 0$ ). It means that we cannot exclude unphysical states from time-longitudinal polarizations of virtual photons, during the intersection from virtual photons to the real ones.

### ELECTRO-BREAK UP AMPLITUDE LIMIT IN PHOTON POINT ( $q^2 \rightarrow 0$ )

In view of raised situation, I wish to show that methods, developed in [5-8] and applied in [3,4,9,10] to describe photodisintegration processes on deuterium,  ${}^3\text{He}$  and  ${}^4\text{He}$  nuclei, give us the opportunity to describe electro-break up processes on light nuclei the same way we did before. To do this, we define gauge-invariant amplitude, which corresponds to photodisintegration process of nonlocal scalar field into two scalar fragments. Such amplitude has the following shape (to be precise we consider scalar deuteron, which consists of two scalar nucleons. Ones have the value of masses and charges corresponding to real particles.):

$$M = e\varepsilon_\mu J^\mu, \quad (1)$$

$$e = \sqrt{4\pi\alpha}, \quad (2)$$

$$J^\mu = J_{pol}^\mu + J_{reg}^\mu, \quad (3)$$

$$J_{pol}^\mu = z_s G_s \frac{(d+d')^\mu}{s-m_d^2} + z_t G_t \frac{(p+p')^\mu}{t-m^2} + z_u G_u \frac{(n+n')^\mu}{u-m^2}, \quad (4)$$

$$J_{reg}^\mu = \frac{k^\mu}{kq} (z_t G_t + z_u G_u - z_s G_s), \quad (5)$$

where  $\alpha = 1/137$ ,  $z_{s,t,u}$  - charges of scalar deuteron, proton and neutron in elementary charge units,  $k_\mu$  - relative space-like 4-momentum of  $pn$ -pair,  $k = (0, \vec{p})$ . Vertex functions  $G_i \equiv G(-k_i^2)$ ,  $i = [s, t, u]$  depend on square of appropriate channel relative 4-momentum:  $k_t = \frac{p'-n}{2} = k_s - \frac{q}{2}$ ;  $k_u = \frac{p-n'}{2} = k_s + \frac{q}{2}$ ;  $q = (\omega, \vec{\omega})$ .

The final expression for amplitude  $J_{reg}^\mu$  regular part does not contain a singularity, when proton and neutron are scattered at right angle ( $\theta = 90^\circ$ ). We consider  $\lim_{k_s, q \rightarrow 0} J_{reg}^\mu$  showing this. We factorize the expression using Taylor functional row at the point  $x = -k_s^2$ . Thus, we receive that the expression is defined by the derivative of vertex function  $G(-k_s^2)$  of strong interaction due to the charge conservation law  $z_s = z_t + z_u$ :

$$\lim_{k_s, q \rightarrow 0} \frac{(z_t + z_u)G(-k_s^2) - z_t G(-k_s^2 + qk_s) - z_u G(-k_s^2 - qk_s)}{k_s q} = \lambda \left. \frac{dG(x)}{dx} \right|_{x=-k_s^2}. \quad (6)$$

It is very easy to make sure that pole part of nonlocal current is not conserved:  $q_\mu J_{pol}^\mu = z_s G_s - z_t G_t - z_u G_u$ , in despite of charge conservation:  $z_s - z_t - z_u = 0$ . Further, I am going to show that regular part of total current reclaims described situation. For this, we have to write down the diagram of selected process ( $q^2 \neq 0$ ) (Fig. 1,2). Virtual photon momentum is  $q = E - E'$ .

We consider the limit of intersection from virtual photon to the real one:  $q^2 \rightarrow 0$ . All calculations performed in center of mass system (Fig. 3). We chose this system the way that virtual photon momentum oriented along  $oZ$  axes.

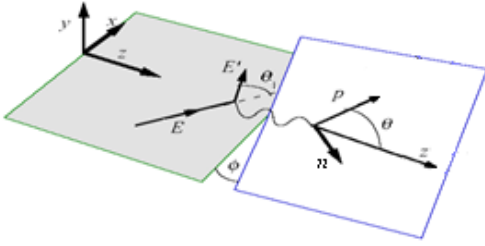


Fig. 1. Electro-break up process of compound scalar system.

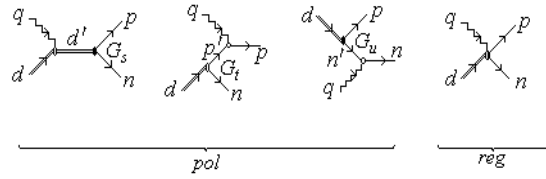


Fig. 2. Gauge invariant electro-break up amplitude of nonlocal scalar field into two scalar fragments.

The angle between virtual photon momentum and proton momentum signed as  $\theta$ . Therefore, coordinates of 4-vectors are:  $\varepsilon_\mu = 1/\sqrt{-q^2}(q_3, 0, 0, \nu)$ ,  $d^\mu = (E_d, 0, 0, -q_3)$ ,  $q^\mu = (\nu, 0, 0, q_3)$ ,  $k^\mu = (0, p \sin \theta, 0, p \cos \theta)$ ,  $p^\mu = (E_p, p \sin \theta, 0, p \cos \theta)$ ,  $n^\mu = (E_n, -p \sin \theta, 0, -p \cos \theta)$ .

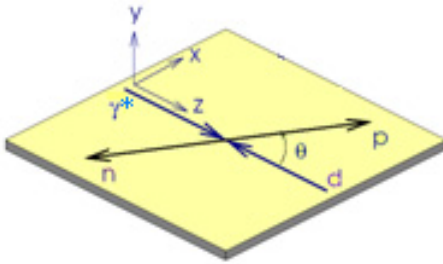


Fig. 3. Center of mass system.

We are going to find amplitudes limit at  $q^2 \rightarrow 0$  in the following way. We split our problem into three identical parts, which depend from the momentums in  $s, t, u$  – channels appropriately. The limit of the amplitude in  $s$  – channel is given by the sum of pole part and the third summand in expression (5):

$$J_{sreg}^\mu + J_{spol}^\mu = z_s G_s \frac{(d + d')^\mu}{s - m_d^2} - \frac{k^\mu}{kq} z_s G_s. \quad (7)$$

From the relation between momentums  $q + d = d'$  (Fig. 2) follows that  $d + d' = q + 2d$ , and, therefore Mandelstam variable  $s = (d + q)^2 = m_d^2 + 2dq + q^2$ . It means that the limit of the expression contained in matrix element of  $s$  – channel can be written down as:

$$\lim_{q^2 \rightarrow 0} q_\mu (J_{sreg}^\mu + J_{spol}^\mu) = z_s G_s \lim_{q^2 \rightarrow 0} \left( \frac{2q_\mu d^\mu + q^2}{m_d + 2dq + q^2 - m_d} - \frac{q_\mu k^\mu}{kq} \right) = z_s G_s \lim_{q^2 \rightarrow 0} \left( \frac{2q_\mu d^\mu}{2dq} - \frac{q_\mu k^\mu}{kq} \right) = 0. \quad (8)$$

In a reciprocal manner one can receive the expressions for  $t, u$  – channels:

$$\lim_{q^2 \rightarrow 0} q_\mu (J_{treg}^\mu + J_{tpol}^\mu) = 0; \quad \lim_{q^2 \rightarrow 0} q_\mu (J_{ureg}^\mu + J_{upol}^\mu) = 0;$$

which also seek to zero, while  $q^2 \rightarrow 0$ . Well then, we satisfy the  $J_{total}^\mu$  conservation requirement:

$$\lim_{q^2 \rightarrow 0} (q_\mu J_{total}^\mu = 0). \quad (9)$$

At this stage, the following question arises. Why the amplitudes final expression does contain just the fragments relative momentum  $k_s$ ? In paper [6] the expression for matrix element of compound system disintegration process was developed:

$$\mathfrak{M}_{reg} = (2\pi)^4 \delta(q + p - p_1 - p_2) \varepsilon_\mu \int_0^1 d\lambda \left\{ e_1 \frac{\partial G(p_1 - \lambda q, p_2)}{\partial (p_1 - \lambda q)^\mu} + e_2 \frac{\partial G(p_1, p_2 - \lambda q)}{\partial (p_2 - \lambda q)^\mu} \right\}, \quad (10)$$

where  $e_i, p_i, i = \{1, 2\}$  are the charges and momentums of fragments appropriately. Let's concretize the form of the vertex function  $G$  argument. In fact, vertex function depends on the square of relative momentum  $k_s = \frac{E_2}{w} p_1 - \frac{E_1}{w} p_2 = \eta_2 p_1 - \eta_1 p_2$ , where  $\eta_i = E_i/w, i = 1, 2$ , a  $w = E_1 + E_2$  – total energy. At fragments center of mass system, relative 4-momentum  $k_s = (0; \vec{p})$  is space-like,  $p_1 = (E_1; \vec{p}), p_2 = (E_2; -\vec{p})$ . Let's consider the first summand under the integral. We regain the dependence from current value of the relative momentum square:

$$\varepsilon_\mu \int_0^1 d\lambda \left\{ e_1 \frac{\partial G(p_1 - \lambda q; p_2)}{\partial (p_1 - \lambda q)_\mu} + \dots \right\} = \varepsilon_\mu \int_0^1 d\lambda \left\{ e_1 \frac{\partial (k - \lambda \eta_2 q)^2}{\partial (p_1 - \lambda q)_\mu} \frac{\partial G[(k - \lambda \eta_2 q)^2]}{\partial (k - \lambda \eta_2 q)^2} + \dots \right\}.$$

Here, argument  $(p_1 - \lambda q; p_2)$  of vertex function in terms of relative momentum square

$k_{st}^2(\lambda) = (\eta_2(p_1 - \lambda q) - \eta_1 p_2)^2 = k_s^2 - 2\lambda\eta_2 k_s \cdot q$ ,  $k_{st}^2(1) = k_t^2$ ,  $k_{st}^2(0) = k_s^2$ , defined as  $G[(k_s - \lambda\eta_2 q)^2]$ . Let's rewrite

the integral taking into account the defined argument:  $\varepsilon_\mu \int_0^1 d\lambda \left\{ e_1 \frac{\partial k_{st}^2}{\partial(p_1 - \lambda q)_\mu} \frac{\partial G[k_{st}^2]}{\partial k_{st}^2} + \dots \right\}$ . Now, we calculate the

derivative  $\varepsilon_\mu \frac{\partial k_{st}^2}{\partial(p_1 - \lambda q)_\mu} = \varepsilon_\mu \frac{\partial[\eta_2(p_1 - \lambda q) - \eta_1 p_2]^2}{\partial(p_1 - \lambda q)_\mu} = 2(k - \lambda\eta_2 q)_\beta \eta_2 g^{\beta\mu} \varepsilon_\mu = 2\varepsilon \cdot k \eta_2$ , accounting transverse

condition  $\varepsilon q = 0$ . Initial integral takes the shape

$\varepsilon_\mu \int_0^1 d\lambda \left\{ e_1 \frac{\partial(k - \lambda\eta_2 q)^2}{\partial(p_1 - \lambda q)_\mu} \frac{\partial G[(k - \lambda\eta_2 q)^2]}{\partial(k - \lambda\eta_2 q)^2} + \dots \right\} = \varepsilon \cdot k \int_0^1 2\eta_2 d\lambda \left\{ e_1 \frac{\partial G[k_{st}^2(\lambda)]}{\partial k_{st}^2(\lambda)} + \dots \right\}$ . Finally, we divide and multiply

this expression by  $k \cdot q$ , therefore integration by  $\lambda$  reduces to the new variable  $d\lambda 2\eta_2 k \cdot q = -dk_{st}^2(\lambda)$ . Totally we

receive:  $\varepsilon \cdot k \int_0^1 2\eta_2 d\lambda \left\{ e_1 \frac{\partial G[k_{st}^2(\lambda)]}{\partial k_{st}^2(\lambda)} + \dots \right\} = -\frac{\varepsilon \cdot k}{q \cdot k} \int_0^1 dk_{st}^2(\lambda) \left\{ e_1 \frac{\partial G[k_{st}^2(\lambda)]}{\partial k_{st}^2(\lambda)} + \dots \right\} = -\frac{\varepsilon \cdot k_s}{q \cdot k_s} \{e_1 G[k_{st}^2(1)] - e_1 G[k_{st}^2(0)]\}$ .

Calculating the integral for the second charge  $e_2$  and generalizing the result taking into account the charge conservation

law  $e = e_1 + e_2$  we receive:  $-(2\pi)^4 \delta(p + q - p_1 - p_2) \cdot \frac{\varepsilon \cdot k_s}{q \cdot k_s} \{e_1 G[k_t^2] + e_2 G[k_u^2] - e G[k_s^2]\}$ . Therefore, the matrix

element final expression is:

$$\mathfrak{M}_{reg} = -(2\pi)^4 \delta(p + q - p_1 - p_2) \cdot e \cdot \frac{\varepsilon \cdot k_s}{q \cdot k_s} \{z_1 G[k_t^2] + z_2 G[k_u^2] - z G[k_s^2]\}. \quad (11)$$

This expression does not contain any kinematical singularities. Moreover, one defined by the sum of the change “velocities” of structure formatting interaction in every point of nonlocality. Total expression for covariant amplitude

after photon “insertation” into strongly connected “threetail” takes the form  $\mathfrak{M}_{tot} = e \cdot \frac{F_{\mu\nu} \cdot J_{pol}^\mu k_s^\nu}{q \cdot k_s}$ , where

$F_{\mu\nu} = \varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu$  – EM-field tensor,  $J_{pol}^\mu$  total EM pole current of s-, t- and u-channels.

The inference is that injection into consideration of additional (regular) part ensures the existence of photon point limit. This property provides the opportunity to describe electro break-up processes applying developed covariant approach [5-7].

### GENERAL ANALYSIS OF COMPOUND SCALAR SYSTEM ELECTRO-BREAK UP PROCESS

To carry out calculations of observed characteristics one has to establish the conditions of concrete model, which corresponds to electromagnetic hadron intersection current. We apply developed approach with this objective. The only distinction with the respect to photo processes is the presence of virtual  $\gamma^*$  – quantum. Extra virtuality (tendered by photon line into electromagnetic vertex) leads to the emergence of form-factors instead of charges in electromagnetic currents.

Matrix element of compound scalar system electro break-up process (corresponding to Feynman diagrams on Fig. 2.) has the shape (1) with the amplitudes of appropriate channels (4, 5). For the numerical calculations one has to use realistic form-factors of target and fragments instead of charges in amplitudes. There is a number of articles containing different  $q^2$  – dependence of form-factors. For example, form-factor dependence can be taken from [11,12]. Such dependence fulfills the agreement to the photo process amplitude:  $F_d(q^2 = 0) = z_d$ ;  $F_p(q^2 = 0) = z_p$ ;  $F_n(q^2 = 0) = z_n$ . Values of  $z_d, z_p, z_n$  – are charges of compound system and its constituents in elementary charge units. Gauge invariance of the amplitude has no hesitation. Providing the substitution:  $\varepsilon_\mu^* \rightarrow q_\mu$ , one receive the expression:

$$\begin{aligned} \mathfrak{M}(\varepsilon_\mu^* \rightarrow q_\mu) &= e \{ F_d(q^2) G_s + [F_p(q^2) + F_n(q^2) - F_d(q^2)] G_s \\ &- F_p(q^2) G_t - F_n(q^2) G_u + F_p(q^2) G_t + F_n(q^2) G_u - [F_p(q^2) + F_n(q^2)] G_s \} = 0. \end{aligned} \quad (12)$$

As the result, we have the opportunity of light nuclei electro-break up covariant description. Analysis of the results of observable quantities calculations and comparison with experimental data will be given in further articles.

### CONCLUSION

Regular part of gauge invariant amplitude defines the value of electrical many-particle mechanisms dynamical deposit in addition to one-particle (pole) deposit. This statement is in precise agreement with the requirement of gauge invariance. Therefore, in developed covariant approach, the photon point limit of the electro-break up amplitude exists and equals to zero. This property gives the opportunity of unified description of photo and electro processes based on general principles. Application of this approach permits us to receive precisely conserved nuclear electromagnetic currents at arbitrary  $q^2$ . The main notice of the article is the description of interaction between electromagnetic field and bound system of strongly interacting particles. As the result, it was shown that conservation of electromagnetic current in processes with electrons is also very important as in photoreactions.

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