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NONLINEAR SELF-FOCUSING OF q-GAUSSIAN LASER BEAMS IN PLASMA WITH RELATIVISTIC AND PONDEROMOTIVE EFFECTS UNDER LINEAR ABSORPTION

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The current study presents a theoretical analysis of the nonlinear self-focusing of a q-Gaussian laser beam propagating through an unmagnetized plasma, incorporating the simultaneous effects of relativistic mass variation and the ponderomotive mechanism. Linear absorption is also included to account for energy dissipation during beam propagation. By applying WKB and paraxial approximations, the problem is reduced to 2nd order differential equation that governs the evolution of the laser beam width as a function of the normalized propagation distance. The resulting equation is solved numerically using 4th order Runge-Kutta method. A systematic analysis is performed to examine the effect of laser intensity, plasma density, absorption coefficient, q-parameter, and initial beam radius on the self-focusing dynamics of a q-Gaussian laser beam. These findings indicate that laser-plasma parameters substantially affect beam dynamics and critically govern the self-focusing process.
Keywords: *Laser-plasma interaction; q-Gaussian beam; Nonlinear self-focusing; Absorption coefficient; Nonlinear dynamics*
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1. INTRODUCTION

The sustained interest has been attracted by laser-plasma interaction in view of its vital role in diverse range of applications including ionospheric modification, charged particle acceleration, laser-driven fusion and nonlinear radiation generation [1-10]. The ability of laser beams to transit through plasma decides efficiency of all these applications. Much deeper penetration of laser beams through plasma media causes beam narrowing, which further causes enhancement in energy confinement and improvement in transfer of laser energy to plasma. However, the natural beam's diffraction imposes fundamental limitation on their transition through plasma environment. Self-focusing provides an effective mechanism to counteract diffraction during laser beam transition through plasma. Under intense laser irradiation, there is excitation of several nonlinear phenomenon including self-focusing, filamentation, and parameteric instabilities [11-19]. Among these nonlinear effects, the central role is played by self-focusing as it directly governs beam confinement and greatly influences the growth of other nonlinear processes. The concept of nonlinear refraction and the possibility of self-focusing of an intense electromagnetic wave was provided by Askar'yan [20]. Since then, theoretical/experimental researchers have extensively explored this phenomenon in view of its importance in nonlinear optics and plasma dynamics [21-32]. It has been demonstrated from numerous investigations that self-focusing critically governs overall behavior of laser-plasma interaction [33-36]. Self-focusing causes contraction of laser beam's transverse dimensions, thereby causing strong localization of intensity. This enhanced localization may disrupt beam uniformity, which is undesirable in applications such as laser-driven fusion. Therefore, achieving effective control over beam transition through plasma is mandatory in order to optimize energy coupling and attaining high energy gain. The first experiment observation of self-focusing was reported by Akhmanov et al. [37], followed by comprehensive studies by Sodha et al. in plasmas, dielectrics, and semiconductors [38-39]. In plasmas, self-focusing occurs due to intensity dependent variations of effective dielectric permittivity, leading to significant changes in plasma optical properties. These changes mainly originate from three mechanisms: ponderomotive, thermal, and relativistic nonlinearities. The ponderomotive self-focusing was first time introduced by Hora [40], who successfully derived critical power condition by balancing ponderomotive and hydrostatic forces. The ponderomotive force expels electrons from high intensity regions, producing plasma density depressions and corresponding reduction in local plasma frequency [41]. The density redistribution alters the plasma dielectric function and leads to self-focusing. Relativistic self-focusing becomes significant at ultra-high laser intensities, where electrons attain relativistic velocities. The associated relativistic mass increase modifies plasma frequency and dielectric permittivity, resulting in intensity dependent refractive index and enhanced beam focusing. Despite extensive studies, relativistic and ponderomotive nonlinearities are often treated separately, although both may exist depending on laser pulse duration. Relativistic effects dominate for $\tau < \tau_e$, whereas for $\tau_e < \tau < \tau_i$, both mechanisms act simultaneously. Here, τ is time scale of laser-plasma interaction, τ_e and τ_i correspond to electron and ion response times. Ponderomotive nonlinearity complements relativistic effects, enhancing self-focusing and plasma density perturbations. Mostly earlier investigations

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of laser-plasma interactions have employed conventional Gaussian beam profiles. The q-Gaussian laser beams form an important class characterized by the irradiance distribution $f(r) = f(0)\left(1 + \frac{r^2}{qr_0^2}\right)^{-q}$, which reduces to the standard Gaussian profile in the limit $q -> \infty$. Here, $f(r)$ is radial profile of laser beam amplitude and $f(0)$ is beam's peak value at $r = 0$. The q-Gaussian beams possess lower total power then Gaussian beams and exhibit distinct propagation features, making them suitable for controlled nonlinear interaction studies. The main origin of q-Gaussian profile is from q-Gaussian distribution which was derived in non-extensive statistical mechanics by Tsallis [42]. In laser-plasma interaction, this form has been adopted by many authors for modeling high power beams transiting through nonlinear plasma media. Motivated by these considerations, the present research investigates nonlinear self-focusing of q-Gaussian laser beams propagating in unmagnetized plasma under the combined influence of relativistic and ponderomotive nonlinearities, with linear absorption included to account for energy dissipation. The organization of paper is as follows: Section 2 presents theoretical formulation, where the paraxial and WKB approximations are employed to derive 2nd order differential equation governing the beam width. Section 3 discusses numerical results and their physical implications; the main conclusions are summarized in Section 4.

2. NONLINEAR EVOLUTION OF BEAM SPOT SIZE

A laser beam with q-Gaussian intensity profile is assumed to be propagating along z-axis in unmagnetized plasma. One can describe the beam intensity at the initial plane $(z = 0)$ by

$$E_0 \cdot E_0^*|_{z=0} = E_{00}^2\left(1 + \frac{r^2}{qr_0^2}\right)^{-q} \tag{1}$$

Where, r_0 represents initial beam radius, E_0 is the complex field amplitude, and E_{00} denotes field amplitude along the beam axis. The beam profile and decay behavior of its tails are governed by the q-parameter. This q-parameter is helpful in distinguishing a q-Gaussian beam from conventional beams. During the transition of the beam through propagation axis $(z > 0)$, its intensity distribution behaves according to equation,

$$E_0 \cdot E_0^* = \frac{E_{00}^2}{f^2}\left(1 + \frac{r^2}{qr_0^2 f^2}\right)^{-q}. \tag{2}$$

Where f denotes beam waist evolution, describing focusing/defocusing of beam during its transition through plasma. Eq.(2) provides complete description of energy distribution of q-Gaussian beam along propagation axis. Maxwell's equation can be used to derive fundamental equation governing propagation of an electromagnetic wave in plasma medium. Faraday's law and Ampere's law can be combined to obtain following wave equation for electric field of laser beam as

$$\nabla^2 E - \nabla(\nabla \cdot E) + \frac{\omega^2}{c^2}\varepsilon E = 0. \tag{3}$$

In Eq.(3), ε represents the effective dielectric response of plasma, ω is angular frequency of incident radiation, and c is speed of light in vacuum. The second term on LHS, $\nabla(\nabla \cdot E)$ arises on account of possible spatial variation in dielectric function of plasma. However, for a slowly varying and weakly inhomogeneous medium, a negligible contribution is made by this term towards overall beam dynamics. This approximation remains valid provided $\frac{c^2}{\omega^2}\left|\frac{1}{\varepsilon}\nabla^2 \ln \varepsilon\right| \leq 1$ condition is satisfied. As a result, $\nabla(\nabla \cdot E)$ term can be safely neglected. So, Eq. (3) reduces to

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E = 0. \tag{4}$$

The transition of high intensity electromagnetic wave through plasma medium causes strong nonlinear behavior of electron dynamics. In such regimes, electrons simultaneously experience relativistic mass enhancement, due to their large quiver velocities, and the ponderomotive force which arises from spatial gradient in electromagnetic field intensity. The combined action of these two mechanisms is referred to as relativistic ponderomotive force (RP force). The electrons are driven away from strong laser intensity regions due to this RP force, thereby causing redistribution of charge density and corresponding modification of optical properties of plasma. The RP force acting on an electron in an intense laser field is given by [43-45]

$$F_{pe} = -m_0 c^2 \nabla(\gamma - 1). \tag{5}$$

Where, m_0, c and γ correspond to electron rest mass, speed of light in vacuum and Lorentz relativistic factor respectively. At sufficiently high laser intensities, electron quiver motion becomes relativistic, resulting in a Lorentz factor that explicitly depends on local electromagnetic field amplitude. It may be expressed as $\gamma = \sqrt{1 + \alpha E E^*}$ with $\alpha = \frac{e^2}{m_0^2 c^2 \omega_0^2}$ representing the nonlinear coefficient. As a direct consequence of RP force, electrons are expelled from high intensity regions of the beam, resulting in a local depletion of electron density. This density modification plays a crucial role in determining plasma's dielectric response. Accounting for both relativistic mass variation and ponderomotive density redistribution, the effective dielectric function of plasma can be written as [43-45]

$$\varepsilon = 1 - \frac{\omega_p^2}{\gamma\omega^2} exp\left(-\frac{m_0c^2}{T_e}(\gamma - 1)\right). \quad (6)$$

Where $\omega_p = \sqrt{\frac{4\pi N_0 e^2}{m_0}}$ is electron plasma frequency corresponding to unperturbed electron density N_0, and T_e denotes electron temperature. The RP force-induced modification of equilibrium electron density is correspondingly given by [45]

$$N_{0e} = \frac{N_0}{\gamma} exp\left(-\frac{m_0c^2}{T_e}(\gamma - 1)\right). \quad (7)$$

Accordingly, total plasma dielectric function may be decomposed in to linear and nonlinear components as

$$\varepsilon = \varepsilon_0 + \Phi(E \cdot E^*). \quad (8)$$

Where, $\varepsilon_0 = 1 - \frac{\omega_p^2}{\omega^2}$ represents the linear dielectric response of cold plasma in absence of RP force. The nonlinear contribution, $\frac{\omega_p^2}{\omega^2}\left[1 - \frac{N_{0e}}{N_0}\right]$, arises from the intensity dependent depletion of electron density and represents the nonlinear susceptibility induced by relativistic mass variation and ponderomotive expulsion of electrons.

Following [37-39], a suitable solution of wave Eq. (4) may be written in the form of a slowly varying envelope modulating a rapidly oscillating carrier wave, namely

$$E = E_0(r, z)\exp[i(\omega t - k(S + z))]. \quad (9)$$

The corresponding intensity distribution of the q-Gaussian beam during propagation is expressed as [38-39]

$$E_0 \cdot E_0^* = \frac{E_{00}^2}{f^2}\left(1 + \frac{r^2}{qr_0^2 f^2}\right)^{-q} exp(-2k_i z). \quad (10)$$

Here, k_i accounts for absorption effects in plasma medium. The phase function $'S'$ which incorporates the effect of beam curvature and axial phase variation is given by

$$S = \frac{1}{2}r^2\frac{1}{f}\frac{df}{dz} + \Phi_0(z). \quad (11)$$

Where, $\Phi_0(z)$ denotes longitudinal phase shift accumulated during propagation. The wave number k in plasma is defined as

$$k = \frac{\omega}{c}\sqrt{\varepsilon_0} \quad (12)$$

The transverse evolution of laser beam is described by beam width function $'f(z)'$ whose variation along direction of propagation is governed by following 2nd order nonlinear ordinary differential equation [38-39];

$$\frac{d^2f}{d\eta^2} = \frac{q+4}{qf^3} - \left(\frac{\omega_p r_0}{c}\right)^2 exp(-2k_i\eta)\frac{\alpha E_{00}^2}{2f^3} exp\left[-\frac{mc^2}{T_e}\left\{\sqrt{1 + \frac{\alpha E_{00}^2 exp(-2k_i\eta)}{f^2}} - 1\right\}\right]\left(1 + \frac{\alpha E_{00}^2 exp(-2k_i\eta)}{f^2}\right)^{-3/2}\left[1 + \frac{mc^2}{T_e}\sqrt{1 + \frac{\alpha E_{00}^2 exp(-2k_i\eta)}{f^2}}\right] \quad (13)$$

Eq. (13) governs evolution of normalized beam waist f as a function of normalized distance η. The first term on right hand side represents diffraction induced beam spreading, which tends to increase the beam width as beam propagates. The second term arises from nonlinear RP force, which introduces an effective focusing mechanism by modifying the plasma dielectric response. The competition between these two opposing effects, diffraction-driven divergence and RP-induced nonlinear convergence dictates the overall propagation behavior of beam. Depending on relative strength of these mechanisms, the beam may undergo continuous expansion, strong self-focusing, or exhibit oscillatory evolution as it propagates through plasma medium.

The initial condition used is, $f = 1$ & $\frac{df}{d\eta} = 0$ at $\eta = 0$.

3. DISCUSSION

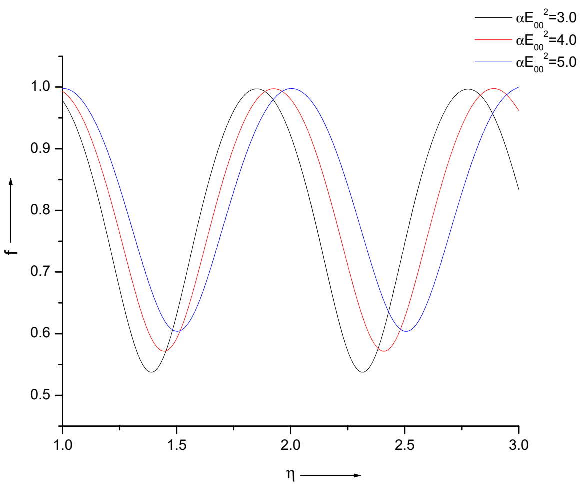
Eq. (13) can't be solved numerically; its solution is obtained through numerical technique using Runge-kutta 4th order method. The computations are carried out for a representative set of experimentally relevant laser-plasma parameters;

$$\alpha E_{00}^2 = 3.0, 4.0, 5.0; \frac{\omega_p^2}{\omega^2} = 0.4, 0.5, 0.6; k_i = 0, 0.3, 0.6\,; q = 1, 2, 3;\ r_0 = 20\ \mu m, 25\ \mu m, 30\ \mu m$$

The evolution of beam width governed by Eq. (13) results from interplay of two distinct mechanisms appearing on right hand side. The first mechanism is connected with diffraction, which reflects beam's tendency to spread as it transits through plasma. The dominance of this mechanism causes beam's divergence, leading to an increase in transverse width. Mathematically, this regime is characterized by $\frac{d^2f}{d\eta^2} > 0$. The second mechanism arises from nonlinear modification plasma's refractive index caused by interaction of an intense laser field. The dominance of this mechanism causes beam's convergence, leading to decrease in transverse width. Mathematically, this regime is characterized by $\frac{d^2f}{d\eta^2} < 0$. A special situation arises, when both mechanisms exactly compensate each other. Under this balance, the beam transits in a steady state manner without any change in its transverse width. This stationary transition condition is characterized by $\frac{d^2f}{d\eta^2} = 0$.

Figure 1 shows evolution of beam width f as a function of normalized distance η for different laser intensities, $\alpha E_{00}^2 = 3.0, 4.0, \text{and } 5.0$, represented by Black, Red, and Blue respectively. In all cases, the beam width exhibits oscillatory behavior due to saturating response of plasma dielectric function. These oscillations arise from continuous interplay between diffraction, which tends to broaden the beam, and nonlinear self-focusing, which acts to contract it. As the laser intensities increases, the diffraction term becomes increasingly dominant over nonlinear focusing term, thereby reducing beam's self-focusing tendency. Consequently, at higher laser intensities, the beam reaches its minimum width at larger propagation distances, clearly indicating diminished self-focusing effect.

Figure 2 shows evolution of beam width f as a function of normalized distance η for different plasma densities, $\frac{\omega_p^2}{\omega^2} = 0.4, 0.5, \text{and } 0.6$, represented by Black, Red, and Blue respectively. In all cases, the beam width exhibits oscillatory behavior due to saturating response of plasma dielectric function. These oscillations arise from continuous interplay between diffraction, which tends to broaden the beam, and nonlinear self-focusing, which acts to contract it. As the plasma density increases, the nonlinear focusing term becomes increasingly dominant over diffraction term, thereby strengthening beam's self-focusing tendency. Consequently, at higher plasma densities, the beam reaches its minimum width at shorter propagation distances, clearly indicating enhanced self-focusing effect.



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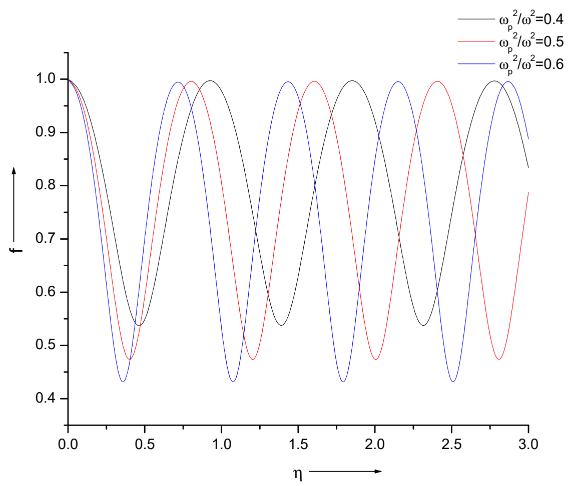

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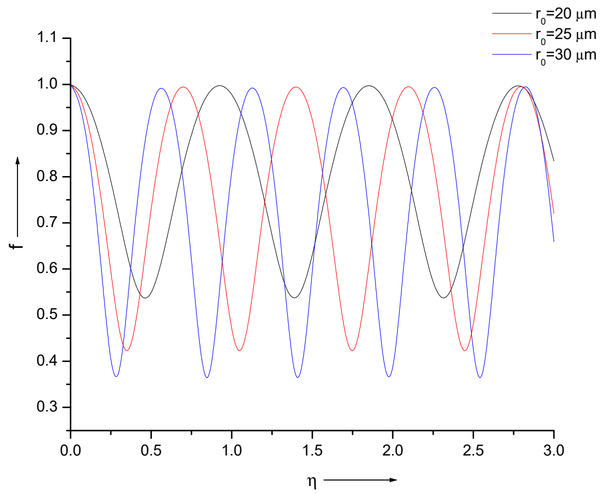
Figure 3 shows evolution of beam width f as a function of normalized distance η for different beam radii, $r_0 = 20\ \mu m, 25\ \mu m, \text{and } 30\ \mu m$, represented by Black, Red, and Blue respectively. In all cases, beam width exhibits an oscillatory behavior, which originates from saturating response of plasma dielectric function. As the beam radius increases, the contribution of nonlinear focusing term becomes increasingly dominant compared to diffraction term. This shift strengthens beam's tendency to self-focus. As a result, for larger beam radii, the beam reaches its minimum width at shorter propagation distances, clearly indicating an enhanced self-focusing effect.

Figure 4 shows evolution of beam width f as a function of normalized distance η for different q parameters, $q = 1,2, \text{and } 3$, represented by Black, Red, and Blue respectively. In all cases, beam width exhibits an oscillatory nature, which arises from saturating response of plasma dielectric function. It is clearly observed that increasing the value of q markedly enhances focusing characteristics of beam. At higher q values, the nonlinear focusing contribution becomes stronger relative to diffraction, enabling the beam to converge more effectively. This leads to improved spatial localization of beam and corresponding increase in central field intensity. The enhanced intensity strengthens the nonlinear interaction beam and the plasma, which further reinforces self-focusing mechanism and results in deeper and more pronounced beam convergence.



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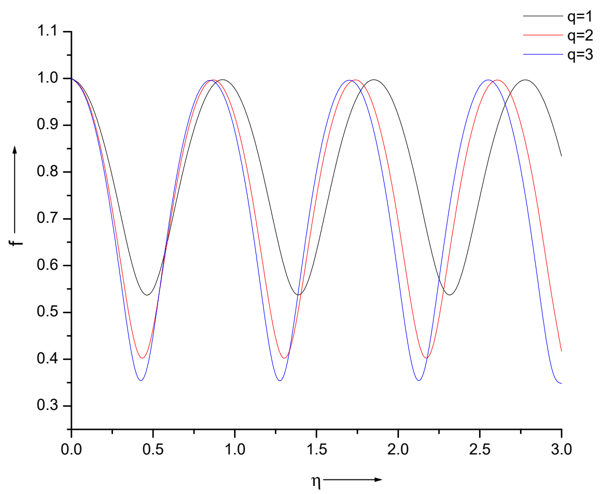
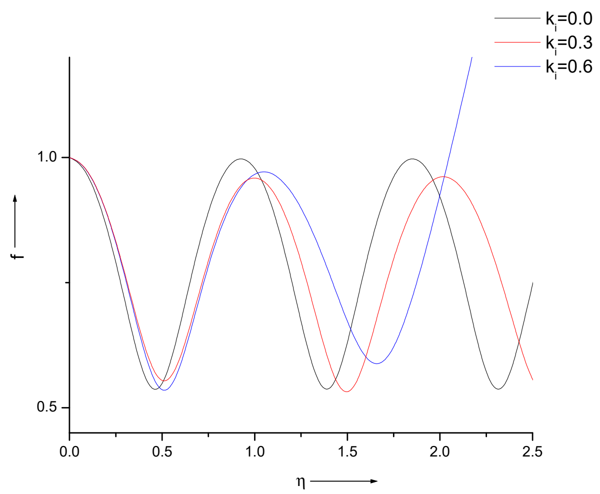


Figure 4. Evolution of beam width f as a function of normalized distance η for different q parameters, $q = 1,2,$ and 3, represented by Black, Red, and Blue respectively

Figure 5 shows evolution of beam width f as a function of normalized distance η for different k_i values, $k_i = 0, 0.3, 0.6$, represented by Black, Red, and Blue respectively. The figure shows that increasing k_i weakens beam's self-focusing due to energy loss through absorption. As absorption grows, the refractive index gradient decreases thereby reducing nonlinear convergence effect. Consequently, diffraction dominates and beam focuses less effectively.

Figure 6 shows evolution of beam width f as a function of normalized distance η for two different plasma regimes. The Black line shows the combined effect of relativistic mass variation and ponderomotive nonlinearities, while Red line represents relativistic nonlinearity alone. When both act together, the beam width f shifts to smaller η, indicating stronger self-focusing. Since, relativistic nonlinearity acts instantaneously, the ponderomotive contribution reinforces it, enhancing overall focusing effect.



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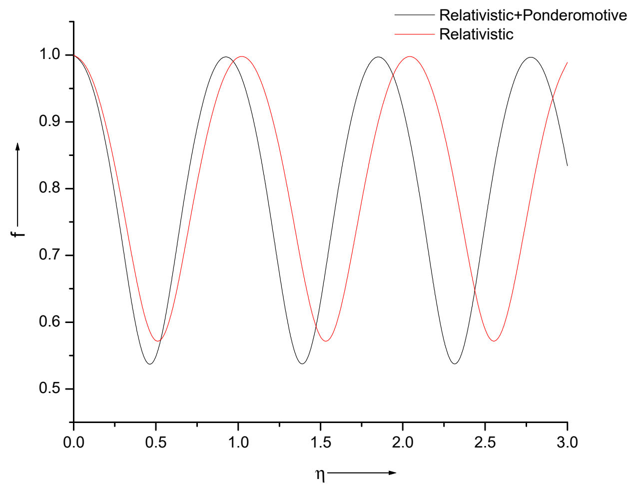

Figure 6. Evolution of beam width f as a function of normalized distance η for two different plasma regimes. The Black line shows the combined effect of relativistic mass variation and ponderomotive nonlinearities, while Red line represents relativistic nonlinearity alone

4. CONCLUSION

This study examines a detailed analysis of nonlinear self-focusing of q-Gaussian laser beams in plasma with relativistic and ponderomotive effects under linear absorption. Using WKB and paraxial approximations, self-focusing equation is derived, revealing that beam's focusing strengthens with higher plasma density, larger q-parameter, increased initial beam radius, combined action of RP force, while higher beam intensity and absorption weakens it. These results provide essential insights for controlling laser propagation in plasma, with direct implications for optimizing laser-driven fusion and other high intensity laser applications.

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Conflict of Interest
The authors declare that there are no conflicts of interest.

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НЕЛІНІЙНЕ САМОФОКУСУВАННЯ q-ГАУСІВСЬКИХ ЛАЗЕРНИХ ПРОМЕНІВ У ПЛАЗМІ З РЕЛЯТИВІСТСЬКИМИ ТА ПОНДЕРОМОТОРНИМИ ЕФЕКТАМИ ПРИ ЛІНІЙНОМУ ПОГЛИНАННІ

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У цьому дослідженні представлено теоретичний аналіз нелінійного самофокусування q-гауссового лазерного променя, що поширюється через немагнічену плазму, враховуючи одночасний вплив релятивістської зміни маси та пондеромоторного механізму. Лінійне поглинання також враховується для врахування дисипації енергії під час поширення променя. Застосовуючи ВКБ-апроксимацію та параксіальне наближення, задача зводиться до диференціального рівняння 2-го порядку, яке описує еволюцію ширини лазерного променя як функції нормалізованої відстані поширення. Отримане рівняння розв'язується чисельно за допомогою методу Рунге-Кутти 4-го порядку. Проведено систематичний аналіз для вивчення впливу інтенсивності лазера, густини плазми, коефіцієнта поглинання, q-параметра та початкового радіуса променя на динаміку самофокусування q-гауссового лазерного променя. Ці результати показують, що параметри лазерної плазми суттєво впливають на динаміку променя та критично керують процесом самофокусування.

Ключові слова: *взаємодія лазера з плазмою; q-гауссовий промінь; нелінійне самофокусування; коефіцієнт поглинання; нелінійна динаміка*