East Eur. J. Phys.Vol.2 No.1 (2015) 23-31

PACS: 44.25.+f, 47.20.Bp

ELEMENTARY CONVECTION CELL IN THE HORIZONTAL LAYER OF VISCOUS INCOMPRESSIBLE LIQUID WITH RIGID AND MIXED BOUNDARY CONDITIONS

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Received March 12, 2015

The result of experimental study of convection cells formation of vacuum oil with mixed boundary conditions is presented. The adding small amounts of dispersed phase (aluminum powder) the viscosity and density of the oil didn't change and under such conditions the boundary conditions for liquid velocity on a rigid boundary can be applied is shown. The experiments demonstrated that exceeding the certain temperature of the container bottom cells of cylindrical shape start to appear in the oil layer with small addition of dispersed phase (oil paint or aluminum powder). The process of appearing of cells finished when the number of cells increases up to the complete filling of oil volume. When amount of the added dispersed phase are small regardless of the its type a rigid boundary conditions can be applied for the lower boundary, i.e. the mixed boundary conditions are present in the layer is shown. The diameter of cells with the mixed boundary conditions varies from 2,65 to 2,83 mm, which is less than the diameter of a cell with free boundary conditions. For a special case there were obtained the analytical solutions of Navier–Stokes equation with rigid boundaries when Rayleigh number is $R \approx 7124.78$ and wave number is $k_r \approx \pi\sqrt{7}$. The expressions of distribution for perturbed velocity and temperature in cylindrical convection cell were received. This distributions were compared to similar property for free convective cell for the main mode $n=1$. It is demonstrated that the diameter of convective cell is inversely related to the value of minimal wave number of the corresponding boundary value problem, i.e. the diameter of a cell with the mixed boundary conditions is less than the diameter of a cell with free boundary conditions, but it's larger than the diameter of a cell with rigid boundary conditions.
KEY WORDS: convection cells, vacuum oil, mixed and a rigid boundary conditions, perturbed temperature, perturbed vertical velocity

ЕЛЕМЕНТАРНИЙ КОНВЕКТИВНИЙ ОСЕРЕДОК З ТВЕРДИМИ ТА ЗМІШАНИМИ ГРАНИЧНИМИ УМОВАМИ В ГОРИЗОНТАЛЬНОМУ ШАРІ В'ЯЗКОЇ, НЕСТИСЛИВОЇ РІДИНІ

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Приведені результати експериментальних досліджень формування конвективних осередків зі змішаними граничними умовами у вакуумній олії. Показано, що малі кількості доданої дисперсної фази (алюмінієва пудра) не змінюють в'язкість і щільність масла, і в таких умовах застосовуються граничні умови для швидкості рідини на твердій стінці. В експериментах показано, що при підвищенні певної температури дна ємкості в шарі масла з добавкою невеликої кількості дисперсної фази (олійної фарби або алюмінієвої пудри) починають з'являтися осередки циліндричної форми. Зі збільшенням температури дна ємкості кількість осередків збільшується аж до повного заповнення ними об'єму масла. Показано, що незалежно від виду доданої дисперсної фази, але при її малій кількості, на нижній границі ємкості застосовуються граничні умови на твердій границі, тобто в шарі реалізуються змішані граничні умови. Величина діаметра осередків зі змішаними граничними умовами варіюється від 2,65 до 2,83, що менше діаметра осередку з вільними граничними умовами. В окремому випадку, для значення числа Релея $R \approx 7124.78$ і хвильового числа $k_r \approx \pi\sqrt{7}$ отримані аналітичні рішення рівняння Нав'є-Стокса з твердими границями. Знайдено вирази для збурених швидкості і температури в циліндричному конвективному осередку. Проведено їх порівняння з аналогічними параметрами вільного конвективного осередку для основної моди $n=1$. Показано, що діаметр конвективного осередку обернено пропорційний значенню мінімального хвильового числа відповідної крайової задачі, тобто діаметр осередку зі змішаними граничними умовами менше діаметра осередку з вільними граничними умовами, але більше діаметра осередку з твердими граничними умовами.
КЛЮЧОВІ СЛОВА: конвективні осередки, вакуумна олія, змішані та тверді граничні умови, збурення температури, збурення вертикальної швидкості

ЭЛЕМЕНТАРНАЯ КОНВЕКТИВНАЯ ЯЧЕЙКА С ТВЕРДЫМИ И СМЕШАННЫМИ ГРАНИЧНЫМИ УСЛОВИЯМИ В ГОРИЗОНТАЛЬНОМ СЛОЕ ВЯЗКОЙ, НЕСЖИМАЕМОЙ ЖИДКОСТИ

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Приведены результаты экспериментальных исследований формирования конвективных ячеек со смешанными граничными условиями в вакуумном масле. Показано, что малые количества добавленной дисперсной фазы (алюминиевая пудра) не изменяют вязкость и плотность масла, и в таких условиях применимы граничные условия для скорости жидкости на твердой стенке. В экспериментах показано, что при превышении определенной температуры дна емкости в слое масла с добавкой небольшого количества дисперсной фазы (масляная краска или алюминиевая пудра) начинают появляться ячейки цилиндрической формы. С увеличением температуры дна емкости количество ячеек увеличивается вплоть до полного заполнения ими объема масла. Показано, что независимо от вида добавленной дисперсной фазы, но при ее малом количестве, на нижней границе емкости применимы граничные условия на твердой стенке, т.е. в слое реализуются смешанные граничные условия. Величина диаметра ячеек со смешанными граничными условиями варьируется от 2,65 до 2,83, что меньше диаметра ячейки со свободными граничными условиями. В частном случае, для значения числа Рэлея $R \approx 7124.78$ и волнового числа $k_r \approx \pi\sqrt{7}$ получены аналитические решения уравнения Навье-Стокса с твердыми границами. Найдены выражения для возмущенных скорости и температуры в цилиндрической конвективной ячейки. Проведено их сравнение с аналогичными параметрами свободной конвективной ячейки для основной моды $n = 1$. Показано, что диаметр конвективной ячейки обратно пропорционален значению минимального волнового числа соответствующей краевой задачи, т.е. диаметр ячейки со смешанными граничными условиями меньше диаметра ячейки со свободными граничными условиями, но больше диаметра ячейки с твердыми граничными условиями.

КЛЮЧЕВЫЕ СЛОВА: конвективные ячейки, ваккумное масло, смешанные и твердые граничне условия, возмущение температуры, возмущение вертикальной скорости

Study of physical processes connected with appearance of convection cells in the layers of liquid heated from below under the conditions when the layer boundaries contact with liquid or gaseous mediums (free boundaries) or with solid thermal control materials (rigid boundaries) is of big scientific and practical interest. It's possible for convection cells to appear in the layers mentioned above in case of their certain geometrical sizes and liquid parameters. The process of such spatiotemporal structures formation takes place due to the medium inability to provide the necessary heat transfer from the bottom boundary to the upper one by means of thermal diffusion, which is corrected by the appearance of ordered convective motion of liquid due to the occurrence of buoyancy force – the difference between Archimedes force and the gravity one [1].

Heat convection – is a phenomenon of mass transfer in the gravity force field and temperature gradient. It can be observed both in nature [2] and in many technological processes as well [3].

Convective mass transfer of air masses, for example, is efficiently used in farming [4]. Convection phenomenon is also used in complex processes of crystals growth for microelectronics [5]. Description of cellular structures formation processes is of great importance for applications in the field of materials laser processing technology [6].

Navier–Stokes equations in Boussinesq approximation (NSBA) are commonly used to describe convective processes. At that there are three types of problems that differ in boundary conditions: both boundaries are free, both boundaries are rigid, one boundary is free and the other one is rigid (mixed boundary conditions) [1,7,8].

Shear stress is absent on the layer boundaries in the problems with free boundary conditions and this leads to the boundary conditions under which NSBA solutions have analytic view in form of normal modes with respect to perturbed velocity and the temperature inside the cell. Lord Rayleigh was the first to obtain these solutions in 1916 [8]. The obtained solutions, as it is shown in [7, 9], allows to form square convection cells or cells consisting of regular polygons (triangle or hexagon) with the help of geometric transformations.

As it appears from [7, 9], the geometric transformations were used to explain the formation of polygon spatially periodic structures which completely fill the volume of convective layer and in this way provide maximum transport of heat between the layer boundaries.

As it was underlined in [10], the explanation of polygon convective structures formation in the liquid layer heated from below should be based not on geometry, but it should have regard to energy advisability or in other words – the explanation should be based on energy principle. This principle sets the correspondence between the number of convective cells and the temperature of lower layer boundary (under the corresponding temperature gradient) when the latter is increased. To perform such an energy principle of polygon convective structures formation it's necessary to introduce the notion of elementary convective cell. Polygon convective cells can be formed from the large number of such elementary convective cells in case of their close packing. It was offered to use cylindrical convective cell, for which there were obtained analytical expressions of perturbed velocity and temperature, as an elementary one [10]. In the papers being referred to it was shown that the value of analytically counted diameter of cylindrical convection cell corresponds numerically to the experimental data obtained. In general, the explanation (from the energy principle) of the principle of liquid surface coverage with cylindrical convective cells was given basing on the suggested concept of elementary convective cell.

Energy principle can turn out to be useful when studying spatially periodic convective structures formation basing on NSBA equations with rigid and mixed boundary conditions.

It is known that solving NSBA equations with rigid and mixed boundary conditions the desired expressions for perturbed velocity and temperature inside the cell don’t have analytical form and they require engagement of numerical methods [6, 8]. That’s why the search of analytical solutions of NSBA equations with rigid and mixed boundary conditions is an actual task.

In the scientific literature devoted to spatially periodic convectional structures study it’s conventional to analyze liquid layers completely filled with convective cells of different geometry. However, these sources don’t have the information as for convective cells nascent stage. The change of their number wasn’t studied under the container bottom temperature increase up to the time when the cells completely fill the liquid volume. Study of such processes basing on NSBA equations with rigid and mixed boundary conditions can give some new theoretical and experimental data in the field of spatially periodic convectional structures formation.

That’s why the goal of the current study is to investigate theoretically and experimentally the properties of elementary convective cell which will appear in the horizontal layer of viscous incompressible liquid heated from below with rigid or mixed boundary conditions.

EXPERIMENTAL STUDIES OF CONVECTIVE CELLS FORMATION

Substance to be studied. To study experimentally the appearance of convection cells with rigid or mixed boundary conditions it should be given to the liquid the properties of single-phase suspension [11]. This is important since it’s necessary that the surface (boundary) layer is not be free dispersion medium and to some extent and not to be grease for the second phase of suspension. It has to have the same properties as the bulk of liquid. This condition sets certain requirements for the parameters of dispersion phase particles [11]:

- either the size of dispersion phase particles has to be small enough (approximately equal or several times thicker than the thickness of liquid surface layer) and in this way it is minimizing the thickness of free dispersion medium interlayers;

- or the volume fraction of dispersion phase solid particles has to be significantly lower than their volume fraction limit, which provides maximum thickness of free dispersion medium interlayers.

The first requirement of rigid or mixed boundary conditions presence is that there was used vacuum oil BM-5 as single-phase viscous medium with the addition of small amount (50-100 mg) of white oil paint (to visualize the process) which particles are rather small – less than 15 μm (GOST 11826-77), that is less than oil surface layer thickness. The second requirement can be met by adding of aluminum pigment powder ПАП-1 into vacuum oil. This powder is made of refined particles of plate-like shaped aluminum which average thickness is about 0,25 - 0,50 μm and average linear size is 20 - 30 μm (GOST 5494 - 95). The aluminum powder is added is such small amount that the visualization is still possible and the suspension viscosity corresponds the viscosity of dispersion medium (vacuum oil BM-5).

In Fig. 1,2 there are given micro photos (taken using МБС-9 microscope) of vacuum oil BM – 5 with addition of 2 ml of oil paint in amount of m=0.05 g (Fig. 1) or aluminum pigment powder ПАП-1 in amount of 0.005 g.

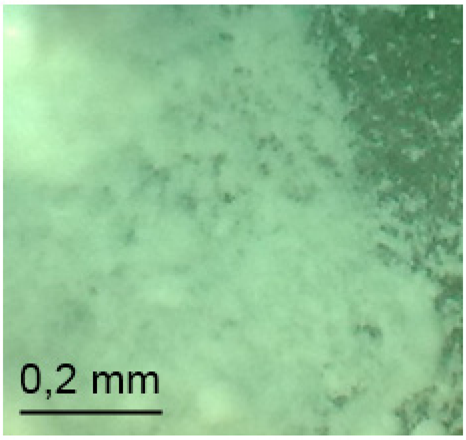


Fig. 1. Distribution of oil paint in vacuum oil.

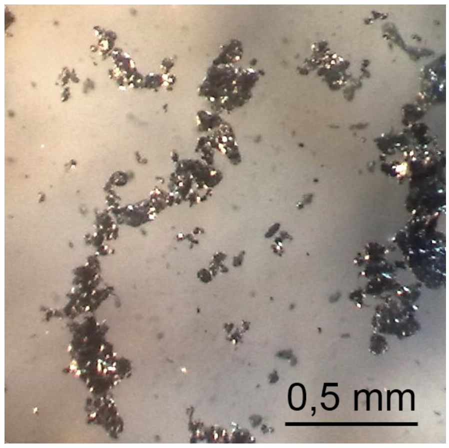


Fig. 2. Distribution of aluminum powder in vacuum oil.

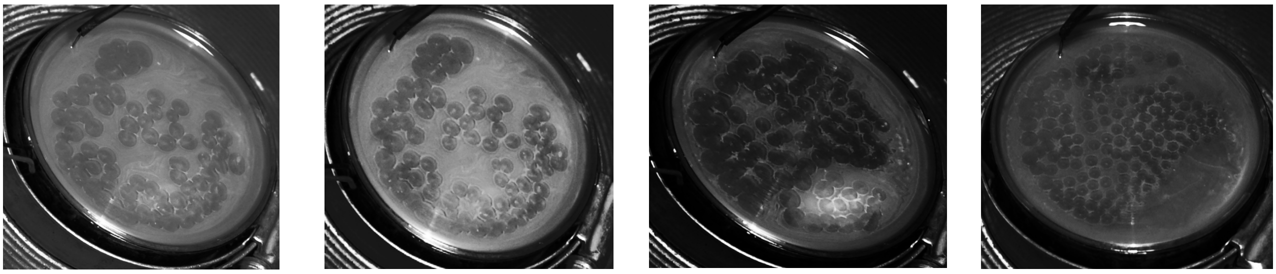
Analysis of the pictures indicates that in case when oil paint was added into vacuum oil (Fig.1) the interlayer of free dispersion medium is absent, while there were observed free dispersion medium interlayers 0.3 - 0.5 mm thick in the spatial distribution of aluminum powder particles (Fig.2).

Thus, the ways to prepare liquid, viscous media described below can be used to perform rigid or free boundary conditions for modeling convection processes in layers of viscous incompressible liquid heated from below.

Description of the experiment. To perform the experiment there was used 2 ml of vacuum oil BM – 5 with the addition of oil paint in the amount of m_1=0.05 g or aluminum pigment powder in the amount of m_2=0.005 g. electric stove

Cylindrical container was filled with oil, the thickness of its layer was chosen empirically and it was equal to h=1.0 mm. The uniform heating of the container bottom was performed by means of electric furnace up to 140±1 °C. The cells were formed in the container filled with oil with addition of some amount of oil paint and aluminum powder. The experiments showed that in the thin layer of heated from below oil the cells of cylindrical shape appeared. Inside each cell the vacuum oil moves up at the cell center and it moves down along its external boundary.

In Fig. 3 there are the photos of cylindrical convective cells the number of which grows when the temperature of the cylindrical container bottom is increasing from 82°C to 110 °C. The process starts from formation of 1-2 cells till the time when they are tightly packed in the container with the formation of polygons including hexagons. The measured diameter of convection cells varied within the interval 2.4 – 2.5 mm.

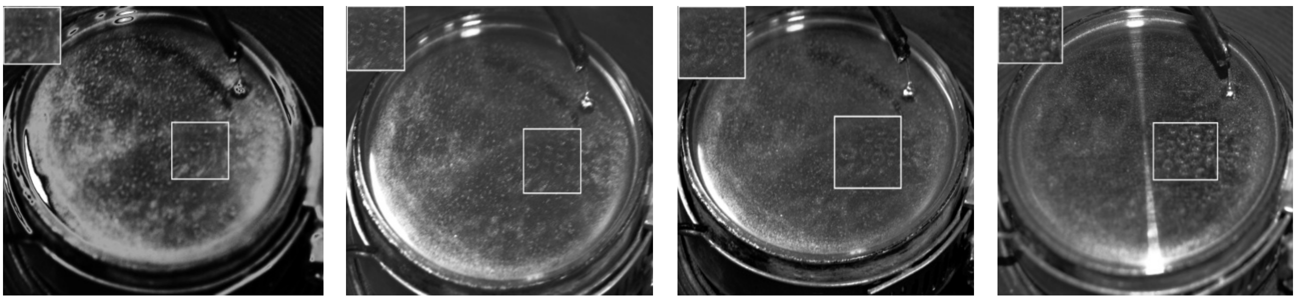


a) b) c) d)

Fig. 3. Dependence of formed cells number on the container bottom temperature with adding oil paint into the vacuum oil. (T_1- the temperature of oil upper boundary, T_2 – the temperature of container bottom)
a) T_1=81ºC, T_2=91ºC; b) T_1=85ºC, T_2=96ºC; c) T_1=92ºC, T_2=104ºC; d) T_1=97ºC, T_2=110ºC

In the experiments the cylindrical container was filled oil with aluminum powder. The thickness of oil layer was chosen empirically and it was equal to h=1 mm. Uniform heating of container bottom was performed using of electric furnace and the temperature was kept on the level of 130±1°C. Cylindrical cells were formed in the oil with the addition of small amount of aluminum powder (0.005 g) just as it was in the case with oil paint adding into the oil. Dynamics of liquid inside of each cell repeated the similar dynamics for oil with addition of oil paint.

In Fig.4 there are the photos of cylindrical convective cells the number of which grows when increasing the temperature of the cylindrical container bottom from 84°C до 130°C. Their number increases from formation of 1-2 cells till the time when they are tightly packed in the container with the formation of polygons including hexagons. The measured diameter of convective cells varied within the interval 2.6 – 2.7 mm.



a) b) c) d)

Fig. 4. Dependence of cells number on the container bottom temperature with adding aluminum powder into the vacuum oil.
a) T_1=84ºC, T_2=113ºC; b) T_1=87ºC, T_2=118ºC; c) T_1=92ºC, T_2=122ºC; d) T_1=95ºC, T_2=126ºC

The areas marked with rectangles in Fig.4 are the areas where cylindrical cells formation is observed: first, 3 cells appear, then 5-6 and in the last photos their number varies from 10-12 to 20. It can be seen that the cells have a cylindrical shape and they are separated with the oil interlayers of certain thickness.

Basing on the experiment results one can make a conclusion that cells of cylindrical shape appear at certain temperature of the container bottom in the oil layer with the addition of small amount of oil paint or aluminum powder and that the boundary conditions correspond boundary conditions on the rigid boundary. When the container bottom temperature increases the number of cells grows too up to the complete filling of oil volume. Regardless of the type of added disperse phase (oil paint or aluminum powder) the cells diameter in the unit of layer thickness varies from 2.65 to 2.83, i.e. the dimensionless cells diameter with the mixed boundary conditions is less than the diameter of free cell which is defined by the value 3.4 [10].

THE THEORY OF CYLINDRICAL CELL WITH RIGID BOUNDARY CONDITIONS

In the problem of heat convection in the viscous medium with rigid boundaries [7-9] there was studied viscous liquid layer with the thickness h and it was infinite in both axis x and y directions. Axis z was directed upwards perpendicularly to the layer boundaries $z=0$ and $z=h$. Distribution of temperature inside the layer $T_0(z)$ was given in such a way that the lower boundary temperature was higher the one of the upper boundary: $T_0(0)=T_2$, $T_0(h)=T_1$, $(T_2>T_1)$. Let's consider, that in equilibrium state the dependence of the layer temperature on z coordinate is described by linear function:

$$\nabla T_0(z) = -\frac{\Theta}{h}\vec{e}_z , \tag{1}$$

where $\Theta = T_2 - T_1$ - the difference of temperatures between lower and upper planes, $\vec{e}_z$ - unit vector directed along the axis z.

Basing on the experiment results mentioned above (Fig. 3,4) one can make two conclusions:

- the shape of convection cells is a cylindrical one;
- the internal structure of convection cells doesn't depend on azimuthal angle ϕ.

On the ground of these conclusions we'll search for the solutions of linearized Navier–Stokes equations [7] in cylindrical geometry. In this case in the layer with flat boundaries the initial equations for perturbed vertical velocity v_z and temperature T have the form:

$$\frac{\partial}{\partial t}\Delta \mathrm{v}_z = \Delta\Delta \mathrm{v}_z + R\Delta_\perp T , \tag{2}$$

$$P\frac{\partial T}{\partial t} = \Delta T + \mathrm{v}_z , \tag{3}$$

where $\Delta = \Delta_\perp + \frac{\partial^2}{\partial z^2}$ - Laplacian operator, $\Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$ - transverse Laplacian in which on the basis of the axial symmetry of the cells is not a term that characterizes the dependence of the perturbations of the azimuthal angle is missing, i.e. everywhere we suppose $\partial .../\partial\phi = 0$, $R = g\beta h^3\Theta/(\nu\chi)$ - Rayleigh number, g – gravitational acceleration directed against axis z, $P=\nu/\chi$ - Prandtl number, ν and χ - coefficients of kinematic viscosity and thermal conductivity of liquid correspondingly, β - volumetric coefficient of thermal expansion of the liquid, v_z, T - perturbations of vertical velocity and temperature correspondingly.

For reducing the system of equations (2) - (3) to the dimensionless type there were used the following characteristic measurement units: unit of length – layer thickness h; unit of time - $\tau = h^2\nu^{-1}$; unit of temperature - Θ. It has to be mentioned that for the chosen unit of length z coordinate changes within the interval $0 \le z \le 1$.

System of equations (2) - (3) can be applied to define "normal" perturbations in the viscous liquid layer heated from below under the condition that this system has to be complemented by the boundary conditions. In the current study we'll consider rigid boundary conditions – the case when on the boundaries at $z=0$ and $z=1$ the perturbed velocity projection, temperature and vertical speed derivatives have the next values [6, 12]:

$$\mathrm{v}_r = \mathrm{v}_z = 0; T = 0; \frac{d\mathrm{v}_z}{dz} = 0 . \tag{4}$$

Solution of initial system of equations. The initial equations (2), (3) have partial solutions which describe temporal dynamics of perturbations of vertical speed and temperature of the cylindrical cell [10-11]:

$$\mathrm{v}_z(r,z,t) = \mathrm{v}(z)J_0(k_r r)\exp(-\lambda t), \tag{5}$$

$$T(r,z,t) = \vartheta(z)J_0(k_r r)\exp(-\lambda t), \tag{6}$$

where λ - eigenvalues characterizing damping ($\lambda>0$), increase ($\lambda<0$) or steady state ($\lambda=0$) of perturbations (4), (5); $\mathrm{v}(z)$ and $\vartheta(z)$ - perturbation amplitudes of the vertical velocity and the temperature correspondingly; $J_0(x)$ - Bessel function of first kind and zero order of the argument x; k_r - radial wave number characterizing the dependence of perturbations on transverse coordinate r.

Let's consider the stable solutions of equations (2), (3), that correspond to the condition $\lambda = 0$. Substitution of solutions (5), (6) into the equations (2), (3) leads to the characteristic equation:

$$\left(q^2 - k_r^2\right)^3 = -k_r^2 R . \tag{7}$$

In equation (7) q — parameter defines the dependence of vertical velocity amplitude $\mathrm{v}(z)$ on z coordinate in the form $\mathrm{v}(z) = A_0\exp(qz)$, where A_0 - an arbitrary constant.

Roots of the characteristic equation (7) are defined by the following expressions:

$$q_{1,2} = \pm\sqrt{b-a};$$
$$q_{3,4} = \sqrt{b+\frac{a}{2}(1+i\sqrt{3})} = \pm\left(X_+ + iX_-\right); \qquad (8)$$
$$q_{5,6} = \sqrt{b+\frac{a}{2}(1-i\sqrt{3})} = \pm\left(X_+ - iX_-\right),$$

where $a = (k_r^2 R)^{\frac{1}{3}}$, $b = k_r^2$, $X_\pm = \left(\frac{1}{2}\left(\sqrt{\left(\frac{1}{2}a+b\right)^2 + \left(\frac{\sqrt{3}}{2}a\right)^2} \pm \left(\frac{1}{2}a+b\right)\right)\right)^{\frac{1}{2}}$, $i = \sqrt{-1}$ - unit imaginary number.

Solutions of the equations (2), (3) describes neutral disturbances for vertical velocity amplitude in the interval $0 \le z \le 1$. We'll search the solution in the form:

$$\mathrm{v}(z) = \sum_{m=1}^{6} C_m \exp\left(q_m z\right), \qquad (9)$$

where C_m - arbitrary constants defined by the boundary conditions

$$\mathrm{v}(0) = \mathrm{v}(1) = 0, \quad \partial\mathrm{v}(0)/\partial z = \partial\mathrm{v}(1)/\partial z = 0, \quad \vartheta(0) = \vartheta(1) = 0. \qquad (10)$$

As it follows from [6] the solutions of equation (7) and expression (9) obtained basing on them define the values of critical Rayleigh numbers and the amplitude of neutral disturbances correspondingly. Critical Rayleigh numbers and neutral disturbances amplitude can't be expressed analytically it's possible to get only the result of approximate numerical solution of transcendental equations. However, under some assumptions, solution (9) can be represented analytically. See below the approach used to obtain the analytical solution.

Similarly to the solution for the vertical velocity of a free convective cell, when $\mathrm{v}_z(z) \propto \sin\left(n\pi z\right), n = 1,2,3,...,$ we'll search the solutions for the system of equations (2), (3) for $\mathrm{v}_z(z)$ in the same form, i.e. we assume $\sqrt{a-b} = in\pi$, where the inequality $a < b$ is always considered to be true. For the special case we assume $X_- = n\pi$ that turns out to be possible only in the case when $a = 8\left(n\pi\right)^2$, $b = 7\left(n\pi\right)^2$.

If in (9) the followings values of constants are given: $C_1 = C_2 = A_1/2$, $C_3 = C_4 = C_5 = C_6 = A_1/4ch\left(X_+/2\right)$, then in the limits in z which are symmetrical with respect to $z_0 = 1/2$ the expression for vertical velocity that meets boundary condition requirements (10) has the following form:

$$\mathrm{v}(z) = A_1\left(1 - ch\left[\left(z-\frac{1}{2}\right)X_+\right]ch^{-1}\left(\frac{X_+}{2}\right)\right)sin\left(z\sqrt{a-b}\right). \qquad (11)$$

For the comparative assessment of heat and mass transfer parameters between the convective cell with a rigid boundary conditions and a free boundary conditions we'll perform comparative analysis of spatial distribution for their perturbed velocity and temperature. When comparing the expressions for vertical velocity we'll assume constant coefficients A_1 and a (in [6]) to be equal to 1.

In Fig. 5 there are shown the dependences of perturbed vertical velocity amplitude of the cell with a rigid boundaries and a free boundaries of convective cell on z coordinate for mode number $n = 1$.

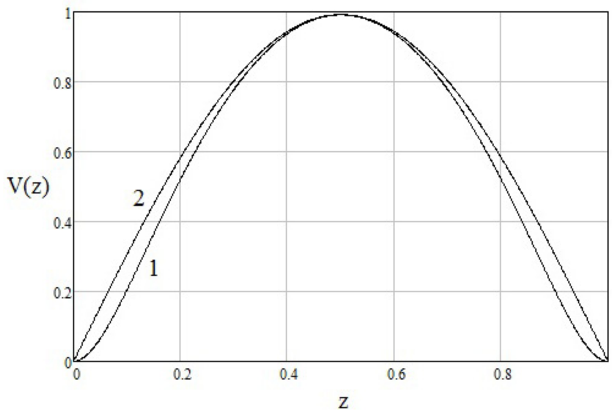


Fig. 5. The dependencies of perturbed vertical velocity amplitude of the convective cell with a rigid (1) and a free boundaries (2) on z coordinate for main mode $n = 1$.

It follows from the picture that a rigid boundary conditions insignificantly (in 1.0087425 times) decreases the amplitude of vertical velocity of liquid mass-transfer compared to a free boundary conditions case. At that close to the boundary (when $z = 0.09$) the decreases value makes up 10.48 % relating to the mass-transfer velocity in a free cell.

Partial solution for the perturbed temperature amplitude $\vartheta(z)$ will be received by method of arbitrary-constant variation from the equation (3):

$$\frac{d^2\vartheta(z)}{dz^2} - b\vartheta(z) = -\mathrm{v}(z). \tag{12}$$

For the perturbed temperature the general solution that meets boundary conditions requirements has the following form:

$$\vartheta(z) = -\frac{1}{\sqrt{b}}\int_0^z \mathrm{v}(\xi) sh\left(\sqrt{b}(\xi - z)\right)d\xi + \frac{1}{\sqrt{b}}\frac{sh\left(z\sqrt{b}\right)}{sh\left(\sqrt{b}\right)}\int_0^1 \mathrm{v}(\xi) sh\left(\sqrt{b}(\xi - 1)\right)d\xi. \tag{13}$$

Let's compare the amplitude of a free convective cell perturbed temperature and the one of the cell with a rigid boundary conditions. At that it's necessary to consider that the amplitude of a free cell perturbed temperature when $a = 1$ is defined by the value $\bar{b} = \frac{1}{n^2\pi^2 + k^2}$. For the mode $n = 1$ in the neutral curve minimum ($R_{\min} = \frac{27}{4}\pi^4, k_{\min} = \frac{\pi}{\sqrt{2}}$) we'll find the value of the perturbed temperature amplitude: $\bar{b} = \frac{2}{3\pi^2} \approx 0.0675$.

The comparison of the amplitudes of perturbed temperature in a free convection cell and the one of a cell with a rigid boundary conditions (Fig. 6) shows that in the latter case its amplitude is in 5.458 times less. So, we can conclude that the presence of a rigid boundary conditions decreases the perturbed temperature amplitude in comparison with a free boundary conditions. At that real (taking into consideration multiplier 5.458) maximum deviation of curve 1 from curve 2 in the point $z = 0.14$ equals to 24.72 %.

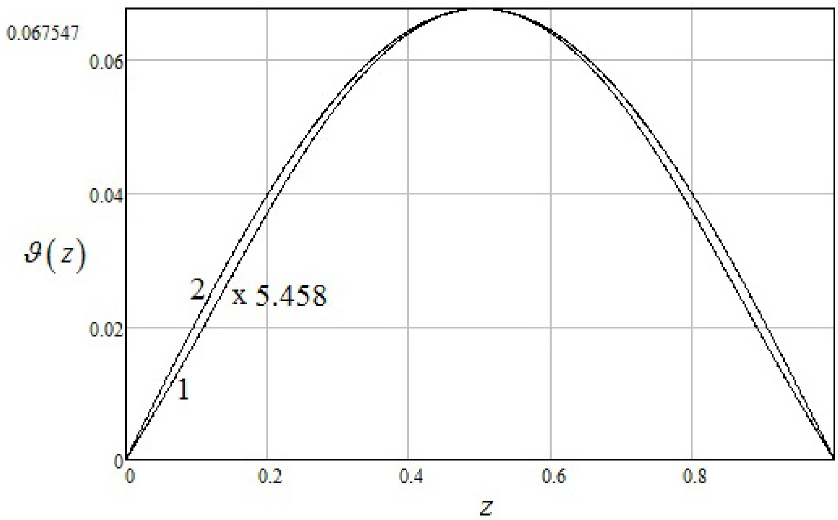


Fig. 6. Dependences of perturbed temperature amplitude of the cell with a rigid boundaries (1) and the one of a free convective cell (2) on z coordinate for the main mode $n = 1$.

The expression for radial velocity of liquid movement in a convective cell $\mathrm{v}_r(r,z)$ follows from the condition of its incompressibility:

$$\mathrm{v}_r(r,z) = -\frac{d\mathrm{v}(z)}{dz}\frac{1}{k_r}J_1(k_r r). \tag{14}$$

Thus in the current section there were defined values of perturbed velocity ($\mathrm{v}_r, \mathrm{v}_z$) and temperature T in a cylindrical convective cell with a rigid boundaries in the special case when Raleigh number $R \approx 7124.78$ and wave number $k_r \approx \pi\sqrt{7}$. These values were compared to the similar parameters of a free convective cell for the main mode $n = 1$.

In particular it was shown that a rigid boundary conditions presence insignificantly (in 1.0087425 times) decreases the vertical velocity amplitude of liquid mass transfer in comparison with vertical velocity of a cell with a free boundary conditions and it decreases perturbed temperature amplitude (in 5,458 times) in comparison with the one of a cell with free boundary conditions.

Definition of a diameter of convection cell with a mixed and a rigid boundaries. Expression (14) fulfills physically based boundary conditions on the axis and on a cell periphery, because when $r=0$ and on cell external boundary $r=R_c$ the radial speed of liquid equals zero. So, the value of radial wave number can be defined by the following relation:

$$k_{r,l}=\sigma_{1,l}R_c^{-1}, \tag{15}$$

where R_c - convection cell radius, $\sigma_{1,l}$ - l-th zero of Bessel function of first kind ($J_1(\sigma_{1,l})=0$), $l=1,2,3,....$. Let's give the values of first five zeroes of Bessel function: $\sigma_{1,1}=3.832$; $\sigma_{1,2}=7.016$; $\sigma_{1,3}=10.173$; $\sigma_{1,4}=13.324$; $\sigma_{1,5}=16.471$.

In the study [10] there was experimentally and theoretically showed that radius of free convection cell in defined by the relation (15), where minimum value of wave number $(k_r)_{\min}=\pi/\sqrt{2}\approx 2.221$ is used as wave number. At that free cell radius is defined by the value:

$$(R_c)_{free}=\sigma_{1,1}2^{\frac{1}{2}}\pi^{-1}\approx 0.45\sigma_{1,1}\approx 1.72. \tag{16}$$

Form the experimental results of the current study it follows that value of radius of a cell with a mixed boundary conditions varies from 2.65 to 2.83, i.e. it has lower value than the one of a free cell. Theoretically calculated diameter value of convective cell with a mixed boundary conditions we'll define from relation (15) similarly to calculation of a free convection cell diameter – by using minimum wave number of the considered boundary value problem $(k_r)_{mix}\approx 2.682$ [6]:

$$(D_c)_{mix}=2(R_c)_{mix}=2\sigma_{1,1}/(k_r)_{mix}\approx 2\cdot 3.832/2.682\approx 2.86. \tag{17}$$

The obtained theoretical value of diameter of convective cell with a mixed boundary conditions corresponds to the experimentally measured values in number. These values range from 2.65 to 2.83.

Basing on the results obtained for convective cell with a free and a mixed boundary conditions one can define diameter of convective cell for the problem with a rigid boundary conditions. Its theoretical value equals to:

$$(D_c)_{rigid}=2(R_c)_{rigid}=2\sigma_{1,1}/(k_r)_{rigid}\approx 2\cdot 3.832/3.116\approx 2.46. \tag{18}$$

To prove experimentally this result let's use the data obtained in the study [13]. Here there was studied the dependence of Nusselt number (the relation of heat exchange intensity at the account of convection and heat transfer to the intensity of heat exchange at the account of heat transfer) on Raleigh number in the layer of viscous incompressible liquid with a rigid boundary conditions heated from below. In the study it was shown that when critical value of Raleigh number increases ($R\ge R_{critical}=1700\pm 51$) there was observed hexagons formation. It's obvious that the diameter of cylindrical convective cell from which a hexagon appeared is equal to the diameter of hexagon circumscribed circle. In Fig. 7,8 there are the results of study of convection structures formation (hexagons and roller) in silicon layer AK - 350 of different thickness under controlled temperature of upper and lower boundary of the layer [13]. In these figures it's seen that hexagons are formed when exceeding critical Raleigh number value (Fig. 7b and Fig. 8c,d).

In Fig. 8c the formation of convective roller is shown. This is typical for the oil layers of higher thickness and for higher Raleigh numbers.

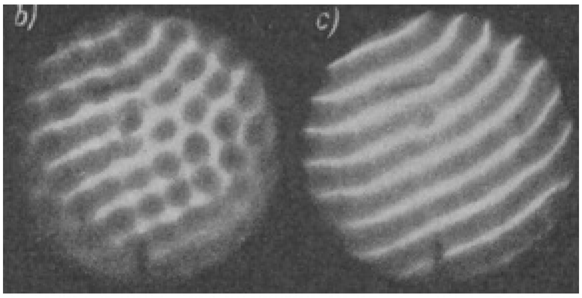


Fig. 7. Formation of hexagons in the layer of silicon oil AK-350 with the layer thickness $h=7$ mm [13]

b) $R=1800$; c) $R=2400$, formation of convective roller.

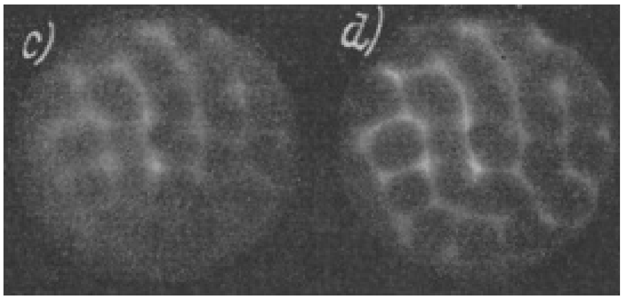


Fig. 8. Formation of hexagons in the layer of silicon oil AK-350 with the layer thickness $h=10$ mm [13]

c) $R=2280$; d) $R=2470$, the beginning of convective roller formation.

As it follows from Fig. 7 the hexagons formed together with Raleigh number increase form stripes with width H. Experimentally measured ratio of the stripe width to the liquid layer depth for Raleigh numbers which value is close to the critical one ($R-R_{крит}<<1$) is equal to $H/h=2$ [13]. Boundaries of the stripes with width H are formed by the parallel sides of hexagons. So, knowing the distance between them one can define the convective cell diameter by means of simple geometrical constructions: $\left(D_c\right)_{rigid} = 2H\left(h\sqrt{3}\right)^{-1}$. Proceeding from the calculations made and basing on the experimental data we can state that the diameter of convection cell with a rigid boundary conditions is equal to 2.31 mm. The given value of the convection cell diameter is the closest to the one obtained analyticallyin (18).

Thus, the diameter of cylindrical convective cell with a mixed and a rigid boundary conditions is inversely related to the minimum value of wave number of the studied boundary value problem. Extension of the obtained data shows that the diameter of elementary convective cell with a mixed boundary conditions is less than the one of the cell with a free boundaries, but it's larger than the diameter of the cell with a rigid boundaries (Table).

Table

Cells diameter value for the different boundary conditions

Type of boundary conditions	Theoretical calculations, mm	Experimental data, mm
A rigid boundaries	2.45	2.2 – 2.4
A mixed boundaries	2.8	2.6 – 2.8
A free boundaries [10]	3.4	3.2 – 3.8

CONCLUSION

There were carried out experimental studies of convective cells formation in vacuum oil under a mixed boundary conditions (the upper boundary is free and the lower one is rigid) for the 2 variants of process visualization. Some amount of oil paint was added into the oil (50 - 100 mg) – the first variant and some aluminum powder was added (5 mg) – the second variant. The first approach to perform the mixed conditions was to use vacuum oil BM-5 as a single-phase viscous medium of vacuum oil with addition of some amount (50-100 mg) of white oil paint into it to visualize the process (paint particles were rather small, less than 15μm). The second approach was to add some amount of aluminum powder (5 mg) into oil. Microscopic studies showed that small amounts of added aluminum powder don't change the oil viscosity and density. Thus, the oil is a single-phase medium in which the boundary conditions are present on a rigid boundary. The boundary conditions for liquid velocity on a rigid boundary can be applied in the studied cases. In the experiments it was shown that cylindrical cells appear at certain temperature of the container bottom in the oil layer with addition on small amount of oil paint or aluminum powder. When increasing the container bottom temperature the number of cells increases up to the moment when they completely fill the oil volume. Regardless of the type of added disperse phase the cells diameter value varies from 2.65 to 2.83. In the special case for Raleigh number $R \approx 7124.78$ and wave number $k_r \approx \pi\sqrt{7}$ there were obtained the analytical expressions for the perturbed velocity $\mathrm{v}_r, \mathrm{v}_z$ and temperature T in a cylindrical convective cell with a rigid boundaries and they were compared with the similar parameters of a free convective cell for the main mode $n = 1$.

It was shown that a rigid boundary conditions presence insignificantly (in 1.0087425 times) decreases the vertical velocity amplitude of liquid mass transfer in comparison with vertical velocity of a cell with a free boundary conditions and it decreases perturbed temperature amplitude (in 5,458 times) in comparison with the one of a cell with a free boundary conditions.

There were obtained analytical expressions for definition of diameter of convection cell with a mixed and a rigid boundary conditions. It was demonstrated that convection cell diameter is inversely related to minimal value of wave number of the corresponding boundary value problem. Basing on the experimental results and theoretical analysis the following conclusion was made – the diameter of elementary convective cell with a mixed boundary conditions is less than the one of the cell with a free boundary conditions, but it's larger than the one of the cell with a rigid boundaries.

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