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EXPLORING COSMOLOGICAL CONSEQUENCES AND VIABILITY OF VARYING G AND Λ WITH DECELERATION PARAMETER

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We give a brief review of a spatially homogeneous and anisotropic Bianchi Type-I cosmological model with varying gravitational constant $G(t)$ and cosmological term $\Lambda(t)$. The Einstein field equations are solved by considering a time-dependent deceleration parameter(DP) and barotropic equation of state (EoS) $p = W\rho$. The model universe is fit with a scale factor of the form $a(t) = \left(e^{A\zeta_c t} - 1\right)^{1/\zeta_c}$ which provides a smooth evolution from a decelerating to an accelerating phase of cosmic expansion. Analytical expressions for the pressure, energy density, $G(t)$ and $\Lambda(t)$ are derived and their variations with redshift are analyzed. The behaviour of cosmological parameters such as the Hubble function $H(z)$, deceleration parameter $q(z)$, jerk parameter $J(z)$ and $Om(z)$ diagnostic are examined. The present values $H_0 = 67.112^{+0.049}_{-0.11}$ km s^{-1} Mpc^{-1}, $q_0 = -0.2926$ and transition redshift $z_t = 0.8626$ are obtained, consistent with recent observations. Overall, the proposed variable G and Λ Bianchi Type-I model provides a coherent description of the universe's transition from deceleration to acceleration, consistent with 46 OHD.

Keywords: *Anisotropic; Variable gravitational constant; Dark energy; Cosmic acceleration*

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1. INTRODUCTION

The question of the universe's size has been persistently explored by many authors, while the vast beauty of the universe often defies explanation even by renowned researchers. Numerous questions remained unanswered after Newton's era. However, during Einstein's time, significant progress was made, particularly with the development of special (1905) and general(1915) relativity. Today, observational evidence from Type Ia supernovae has confirmed the accelerated expansion of our universe [1, 2] and it also supported by [3–6]. Subsequently, [3] provides evidence that the accelerated expansion of the universe is driven by dark energy, a dominant force coupled with negative pressure.

The study of cosmological models has seen considerable advancements, particularly with the integration of the cosmological constant (Λ) and the gravitational constant (G) as dynamic variables, extensively explored in the FRW framework and other models by numerous researchers. The cosmological constant (Λ), initially introduced by Einstein to describe a static universe [7], has since been reinterpreted as a representation of dark energy, driving the universe's accelerated expansion, as evidenced by Type Ia supernova observations [1, 2]. Dirac [8] first introduced the idea of a variable G in what he termed the Large Number Hypothesis. Since then, various studies have been conducted on modified general relativity theories incorporating this variation in G. In evolving models, Λ is often treated as a time-dependent variable, allowing for a more flexible representation of cosmic dynamics [9, 10]. Similarly, changes to the gravitational constant (G), traditionally assumed to be invariant in Newtonian and Einsteinian physics, have been proposed within scalar-tensor theories and other alternative gravitational models [11, 12]. Allowing G to vary over time in anisotropic models, such as Bianchi Type I, provides profound insights into the dynamics of the early universe [13]. Singh and Meitei [14] investigated optimal dynamics of the evolution of viscous fluid string universes in the presence of a variable Λ cosmological term in a anisotropic higher dimensional model.

Furthermore, numerous authors have contributed extensively to the study of G and Λ in various types of models. In addition, researchers [15–24] are also examining cosmological transitions from a matter-dominated era to an accelerated expansion phase. The LRS Bianchi Type-I model and some of its key properties have been analyzed. Recently, cosmological models with domain walls in $f(R,T)$ gravity have been investigated. Furthermore, dark energy models in $f(R,T)$ theory with a variable deceleration parameter have been explored. Cosmological models in $f(R,T)$ gravity with $\Lambda(T)$ in a general class of Bianchi space-times have also been discussed. Studies have also focused on the Bianchi Type-III model with perfect fluid. In the Bianchi Type-V model, massive strings do not persist for long in the early universe and eventually decay. New cosmological models have been proposed within the modified $f(R,T)$-gravity theory in a variable $\Lambda(T)$ scenario.

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2. MODEL AND SOLUTION OF FIELD EQUATIONS

The spatially homogeneous and anisotropic Bianchi-I space-time is characterized by the following line element

$$ds^2 = -dt^2 + S_1^2(t)dx^2 + S_2^2(t)dy^2 + S_3^2(t)dz^2, \tag{1}$$

where $S_1(t)$, $S_2(t)$ and $S_3(t)$ are the metric functions of cosmic time t.

We denote $a = (S_1S_2S_3)^{\frac{1}{3}}$ as the mean scale factor, allowing the generalized Hubble parameter in anisotropic models to be expressed as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{S_1}}{S_1} + \frac{\dot{S_2}}{S_2} + \frac{\dot{S_3}}{S_3}\right), \tag{2}$$

where an over dot denotes derivative with respect to the cosmic time t.

The directional Hubble parameters along the x, y and z axes can be expressed as

$$H_1 = \frac{\dot{S_1}}{S_1},\ H_2 = \frac{\dot{S_2}}{S_2},\ H_3 = \frac{\dot{S_3}}{S_3}. \tag{3}$$

The Einstein's field equations with time-dependent G and Λ are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}, \tag{4}$$

where T_{ij} represents the stress-energy tensor of matter, which for a perfect fluid, takes the form

$$T_{ij} = (\rho + p)\,u_iu_j + pg_{ij}, \tag{5}$$

where ρ denotes the matter density, p represents the thermodynamic pressure, and u^i is the four-velocity vector satisfying $u^iu_i = -1$. In the field equations (4), Λ represents the vacuum energy.

In a co-moving coordinate system, the field equations (4) for the anisotropic Bianchi type-I spacetime (1), under the conditions of (5), are given by

$$\frac{\ddot{S_2}}{S_2} + \frac{\ddot{S_3}}{S_3} + \frac{\dot{S_2}\dot{S_3}}{S_2S_3} = -8\pi Gp + \Lambda, \tag{6}$$

$$\frac{\ddot{S_3}}{S_3} + \frac{\ddot{S_1}}{S_1} + \frac{\dot{S_3}\dot{S_1}}{S_3S_1} = -8\pi Gp + \Lambda, \tag{7}$$

$$\frac{\ddot{S_1}}{S_1} + \frac{\ddot{S_2}}{S_2} + \frac{\dot{S_1}\dot{S_2}}{S_1S_2} = -8\pi Gp + \Lambda, \tag{8}$$

$$\frac{\dot{S_1}\dot{S_2}}{S_1S_2} + \frac{\dot{S_2}\dot{S_3}}{S_2S_3} + \frac{\dot{S_3}\dot{S_1}}{S_3S_1} = 8\pi G\rho + \Lambda \tag{9}$$

The covariant divergence of (4) yields

$$\dot{\rho} + 3(\rho + p)H + \rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0 \tag{10}$$

The usual energy conservation equation $T^{ij}_{;j} = 0$, leads to

$$\dot{\rho} + 3(\rho + p)H = 0. \tag{11}$$

The field equations (6)-(9) involve seven unknown variables, namely S_1, S_2, S_3, ρ, p, G, and Λ. Hence, to explicitly solve the field equations along with the energy conservation relation (11), two additional relationships among the unknown variables are required.

Using $a = (S_1S_2S_3)^{\frac{1}{3}}$, equations (6)-(8) yields

$$\frac{S_1}{S_2} = d_1exp\left(x_1\int a^{-3}dt\right), \tag{12}$$

$$\frac{S_1}{S_3} = d_2exp\left(x_2\int a^{-3}dt\right), \tag{13}$$

$$\frac{S_2}{S_3} = d_3exp\left(x_3\int a^{-3}dt\right), \tag{14}$$

where d_1, x_1, d_2, x_2, d_3 and x_3 are constants of integration.

We assumed the deceleration parameter in the form[25]

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\zeta_c}{1 + a^{\zeta_c}}, \tag{15}$$

where 'a' is the scale factor and ζ_c is arbitrary constant. So, equation (15) becomes

$$a = (e^{A\zeta_c t} - 1)^{\frac{1}{\zeta_c}}, \tag{16}$$

where A is the integration constant.

Using equation (16), equations (12)-(14) becomes (by taking $d_1 = d_2 = d_3 = d$ and $x_1 = x_2 = x_3 = x$)

$$S_1 = \sqrt[3]{d^2 (h(t))^{1/\zeta_c} \exp\left(\frac{2x (h(t))^{\frac{\zeta_c - 3}{\zeta_c}} f(t)}{A(\zeta_c - 3)}\right)} \tag{17}$$

$$S_2 = \sqrt[3]{d^2 (h(t))^{1/\zeta_c} \exp\left(\frac{2x (h(t))^{\frac{\zeta_c - 3}{\zeta_c}} f(t)}{A(\zeta_c - 3)}\right)} \tag{18}$$

$$S_3 = \frac{1}{d} \exp\left(\frac{-x (h(t))^{\frac{\zeta_c - 3}{\zeta_c}} f(t)}{A(\zeta_c - 3)}\right) \sqrt[3]{d^2 (h(t))^{1/\zeta_c} \exp\left(\frac{2x (h(t))^{\frac{\zeta_c - 3}{\zeta_c}} f(t)}{A(\zeta_c - 3)}\right)} \tag{19}$$

where, $f(t) = {}_2F_1\left(1, \frac{\zeta_c - 3}{\zeta_c}; 2 - \frac{3}{\zeta_c}; 1 - e^{At\zeta_c}\right)$ and $h(t) = e^{A\zeta_c t} - 1$

Another assumption in our model is the equation of state (EoS) of the form $p = W\rho$. By using this relation, we can obtain the pressure and density through the equations (11) and (16).

$$p = W\rho = W\left(1 - e^{A\zeta_c t}\right)^{-\frac{3k(W+1)}{\zeta_c}}, \tag{20}$$

where k is arbitrary constant.

Then we get $G(t)$ and $\Lambda(t)$

$$G(t) = \frac{1}{12\pi(W+1)}\left[(h(t))^{-\frac{2(\zeta_c+3)}{\zeta_c}} \left(A^2 \zeta_c e^{A\zeta_c t} (h(t))^{6/\zeta_c} - 2x^2 (h(t))^2 + 5Axe^{A\zeta_c t} (h(t))^{\frac{3}{\zeta_c}+1} \right) \left(1 - e^{A\zeta_c t}\right)^{\frac{3k(W+1)}{\zeta_c}} \right] \tag{21}$$

$$\Lambda(t) = \frac{1}{3(W+1)}\left[(h(t))^{-\frac{2(\zeta_c+3)}{\zeta_c}} \left(A^2 e^{A\zeta_c t} (h(t))^{6/\zeta_c} \left((W+1)e^{A\zeta_c t} - 2\zeta_c\right) + 2A(W-4)xe^{A\zeta_c t} (h(t))^{\frac{\zeta_c+3}{\zeta_c}} + 4x^2 (h(t))^2 \right) \right] \tag{22}$$

3. ANALYSIS OF COSMOLOGICAL PARAMETERS:

The relationship between the scale factor $a(t)$ and the redshift z is given by

$$1 + z = \frac{a_0(t)}{a(t)}, \tag{23}$$

where $a_0(t) = 1$ is the present value.

Using equation (16) we get

$$t = \frac{\log\left((z+1)^{-\zeta_c} + 1\right)}{A\zeta_c} \tag{24}$$

Using equation (24), the variations of pressure (p), energy density (ρ), gravitational term ($G(t)$), and cosmological term ($\Lambda(t)$) with respect to redshift (z) are shown in Figures 1 to 4 for the values $A = 33.2$, $\zeta_c = 1.4148$, $k = 7$, $W = -0.34$, $W = -0.35$, and $W = -0.36$. The pressure analysis reveals that our model universe expands with dark energy, and at present ($z = 0$), the pressure is negative and varies with different values of W. In the later stages, the pressure tends to

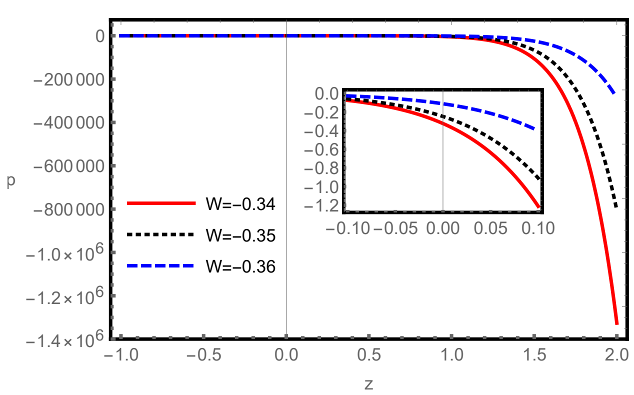

Figure 1. Variation of p vs z.

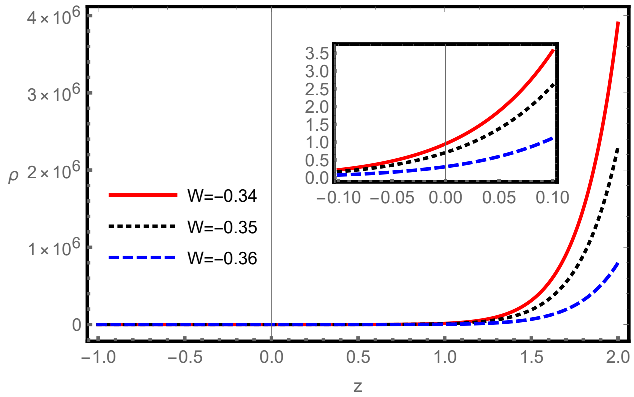

Figure 2. Variation of ρ vs z.

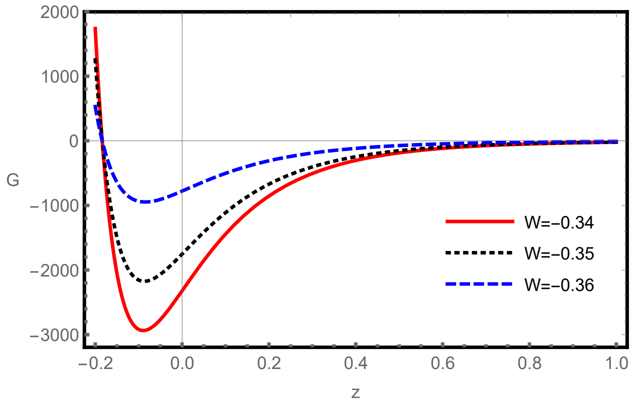

Figure 3. Variation of G vs z.

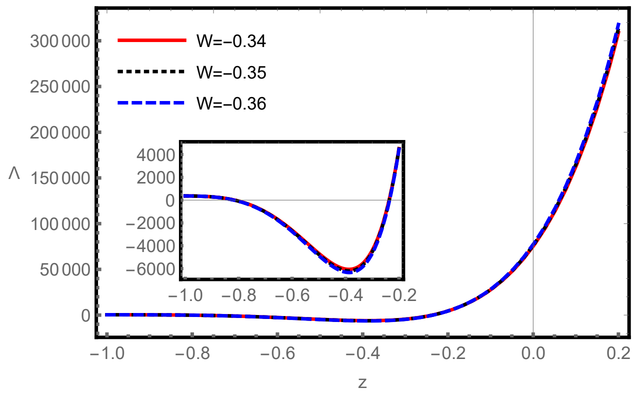

Figure 4. Variation of Λ vs z.

zero. Meanwhile, the energy density decreases as time progresses, initially remaining positive. At present ($z = 0$), the energy density ρ is a small positive value and eventually approaches zero.

In our proposed model, the gravitational coupling is more appropriately interpreted as an effective quantity $G_{\text{eff}}(t) = G_0 + G(t)$, where G_0 is a constant baseline gravitational coupling and $G(t)$ is a time-dependent contribution arising from the underlying dynamics of the theory. At the initial stage, the dynamical part $G(t)$ tends to zero, so that gravitational effects are effectively suppressed relative to other interactions, without implying the literal absence of gravity as a fundamental interaction. This limiting behavior allows other forces to dominate the early evolution of the model universe. After some time, including the present epoch, $G(t)$ becomes negative, leading to a reduced or partially repulsive effective gravitational coupling and suggesting a "negative gravity"–like phase associated with exotic matter and dark energy. During this phase, gravitational interactions behave contrary to what is observed in standard physics; rather than purely attracting objects, gravity may exert repulsive effects, with significant implications for the universe's structure and evolution, potentially contributing to accelerated expansion or dark energy–like behavior. Finally, at later times, $G(t)$ becomes positive again, so that $G_{\text{eff}}(t)$ is dominated by the baseline term G_0 and remains positive; in this phase, gravity is attractive and shapes the formation of cosmic structures such as galaxies, stars, and planets, and the present positive value of $G_{\text{eff}}(t)$ leads to gravitational collapse of matter and regulates the dynamics of the universe on both small and large scales, as shown in Fig. 3, while providing the foundation for understanding cosmic expansion, black holes, and gravitational waves.

In our model universe, the value of Λ is positive during the early stages and decreases when time increases, representing a repulsive force that pushes accelerated expansion. The behavior of Λ is linked to a high energy density, similar to the inflationary field. In the current time, Λ remains a positive value, indicating the dominance of dark energy and ongoing model of the universe's accelerated expansion. However, during certain late epochs shown in Fig. 4, Λ becomes negative, presenting an attractive force that slows the expansion. At later times, Λ returns to a positive value, reflecting the dominance of dark energy and the resulting accelerated expansion of the proposed model universe.

3.1. Analysis of Hubble Function:

To study the rate of cosmic expansion of the universe, we examine the behavior of the Hubble parameter. Since the Hubble parameter as a function of redshift (z) provides essential insights into the universe's expansion rate over cosmic epochs, it reveals key aspects of its dynamic evolution. Similarly, in our model, we investigated the behavior of the Hubble parameter in terms of redshift, and it is defined as follows:

$$H(z) = -\frac{1}{(1+z)}\frac{dz}{dt}, \tag{25}$$

by using eqn. (24)

$$H(z) = \frac{1}{2}H_0\left((z+1)^{\zeta_c} + 1\right), \tag{26}$$

where H_0 is the present value of Hubble parameter

Fig. 5 shows that the error bar plots of $H(z)$ dataset the best fit vs. redshift z of the proposed model. And in Fig. 6, we illustrate the variation of the Hubble parameter concerning z as described by equation (26). In our proposed model, $H(z)$ is a decreasing function, indicating a slowing expansion rate. The present value of the Hubble parameter is determined to be $H_0 = 67.112^{+0.049}_{-0.11} kms^{-1}Mpc^{-1}$. Additionally, we have utilized a dataset of 46 Hubble parameter measurements, $H(z)$, over redshift z, including their associated errors, as detailed in Table 1.

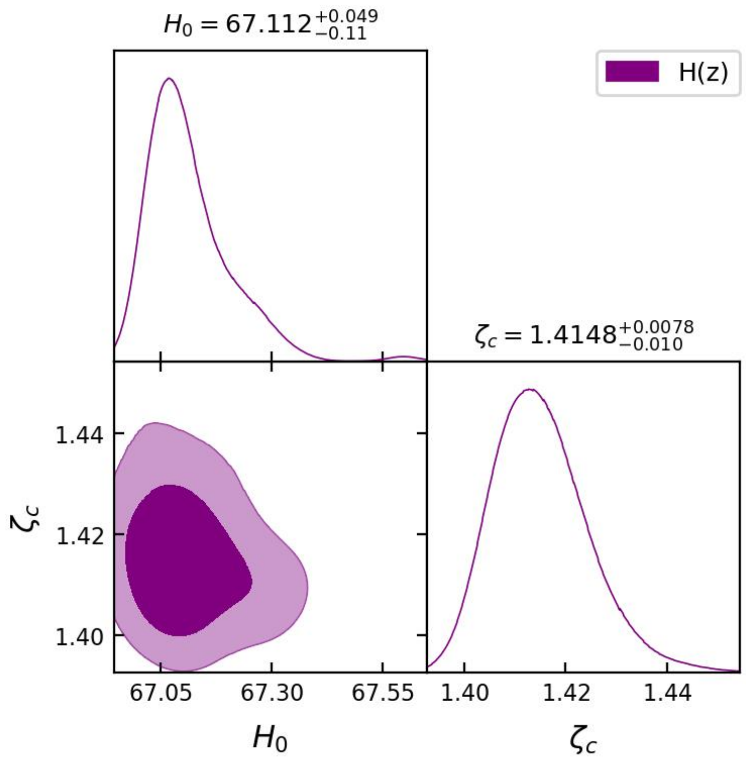

Figure 5. $1-\sigma$ and $2-\sigma$ likelihood contours for the model parameters using 46OHD

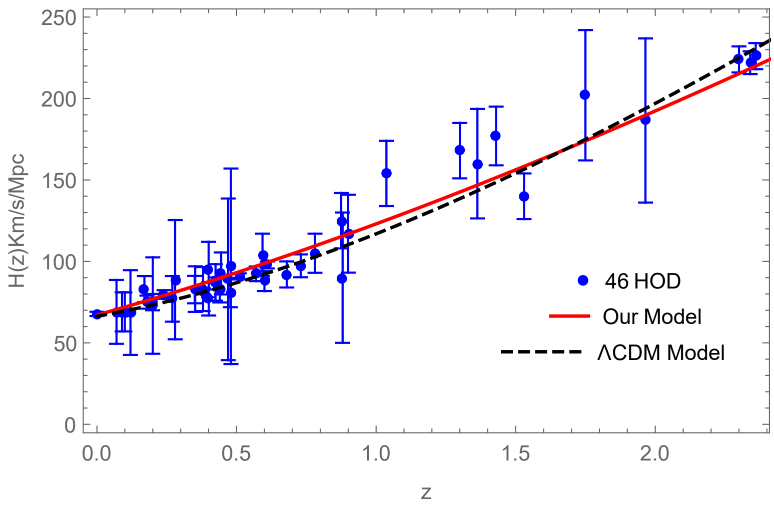

Figure 6. Variation of H vs z.

Table 1. $46H(Z)$ data Hubble Chart

z	$H^{obs}(z)$	σ_H	Reference
0	67.77	1.30	[27]
0.07	69	19.6	[28]
0.09	69	12	[29]
0.1	69	126	[27]
0.12	68.6	26.02	[28]
0.17	83	8	[27]
0.179	75	4	[30]
0.1993	75	5	[30]
0.20	72.9	29.6	[28]
0.24	79.69	2.65	[28]
0.27	77	14	[27]
0.28	88.8	36.6	[28]
0.35	82.7	8.4	[31]
0.352	83	14	[30]
0.38	81.9	1.9	[32]
0.3802	83	13.5	[31]
0.40	95	17	[29]
0.4004	77	10.2	[33]
0.4247	87.1	11.2	[33]
0.43	86.45	3.68	[28]
0.44	82.6	7.8	[34]
0.44497	92.8	12.9	[33]
0.47	89.0	49.6	[35]
0.4783	80.9	9	[33]
0.48	97	60	[27]
0.51	90.8	1.9	[32]
0.57	92.4	4.5	[36]
0.593	104.0	13.0	[37, 38]
0.60	87.9	6.1	[34]
0.61	97.8	2.1	[32]
0.68	92	8	[30]
0.73	97.3	7	[34]
0.781	105	12	[30]
0.875	125	17	[30]
0.88	90	40	[27]
0.9	117	23.9	[27]
1.037	154	20	[28]
1.3	168	17	[27]
1.363	160	33.6	[39]
1.43	177	18	[27]
1.53	140	14	[27]
1.75	202	40	[39]
1.965	186.5	50.4	[28]
2.3	224	8	[40]
2.34	222	7	[41]
2.36	226	8	[42]

Our model has been compared with the standard ΛCDM model using the recent Hubble constant measurement, $H_0 = 67.36 \pm 0.54 kms^{-1}Mpc^{-1}$, from the Planck 2018 results [26]. To constrain the model parameters, we minimize the Chi-square value, $\chi^2_{\min}$, which corresponds to the maximum likelihood analysis and is expressed as follows:

$$\chi^2_{OH} = \sum_{i=1}^{46} \left[\frac{H^{obs}(z_i) - H^{th}(z_i)}{\sigma(z_i)} \right]^2, \tag{27}$$

here, OH refers to the observational dataset of Hubble parameters. H^{obs} and H^{th} denote the observed and theoretical values of H, respectively. $\sigma(z_i)$ represents the standard error associated with the measurement of H at redshift z_i. Based on the data, the Hubble error bar plots for the $H(z)$ dataset show the best-fit comparison against redshift z for the proposed model and the ΛCDM model. In our model universe, the Chi-square value is $\chi^2_{OH} = 34.084162$, with a minimum Chi-square value of $\chi\text{min}^2 = 0.000004$.

3.2. Analysis of q(z) of the model:

The deceleration parameter is expressed as a function of redshift:

$$q = \frac{\zeta_c}{(z+1)^{-\zeta_c}+1} - 1 \tag{28}$$

The geometric evolution of $q(z)$ is illustrated in Fig. 7, with its mathematical expression given by equation (28). It is evident that $q(z)$ is an increasing function of z, featuring a signature-flipping point (transition point) within the redshift range $0 \leq z \leq 4$. The present value of the deceleration parameter is estimated as $q_0 = -0.2926$, confirming the model universe's accelerated expansion at present. We have determined the transition redshift $z_t = 0.8626$, which indicates that our proposed universe starts accelerating its expansion for $z < z_t$ while it was in a decelerating phase of expansion for $z > z_t$. Here, the universe's past evolution is represented by a positive value of $z > 0$, its present state is marked by $z = 0$, and its predicted future evolution is depicted by a negative redshift, $z < 0$.

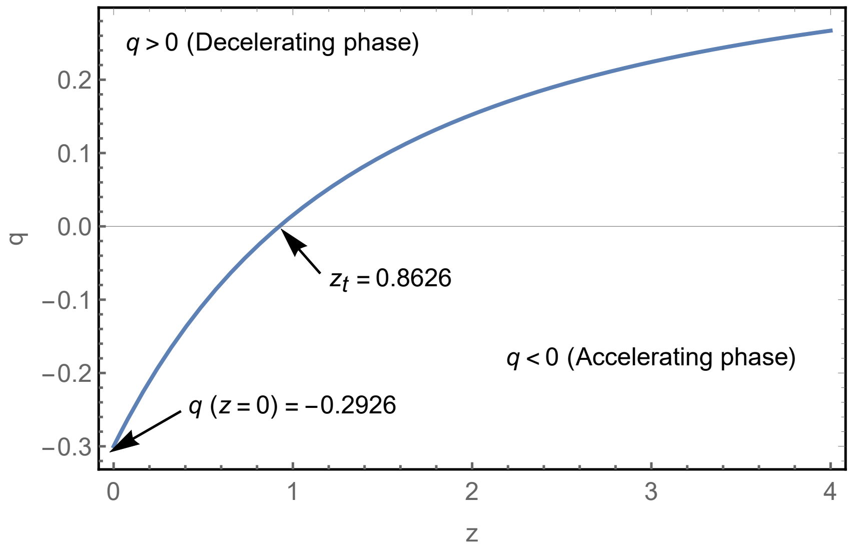

Figure 7. Variation of q vs z.

3.3. Analysis of Jerk parameter J(Z):

The third time derivative of the universe's scale factor with respect to cosmic time is defined in [43] as:

$$J(z) = q(z) + 2q(z)^2 + (1+z)\frac{dq(z)}{dz}. \tag{29}$$

Using equations(28), equation (29) becomes

$$J(z) = \frac{\zeta_c\left(\zeta_c + (2\zeta_c - 3)(z+1)^{\zeta_c} - 3\right)(z+1)^{\zeta_c}}{\left((z+1)^{\zeta_c}+1\right)^2} + 1. \tag{30}$$

Using jerk parameterization, [37, 43–45] propose an alternative approach to describe cosmological models within the ΛCDM framework. The constant jerk parameter $J = 1$ characterizes the ΛCDM model. Deviations from $J = 1$ can serve as a criterion to distinguish between dark energy models, as any divergence from $J = 1$ would support models other than ΛCDM. Consequently, the jerk formalism provides an effective means to quantify departures from ΛCDM. From Fig. 8 and equation (30), our model suggests $J = 0.37$ at $z = 0$ and $J = 1$ at $z = -1$, and indicating that our universe is undergoing late-time expansion, as also supported by [43, 44]. Therefore, we conclude that our model aligns with ΛCDM at late times.

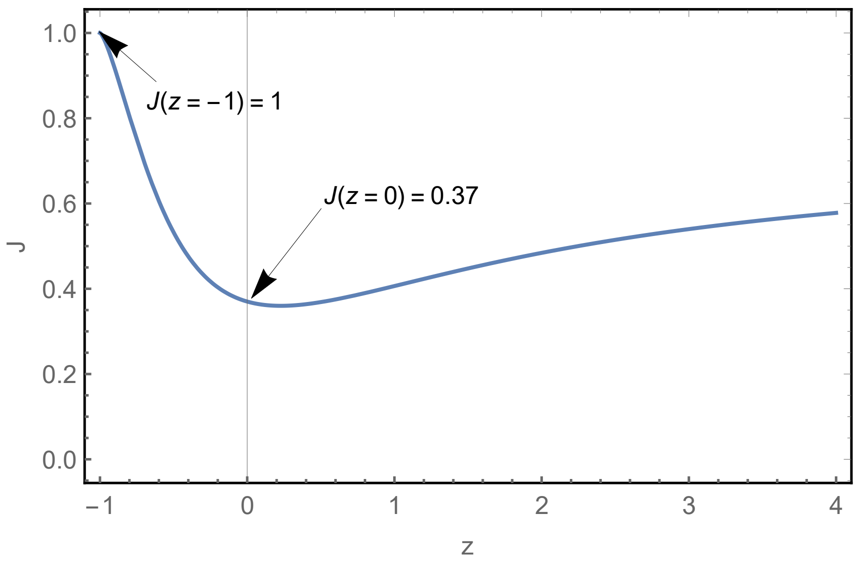

Figure 8. Variation of J vs z.

3.4. Analysis of Om(z) parameter:

The $Om(z)$ parameter is a diagnostic tool in cosmology, aiding in analyzing the universe's expansion history and distinguishing between dark energy models and measure deviations from a cosmological constant Λ. As described in [46], the $Om(z)$ parameter is defined as:

$$Om(z) = \frac{\left[\frac{H(z)^2}{H_0^2} - 1\right]}{(z+1)^3 - 1} \tag{31}$$

Using equation (26),

$$Om(z) = \frac{\frac{1}{4}\left((z+1)^{\zeta_c} + 1\right)^2 - 1}{(z+1)^3 - 1} \tag{32}$$

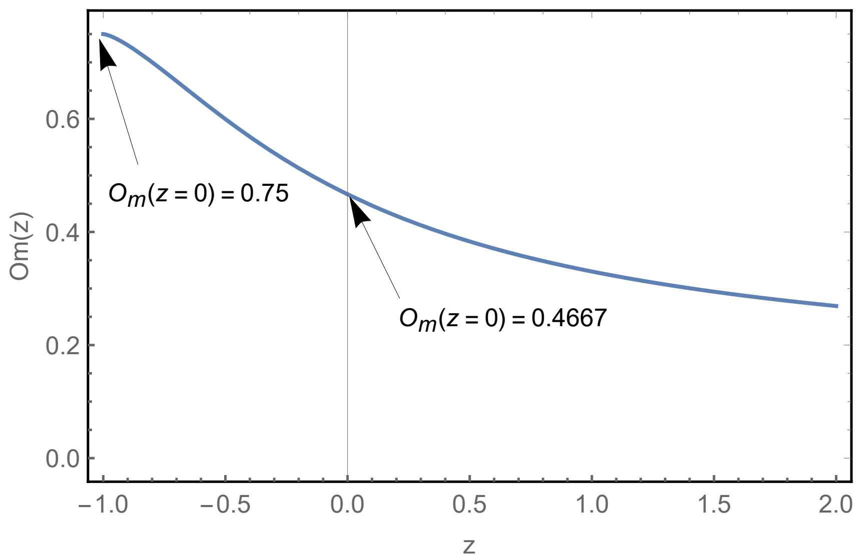

Figure 9. Variation of $O_m(z)$ vs z.

Fig. 9 and equation (32) illustrate the variation of the $Om(z)$ parameter with respect to redshift z. It is observed that in our model, the $Om(z)$ parameter evolves with positive values in the redshift range $z \in [-1, \infty)$. For the standard ΛCDM model, $Om(z)$ remains constant and increases from a continuous positive value to a higher positive value as the redshift decreases [47]. In this study, we find that $Om(z)$ increases with decreasing redshift z, and at $z = 0$, $Om(z) = 0.4667$, showing a positive value. The positive trajectory of the Om-diagnostic signifies a dark energy-dominated era, resembling a phantom-like behavior, while a quintessence-like era is characterized by a negative trajectory [48, 49].

3.5. Age of the universe

Current estimates of the universe's age are based on various cosmological observations, including measurements of the CMB, the Hubble constant, and the standard model of cosmology ΛCDM. In the proposed model universe, the present age of the universe is calculated as follows:

$$\begin{aligned} H_0(t_0 - t) &= \int_0^z \frac{dz}{(1+z)h(z)}; \quad h(z) = \frac{H(z)}{H_0} \\ &= \frac{2}{\zeta_c} \log\left(2 - \frac{2}{(z+1)^{\zeta_c} + 1}\right), \end{aligned} \tag{33}$$

where

$$H_0 t_0 = \lim_{z\to\infty} \int_0^z \frac{dz}{(1+z)h(z)} \tag{34}$$

by taking $\zeta_c = 1.4148$ equation (34) becomes

$$t_0 \approx \frac{0.97985}{H_0}. \tag{35}$$

A variation of $H_0(t_0 - t)$ with respect to redshift z is depicted in Fig. 10, where t_0 denotes the present age of the universe. At large z, we find $H_0 t_0 \approx 0.97985$, indicating that $t_0 \approx 0.97985 H_0^{-1}$. For the observational Hubble data (OHD), we obtained $H_0 = 67.112^{+0.049}_{-0.11} kms^{-1}Mpc^{-1}$ when $\zeta_c = 1.4148^{+0.0078}_{-0.010}$. According to the proposed model, the present age of the universe is $t_0 = 14.3 \pm 0.5$ Gyr, which aligns well with the results of [26]. Thus, the derived model demonstrates strong consistency with recent astrophysical observations.

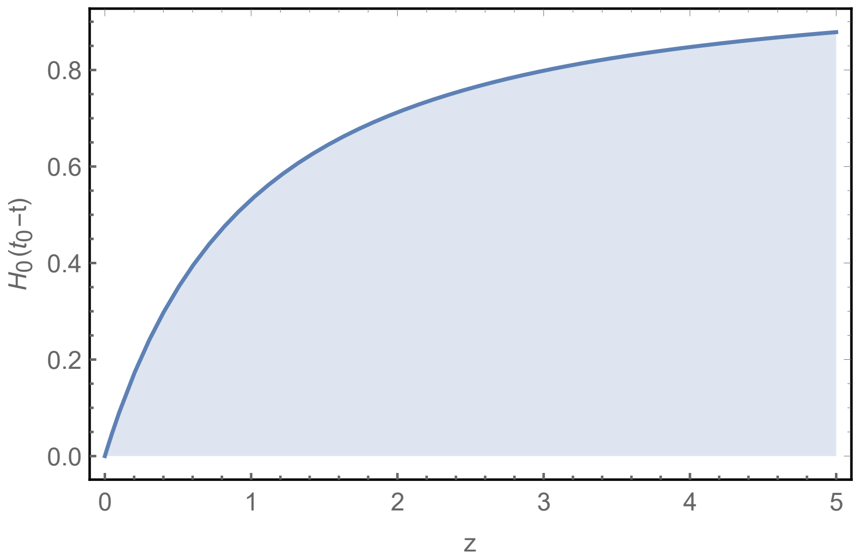

Figure 10. Variation of $H_0(t_0 - t)$ vs z.

3.6. Analysis of state-finder diagnosis of the model

A model-independent approach for distinguishing between the various contenders is highly sought after, as an increasing number of models are being proposed to explain cosmic acceleration. The cosmological diagnostic pair (r, s), introduced by [50, 51], is known as the statefinder, and is defined as:

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r-1}{2(q - \frac{1}{2})}. \tag{36}$$

Using equations (16), equation (36) becomes

$$\begin{aligned} r =&\zeta_c + e^{-2A\zeta_c t}\left((\zeta_c - 3)e^{A\zeta_c t} + \zeta_c\right), \\ s =&\frac{\zeta_c\left(\zeta_c\left(-e^{-A\zeta_c t} - 1\right) + 3\right)}{3e^{A\zeta_c t} - 2\zeta_c} \end{aligned} \tag{37}$$

In this model, the state-finder parameters are dependent on the cosmic time t. The dynamics of the model universe, as characterized by the geometric structures of the model, are represented by the $r - s$ trajectory in Fig. 11. We adopt the values $A = 33.2$ and $\zeta_c = 1.4148$ for our numerical calculations. Fig. 11 illustrates that, within the framework of the present model universe, which includes the deceleration parameter, the universe passes through a phase near the ΛCDM model at the point $\{$r = 1, s = 0$\}$. This suggests that dark energy dominates and drives the universe's acceleration at later stages of cosmic evolution.

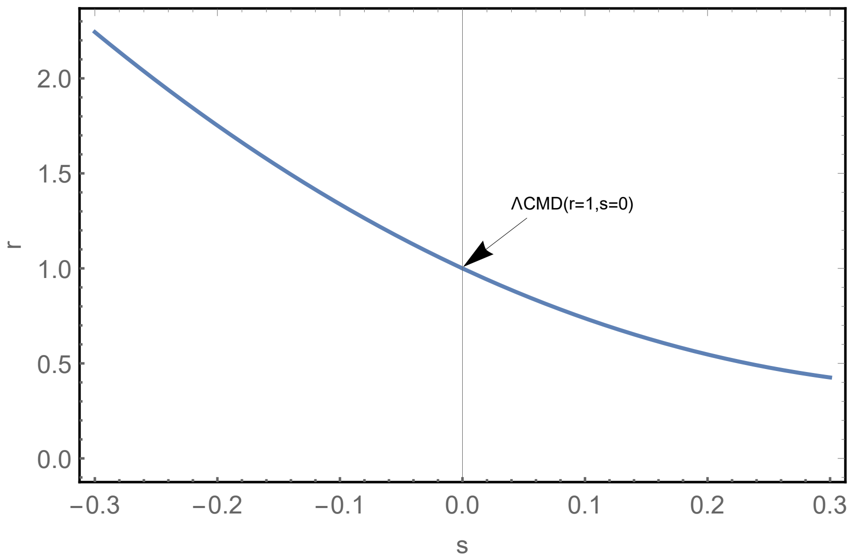

Figure 11. Variation of $r - s$.

3.7. Analysis of Energy condition:

The well-known Raychaudhuri equations [52, 53] which are expressed as follows:

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma^2 + \omega^2 - R_{ij}u^i u^j \tag{38}$$

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma^2 + \omega^2 - R_{ij}\eta^i \eta^j \tag{39}$$

Here, θ represents the expansion factor, η^i is the null vector, and σ and ω denote the shear and rotation associated with the vector field u^i.

Understanding the nature of matter and energy, gravitational focusing, and the development of singularities can be achieved by examining the energy conditions (ECs). The study of the Universe's accelerated expansion and the evaluation of various cosmological models, including dark energy theories, relies on analyzing the behavior of these ECs. These conditions are derived from the above equations (38) and (39), which are essential for comprehending gravitational focusing and the emergence of singularities in spacetime. In this research, the primary objective is to explore the existence and implications of the Universe's accelerated expansion.

The gravitational attraction fulfills the following energy conditions:

- **Strong Energy Conditions (SEC)**: $\rho + 3p \geq 0$
- **Weak Energy Conditions (WEC)**: $\rho \geq 0,\ p + \rho \geq 0$
- **Null Energy Conditions (NEC)**: $p + \rho \geq 0$

- **Dominant Energy Conditions (DEC)**: If $\rho \geq 0$, $|p| \leq \rho$

The ECs of the model universe can expressed in terms of redshift z are as folows:

$$\rho + p = (W+1)\left(-(z+1)^{-\zeta_c}\right)^{-\frac{3k(W+1)}{\zeta_c}} \tag{40}$$

$$\rho - p = (W-1)\left(-(z+1)^{-\zeta_c}\right)^{-\frac{3k(W+1)}{\zeta_c}} \tag{41}$$

$$\rho + 3p = (W+3)\left(-(z+1)^{-\zeta_c}\right)^{-\frac{3k(W+1)}{\zeta_c}} \tag{42}$$

The graphs depicting the NEC, DEC, and SEC are presented in Fig. 12, Fig.13, and Fig. 14, respectively. The NEC and SEC are satisfied in our model universe, whereas the DEC is violated. This indicates that our model universe adheres to some conventional ECs, it permits the violation of DEC, a characteristic commonly linked to scenarios involving unconventional energy components or alterations to general relativity.

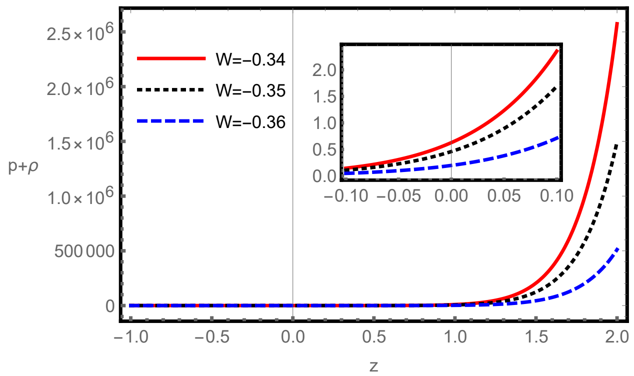

Figure 12. Variation of $p + \rho$ vs z.

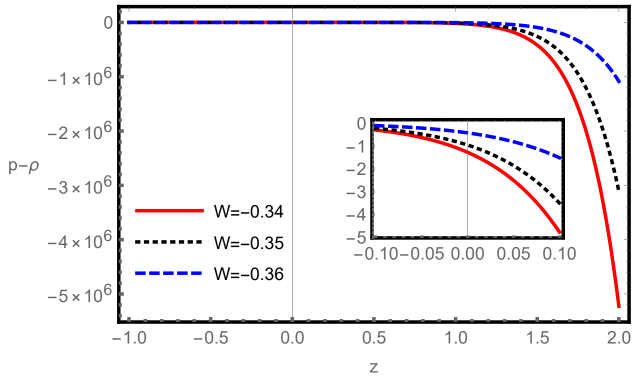

Figure 13. Variation of $p - \rho$ vs z.

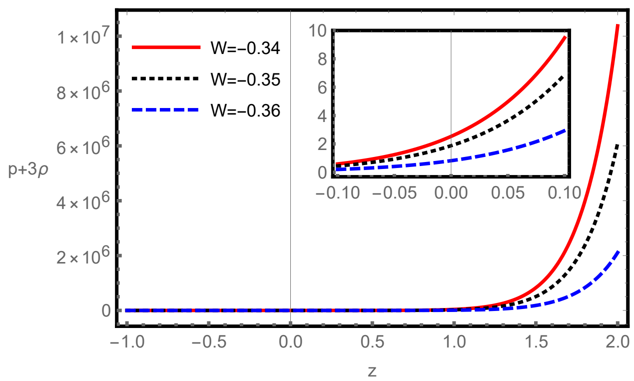

Figure 14. Variation of $p + 3\rho$ vs z.

4. CONCLUDING REMARK

In this study, we derive the equations of motion for the anisotropic LRS Bianchi type-I cosmological model, considering a perfect fluid with variable gravitational constant G and cosmological parameter Λ. By adopting a specific form of the deceleration parameter and an equation of state $p = W\rho$ with $W = -0.34$, $W = -0.35$, and $W = -0.36$, we obtain cosmological solutions consistent with the dark energy ΛCDM model. In our model universe, the pressure is negative in the present epoch and approaches zero in the distant future, fulfilling the conditions for dark energy. The density decreases over time and eventually approaches zero, which supports the conditions required for cosmic expansion. The variables G and Λ contribute to the gravitational collapse of matter and regulate the dynamics of the universe across both small and large scales. Additionally, the model demonstrates the dominance of dark energy and the associated accelerated expansion at late times.

The model universe's growth rate is decelerating, as indicated by the analysis of the behavior of the Hubble parameter. The present value of the Hubble parameter is $H_0 = 67.112^{+0.049}_{-0.11}$ km s^{-1} Mpc^{-1}, which is consistent with the ΛCDM model.

Analyzing the 46 Hubble data points, we obtain a chi-square value of $\chi^2_{OH} = 34.084162$, with a minimum chi-square value of $\chi^2_{\min} = 0.000004$. To support the accelerating universe, we observe that the present value of the deceleration parameter q is negative. Furthermore, the current values of the jerk and $\Omega(z)$ parameters confirm that the proposed model universe is experiencing accelerated expansion both now and in the future, driven by dark energy. According to our model, the universe's present age is $t_0 = 14.3 \pm 0.5\,\text{Gyr}$, consistent with recent astrophysical observations. Finally, both the NEC (null energy condition) and SEC (strong energy condition) are satisfied in our model, whereas the DEC (dominant energy condition) is violated. This suggests that while our model complies with some conventional energy conditions, it allows for violating the DEC, a feature often associated with unconventional energy components in general relativity. Thus our model successfully describes an anisotropic cosmological universe where dark energy drives accelerated expansion, consistent with observations and the ΛCDM framework, while allowing for the violation of the dominant energy condition, a hallmark of unconventional energy components.

DECLARATION OF COMPETING INTEREST

In this paper the authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported.

A. (METHODOLOGY)

The present work is based on the spatially homogeneous and anisotropic Bianchi Type-I cosmological model in the framework of general relativity with variable gravitational constant $G(t)$ and cosmological term $\Lambda(t)$.

$$ds^2 = -dt^2 + S_1^2(t)dx^2 + S_2^2(t)dy^2 + S_3^2(t)dz^2, \tag{43}$$

where $S_1(t)$, $S_2(t)$ and $S_3(t)$ are the scale factors along the x, y, and z axes, respectively. The energy momentum tensor for a perfect fluid is defined by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}, \tag{44}$$

with ρ representing the energy density, p the isotropic pressure, and u^i the four-velocity vector.

Einstein's field equations with variable G and Λ are expressed as

$$R_{ij} - \frac{1}{2}R g_{ij} = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}. \tag{45}$$

For the Bianchi Type-I metric, these equations yield a set of coupled nonlinear differential equations in $S_1(t)$, $S_2(t)$ and $S_3(t)$. To obtain a determinate solution, we define the average scale factor $a(t)$ as $a = (S_1 S_2 S_3)^{1/3}$ and the mean Hubble parameter $H = \dot{a}/a$. The deceleration parameter is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\zeta_c}{1 + a^{\zeta_c}}. \tag{46}$$

To model a universe that evolves from deceleration to acceleration, we assume a time-dependent deceleration parameter and adopt the scale factor

$$a(t) = \left(e^{A\zeta_c t} - 1\right)^{1/\zeta_c}, \tag{47}$$

where $A > 0$ and ζ_c are constants. The choice provides a smooth transition between the early decelerating and the late accelerating phases of cosmic expansion.

The cosmic fluid obeys a barotropic equation of state

$$p = W\rho, \qquad -1 \le W \le 0, \tag{48}$$

and the conservation equation

$$\nabla_j \left(8\pi G T^{ij} + \Lambda g^{ij}\right) = 0, \tag{49}$$

which leads to the differential relation

$$8\pi\dot{G}\rho + 8\pi G(\dot{\rho} + 3H(\rho + p)) + \dot{\Lambda} = 0. \tag{50}$$

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ДОСЛІДЖЕННЯ КОСМОЛОГІЧНИХ НАСЛІДКІВ ТА ДОЦІЛЬНОСТІ ЗМІНИ G ТА Λ ЗАЛЕЖНО ВІД ПАРАМЕТРА УПОВІЛЬНЕННЯ

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Ми коротко розглядаємо просторово однорідну та анізотропну космологічну модель Біанкі I типу зі змінною гравітаційною сталою $G(t)$ та космологічним членом $\Lambda(t)$. Рівняння поля Ейнштейна розв'язуються з урахуванням залежного від часу параметра уповільнення (DP) та баротропного рівняння стану (EoS) $p = W\rho$. Модельний всесвіт узгоджується з масштабним коефіцієнтом виду $a(t) = \left(e^{A\zeta_c t} - 1\right)^{1/\zeta_c}$, який забезпечує плавну еволюцію від уповільнюючої до прискорюючої фази космічного розширення. Отримано аналітичні вирази для тиску, густини енергії, $G(t)$ та $\Lambda(t)$, а також проаналізовано їх зміни з червоним зміщенням. Досліджено поведінку космологічних параметрів, таких як функція Хаббла $H(z)$, параметр уповільнення $q(z)$, параметр ривка $J(z)$ та діагностика $Om(z)$. Отримано поточні значення $H_0 = 67.112^{+0.049}_{-0.11}$ km s^{-1} Mpc^{-1}, $q_0 = -0.2926$ та перехідне червоне зміщення $z_t = 0.8626$, що узгоджується з нещодавніми спостереженнями. Загалом, запропонована модель змінної G та Λ типу Б'янкі забезпечує узгоджений опис переходу Всесвіту від уповільнення до прискорення, що узгоджується з 46 OHD.

Keywords: *анізотропна; змінна гравітаційна константа; темна енергія; космічне прискорення*