East European Journal of Physics. 1. 548-557 (2026)
DOI:10.26565/2312-4334-2026-1-63 ISSN 2312-4334

COMPREHENSIVE ANALYSIS OF BIANCHI TYPE V MODEL IN *f*(*R*, *L*$_m$) THEORY OF GRAVITY

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Received December 8, 2025; revised February 3, 2026; accepted February 27, 2026

In this article, a homogenous Bianchi Type V cosmological model has been investigated within the framework of $f(R,L_m)$ gravity. The solution of the field equations has been obtained by considering the special case $f(R,L_m)=R/2+L_m^n$, where n is free model parameter. The physical as well as the dynamical properties of the model have been analyzed, and graphical representations are provided to illustrate the properties of these parameters.
Keywords: *Biachi Type – V; Stability analysis; f*(*R*, *L*$_m$) *gravity, Perfect Fluid*
PACS: 04.50.Kd, 98.80.-k, 98.80.Jk

1. INTRODUCTION

Harko *et al.* (2010) introduced an advanced form of matter curvature coupling theory, known as $f(R,L_m)$ gravity, where f is a variable function that depends on the matter Lagrangian L_m and the Ricci scalar R. This theory represents a comprehensive extension of gravitational models in Riemannian space, where the trajectory of the test particle deviates from the standard geodesic path, resulting in an additional force that acts perpendicular to its four velocity vector. Functional expressions for $f(R)$ gravity incorporate logarithmic, exponential and power law models. An extension of this framework is known as $f(R,L_m)$ gravity which has recently emerged as $f(R,L_m)=f_1(R)+f_2(R)G(L_m)$, where f_1,f_2 and G are arbitrary functions of the Ricci scalar and the matter Lagrangian density respectively.

The Kasner-type static, cylindrically symmetric interior solutions were studied in the $f(R,L_m)$ theory of gravity by Harko *et al*. (2015), with explicit derivation of the thermodynamic parameters of the string. In the article, “Cosmology in $f(R,L_m)$ Gravity” Jaybhaye *et. al*. (2022a) analysed the model using the Pantheon ranges for the model and explored the variation in cosmological parameters based on the constraints set by these datasets. Additionally, the stability of the resulting model was also investigated.

Jaybhaye *et al.* (2022b) discussed about the constraints on energy conditions by employing cosmographic parameters like mean Hubble parameter, deceleration parameter, jerk parameter and snap parameter whereas Solanki *et al.* (2023) studied $f(R,L_m)$ gravity by considering non-linear models. They obtained the Wormhole solutions by assuming three different cases viz. linear barotropic, anisotropic and isotropic equation of state (EoS).

Singh *et al.* (2023) studied a constrained cosmological model in $f(R,L_m)$ gravity. Shukla *et al.* (2023) used equation of state parameter and Garg *et al.* (2023) used a linear equation of state parameter to study the expansion of the universe. Lobato *et al.* (2021) investigated Neutron stars with realistic equation of state, Patil *et al.* (2023) analyzed FLRW cosmology with Hybrid scale factor, Pawde *et al.* (2023) studied anisotropic behavior of universe with varying deceleration parameter and Jaybhaye *et al.* (2024) derived bouncing cosmological models in $f(R,L_m)$ gravity.

Avelino *et.al.* (2018) demonstrated that if the fluid is constituted by localized concentrations of energy with fixed rest mass and structure (solitons) then the average on-shell Lagrangian of a perfect fluid is given by $L_m=T$, where T is the trace of the energy-momentum tensor. The results give profound implications for theories of gravity where the matter Lagrangian appears explicitly in the equations of motion of the gravitational and matter fields, potentially leading to observable deviations from a nearly perfect cosmic microwave background black body spectrum. Sharif *et. al*. (2019) investigated the dynamics of perfect fluid spherical collapse in curvature-matter coupled $f(G,T)$ gravity.

To obtain the solution, Carvalho *et al.* (2021) used the Zel’dovich approximation and also explored the propagation of light in $f(R)$ cosmic string. Also, they compared the results with wakes formed by cosmic string solutions obtained in General Relativity and Scalar Tensor Theories of Gravity. da Silva *et al.* (2021) studied cosmic string in modified theories of gravitation. Also, several authors studied $f(R)$ theory of gravity in different content [Adhav *et al.* 2012; Hatkar *et al*. 2018; Agrawal and Nile 2024; Malik 2024]. Bishi *et al.* (2015) studied Bianchi type V

Cite as: P.R. Agrawal, H.G. Paralikar, A.P. Nile, East Eur. J. Phys. 1, 548 (2026), https://doi.org/10.26565/2312-4334-2026-1-63

string cosmological model with bulk viscosity in $f(R,T)$ Gravity by considering a special form and linearly varying parameter.

In line with the above discussion, we investigate, Bianchi type-V cosmological model in the context of $f(R,L_m)$ theory of gravity. The paper is structured as follows: Section 2, gives the basic theoretical approach of $f(R,L_m)$ gravity. Section 3 presents the metric and the solution of field equation for Bianchi type-V model. In section 4, the cosmological models of $f(R,L_m)$ gravity are derived whereas section 5 examines the stability of the derived models. The last section, provides discussions and concluding remarks.

2. $f(R,L_m)$ THEORY OF GRAVITY

The action for the $f(R,L_m)$ gravity given by Harko *et al.* (2010) is as

$$S=\int f(R,L_m)\sqrt{-g}d^4x\,, \tag{1}$$

where, $f(R,L_m)$ represents an arbitrary function of the Ricci scalar R and the matter Lagrangian term L_m.
The field equation can be acquired by varying action (1) for the metric tensor g_{uv},

$$f_R R_{\mu\nu}+(g^{\mu\nu}-\nabla_\mu\nabla_\nu)f_R-\frac{1}{2}(f-f_{L_m}L_m)g_{\mu\nu}=\frac{1}{2}f_{L_m}T_{\mu\nu}\,, \tag{2}$$

here $f_R\equiv\frac{\partial f}{\partial R}$, $f_{L_m}\equiv\frac{\partial f}{\partial L_m}$ and $T_{\mu\nu}$ represents the energy-momentum tensor for the perfect fluid, defined by

$$T_{\mu\nu}=\frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}\,. \tag{3}$$

3. METRIC AND FIELD EQUATIONS IN $f(R,L_m)$ GRAVITY

The Bianchi type-V line element is given by

$$ds^2=dt^2-A^2dx^2-B^2e^{-2mx}dy^2-C^2e^{-2mx}dz^2\,, \tag{4}$$

here, A,B and C are functions of the cosmic time t.
The energy momentum tensor for perfect fluid is given by

$$T^\mu_\nu=(\rho+p)u_\mu u^\nu-pg^\mu_\nu\,, \tag{5}$$

where ρ is energy density, p is pressure and $u^\mu=(1,0,0,0)$ are components of four velocities of the cosmic time.
Since, we are studying the universe filled with the perfect fluid, which leads to (Chawla and Mishra 2013),

$$p=\gamma\rho\,, \tag{6}$$

where, the constant γ lies in $[0,1]$.
The dynamical parameters for the line element (4) are defined as follows.
The directional Hubble parameters are given by

$$H_x=\frac{\dot a_1}{a_1},\quad H_y=\frac{\dot a_2}{a_2},\quad H_z=\frac{\dot a_3}{a_3}\,. \tag{7}$$

The mean Hubble parameter H is given by

$$H=\frac{1}{3}\theta=\frac{1}{3}\frac{\dot V}{V}=\frac{1}{3}\left(H_x+H_y+H_z\right). \tag{8}$$

The deceleration parameter q is defined as

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{9}$$

The anisotropy parameter Δ of the expansion. shear scalar σ and the expansion parameter θ are respectively defined as,

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{10}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{11}$$

$$\theta = 3H \tag{12}$$

4. COSMOLOGICAL MODEL OF $f(R, L_m)$:

Here, we consider the following form of $f(R, L_m)$ for our further investigation (Harko *et al.* 2014),

$$f(R, L_m) = \frac{R}{2} + L_m^n \tag{13}$$

where n is free model parameter.

For the particular $f(R, L_m)$ model given in equation (13), we take $L_m = \rho$ (Harko *et al.* 2014), the equation (2), (4) and (5) gives,

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = np\rho^{n-1} + (n-1)\rho^n - \Lambda \tag{14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = np\rho^{n-1} + (n-1)\rho^n - \Lambda \tag{15}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = np\rho^{n-1} + (n-1)\rho^n - \Lambda \tag{16}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3m^2}{A^2} = -\rho^n - \Lambda \tag{17}$$

where the over dot represents the derivative with respect to cosmic time *t*.

Here, while studying, it is observed that the system of equations (14) - (17) contains four equations and six unknowns $A, B, C, p, \rho, \Lambda$. Hence, to solve them we need one more condition. Therefore, we consider relation between scale factor *A* and the spatial volume V as

$$A = V^{1/3} \tag{18}$$

Solving above field equations (14) to (17), using (18), we get

$$B = DV^{1/3} e^{\left[M\int \frac{dt}{V}\right]} \tag{19}$$

$$C = D^{-1}V^{1/3} e^{\left[-M\int \frac{dt}{V}\right]} \tag{20}$$

where D and M are constants of integration.

4.1 Model-I: Power Law

We consider power law given by as Sharif and Zubair (2012)

$$a = \alpha t^{\chi} \tag{21}$$

where, α and χ are the positive constant.

On using equations (21), the Hubble parameter, deceleration parameter, the spatial volume, the expansion parameter in terms of cosmic time *t* are obtained as

$$H = \frac{\chi}{t} \tag{22}$$

$$q = \frac{1}{\chi} - 1 \tag{23}$$

$$V = a^3 = (\alpha t^{\chi})^3 \tag{24}$$

$$\theta = 3H = \frac{3\chi}{t} \tag{25}$$

Using equations (14) to (21) the energy density obtained as

$$\rho = \frac{1}{n^{1/n}(\gamma+1)^{1/n}}\left[\frac{2\chi(\chi-1)-\chi^2}{t^2}+\frac{2M^2}{\alpha^6 t^{6\chi}}-\frac{M}{\alpha^2 t^{3\chi+1}}+\frac{\chi}{\alpha^3 t^{3\chi+1}}-\frac{2\chi}{t^{\chi-2}}+\frac{2m^2}{\alpha^2 t^{2\chi}}\right]^{1/n} \tag{26}$$

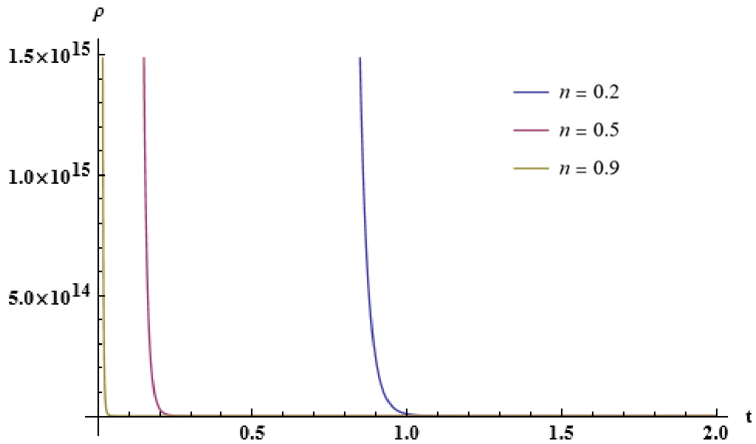

Figure 1. Energy density ρ verses cosmic time *t* is plotted by assuming $\chi = 1.1, M = 1, \alpha = 0.5, m = 1.5$

Using equations (6) and (26) the pressure is obtained as

$$p = \frac{\gamma}{n^{1/n}(\gamma+1)^{1/n}}\left[\frac{2\chi(\chi-1)-\chi^2}{t^2}+\frac{2M^2}{\alpha^6 t^{6\chi}}-\frac{M}{\alpha^2 t^{3\chi+1}}+\frac{\chi}{\alpha^3 t^{3\chi+1}}-\frac{2\chi}{t^{\chi-2}}+\frac{2m^2}{\alpha^2 t^{2\chi}}\right]^{1/n} \tag{27}$$

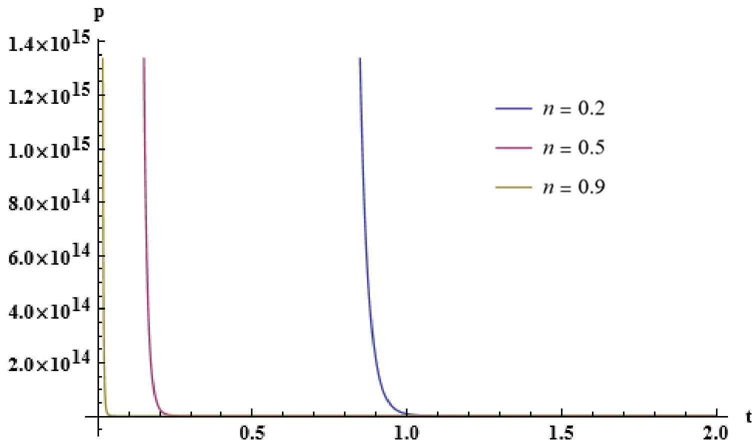

Figure 2. Pressure versus cosmic time *t* is plotted by assuming $\chi = 1.1, M = 1, \alpha = 0.5, m = 1.5$

Here, it is noted that the energy density depends on γ. Therefore, by selecting an appropriate numerical value for γ, three distinct types of universes can be identified as: the dust universe (γ = 0), the radiation-dominated universe (γ = 1/3), and the Zeldovich universe (γ = 1), as outlined below.

i) Dust Universe $(\gamma = 0)$:

For $\gamma = 0$, the model-I corresponds to the equation of state given in equation (6) which leads to $p = 0$. Hence the energy density for the dust universe is given by

$$\rho = \frac{1}{n^{1/n}}\left[\frac{2\chi(\chi-1)-\chi^2}{t^2}+\frac{2M^2}{\alpha^6 t^{6\chi}}-\frac{M}{\alpha^2 t^{3\chi+1}}+\frac{\chi}{\alpha^3 t^{3\chi+1}}-\frac{\alpha\chi}{t^{\chi-2}}+\frac{2m^2}{\alpha^2 t^{2\chi}}\right]^{1/n} \tag{28}$$

ii) Radiation-Dominated Universe ($\gamma = 1/3$)

For $\gamma = 1/3$, the model-I corresponds to the equation of state given in equation (6) which leads to $3p = \rho$. Hence the energy density for the Radiation Dominated Universe is given by

$$\rho = \frac{1}{\left(\frac{4}{3}n\right)^{1/n}}\left[\frac{2\chi(\chi-1)-\chi^2}{t^2} + \frac{2M^2}{\alpha^6 t^{6\chi}} - \frac{M}{\alpha^2 t^{3\chi+1}} + \frac{\chi}{\alpha^3 t^{3\chi+1}} - \frac{\alpha\chi}{t^{\chi-2}} + \frac{2m^2}{\alpha^2 t^{2\chi}}\right]^{1/n} \tag{29}$$

iii) Zeldovich Universe ($\gamma = 1$)

For $\gamma = 1$, the model-I corresponds to the equation of state given in equation (6) which leads to $p = \rho$. Hence the energy density for the Zeldovich universe is given by

$$\rho = \frac{1}{(2n)^{1/n}}\left[\frac{2\chi(\chi-1)-\chi^2}{t^2} + \frac{2M^2}{\alpha^6 t^{6\chi}} - \frac{M}{\alpha^2 t^{3\chi+1}} + \frac{\chi}{\alpha^3 t^{3\chi+1}} - \frac{2\chi}{t^{\chi-2}} + \frac{2m^2}{\alpha^2 t^{2\chi}}\right]^{1/n}, \tag{30}$$

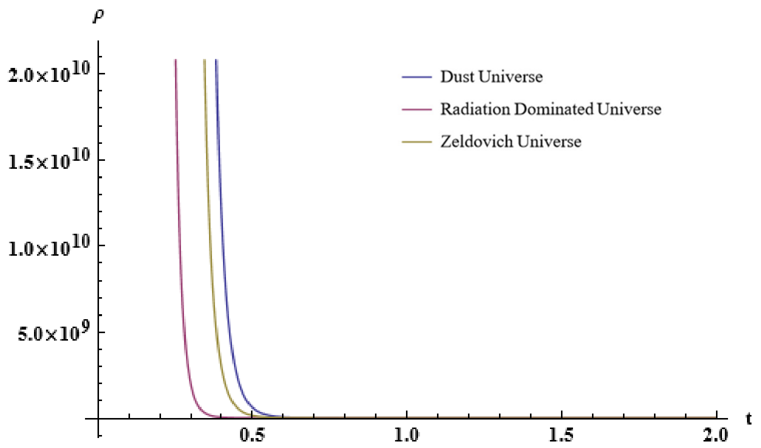

Figure 3. Energy density ρ for Dust Universe, Radiation Dominated Universe and Zeldovich Universe verses cosmic time *t* is plotted by assuming $\chi = 1.1, M = 1, \alpha = 0.5, n = 0.5, m = 1.5$

The Cosmological constant is obtained by using the equations (17) – (21) as

$$\Lambda = \frac{-3\gamma^2}{t^2} - \left(\left(n^{\frac{-1}{n}}(1+\gamma)^{\frac{-1}{n}}\right)\left[\frac{(-2+\chi)\chi}{t^2} + \frac{2\chi}{t^{\chi+2}} + \frac{2m^2 t^{-2\chi}}{\alpha^2} + \frac{2M^2}{\alpha^6 t^{6\chi}} + \frac{\chi - \alpha M}{\alpha^3 t^{(1+3\chi)}}\right]^{\frac{1}{n}}\right)^n - \frac{4\gamma M}{\alpha t^{1+n}} + \frac{3m^2 - M^2}{\alpha^2 t^{2\gamma}} \tag{31}$$

4.2. Model-II: Exponential Law

The exponential law is given by (reference)

$$a = e^{mt} \tag{32}$$

where *m* is the positive constant.

On using equations (8), (9), and equation (32), we obtained the Hubble parameter and the deceleration parameter in terms of cosmic time *t* as

$$H = m \tag{33}$$

$$q = -1,\ a = e^{mt} \tag{34}$$

The spatial volume, using equations (18) and (32) is obtained as

$$V = a^3 = e^{3mt} \tag{35}$$

By using equation (12), we obtain the expansion parameter as

$$\theta = 3m \tag{36}$$

On using equations (16) to (20) and equation (31), the energy density of the model is obtained as

$$\rho = \frac{1}{n^{1/n}(\gamma+1)^{1/n}}\left[\frac{18m^2M^2}{(-3m+1)^2 e^{6mt}} + \frac{2m^2}{e^{2mt}}\right]^{1/n} \tag{37}$$

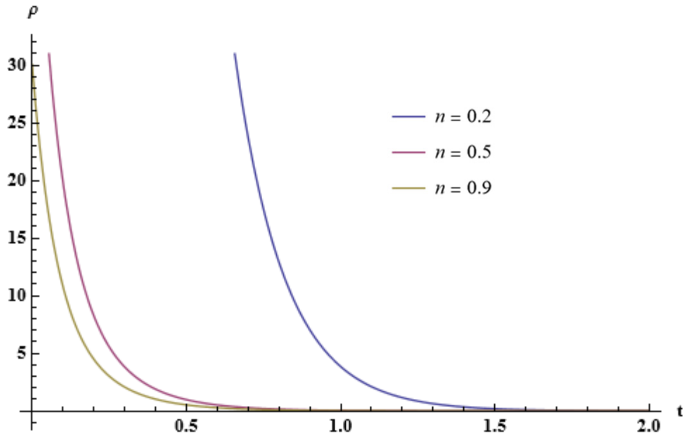

Figure 4. Energy density verses cosmic time *t* is plotted by assuming $\chi = 1.1, M = 1, \alpha = 0.5, m = 1.5$

On using the equations (6) & (37) the pressure of the model is obtained as

$$p = \frac{\gamma}{n^{1/n}(\gamma+1)^{1/n}}\left[\frac{18m^2M^2}{(-3m+1)^2 e^{6mt}} + \frac{2m^2}{e^{2mt}}\right]^{1/n} \tag{38}$$

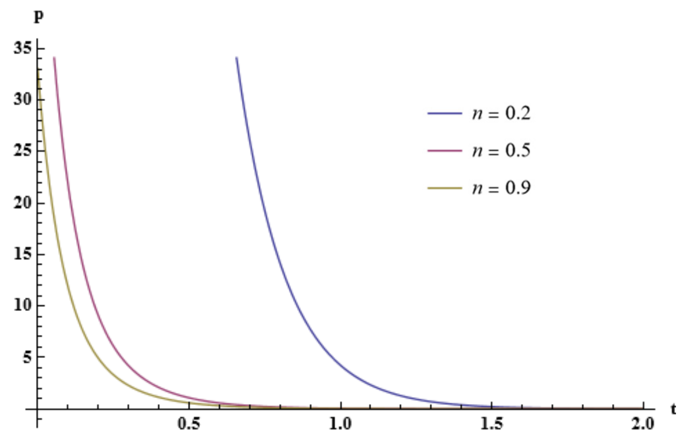

Figure 5. Pressure versus cosmic time *t* is plotted by assuming $\chi = 1.1, M = 1, \alpha = 0.5, m = 1.5$

In the case of Model-II, we also observe that the energy density depends on γ. Therefore, by selecting an appropriate numerical value for γ, three distinct types of universes can be identified as: the dust universe (γ = 0), the radiation-dominated universe (γ = 1/3), and the Zeldovich universe (γ = 1), $\gamma = 1/3$ as outlined below.

i) Dust Universe $(\gamma = 0)$:

For $\gamma = 0$, the model-II corresponds to the equation of state given in equation (6), which leads to $p = 0$. Hence, the energy density for the dust universe is given by

$$\rho = \frac{1}{n^{1/n}}\left[\frac{18m^2M^2}{(-3m+1)^2 e^{6mt}} + \frac{2m^2}{e^{2mt}}\right]^{1/n}. \tag{39}$$

ii) Radiation-Dominated Universe $(\gamma = 1/3)$

For $(\gamma = 1/3)$, the model-II corresponds to the equation of state given in equation (6), which leads to $3p = \rho$. Hence, the energy density for the Radiation Dominated Universe is given by

$$\rho = \frac{1}{n^{1/n}(4/3)^{1/n}}\left[\frac{18m^2M^2}{(-3m+1)^2e^{6mt}} + \frac{2m^2}{e^{2mt}}\right]^{1/n}. \tag{40}$$

iii) Zeldovich Universe $(\gamma = 1)$

For $\gamma = 1$, the model-II corresponds to the equation of state given in equation (6), which leads to $p = \rho$. Hence, the energy density for the Zeldovich universe is given by

$$\rho = \frac{1}{n^{1/n}(2)^{1/n}}\left[\frac{18m^2M^2}{(-3m+1)^2e^{6mt}} + \frac{2m^2}{e^{2mt}}\right]^{1/n}. \tag{41}$$

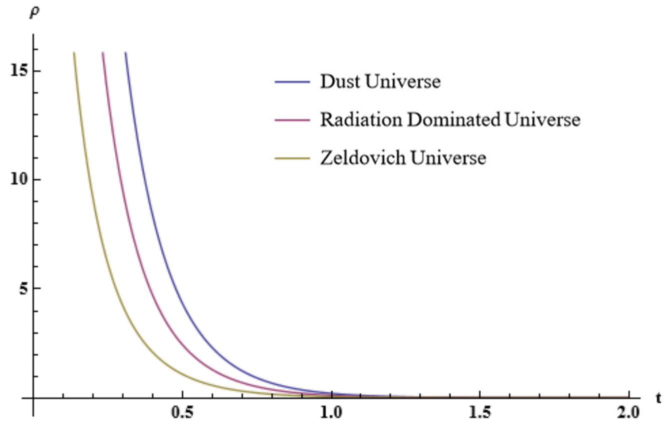

Figure 6. Energy density ρ for the Dust Universe, Radiation Dominated Universe, and Zeldovich Universe versus cosmic time *t* is plotted by assuming $\chi = 1.1, M = 1, \alpha = 0.5, n = 0.5, m = 1.5$

The Cosmological constant is obtained by using equations (17) – (20) and (32) as

$$\Lambda = \frac{1}{n(\gamma+1)}\left[\frac{18m^2M^2}{(-3m+1)^2e^{6mt}} + \frac{2m^2}{e^{2mt}}\right] + \frac{3m^2(e^{2mt}-1)}{e^{2mt}} - \frac{9m^2M^2}{(-3m+1)^2e^{6mt}} \tag{42}$$

Common Physical Parameter for Model-I and Model-II:

On using equation (10), the anisotropic parameter is given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right) = 0. \tag{43}$$

Using equation (11), the shear scalar is obtained as

$$\sigma^2 = \frac{3}{2}\Delta H = 0 \tag{44}$$

Graphical Representation of Physical and Kinematical Parameters

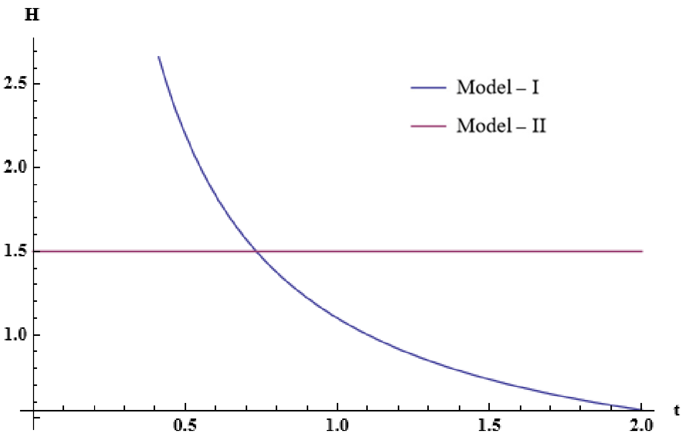

Figure 7. Hubble parameter verses cosmic time *t* is plotted by assuming $\chi = 1.1, \alpha = 0.5, m = 1.5$

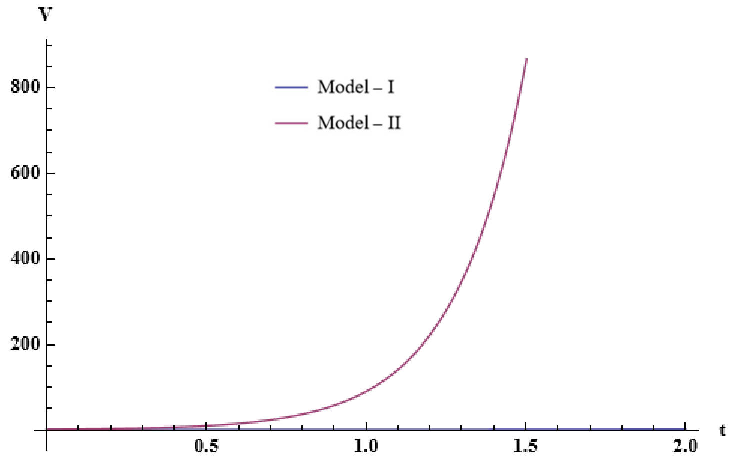

Figure 8. Spatial volume verses cosmic time *t* is plotted by assuming $\chi = 1.1, \alpha = 0.5, m = 1.5$

5. Stability of Models

To assess the stability of the models, the ratio of the sound speed $\frac{dp}{d\rho} = C_s^2$ is analyzed. According to Nimkar et al. (2023), the model is considered stable when this ratio satisfies $\frac{dp}{d\rho} > 0$, and unstable when $\frac{dp}{d\rho} < 0$. In the context of the present model, the sound speed ratio is derived as

$$\frac{dp}{d\rho} = \gamma, \tag{45}$$

Since, it is observed from equation (45) that $\frac{dp}{d\rho} = C_s^2$ remains positive for the selected values of the model parameters, indicating that the models are stable under the given conditions.

6. DISCUSSION AND RESULTS

In this work, we have studied the homogeneous Bianchi Type V cosmological model in the context of $f(R, L_m)$ gravity by considering two specific models as, Model-I is power law and Model-II is exponential law. The solutions of the modified field equations were obtained under the assumption of a functional form $f(R, L_m)$ as $f(R, L_m) = \frac{R}{2} + L_m^n$, where n is free model parameters and the findings are given as below.

In both the models,

- The Hubble parameter H decreases with cosmic time t, showing a accelerated expansion phase (Fig. 7).

- The deceleration parameter q remains constant, depending on the exponent.
- The spatial volume V increases exponentially with cosmic time t, confirming cosmic expansion (Fig. 8).
- Dust Universe ($\gamma = 0$): The energy density decreasing as $\rho \propto a^{-3n}$ (Fig.3 & Fig. 6).
- Radiation Dominated Universe $(\gamma = 1/3)$: Displays steeper decline in energy density.
- Zeldovich Universe $(\gamma = 1)$: Demonstrates even more rapid decay in energy density.
- Anisotropy Parameter (Δ) and Shear Scalar (σ^2) were calculated in equations (43), (44) and both were observed to decrease with time, which implies that the universe transits toward isotropy on late times.
- Expansion scalar and spatial volume both exhibits increasing trends, supporting an expanding universe.
- The square of the sound speed is found to be positive, which ensures that both cosmological models are dynamically stable for the selected values of parameters.
- In case of Model –I (Figs. 1 and 2), there is a sudden fall in both, the energy density ρ and pressure p and after some time they decrease gradually with respective cosmic time whereas, in Model-II (Figs. 4 and 5), the energy density ρ and pressure p both decrease monotonically over time, indicating that matter thins out as the universe expands.

CONCLUSIONS

In this study, we have examined the homogeneous Bianchi Type-V cosmological model within the framework of $f(R, L_m)$ gravity by formulating and analyzing two distinct scenarios. Model-I governed by a power-law expansion and Model-II governed by an exponential-law expansion. The exact solutions of the modified field equations are obtained under suitable assumptions on the functional form of the scale factor.

The analysis reveals that in both models, the Hubble parameter decreases with cosmic time while the deceleration parameter remains constant, corresponding to an accelerated expansion phase. The spatial volume exhibits exponential growth, confirming the overall expanding nature of the universe. For different models, the energy density was found to decay with time, with the sudden decline observed in the radiation $(\gamma = 1/3)$ and Zeldovich $(\gamma = 1)$ dominated epochs, while the dust case shows a slower decrease.

The anisotropy parameter and shear scalar both diminish as time progresses, demonstrating that the anisotropic Bianchi Type-V universe evolves towards isotropy at late times. Furthermore, the expansion scalar and spatial volume increase monotonically, further supporting the expansion scenario. The positivity of the squared sound speed guarantees that the models remain dynamically stable for the considered parameters.

A distinction between the two models lies in the behavior of energy density and pressure, while Model–I exhibits an initial sudden fall before approaching gradual decay, Model–II displays a monotonic decrease throughout the cosmic evolution. This comparative behavior highlights the role of the underlying law of expansion in shaping the dynamics of the evolving universe.

Compared with our, earlier FLRW study in $f(R, L_m)$ gravity, Katore *et. al.* (2025), which showed a constant deceleration parameter and sustained acceleration, the present Bianchi Type – V model extends the analysis to an anisotropic framework. While both models support late-time acceleration, the current work additionally demonstrates the evolution toward isotropy, thereby generalizing the earlier results within the same modified gravity.

Overall, the results suggest that the considered Bianchi Type-V cosmological model not only accommodate an accelerating universe but also successfully describes the dynamical evolution from anisotropy to isotropy at late times. Together with our previous FLRW analysis, the present study strengthens the viability of $f(R, L_m)$ gravity in explaining different cosmological phases within both isotropic and anisotropic frameworks, thereby reinforcing its relevance in modern cosmology.

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КОМПЛЕКСНИЙ АНАЛІЗ МОДЕЛІ БІАНКІ ТИПУ V В ТЕОРІЇ ГРАВІТАЦІЇ $f(R, L_m)$

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У цій статті досліджено однорідну космологічну модель Б'янкі типу V в рамках $f(R, L_m)$ гравітації. Розв'язок рівнянь поля отримано шляхом розгляду окремого випадку $f(R, L_m) = R/2 + L_m^n$, де n вільний параметр моделі. Було проаналізовано фізичні та динамічні властивості моделі, а також наведено графічні зображення для ілюстрації властивостей цих параметрів.

Ключові слова: *тип Біанкі – V; аналіз стійкості; $f(R, L_m)$ гравітація; ідеальна рідина*