EAST EUROPEAN JOURNAL OF PHYSICS. 4. 172-182 (2025)
DOI:10.26565/2312-4334-2025-4-15 ISSN 2312-4334

ON SYNCHRONIZATION OF AN ENSEMBLE OF OSCILLATORS UNDER SUPERRADIANCE CONDITIONS

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Received August 29, 2025; revised September 30, 2025; accepted November 6, 2025

The problems of phase synchronization in an ensemble of oscillators or dipoles, and the mechanisms of coherent field generation in superradiance mode, are discussed. It is shown that an increase in the spread of the initial amplitudes of an ensemble of oscillators suppresses their phase synchronization and reduces the efficiency of field generation. The influence of noise is discussed; it is shown that, below the generation threshold, even an external initiating field cannot synchronize the phases of an ensemble of particles. When the generation threshold is exceeded, the initiating field may not be required. It is shown that the convergence of the oscillator phases with the field phases at the locations of moving oscillators is noticeable only near their exit from the system. At the same time, a complete coincidence of the phases of synchronized oscillators and the field phases in the region of their localization is not observed. Nevertheless, the intensity of the generation field in the superradiance mode is significantly above the spontaneous level, allowing us to speak of induced radiation. The features of the development of the quantum process of superradiance of an ensemble of dipoles are discussed, and a system of equations for its description is given. The features of the quantum analog of superradiance are qualitatively modeled, and the role of the Rabi frequency determining the dynamics of the population inversion is noted. The nutation of the population inversion in the region occupied by the field affects the field intensity not only in this local zone, but also in the subsequent areas of the active zone. This explains the unusual nature of the generation development: the field growth in a particular region of the active zone first stabilizes and then decreases significantly. This decrease in intensity also occurs along the direction of radiation in the peripheral areas of the active zone, despite the large energy reserve there in the form of an unperturbed population inversion.
Keywords: *Classical and quantum emitters; Superradiance regime; Phase synchronization conditions in the classical model; Influence of population inversion nutation on field generation*
PACS: 03.65.Sq;05.45.Xt; 41.60.–m

INTRODUCTION

The phenomenon of superradiance, discovered in the well-known work of Dicke [1], was caused by the overlap of the wave functions of many particles collected in a small volume. In this case, the radiation acquired the features of induced radiation, its coherence increased significantly. A similar phenomenon could be observed in the case of phased classical oscillators or emitters collected in a small region, the dimensions of which are significantly smaller than the wavelength of the radiation. Later, works appeared in which the phenomenon of superradiance was discovered for excited quantum emitters distributed in space. An ensemble of such excited emitters in the field of initiating, comparatively weak external radiation also demonstrated effective field generation in the superradiance mode. Initially, this process was associated with spontaneous emission, in which the synchronization of emitters occurred either forcedly or spontaneously, but later it was realized that superradiance is a form of induced emission emission [2].

Interest in superradiance was also associated with the peculiarities of electromagnetic field generation in open resonators and waveguides, the openness of which in the longitudinal direction was due to the energy release and the exit of particles of the active medium, which gave up part of their energy [3]. Previously, it was believed that the proper field of particles (generators and emitters) in the active zone was very small, practically at the level of spontaneous, and all methods for calculating electronic devices were based on the paradigm of the interaction of each of the active particles of the ensemble only with the field of the waveguide or resonator, and the interaction of active particles with each other was neglected. The closed volume of electronic devices also repeatedly amplified their resonant and waveguide fields, which also provided grounds for neglecting the total field of particles interacting with each other in the active zone. The nonlinear theory of such interaction of active particles only with the field of the resonator or waveguide under conditions of neglecting their interaction with each other was first presented in [4], and this approach to describing the amplification and generation modes became traditional. At the same time, works appeared on the description of superradiance, although initially for ensembles of quantum emitters. The study of the behavior of the total summary particle field - the superradiance field both outside and inside open resonators and waveguides in the classical representation showed that in the absence of noise, the same amplitude and random distribution of phases in the ensemble of oscillators, the development of the generation process is similar to the traditional description of the excitation of the resonator and waveguide fields in similar open systems with the same ensemble of emitters, that is, the increments and maximum achievable field amplitudes in these two cases, considered independently, are similar [5].

Cite as: V.M. Kuklin, E.V. Poklonskiy, East Eur. J. Phys. 4, 172 (2025), https://doi.org/10.26565/2312-4334-2025-4-15

Note that in completely open systems (there is no reflection from the ends), only the sum of fields interacting with each other active particles, in fact the superradiance field, can exist if the conditions for its generation are met [6, 7], which will be discussed below [8]. It is obvious that each active particle is capable of emitting only waves for which the medium is transparent, i.e. the eigenwaves of resonators and waveguides formed by the boundary conditions on their lateral surface. However, the field of the resonator or waveguide does not occur in the absence of reflection from its ends. Only the reflection of the field from the ends can form reflected waves, the superposition of which will be the field of the resonator or waveguide. However, if the sum of the field of interacting particles (which always exists) - is large enough, then even its weak reflection from the ends can create reflected waves, the superposition of which will form the field of the resonator or waveguide. This process is discussed in detail in [9,18].

Therefore, the *goal of this study* is to attempt to answer the question of the magnitude of this total field of an ensemble of interacting particles. Outside resonators and waveguides, as well as in similar open systems without reflections from the ends (or with very weak reflections), this field is clearly a superradiance field. Therefore, studying the excitation and synchronization of radiation from an ensemble of active interacting particles, both in free space and in open systems, is of considerable interest, as it may provide an answer to this question.

FIELD GENERATION BY AN ENSEMBLE OF EXCITED OSCILLATORS

Let us consider the processes of electromagnetic wave generation by a system of oscillators in a one-dimensional case [10]. Let the wave frequency and the oscillator frequency coincide and be equal to ω. The wave vector of oscillations is $\vec{k}=(0,0,k)$, the field components are $\vec{E}=(E,0,0)$, $\vec{B}=(0,E,0)$, and $E=|E|\cdot\exp\{-i\omega t+ikz+i\varphi\}$. The N oscillators are located along the axis OZ in the amount of at the wavelength $2\pi/k$. The mass of the oscillator is equal to m, the charge is equal to $-e$, the oscillator frequency coincides with the wave frequency ω. The initial amplitude of the oscillator oscillations is equal to a. We will assume that the oscillator moves only in the direction of the axis OX. In this case, the influence of the magnetic field of the wave on the oscillator dynamics can be neglected.

The equations describing the excitation of the field by the oscillator current in such a one-dimensional representation $j_x=-ea\omega\cdot\cos(\omega t-\psi)\cdot\delta(z-z_0)$, the coordinates of which can be written as $\vec{r}=(a\cdot\sin(\omega t-\psi),0,z_0)$.

$$\frac{\partial^2E_x}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2D_x}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial J_x}{\partial t}=\frac{4\pi}{c^2}\cdot e\cdot a\cdot\omega_0{}^2\cdot i\cdot\exp\{-i\omega t+i\psi\}\cdot\delta(z-z_0), \tag{1}$$

We will seek a solution for the amplitude of the electric field of the wave in the form $E_x=E\cdot\exp(-i\omega t+ikz)$. For a slowly changing in space amplitude of the radiation field E , the equation is valid

$$\frac{\partial E}{\partial z}=2ea\omega^2\frac{\pi}{c^2k}\cdot\exp\{i\psi+ikz\}\cdot\delta(z-z_0)=\lambda\cdot\delta(z-z_0) \tag{2}$$

the solution of which is

$$E=C+\lambda\cdot\theta(z-z_0)\,, \tag{3}$$

where $\theta(\mathrm{z}<0)=0,\ \ \theta(\mathrm{z}\geq 0)=1$, C – is a constant that should be determined. Since the equation $D(\omega,k)\equiv(\omega^2\varepsilon_0-k^2)=0$, $\varepsilon_0=1$, whose roots are $k_{1,2}=\pm(\omega\mathrm{Re}\,\varepsilon_0/c)(1+i\,\mathrm{Im}\,\varepsilon_0/\mathrm{Re}\,\varepsilon_0)\approx\pm(\omega/c\varepsilon_0)(1+i0)$, are valid for the wave emitted by the oscillator , then for a wave propagating in the direction $z>z_0$ the wave number $k=k_1>0$ and the value of the constant *C* should be chosen equal to zero in order to avoid unlimited growth of the field at infinity. For a wave propagating in the direction $z<z_0$, the wave number $k=k_2<0$ and the value of the constant *C* should be chosen equal to $-\lambda$ for the same reasons. The amplitude of the electric field in this case

$$E_x=2\pi ea\omega_0M\cdot c^{-1}\exp\{-i\omega t+i\psi\}[\exp\{ik_0(z-z_0)\cdot U(z-z_0)+e\,\mathrm{xp}\{-ik(z-z_0)\cdot U(z-z_0)\}, \tag{4}$$

where $U(\mathrm{z}<0)=0,\ \ U(\mathrm{z}\geq 0)=1$, while $M=n_0b$, n_0 is the density of particles per unit volume, *b* is the length of the space under consideration in the longitudinal direction. For one particle in such a volume of unit cross-section and length *b*, M is numerically equal to unity. In the region $z_j\in 0\div b$ occupied by the ensemble of oscillators, the equations of motion for an individual oscillator take the form

$$\frac{dx_i}{dt}=v_i\,,\ \frac{d}{dt}\frac{v_i}{\sqrt{1-\frac{|v_i|^2}{c^2}}}+\omega_0{}^2x_i=-\frac{e}{m}E_x(z_i,t)\,, \tag{5}$$

where $x_i(t)=i\cdot a_i\cdot\exp\{-i\omega t+i\psi\}=iA\cdot\exp\{-i\omega t\},\quad v_{i\perp}=\omega\cdot a_i\cdot\exp\{-i\omega t+i\psi\}=\omega A\cdot\exp\{-i\omega t\}$.
Then the equation of motion describing the change in the amplitudes of the ensemble of oscillators takes the form.

$$\frac{d\,\mathrm{A}_j}{d\tau}=\frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\mathrm{A}_j-\mathrm{E}(Z_j,\tau)\,. \quad 6)$$

Here, the term proportional to α take into account the weak relativism of the oscillator. By the way, in the theory of cyclotron generators, such nonlinearity, proportional to α, is a consequence of the so-called negative mass effect. Taking such nonlinearity into account may be significant, since in [11] it is noted that in a system of linear oscillators the generation efficiency is insignificant. To maintain the field in the volume occupied by the ensemble of oscillators, we will inject them with a random phase from the left edge of the system $z=0$ and remove them upon reaching the right edge of the system $z=b$, the longitudinal velocity will be considered constant $v_z=Const$.

For the superradiance field of the ensemble of particles, we can write the expression:

$$\mathrm{E}_{sr}(Z,\tau)=\frac{1}{\Theta N}\sum_{s=1}^{N}\mathrm{A}_s\cdot e^{i2\pi|Z-Z_s|}\,. \quad (7)$$

We can add a second term to (7) $\mathrm{E}_{ex}(Z,\tau)=\mathrm{E}_{0+}\,e^{i2\pi Z}+\mathrm{E}_{0-}\,e^{-i2\pi Z}$ - this is the external initiating field, often necessary to accelerate the process. Dimensionless variables and parameters were used above

$$\mathrm{A}=A/a_0,\quad kz=2\pi Z,\quad V=kv_z/2\pi\gamma,\quad \gamma_0^2=\omega_{pe}^2/4=\frac{\pi ne^2}{m},\quad \gamma=\gamma_0^2/\delta,\quad \gamma t=\tau,\quad kb=2\pi\bar{b},$$

$$\delta=\frac{c}{b},\quad \Theta=\delta/\gamma_0,\quad E_{01}=\frac{2m\cdot\gamma_0\cdot\omega\cdot a_0}{e},\quad \mathrm{E}=E/E_{01},\quad \alpha=\frac{3\omega}{4\gamma_0}(ka_0)^2.$$

Expression (7) is a slowly changing envelope of the HF oscillations of the field. Due to the short system and the effective removal of energy from it, the accumulation of the field in the volume of the active zone does not occur, as in the case of radiation of a short electron bunch moving in the plasma, considered in [12].

PHASE SYNCHRONIZATION OF AN ENSEMBLE OF CLASSICAL OSCILLATORS

Let us discuss the possibility of synchronizing oscillators in the superradiance mode. The question arises whether phase synchronization is possible in a system of oscillators, which can ensure the transition to the appearance of sufficiently intense induced radiation? Let us return to equation (6), which can be written differently

$$\frac{d[|\mathrm{A}_j|\exp(i\psi_j)]}{dt}=\frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2|\mathrm{A}_j|\cdot\exp(i\psi_j)-|\mathrm{E}(Z_j,\tau)|\cdot\exp(i\varphi)\,. \quad (8)$$

Then the equation for the oscillator phase, which follows from (10), takes the form

$$\frac{d\psi_j}{dt}-\frac{\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2=-\{|\mathrm{E}(Z_j,\tau)|/|\mathrm{A}_j|\}\cdot\sin(\varphi-\psi_j)\,. \quad (9)$$

The right-hand side of the last equation is large enough $|\mathrm{E}(Z_j,\tau)/\mathrm{A}_j|>>1$.. This is what can force the phase of an individual oscillator to synchronize with the phase of the total field of the ensemble $\psi_j(Z_j)\to\varphi(Z_j)$ at the point, where the oscillator is located [13].

For the number of oscillators N=2500, the amplitudes of which are equal to unity, and the phases are randomly distributed, the average integral field $|\mathrm{E}|$ (7) (which can be considered spontaneous in the classical case [2]) is approximately $1/\sqrt{N}$ times smaller than the maximum possible field value in the case when all the phases of the oscillators are close to the phase of the total field at the point where the oscillator is located. That is, for the fields of spontaneous and absolutely coherent induced radiation, the relation is satisfied $(1/N)^{1/2}:1$. For the squares of the amplitudes $|\mathrm{E}|^2$, this relation takes the form $1/N$. It is not difficult to estimate the last relation; it is enough to take the square of the modulus of the right and left parts of expression (7) at an average over fast oscillations. In Fig. 1, the maximum field amplitude is 0.22, which is an order of magnitude greater than the average amplitude of the spontaneous field and five times smaller than the maximum possible field value in the case when all oscillators are synchronized in phase.

The influence of the dispersion of the initial amplitudes of the oscillators on the efficiency of their phase synchronization. It turns out that the spread of the oscillator amplitudes significantly affects the synchronization process. The squares of the initial amplitudes were randomly distributed so that the average value of the amplitudes at the initial moment was equal to one. The calculations of system (6)–(7) were carried out with the following parameters

$N = 900$, $\alpha = 1$, $A_0 = 1$, $\Delta A^2 = 1; 0.4; 0.65; 0.75; 0.8; 1$. The spread of the oscillator amplitudes with a random distribution of their phases at the initial moment is shown in Fig. 3

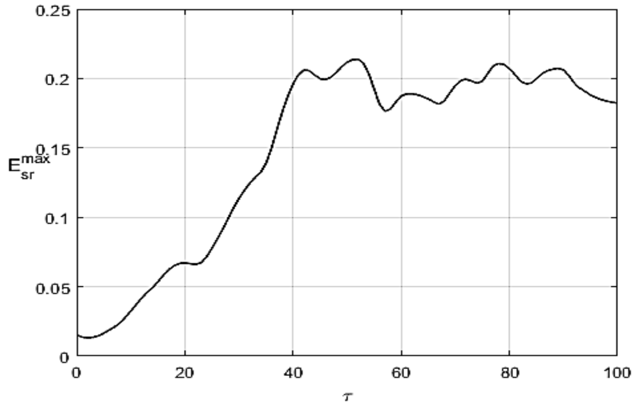

Figure 1. Time dependence of the maximum amplitude modulus of the oscillator field for the particle velocity $V = 0.15$ and number of particles N=2500 for $\alpha = 1$ and $\theta = 1$ in the absence of reflection from both ends of the resonator

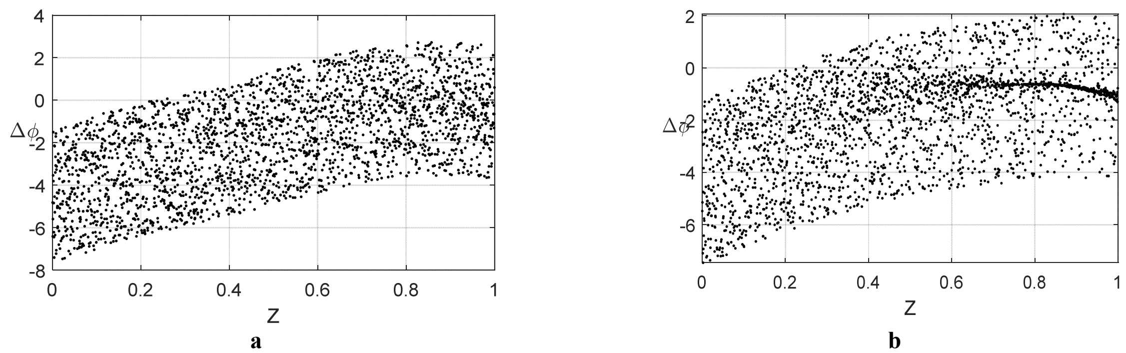

Figure 2. Distribution of the phase difference of the oscillators with the phases of the total field of the ensemble at the points where the oscillators were located $\psi_j(Z_j) - \varphi(Z_j)$. a) $\tau = 20$, b) $\tau = 40$ for the velocity $V = 0.15$, the number of particles N=2500, at $\alpha = 1$ and $\theta = 1$ in the absence of reflection from both ends of the resonator

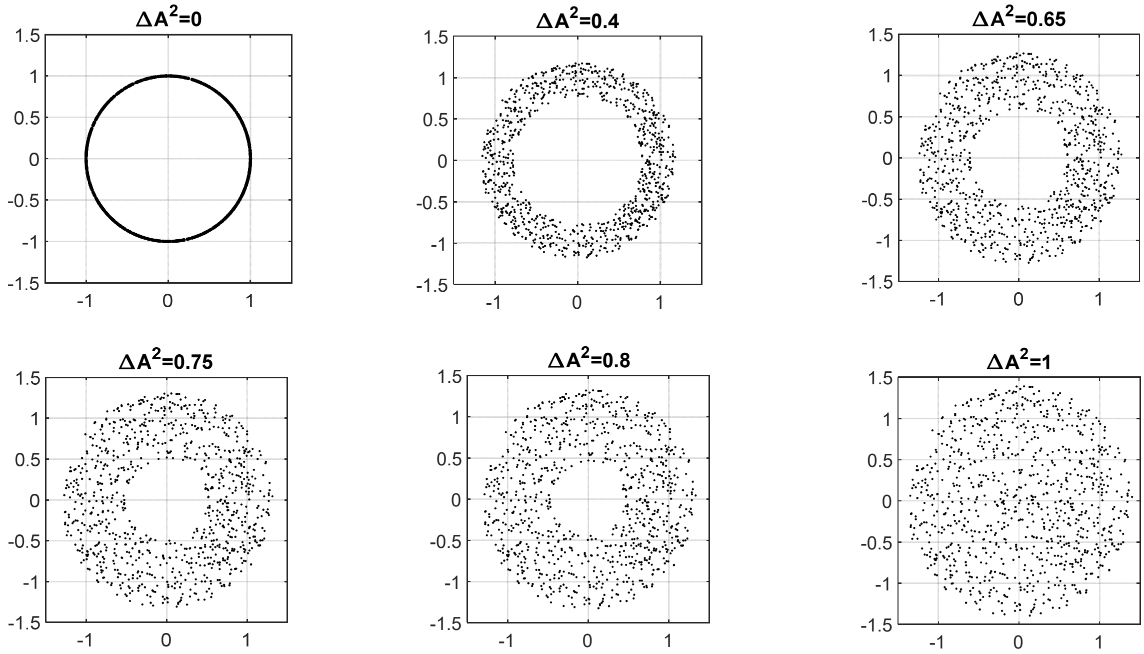

Figure 3. Phase planes "amplitude-phase" for different cases of the initial distribution of oscillators

For these cases, we can present the nature of the change in the energy of the oscillator system and the behavior of the field amplitude (see Fig. 4). The influence of the initial oscillator amplitude spread on synchronization efficiency, as noted by V. A. Buts, is related to the oscillator relativism. That is, the initial amplitude spread is equivalent to the frequency spread of the oscillator ensemble. Let us recall that the spontaneous emission level $|E_{sp}|^2 \propto (1/N) \approx 10^{-3}$.

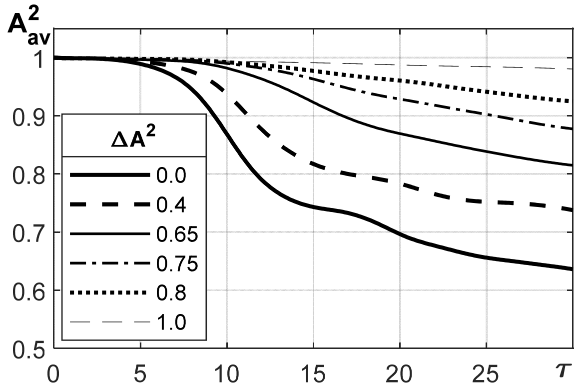

Figure 4a. Changes in the energy of the oscillator system $N = 900$, $\alpha = 1$ for different levels of spread of initial amplitudes, $A_0 = 1$, $\Delta A^2 = 1; 0.4; 0.65; 0.75; 0.8; 1$.

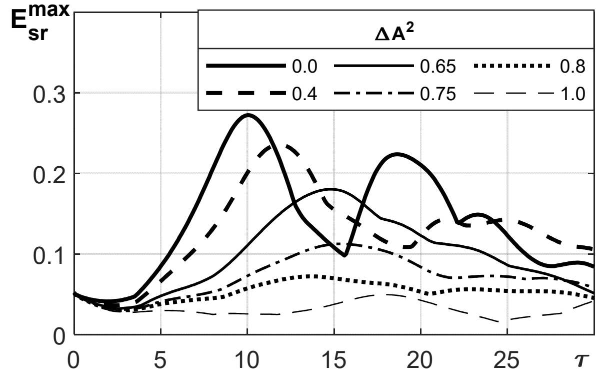

Figure 4b. Behavior of the field amplitude for different levels of scatter of initial amplitudes. For each moment of time, the maximum intensity $|E|^2 \propto |E_{\max}|^2$ is selected.

It is evident from Fig. 3 and Fig. 4 that for efficient generation, in particular in the superradiance mode, it is necessary to achieve an insignificant spread of the amplitudes (or energy) of the oscillators. Note that the value $\frac{i\alpha}{2} \cdot |A_j|^2$, which characterizes the nonlinearity of the oscillator, thereby ensures regularization, that is, some spread of the phase values. With a large spread of the initial amplitudes of the oscillators, it is necessary to use a starting initiating field to accelerate the development of the superradiance process. In this case, the amplitude of such a field should significantly exceed the amplitude of spontaneous emission of an individual oscillator. However, the use of an initiating field not only to accelerate the process, but also to generate coherent superradiance may have another reason, which will be discussed below.

Conditions for the development of generation in the superradiance mode in the presence of noise [8]. The equation describing the change in the complex amplitude of an individual oscillator in the presence of noise takes the form

$$\frac{d\,A_j}{d\tau} = \frac{i\alpha}{2} \cdot |A_j|^2 A_j - E(Z_j, \tau) + i \cdot \delta \cdot r_j(\tau) \cdot A_j, \qquad (10)$$

where the last term in the right-hand side of (10) is additionally introduced, which takes into account the influence of external noise. Here $r_j(\tau)$ – takes random values from –1 to +1, changing through time intervals $\Delta\tau$ on the selected time scale, δ – is the maximum value of this effect. Expression (7) is valid for the field. Random effect, switched through intervals $\Delta\tau = 0.4$, leads to weakening of synchronization or even complete phase chaos. The following parameters are used in the calculation results: number of particles $N = 900$, nonlinearity parameter $\alpha = 1$, noise switching interval $\Delta\tau = 0.4$, system length $b = 1$ (one wavelength).

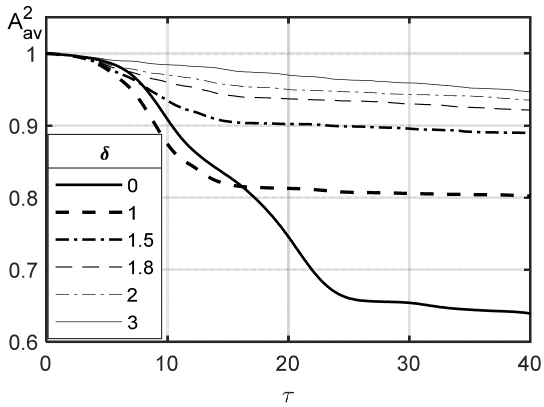

Figure 5a. Change in the average value of the square of the amplitude in the system $A^2_{av} = \frac{1}{N}\sum_{s=1}^{N} |A_s|^2$ over time in the presence of additive noise δ

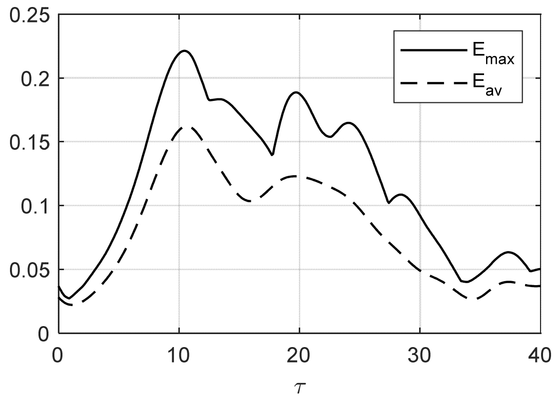

Figure 5b. Time dependence of the field amplitude in the system in the absence of noise ($\delta = 0$). Note that for each moment of time the maximum intensity $|E|^2 \propto |E_{\max}|^2$ is selected

Fig. 5a shows the change in the average value of the squared amplitude $A_{av}^2 = \frac{1}{N}\sum_{s=1}^{N}\left|A_s\right|^2$ in the system over time for different values of the external noise level δ. Fig. 5b shows the behavior of the maximum field amplitude $E_{max} = \max_{Z\in(0,1)} E_{sr}(Z)$ and the average field amplitude $E_{av} = \sqrt{\frac{1}{b}\int_0^b \left|E_{sr}\right|^2 dZ}$ in the volume in the oscillator system in the absence of noise.

Even from these figures, one can see the existence of a threshold: in case of $\delta = 0$, the value of the field amplitude reaches 0.22 in the selected scale. Below the threshold $\delta_{thr} \propto 1{,}5$, there is practically no field growth, and the energy extraction from the oscillators is weakened (the average energy remains at the level of 96% of the initial). In this case, a turbulent state is formed with an average field value E_{av} close to the spontaneous level of electric field strength of 0.02–0.03. The peak level of fluctuations $E_{\max}$ exceeds the average level by two or more times (see Fig. 6).

Far from the threshold, the average oscillator amplitude values (at $\delta \propto 1.8$, $A_{av}^2 < 0.97$) change slightly, i.e. no noticeable energy extraction from the oscillator system is observed. However, as the threshold is approached, the peak fluctuation values increase (see Fig. 7)

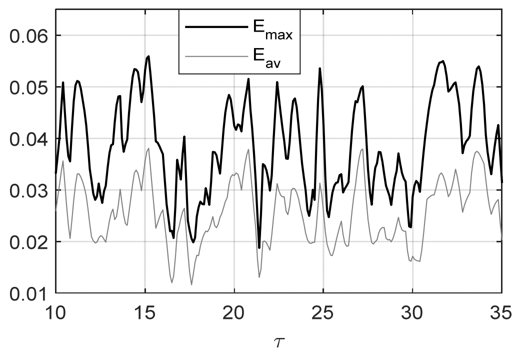

Figure 6. Average field values and peak fluctuation values at noise level $\delta = 3$

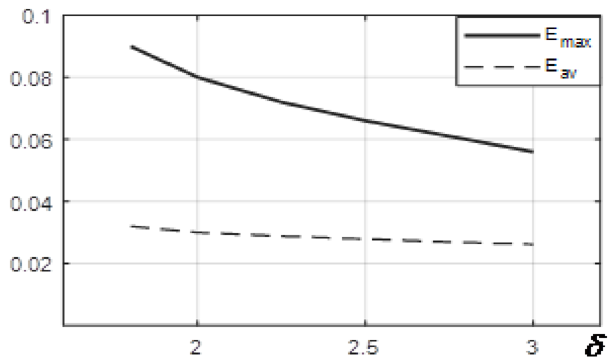

Figure 7. Growth of fluctuations when approaching the threshold of generation development under superradiance conditions

Considering the region near and below the threshold, we can find out how the external field affects the occurrence and development of generation under superradiance conditions. The time dependence of the maximum generation field of the oscillator system for different amplitudes of the external field is shown in Fig. 8.

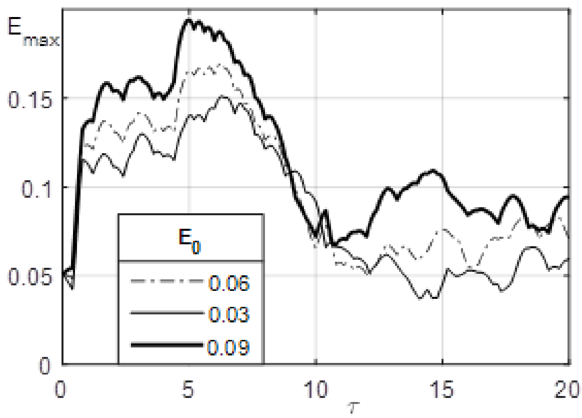

Figure 8. Effect of the external initiating field $E_0 = 0.03; 0.06; 0.09$ on generation under superradiance conditions $\delta = 1.8$. Note that in all cases, the maximum intensity $|E|^2 \propto |E_{\max}|^2$ .is selected for each time moment

It is evident from Fig.8 that an increase in the amplitude of the external initiating field $E_0 = 0.03; 0.06; 0.09$, even in the case of noise ($\delta \approx 1{,}8$), leads to an increase in the maximum generation field up to values that are realized in the absence of noise. Thus, the noise in the system forms the generation threshold. When this threshold is exceeded, even in the absence of an external initiating field, a significant part of the excited oscillators is capable of generating fields whose maximum amplitudes are comparable with the generation amplitudes in the absence of noise. Below the presented threshold, no field growth is observed.

ON SYNCHRONIZATION OF QUANTUM EMITTERS

To describe the generation processes of an ensemble of quantum emitters – dipoles, one can use the semiclassical theory, in particular, previously used in the works of Yu. L. Klimontovich [14] and his colleagues.
The system of one-dimensional equations of the semiclassical theory for the amplitudes of the electric field perturbations E, polarization P ($E,P = \mathrm{E},\mathrm{P}\cdot\exp(-i\omega t + ikx)$), d_{ab} is the matrix element of the dipole moment of emitters δ, $\delta\cdot c$ – the temporal and spatial decrements of the field absorption in the medium, describing the excitation of electromagnetic oscillations in a two-level active medium, can be represented in the following form:

$$\frac{\partial^2 E}{\partial t^2} + \delta c\frac{\partial E}{\partial x} - c^2\frac{\partial^2 E}{\partial x^2} = -4\pi\frac{\partial^2 P}{\partial t^2}, \tag{13}$$

$$\frac{\partial^2 P}{\partial t^2} + \omega^2\cdot P = -\frac{2\omega|d_{ab}|^2}{\hbar}\mu E, \tag{14}$$

to which we must add the equation for the slowly changing population inversion with time

$$\frac{\partial\mu}{\partial t} = \frac{2}{\hbar\omega} < E\frac{\partial P}{\partial t} >. \tag{15}$$

A system of semiclassical equations for a quantum ensemble. We will assume that the frequency of the transition between levels corresponds to the field frequency, the line width in the equation for polarization and the relaxation of the inversion due to external causes are neglected, δ – is the decrement of field absorption in the medium, d_{ab} – is the matrix element of the dipole moment (more precisely, its projection onto the direction of the electric field), $\mu = n\cdot(\rho_a - \rho_b)$ the difference in populations per unit volume, and ρ_a and ρ_b the relative populations of the levels in the absence of a field. The fields are represented as $E = [E(t)\cdot\exp\{-i\omega t\} + E^*(t)\cdot\exp\{i\omega t\}]/2$ and $P = [P(t)\cdot\exp\{-i\omega t\} + P^*(t)\cdot\exp\{i\omega t\}]/2$. In this case $< E^2 >= 2|E(t)|^2 = 4\pi\omega\hbar N.$ For slowly changing amplitudes field $E(t)$ and polarization $P(t)$, the equations are valid

$$\frac{\partial E(t)}{\partial t} + (\delta/2)\cdot E(t) = 2i\pi\omega P(t), \tag{16}$$

$$\frac{\partial P(t)}{\partial t} = |d_{ab}|^2 (\mu/\hbar)\frac{E(t)}{i}, \tag{17}$$

$$\frac{\partial\mu}{\partial t} = \frac{2i}{\hbar}[E(t)P^*(t) - E^*(t)P(t)]. \tag{18}$$

From equations (16) – (18) we obtain the law of conservation of energy

$$\frac{\partial N}{\partial t} + 2\delta N + \frac{\partial\mu}{\partial t} = 0. \tag{19}$$

For sufficiently large losses of field energy in the medium $\delta > \gamma$, the equation for the field takes the form

$$\frac{\partial N}{\partial t} + (8\pi\omega\frac{|d_{ab}|^2}{\hbar\delta}\mu)N = 0, \tag{20}$$

from which it follows that for $\delta > \gamma$ the field can increase with an increment $\gamma = \gamma_0^2/\delta$, where the maximum possible increment in the absence of losses $\gamma_0 = \Omega_0 = (\frac{8\pi|d_{ab}|^2\omega\mu_{t=0}}{\hbar})^{\frac{1}{2}} = (\frac{8\pi|d_{ab}|^2\omega\mu_0}{\hbar})^{1/2}$, $N_k = 2|E|^2/4\pi\omega\hbar$ is the number of field quanta per unit volume. In this approximation, the following expressions should be used $\left(\frac{\delta}{2}\right)\cdot E = 2i\pi\omega P$, $\frac{\partial P(t)}{\partial t} = -\frac{|d_{ab}|^2}{i\hbar}\mu E,\ \gamma = \frac{\Omega_0^2}{\delta} = \Omega_0/\theta.$ which are a consequence of the above system of equations, while the equation describing nutations – oscillations of the population inversion is the following

$$\frac{\partial^2\mu}{\partial t^2} + (8\frac{|d_{ab}|^2}{\hbar^2}|E|^2)\mu = 0, \tag{21}$$

where $\Omega_N = [\frac{4|d_{ab}|^2}{\hbar^2}|E|^2]^{1/2}$ – is the Rabi frequency, which has the meaning of the inverse time of the change in inversion and the probability of an induced transition under the influence of the field [15,16]. It is at this frequency that periodic changes in the inversion – nutation occurs, and the conservation law (19) takes the form $2\delta N + \partial\mu/\partial t = 0$.

The increase in the probability of radiation of excited dipoles at each point in space in the semiclassical description occurs under the influence of a growing integral electric field. The transition to the space-time problem transforms (20) into the following equation

$$\frac{\partial N}{\partial x} + \left(8\pi\omega \frac{|d_{ab}|^2}{\hbar\delta c}\mu\right)N = 0, \quad (22)$$

At the initial moment, the field energy density is small, and this initial period corresponds to spontaneous radiation of oscillators, in the developed mode, the radiation of the ensemble of emitters acquires the features of induced radiation $N_k \propto \mu_0 = \mu(t=0)$. To model the process of spatial growth of the field in an inverse medium, we rewrite equations (21) and (22) in dimensionless form. Equation (23) describes the growth of the field in space, equation (24) - oscillations of the population inversion in a local region, taking into account the conservation law in the form (25)

$$\frac{\partial \mathrm{N}}{\partial Z} = \frac{1}{\theta}\mathrm{MN}, \quad (23)$$

$$\frac{\partial^2 \mathrm{M}}{\partial T^2} + \mathrm{MN} = 0, \quad (24)$$

$$2\delta N + \frac{\partial \mu}{\partial t} = 0. \quad (25)$$

where

$\mu/\mu_0 = \mathrm{M}$, $(8\pi\omega \frac{|d_{ab}|^2}{\hbar}\mu_0)^{1/2} t = T$, $\theta = \delta/(8\pi\omega \frac{|d_{ab}|^2}{\hbar}\mu_0)^{1/2}$, $N_k/\mu_0 = \mathrm{N}$, $\mu_0 = \mu(T=0)$, $\left(8\pi\omega \frac{|d_{ab}|^2}{\hbar}\mu_0\right)^{1/2} x/c = Z/2$.

In the quantum case, when it is impossible to speak about the field phase, synchronization of the ensemble of oscillators-dipoles can be understood only as the transition from spontaneous emission to induced emission. Since the phase of the emitted field in a given quantum system is equal to the phase of the external field, that is, the problem of synchronization in the local sense is removed here. However, the change in the population inversion has an oscillatory character, known as population inversion nutation in an electric field [15, 16]. It is important to note that the Rabi frequency increases proportionally to the magnitude of the increasing external electric field described by equation (20), that is, the rate of change of oscillations of the inversion according to (21) accelerates.

Below, we present the results of numerical modeling of the field growth process in the active zone according to the system of equations (23)–(24). The working region of length L consists of S cells of length $DZ = L/S$. The coordinates of the middle of the cells are $Z_j = DZ \cdot (j - 0.5)$, $j = 1, 2, ..., S$. Each cell is characterized by inversion M_j and the number of quanta N_j. At the initial moment, a constant value of inversion is specified in all cells $\mathrm{M}_j(T=0) = 1$. The number of quanta is set equal to zero in all cells except the first one $\mathrm{N}_j(T=0) = 0$, $j = 2, 3, ..., S$. Note that the inversion does not change in these cells initially. The first cell, in which a small value of the field is initially set $\mathrm{N}_1 = 0.0001$, is the source of the field's initial growth in the active zone. The active zone expands over time at a given constant speed c, i.e., the following cells join the active zone at intervals $DT = DZ/c$. In the model calculation below, the values $DZ = 0.1$, $DT = 0.1$, $c = 1$ were used.

Since the system is open (θ is not small here and is equal to 1), there will be losses of electromagnetic energy from the active zone due to radiation. The field growth occurs due to a decrease in the population inversion. It is the population inversion that generates the field. On the other hand, the inversion, as the field intensity increases, goes into oscillation mode (which is the nutation of the population inversion), and the frequency of these oscillations (the Rabi frequency) increases with increasing field amplitude.

The field growth at the boundary of the active zone (the active zone here is defined by the presence of non-zero values of the population inversion) at a small nutation amplitude acquires an exponential character. However, as the generation front moves, the population inversion value decreases in all regions participating in the generation, both due to the nutation of the population inversion and due to the excitation of field quanta. The field amplitude, in turn, decreases with a decrease in inversion along the entire length of the active generation zone, as well as due to radiation losses (θ =1).

These phenomena lead to stabilization and even to a decrease in the field intensity in the generation region. It is interesting that stabilization and then a decrease in the field intensity first occurs in the region where its maximum was reached and then begins to decrease in the direction of field propagation (see Fig. 9). Although in this direction the population inversion values still remain large. The nature of this stabilization of the field growth is associated not so much with energy losses due to radiation, but to a much greater extent with a decrease in population inversion in the regions located before the point where the maximum radiation was reached, which is illustrated in Fig. 10.

Returning to equation (20), we can understand that the field growth is limited not only by energy losses due to radiation, but also by a decrease in the inversion in the generation region as a whole. Therefore, the development of the superradiance process in the ensemble of quantum dipole emitters passes from a monotonic growth of the field to stabilization and decrease, despite the presence of regions with large values of population inversion ahead.

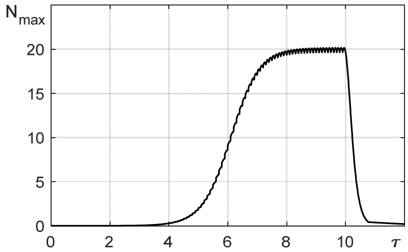

Figure 9. Dependence on time of the maximum value of N cells (N_{max}=max(N_j))

Below Fig. 10 is shown diagrams of distribution N_j and M_j by cells and at different moments in time.

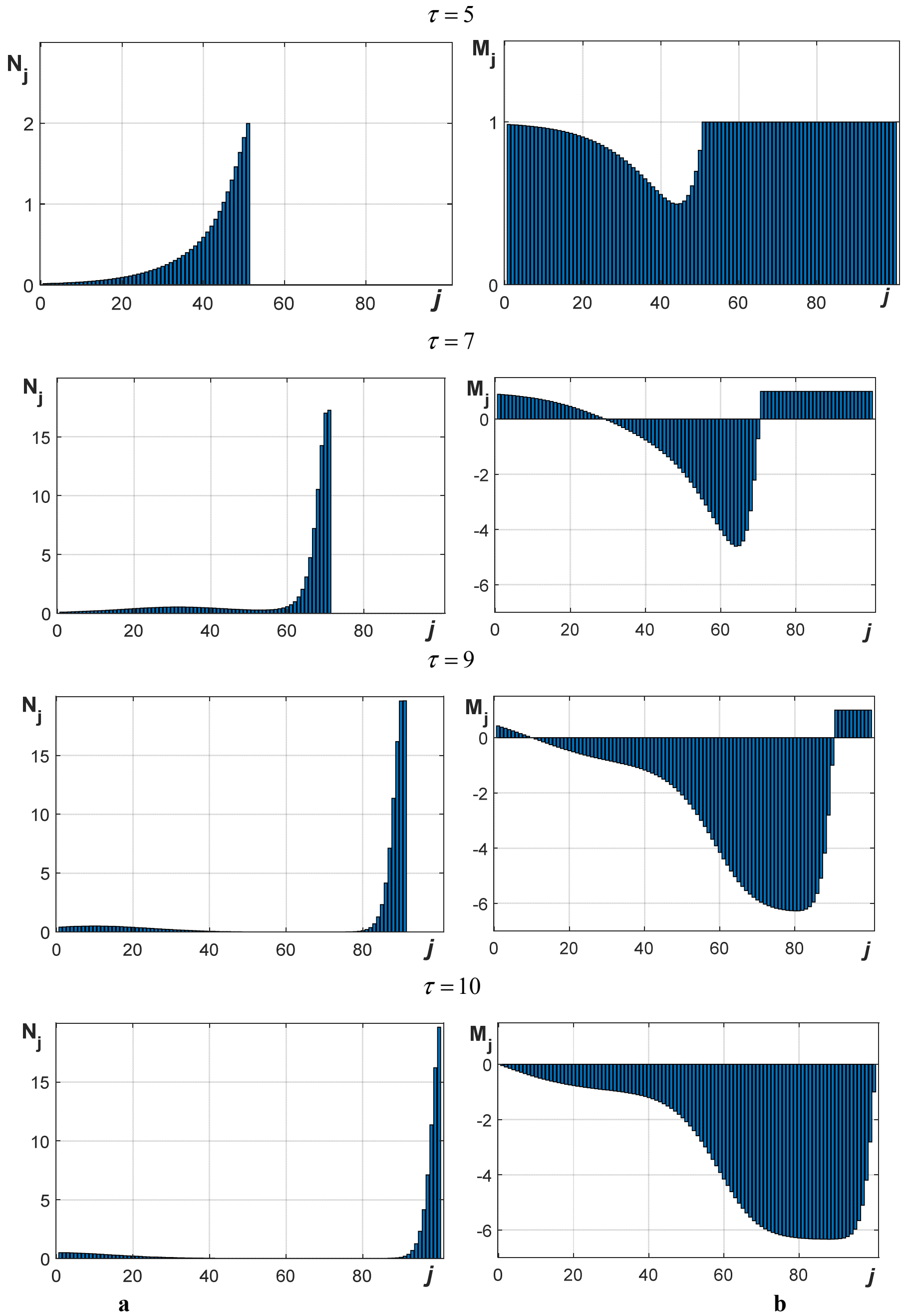

Figure 10. Values of N_j and M_j in cells at times 5, 7, 9, 10, 11; a) N_j ; b) M_j

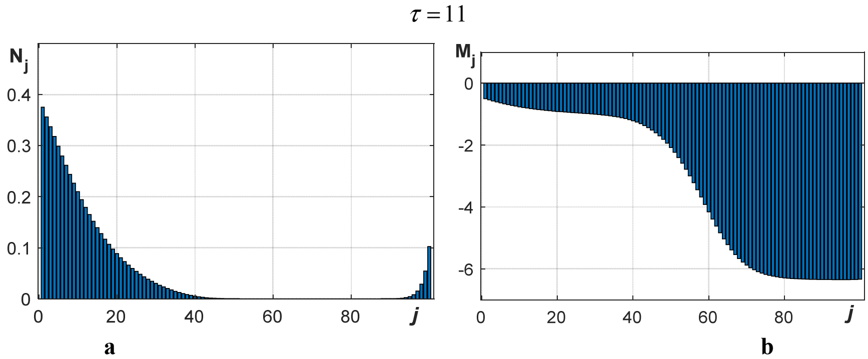

Figure 10. Values of N_j and M_j in cells at times 5, 7, 9, 10, 11; a) N_j ; b) M_j (*continued*)

CONCLUSIONS

As noted above, the volume of a resonator or waveguide always contains the sum of the fields of interacting oscillators, which corresponds to a superradiance field. It is this type of radiation from an ensemble of active emitter-oscillators, whether in the absence of a resonator or waveguide or in open systems of this type, that is discussed in this article. The role of this type of radiation is quite important, although it has often been overlooked, considered unimportant and spontaneous. Primary attention has been paid to the radiation of resonators and waveguides arising from field reflection from the ends of the system. The interaction of active zone particles with each other was ignored; it was assumed that the particles interact only with the fields of the resonator or waveguide. However, this superradiance, i.e., the sum of the fields of interacting oscillators, even with partial reflection from the ends of the system, is capable of generating reflected waves, the superposition of which always forms a resonant or waveguide field, as demonstrated in [9, 18].

It has been shown that, when the above-mentioned phase-locking conditions are met, the superradiance amplitude can be significant, effectively stimulating the formation of a resonant or waveguide field. With significant reflection from the system ends, the amplitudes of the resonant or waveguide field can significantly exceed the sum of the fields of the interacting oscillators, i.e., the actual superradiance field.

In the classical case, the transition to an induced (and, as noted by K. Townes, largely coherent [17]) radiation regime occurs due to the phase synchronization of some of the oscillator emitters by the integral field. However, the nature of the initial oscillator energy distribution can significantly alter the efficiency of phase synchronization and the rate of growth of the superradiance field. It is also important to consider the presence of a threshold due to noise present in the system. As noted, with a small spread of the initial oscillator amplitudes and under conditions of insignificant external noise, the process of oscillator synchronization and the growth of the superradiance field can develop even in the absence of an external initiating field. If these conditions are violated, an external initiating field is necessary for the formation of the superradiance mode. It is important to note that it is precisely the small spread of the initial oscillator amplitudes and insignificant external noise that make it possible to realize the superradiance mode of gyrotrons, the occurrence of which was noted by the authors of [6] and subsequently studied in detail [18]. It was shown that near the injection region of random-phase oscillators in the superradiance mode, their phase synchronization does not occur. The convergence of the oscillator phases with the field phases at the localization sites of moving oscillators is noticeable only near their exit from the system. Moreover, apparently due to the relativism (nonlinearity) of the oscillator dynamics, a complete coincidence of the phases of synchronized oscillators with the phases of the field in the region of their localization does not occur (i.e., forced regularization of the synchronization process occurs). Nevertheless, the intensity of the generation field in the superradiance mode significantly exceeds the spontaneous level, which indicates the presence of stimulated emission.

In the quantum case of the superradiance regime, phase synchronization of the dipole and the external field occurs locally in accordance with the principles of quantum description [15, 16]. Therefore, the main attention is paid to the nature of the field excitation by the population inversion in the active zone, where the initial values of this inversion are positive and sufficiently large. Since the field acts on the inverted population, causing it to oscillate at the Rabi frequency, this additionally affects the nature of the quantum radiation in this region. Nutations, or the oscillatory behavior of the inverted population in the region occupied by the field, change its intensity not only in this local zone, but also in subsequent regions of the active zone. This explains the unusual nature of the development of the superradiance regime: the growth of the field in a certain region of the active zone first stabilizes and then decreases significantly. Moreover, this decrease in intensity also occurs in the direction of radiation to nearby peripheral regions, despite the large, virtually unused energy reserve in them in the form of an unperturbed population inversion.

Acknowledgements

In conclusion, the authors express their sincere gratitude to prof. V. O. Buts for valuable comments and attention to the work.

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ПРО СИНХРОНІЗАЦІЮ АНСАМБЛЯ ОСЦИЛЯТОРІВ В УМОВАХ НАДВИПРОМІНЮВАННЯ

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Обговорюються проблеми фазової синхронізації ансамблю осциляторів або диполів та механізми генерації в режимі надвипромінювання. Показано, що збільшення розкиду початкових амплітуд ансамблю осциляторів пригнічує фазову синхронізацію і знижує ефективність генерації поля. Обговорюється вплив шумів, показано, що нижче за поріг генерації навіть зовнішнє ініціююче поле не здатне синхронізувати фази ансамблю частинок. При перевищенні порога генерації ініціююче поле може не знадобитися. Показано, що зближення фаз осциляторів з фазами поля в місцях розташування осциляторів, що рухаються, помітно лише поблизу їх виходу з системи. При цьому повного збігу фаз синхронізованих осциляторів та фаз поля в області їхньої локалізації не спостерігається. Тим не менш, інтенсивність поля генерації в режимі надвипромінювання суттєво перевищує спонтанний рівень, що дозволяє говорити про ознаки індукованого випромінювання. Обговорюються особливості розвитку квантового процесу надвипромінювання ансамблю диполів та наводиться система рівнянь для його опису. Якісно моделюються особливості квантового аналога надвипромінювання, відзначається роль частоти Рабі, що визначає динаміку інверсії населеності. Нутації інверсії населеності в області, що займає поле, впливають на інтенсивність поля не тільки в цій локальній зоні, але і в наступних областях активної зони. Це пояснює незвичайний характер розвитку генерації: зростання поля певної області активної зони спочатку стабілізується, та потім істотно зменшується. Це зменшення інтенсивності відбувається й у напрямку випромінювання в периферійних областях активної зони, незважаючи на великий запас енергії в них у вигляді незбуреної інверсії населеності.
Ключові слова: *класичні та квантові випромінювачі; режим надвипромінювання; умови фазової синхронізації у класичній моделі; вплив нутацій інверсії населеності на генерацію поля*