EAST EUROPEAN JOURNAL OF PHYSICS. 1. 76-87 (2026)
DOI:10.26565/2312-4334-2026-1-06 ISSN 2312-4334

CONCEPTUAL VIOLATION OF ENERGY CONDITIONS IN BOUNCING COSMOLOGY

A.Y. Shaikh[1#], S.M. Shingne[2*], A.P. Jenekar[3†]

[1]*Department of Mathematics, Indira Gandhi Kala Mahavidyalaya, Ralegaon-445402. (M.S.) India.*
[2]*Department of Mathematics, G. S. Science, Arts and Commerce College, Khamgaon-444312. (M.S.) India.*
[3]*Department of Mathematics, Arts, Commerce and Science College, Maregaon-445303. (M.S.) India.*
**Corresponding Author Email: smshingne131@gmail.com; #Email: shaikh_2324ay@yahoo.com; †Email: apjenekar@gmail.com*

Received November 5, 2025; revised January 23, 2026. accepted January 27, 2026

This paper focuses on examining the bouncing model within a flat Friedmann–Robertson–Walker (FRW) Universe. The equation of state (EoS) parameter pertaining to the considered model under consideration specifies the Universe's peculiar functioning. The kinematics along with the physical attributes of the model's dynamic parameters are investigated comprehensively. We explored several energy conditions (ECs) in this scenario. The diagnostic pair $\{r,s\}$ of statefinder and the jerk parameter $j(t)$ are investigated to identify significantly different cosmic phases. We used the squared sound speed parameter C_s^2, designed to meet the needs of our model's stability analysis. Our analysis revealed that outcomes of our study are in accordance with patterns observed in the bouncing scenarios, offering a method to explain the cosmic acceleration as well as the singularity problem in our Universe.

Keywords: *Bouncing scenario; FRW metric;* $f(R,T)$ gravity; *Cosmic acceleration*

PACS: 98.80.-k, 04.20.-q, 04.50.Kd, 98.80.Cq

1. INTRODUCTION

In pursuance of an intuitive essence of the Universe and astrophysical mechanisms, General Relativity (GR), pioneered by Einstein, is widely considered the most significant theory offering explanations for many mysterious factors of gravitational dynamics. According to observational cosmic evidence, our Universe came across an early inflation accompanied by late-time expansion, which is presently occurring more rapidly [1–4]. It is believed that the force propelling the expanding cosmos appears to be dark energy (DE), characterized by its negative pressure, thereby allocating 68.3% of the peculiar ingredient of the Universe, with an additional 25% being dark matter (DM) [5–10]. Modified gravity (MG) represents a method that alters the action principle GR and serves as an alternative approach to remedying DE, whereas the other method introduces a number of dynamical DE candidates, such as the cosmological constant [11], Chaplygin gas (CG) [12], X-matter [13,14], and quintessence [15,16] for better comprehension of the nature of DE. In GR, the accelerating cosmos is typically ascribed to a cosmological constant, leading to the standard ΛCDM model. Notwithstanding its observable success, ΛCDM encounters fine-tuning and cosmic coincidence issues [17,18]. Numerous researchers have investigated MG in diverse cosmological frameworks; for a comprehensive survey, see ref. [19,20]. Several kinds of MG theories, starting with $f(R)$ gravity, $f(T)$ gravity, $f(G)$ gravity, and $f(Q)$ gravity, are being put forward in this setting. Out of the many MG theories, $f(R)$ gravity is believed to be the best fit for cosmologically useful models, though it faces considerable problems [19,20]. The $f(R,T)$ gravity was presented as an extension, wherein the gravitational Lagrangian is contingent upon the Ricci scalar (R) along with the trace (T) of the matter energy–momentum tensor. In recent decades, much research has been undertaken in a promising category of $f(R,T)$ gravity, with numerous functional expressions being extensively investigated to explain late-time acceleration [21–27]. Even while certain models raise questions about their cosmic viability [28,29], $f(R,T)$ gravity is still a major way to bring together matter and geometry in cosmological dynamics.

The important issue of initial singularity was encountered by GR, among other issues, during the early Universe. In an attempt to tackle the concern of initial singularity, bouncing cosmological models are currently being studied, which prescribe that the cosmos initially contracted leading up to expanding, shorn of coming across a singularity [30]. For a smooth bouncing approach, Steinhardt and Ijjas [31] put together a wedge diagram to gain insight into the detrimental impacts of certain cosmological challenges. Nonetheless, the non-singular bounce might not be able to meet the null energy condition (NEC) within the system associated with a flat Universe. Moreover, the widely accepted Galileon theories [32] addressed non-singular cosmology, which fails to uphold the NEC. In recent years, bouncing cosmology has become more prevalent because it presents a distinct speculative framework compared to other common cosmic theories. For the systematic review regarding conventional bouncing cosmologies, see ref. [33]. Brandenberger and Peter [34] have looked into the current circumstances of bouncing cosmologies as possible alternatives to cosmic inflation to offer an explanation of the very early cosmos. Relevant bounce arrangements founded on the concept of GR are examined in [35-40], while the appropriate bouncing scenarios integrated within MG are analyzed in [41–50]; interestingly, in $f(R,T)$

Cite as: A.Y. Shaikh, S.M. Shingne, A.P. Jenekar, East Eur. J. Phys. 1, 76 (2026), https://doi.org/10.26565/2312-4334-2026-1-06

gravity [51–56], featuring the role of the big bang singularity. The main motivations for this study on the bouncing model within in the light of the GR and $f(R,T)$ gravity are to examine the cosmic speed singularity over a prolonged phase along with exploration of the bouncing conduction appearing in its earliest stages. The structure of the paper follows this approach. Sec. 2 contains the cosmological framework of the FRW world and its associated field equations, whereas its solution via the bouncing scenario is established in sec. 3. The dynamic parameters of our model with the inclusion of DE are analyzed in sec. 4. Sec. 5 covers the exploration of various energy conditions (ECs) of the model. The statefinder diagnostics and jerk parameter, along with the stability of our model, are investigated in secs. 6 and 7, respectively, and at last, the conclusions together with discussion are covered in sec. 8.

2. THE FRW UNIVERSE AND FIELD EQUATIONS

The majority of cosmological models within the confines of contemporary cosmology are predicated on the most underlying aspects of the cosmological principle, claiming that the cosmos is homogeneous as well as isotropic over the cosmological extent. It is widely accepted that the observable cosmos is nearly isotropic and homogeneous. Consequently, cosmologists have focused a considerable amount of emphasis on a flat FRW model, which is articulated as

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \tag{1}$$

with $a(t)$ representing the scaling factor, a function that depends only on t. The Ricci scalar R for the metric (1) arrives to be $R = -6\left(\dot{H} + 2H^2\right)$, with $H = \dot{a}/a$ means the Hubble parameter, while '$\cdot$' reflects the conventional time derivative.

2.1. FIELD EQUATIONS IN GR

The Einstein's field equations in GR are expressed as

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij} \tag{2}$$

where T_{ij} stands for the standard matter energy momentum tensor, which is defined as $T_{ij} = (\rho + p)u_i u_j - g_{ij} p$ with ρ representing energy density, and p is the pressure.
The metric (1) yields the field equations in the form:

$$2\dot{H} + 3H^2 = -p\ , \tag{3}$$

and

$$3H^2 = \rho\ . \tag{4}$$

2.2. FIELD EQUATIONS IN $f(R,T)$ GRAVITY

The action of $f(R,T)$ gravity has the form [57],

$$S = \int \left[\frac{f(R,T)}{16\pi} + L_m \right] \sqrt{-g}\, d^4x\ , \tag{5}$$

with L_m accounting the matter Lagrangian and the stress- energy tensor of matter is specified as

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\ L_m)}{\delta g^{ij}}\ . \tag{6}$$

Following the work of Agrawal et al.[55] in $f(R,T)$ gravity for the bouncing scenario,

We looked into the non-minimal dependence on matter and geometry as, $f(R,T) = f_1(R) + f_2(T)$. The field equations of $f(R,T)$ gravity concerning non-minimal matter interaction can be acquired by tailoring the action pertaining the metric tensor g_{ij} as,

$$f_R(R) R_{ij} - \frac{1}{2} f(R) g_{ij} - (\nabla_i \nabla_j - g_{ij}) f_R(R) = 8\pi T_{ij} + f_T(T) T_{ij} + [f_T(T) p + \frac{1}{2} f(T)] g_{ij,} \tag{7}$$

In Eq. (7), we denote $f_R(R) = \partial f_1(R)/\partial R$ and $f_T(T) = \partial f_2(T)/\partial T$, while p reflects the pressure of the matter.

Based on the three options of $f(R,T)$ provided by Harko et. al. [57], we have selected $f(R,T)=R+2f(T)$. A substantial assortment of cosmological scenarios is described in the field, with $f_1(R)=R$ and $f_2(T)=\beta T$, β as coupling constant. We intend to embrace the timeless cosmological constant Λ_0 here in a way that $f(R,T)=R+2\beta T+2\Lambda_0$. Consequently, the field equations (7) become

$$G_{ij}=(8\pi+2\beta)T_{ij}+\Lambda(T)\, g_{ij}\,, \tag{8}$$

with $\Lambda(T)=(2p+T)\beta+\Lambda_0$ specifying effective cosmological constant.

The discovery of supernovae has significantly enhanced the relevance of the cosmological constant Λ, previously considered negligible, in the examination of accelerating cosmological models. As the Universe evolves, this changes too. The current extended gravity hypothesis that it happens over cosmic time. It turns out that Λ becomes a pure constant Λ_0 for a vanishing β. Moreover, the field equations (8) are simplified to

$$G_{ij}=(8\pi+2\beta)T_{ij}+[(2\rho+T)\beta+\Lambda_0]\, g_{ij}\,. \tag{9}$$

And then the field equations of $f(R,T)$ gravity arrive at

$$2\dot{H}+3H^2=-\eta\, p+\beta\rho+\Lambda_0\,, \tag{10}$$

and

$$3H^2=\eta\rho-\beta\, p+\Lambda_0\,, \tag{11}$$

with $\eta=8\pi+3\beta$. The above field equations reduce to GR case for a vanishing β. Performing some algebraic manipulations among eqns. (10) and (11), we can derive the pressure p, energy density ρ and equation of state (EoS) parameter in terms of Hubble parameter as,

By executing algebraic modifications on Eqs. (10) and (11) as it relates to the Hubble parameter, we can formulate pressure p, energy density ρ, and EoS parameter ω as

$$p=-\frac{1}{(\eta^2-\beta^2)}[2\eta\dot{H}+3(\eta-\beta)H^2-(\eta-\beta)\Lambda_0] \tag{12}$$

$$\rho=\frac{1}{(\eta^2-\beta^2)}[-2\beta\dot{H}+3(\eta-\beta)H^2-(\eta-\beta)\Lambda_0] \tag{13}$$

and

$$\omega=-1+\left[\frac{2(\eta+\beta)\dot{H}}{2\beta\dot{H}-3(\eta-\beta)H^2+(\eta-\beta)\Lambda_0}\right] \tag{14}$$

Given the present circumstances, including yet another constraint equation is absolutely necessary in fully resolving the set of field equations (3)-(4) and (12)-(14) that are formulated with the Hubble term. We want to examine the model's bouncing behavior using an established bouncing scale factor, thus explaining why we're presenting its settings in Hubble terms. It is necessary to prefigure a particular form for the Hubble parameter to acquire the expressions for the dynamical components. Given that bouncing cosmology, an adversary substitute for the inflationary paradigm [58], is being vigorously researched to solve the singularity problem, we are brought about to look into the model within the matter bounce context. Consequently, our next step involves establishing an appropriate bouncing model to explore a particular cosmological solution to the system of field equations.

3. BOUNCING COSMOLOGICAL SOLUTIONS

This section intends to examine a cosmological solution that ultimately results in a bouncing scenario. The cosmos undergoes a non-singular bounce during its initial matter-dominated segregation phase in the bouncing state, as explained in [34,52,54,59–63]. An investigation conducted in [64] suggests that the Universe, which expands following a minimal collapse, represents the bouncing Universe. In bouncing cosmology, the scale factor declines $(\dot{a}<0)$ in the negative time region, indicating the contracting era, and then again it elevates $(\dot{a}>0)$ in the positive time region, representing the expanding era, whereas it vanishes $(\dot{a}=0)$ close to the point $t=0$ where the bounce occurs. The Hubble parameter's positive value $(H>0)$ signifies the expanding period of the cosmos and it is negative $(H<0)$ during the contracting period of the cosmos, as investigated in [65]. At the site of the bounce, the scale factor must attain a minimal, non-zero value, whereas the Hubble parameter must vanish for the effective functioning of the bouncing model without encountering singularity [66]. Additional requirements at the bouncing point consist of violating NEC and transitioning the EoS parameter across the phantom split line $(\omega = -1)$ as discussed in [67]. In order to satisfy these conditions and to have a congruous bounce over the scale factor, we consider $a(t) = \sqrt{1+\gamma^2 t^2}$ with $\gamma > 0$ representing a constant parameter that regulates the cosmic expansion, adhering to the scale factor specified in [56]. Consequently, the Hubble parameter $H(t)$ becomes

$$H = \frac{\dot{a}}{a} = \frac{\gamma^2 t}{1+\gamma^2 t^2} \tag{15}$$

The deviations of the scale factor $a(t)$ and how the Hubble parameter $H(t)$ has evolved across cosmic time (t) are illustrated in Figs. 1 and 2, respectively, which specify the orientation of its curvature for three assigned values of the parameter $\gamma = 0.7, 0.8, 0.9$. As demonstrated in Fig. 1, the scale factor $a(t)$ experiences symmetrical progression from the contracting era of the cosmos towards the expanding Universe and stays finite with a non-zero value, $a(0) = 1$ at the bouncing spot $t = 0$, enabling the Hubble parameter $H(t)$ to potentially be zero during this epoch as shown in Fig. 2, thereby satisfying the bouncing conditions [67]. To visualize a bounce, one can envision the cosmos contracting and then expanding, with this behavior being evident at and near the bounce point. Eventually, the Hubble parameter evolves rectilinearly from the contracting to the expanding era with the assistance of a bounce by passing through zero as depicted in Fig. 2, thereby constituting the bouncing model.

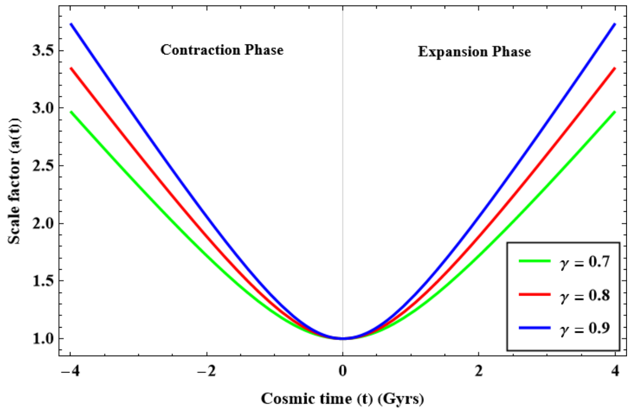


Figure 1. Inspection of the scale factor $a(t)$ versus cosmic time (t).

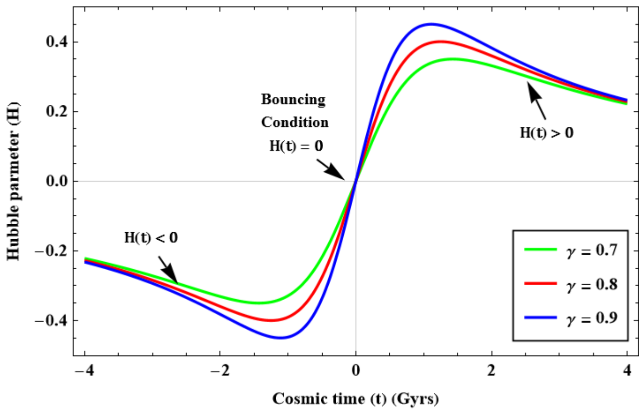


Figure 2. Displays the approach of the Hubble parameter $H(t)$ against cosmic time (t).

The deceleration parameter (q) elucidates the evolution of the Universe's expansion and is found to be

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{1}{\gamma^2 t^2}. \tag{16}$$

In scenarios in which the Universe experiences deceleration throughout its existence, the deceleration parameter remains positive $(q > 0)$; whereas, it becomes negative $(q < 0)$ as the cosmos expands rapidly. Fig. 3 outlines the way the deceleration parameter behaves symmetrically over the bouncing point. It's noteworthy that the deceleration parameter encounters a negative trend $(q < -1)$ with regard to both contracting and expanding Universe, indicating super exponential expansion, and showcases a large negative value in close proximity to the bouncing point. Consequently, the estimated deceleration parameter's value lined up with topical cosmological inspection associated with Type Ia Supernovae [68–70], emphasizing that our cosmos continues to expand at an accelerating pace throughout its evolution. Based on the classification of cosmos explored in [65], our model exhibits a contracting and accelerating phase $(H<0, q<0)$ followed by an expanding and accelerating stage $(H>0, q<0)$, as illustrated in Figs. 2 and 3.

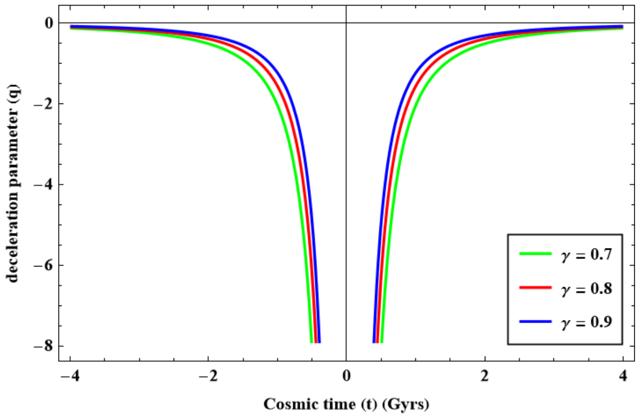


Figure 3. Displays the approach of the deceleration parameter (q) against cosmic time (t).

4. ANALYSIS OF MODEL'S DYNAMICAL PARAMETERS

The appropriate functioning of the dynamical parameters is essential for the sustainability of any cosmological model, and its results ought to align with an equitable agreement. Disparities point to systemic errors arising from varying analytical methods or cosmological observations. The expression's free parameters and the selected representative values for the model might also be linked with its origin. Recent findings of cosmic acceleration reveal that the EoS parameter and pressure must be negative, preferably in both the immediate and distant future. Moreover, a non-singular bounce requires both a positive energy density and negative pressure. Due to this, we now examine the model's dynamic features concerning the patterns of behavior exhibited during bouncing in the light of the scale factor that is taken into account in the present investigation. From field equations (3)-(4), the energy density and pressure for GR are expressed as

$$\rho = \frac{3\gamma^4 t^2}{\left(1+\gamma^2 t^2\right)^2} \tag{17}$$

and

$$p = -\frac{\left(2+t^2\gamma^2\right)\gamma^2}{\left(1+t^2\gamma^2\right)^2}. \tag{18}$$

Using barotropic EoS, $p=\omega\rho$, the EoS parameter (ω) can be obtained as

$$\omega = -\frac{1}{3} - \frac{2}{3\gamma^2 t^2}. \tag{19}$$

From the field equations (12)-(14), the pressure, the energy density and the EoS parameter for $f(R,T)$ gravity are expressed as

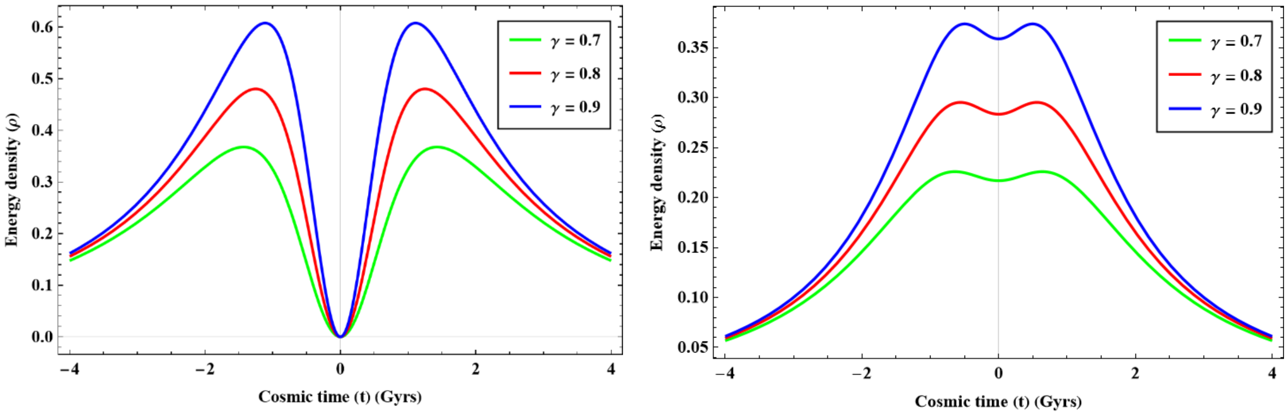
$$p = \frac{1}{\left(\beta^2-\eta^2\right)\left(1+t^2\gamma^2\right)^2}[2\eta\gamma^2 + (\eta-3\beta)\gamma^4 t^2 + \left(1+\gamma^2 t^2\right)^2(\beta-\eta)\Lambda_0], \tag{20}$$

$$\rho = \frac{1}{(\beta^2-\eta^2)\left(1+\gamma^2 t^2\right)^2}[\beta\gamma^2\left(2+\gamma^2 t^2\right) - 3\eta\gamma^4 t^2 - \left(1+\gamma^2 t^2\right)^2(\beta-\eta)\Lambda_0], \tag{21}$$

and

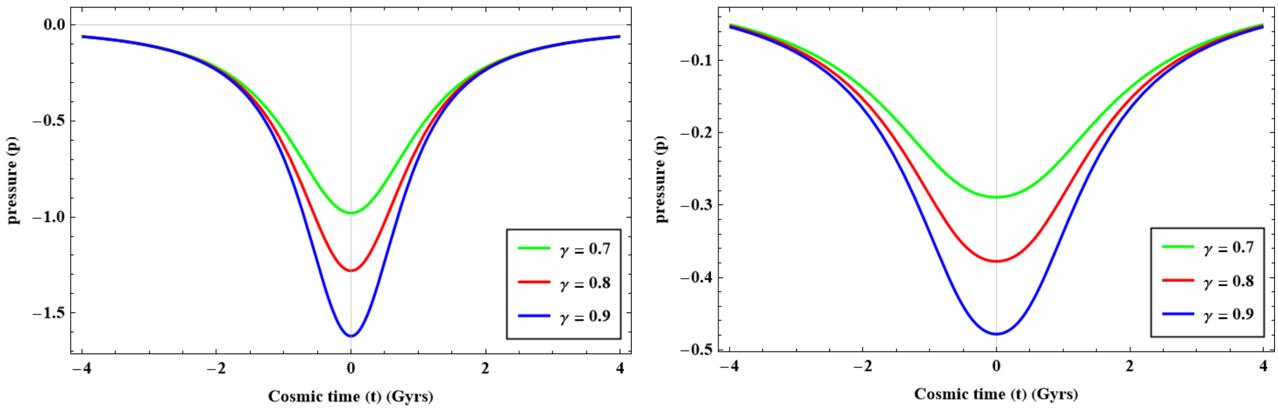
$$\omega = -1 + \frac{2\gamma^2(\beta+\eta)\left(\gamma^2 t^2-1\right)}{3\eta\gamma^4 t^2 - \left(2+\gamma^2 t^2\right)\beta\gamma^2 + \left(1+\gamma^2 t^2\right)^2(\beta-\eta)\Lambda_0}. \tag{22}$$

For graphical illustration, we have used the considered values of the parameter $\gamma = 0.7, 0.8, 0.9$ with $\beta = -5.8$ and $\Lambda_0 = 0.001$ [55]. The progression of energy density across cosmic time for both cases of GR and $f(R,T)$ gravity is depicted in Fig. 4 for three assigned values of parameters γ.The graphical representations in Fig. 4 illustrate the energy density over cosmic time related to the bouncing model. It shows a downward trajectory (indicating contraction) before the bounce and a subsequent rising tendency (indicating expansion) after the bounce, with a minimum occurring at the bounce point. Throughout cosmic time, both the contracting and expanding models maintain positive energy density, although there is a negative trend that culminates at the bounce site. The behavior of pressure (p) up against cosmic time is illustrated in Fig. 5, which validates an assertion that the pressure continues to be negative in both the temporal realms of the contracting as well as the expanding Universe, thereby providing additional backing for the Universe's cosmic acceleration.



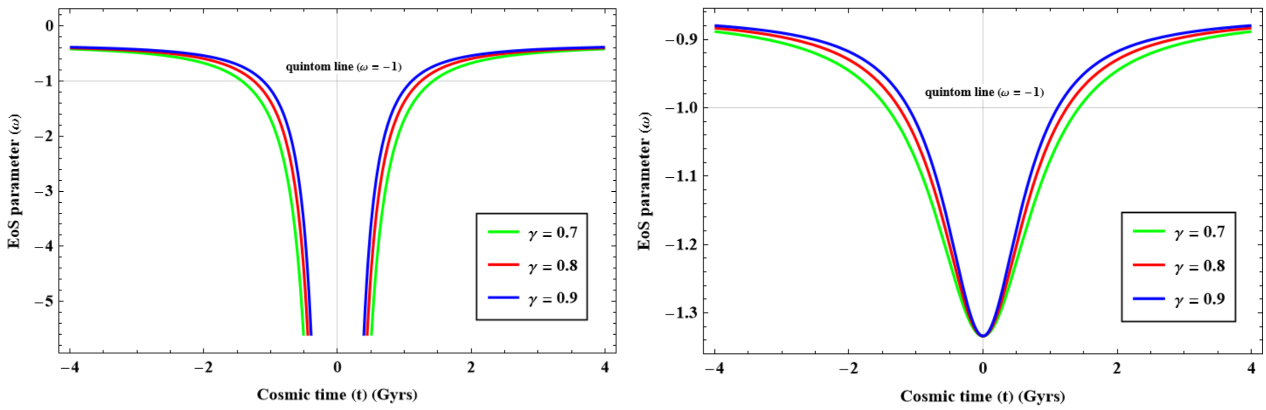
(a) For GR (b) For $f(R,T)$ gravity

Figure 4. Displays the energy density (ρ) for three assigned values of γ.



(a) For GR (b) For $f(R,T)$ gravity

Figure 5. Displays the pressure (p) for three assigned values of γ.



(a) For GR (b) For $f(R,T)$ gravity

Figure 6. The advancement of the EoS parameter (ω) for three assigned values of γ.

Fig. 6 demonstrates the aspects of the EoS parameter's advancement associated with the bouncing model for both the cases of GR and $f(R,T)$ gravity. The EoS parameter downscales all via the phantom divide line $(\omega = -1)$ to the underlying negative value in each respective temporal realm of the contracting and expanding Universe, thereby entering the phantom region $(\omega < -1)$, and it showcases a symmetric trend over the two faces of the bouncing epoch. This indicates that the cosmos is presently undergoing a phantom DE phase in symmetric bouncing cosmology. Moreover, the intriguing traversal of the EoS parameter along the quintom line close to the bouncing spot, as illustrated in Fig. 6, for three conceivable values of these model parameters γ, provides additional supporting evidence for the efficiency and credibility of the suggested bouncing model [71,72].

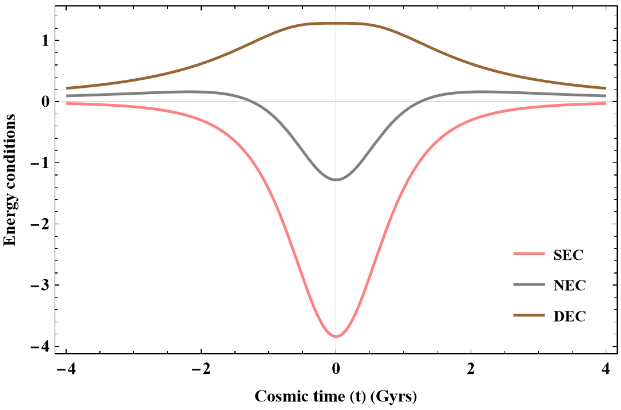
5. ENERGY CONDITIONS

A system of linear equations incorporating density along with pressure referred to as energy conditions (ECs), which points out that gravity remains constantly repulsive; additionally, the energy density is unlikely to be negative. Building feasible bouncing models and investigating the kind of dark energy that sustains them depend on insights from ECs. Each of these ECs which are assimilated into GR and $f(R,T)$ gravity as follows:

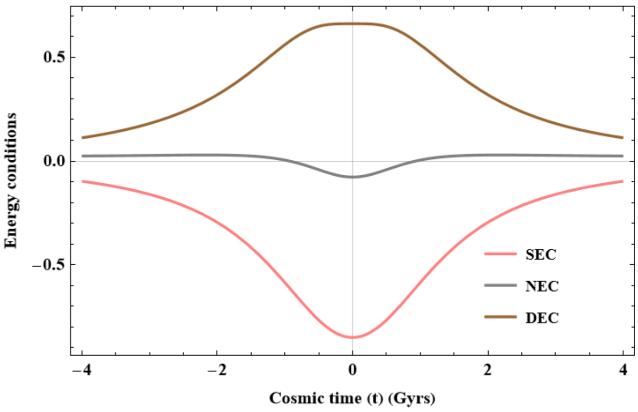
(1) Null Energy condition (NEC) $\Leftrightarrow \rho + p \geq 0$

(2) Strong Energy Condition (SEC) $\Leftrightarrow \rho + 3p \geq 0$

(3) Dominant Energy Condition (DEC) $\Leftrightarrow \rho_{de} \geq |p_{de}|$



(a) For GR



(b) For $f(R,T)$ gravity

Figure 7. Approach of ECs varying over the cosmic time.

For the present bouncing scenario, the graphical representations of NEC, DEC, and SEC of our model varying with the passage of cosmic time for both the cases of GR and $f(R,T)$ gravity are demonstrated in Fig. 7. An essential aspect of the bouncing situation is non-compliance with the ECs. It is abundantly clear from an investigation conducted in [59] that one must account for the violation of NEC and SEC while attempting to accomplish non-singular bouncing solutions within GR. Astrophysical observations [69, 70] must go against the SEC while complying with the persisting statistics of the speeding cosmos. It is noteworthy that an outbreak of dark energy is initiated by an invalidation of the SEC, which thereby validates the Universe's fast expansion. Furthermore, the spatially flat setting that we focused on herein demands NEC violation at the bouncing point for a successful bounce to occur [67]. As per the illustration in Fig. 7, we noticed the overall validation of DEC, whereas SEC is violated for intermediate values of γ. Consequently, our model meets the necessary criteria of NEC violation with constant negative value to enable a successful bouncing mechanism, while averting singularities, and the violation of SEC culminates in the Universe’s accelerating expansion, which is in concurrence with the current scenario of the cosmos.

6. STATEFINDER DIAGNOSTIC

The statefinder diagnostic assists in the investigation of bouncing model, as their time-dependent behavior close to the bounce indicates departures from conventional expansion and aids in finding the consequences of the DE model. These are therefore critically essential for grasping the dynamics of a non-singular, bounce-driven Universe. Sahni et al. [73] accompanied by Alam et al. [74] established the statefinder diagnostic pair $\{r,s\}$ by exploiting the derivatives of the scaling factor with r as the jerk parameter whereas s represents the physical entities of DE, which are described by $r \equiv \frac{\dddot{a}}{aH^3}$ and $s \equiv \frac{r-1}{3(q-1/2)}$. For $(r,s)=(1,1)$, the model reflects the cold dark matter (CDM) limit, while $(r,s)=(1,0)$ belongs to the ΛCDM limit. When $r<1$, it contributes to the quintessence DE era, the phantom DE region for $s>0$

while the trajectory for the Chaplygin Gas (CG) model arrives for $r>1$ with $s<0$. These two statefinder settings turn out to be $r=-\frac{3}{\gamma^2 t^2}$ with $s=\frac{2}{3}\left(1+\frac{1}{2+\gamma^2 t^2}\right)$ and their relation is expressed as

$$r=\frac{3s-2}{2(s-1)} \tag{23}$$

The deviation of r against s is depicted in Fig. 8 which signifies that our model approaches the ΛCDM model as it crosses the point $(r,s)=(1,0)$, while serving variations thereafter by covering the DE regions, including CG, quintessence, and phantom over the course of the Universe's development, which further supports the model's consistency with our earlier studies [75–77].

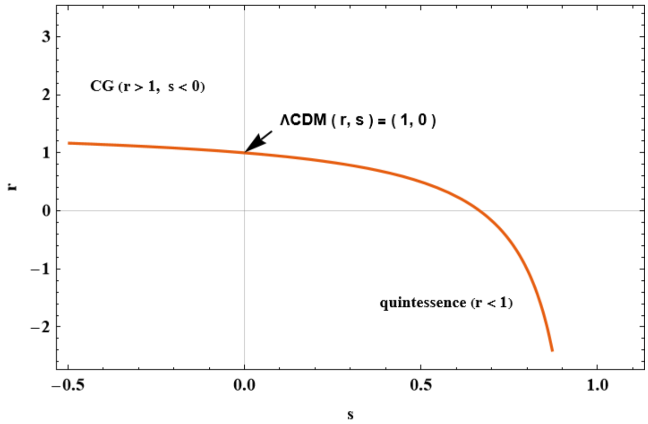
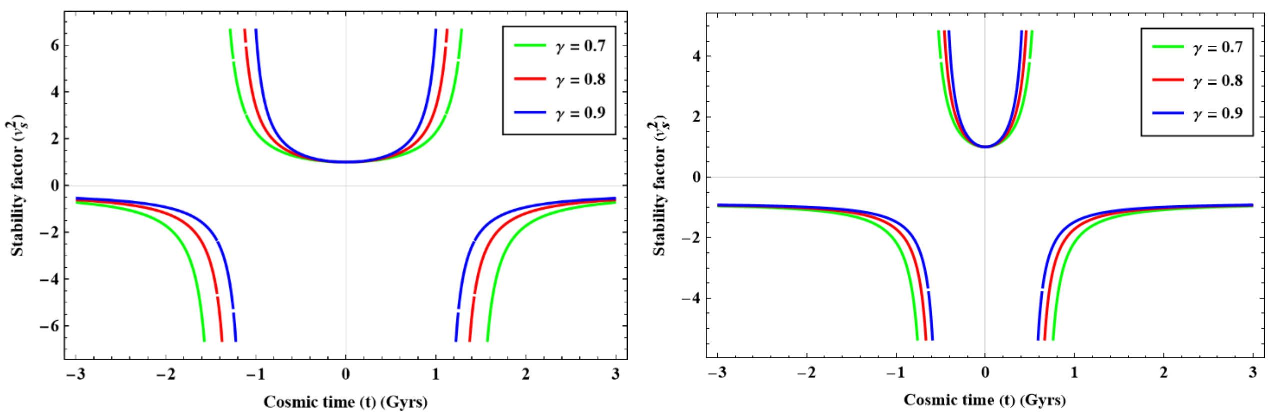


Figure 8. The deviations of r against s.

7. STABILITY OF THE UNIVERSE

Figuring out the stability of an observatory model is crucial for examining distorting behavior while forecasting the progression of the cosmos. The worthwhile indicator for assessing a dark energy model's stability in opposition to perturbations is the square of sound speed v_s^2 as studied in [49] which is prescribed by $v_s^2=dp/d\rho$. The model's stability is specified by the sign of v_s^2. If v_s^2 is positive, the model is rendered stable; otherwise, it grows into unstable. The expressions for v_s^2 for our GR and $f(R,T)$ gravity framework are obtained, respectively, as

$$v_s^2=\frac{3+\gamma^2 t^2}{3(1-\gamma^2 t^2)} \text{ and } v_s^2=\frac{3\beta(1-\gamma^2 t^2)+\eta(3+\gamma^2 t)}{\beta(3+\gamma^2 t^2)+3\eta(1-\gamma^2 t^2)}. \tag{24}$$



(a) For GR (b) For $f(R,T)$ gravity

Figure 9. Approach of stability factor v_s^2 varying over the cosmic time.

For three specified values of γ, Fig. 9 displays the variations of the stability factor C_s^2 over the cosmic time. At the bouncing site and its neighbourhood, the stability factor $C_s^2>0$ which foresees stable behaviour for the bouncing period, while the stability factor $C_s^2 < 0$ thereafter indicates instability over a prolonged time frame. Our model showcases bouncing conduct near $t = 0$, we draw the inference that our model is stable in the close proximity of the bouncing point.

8. CONCLUSIONS

In this study on the bouncing model within the light of the GR and $f(R,T)$ gravity, we examined the cosmic speed singularity over a prolonged phase along with the exploration of the bouncing conduction. The incorporation of our model aims to elucidate the dynamics of the early Universe, particularly through the bounce, as well as the current era of accelerated expansion. Our approach was to prevent singularities and facilitate a smooth progression from contraction to expansion. We consider three conceivable values of model parameters as $\gamma = 0.7, 0.8, 0.9$ for obtaining the graphical presentations of various dynamic parameters of our model in accordance with the patterns of behavior observed during bouncing. The illustrations in the Figs. 1 and 2 display that the scale factor and Hubble parameter are respectively certain to be constrained by an essential condition of minimum non-zero scale factor $a(t)$ and Hubble parameter H being zero at the bouncing point owing to its symmetrical trajectory that transitions from contraction to expansion through a process of bounce whereas Fig. 3 shows that the deceleration parameter propels from $q = -1$ to significant negative value while approaching to the point where the bounce occurs. The following presents a snapshot of the findings we observed for both the cases of the GR and $f(R,T)$ gravity:

- Our bouncing model has effectively shown an identifiable phase change from contraction to expansion, as evidenced by the graphical representations of both the energy density (ρ) over cosmic time for all assigned values of model parameters γ (see Fig. 4). The model's effectiveness in illustrating this pivotal shift is highlighted by the consistent downward trajectory leading up to the bounce, followed by a rising trend post-bounce. Additionally, the behavior of pressure (p) throughout the cosmic timeline as reflected in the Fig. 5 reinforces the model's viability, as it remains negative during both the Universe's temporal regions which makes up an additional foundation for the Universe's rapid expansion. This combination of positive energy density along with negative pressure is crucial for achieving a non-singular bounce, further underscoring the attainment of our model.
- The intriguing traversal of the EoS parameter along the quintom line $(\omega = -1)$, close to the bouncing spot, thereby entering the phantom region $(\omega < -1)$ and a symmetric trend over the two faces of the bouncing epoch, as illustrated in Fig. 6 for all assigned values of model parameters γ, provides additional supporting evidence for the efficiency and credibility of the suggested bouncing model [71,72].
- Our study points out how important the violations of energy condition are in establishing a non-singular bouncing model that takes cosmic acceleration into account, especially the NEC near the bouncing spot and SEC thoroughly as displayed in Fig. 7. The SEC breach offers reliable evidence of the rapid expansion of the cosmos. Our model accomplishes the required criteria of the NEC violation near the bouncing spot, thereby enabling an efficient bounce mechanism that refrains from singularities.
- The statefinder diagnostic suggests that our model covers the DE components such as quintessence, phantom, and Chaplygin gas, thereby approaching subsequently to the ΛCDM model during cosmic progression as demonstrated in Fig. 8. At the bouncing spot, the stability factor C_s^2 supports the model's requirements for a bounce, as shown in Fig. 9.

ORCID

A.Y. Shaikh, https://orcid.org/0000-0001-5315-559X; **A.P. Jenekar**, https://orcid.org/0009-0005-8928-3725

REFERENCES

[1] A.G. Riess, et al., "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," Astron. J. **116**, 1009–1038 (1998). https://doi.org/10.1086/300499

[2] S. Perlmutter, *et al.*, "Measurements of Ω and Λ from 42 High-Redshift Supernovae," Astrophys. J. **517**, 565–586 (1999). https://doi.org/10.1086/307221

[3] M. Tegmark, *et al.*, "Cosmological parameters from SDSS and WMAP," Phys. Rev. D, **69**, 103501 (2004). https://doi.org/10.1103/PhysRevD.69.103501

[4] E. Komatsu, *et al.*, "Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation," Astrophys. J. Suppl. Ser. **192**, 18 (2011). https://doi.org/10.1088/0067-0049/192/2/18

[5] S. Nojiri, and S.D. Odintsov, "The new form of the equation of state for dark energy fluid and accelerating universe," Phys. Lett. B, **639**, 144–150 (2006). https://doi.org/10.1016/j.physletb.2006.06.065

[6] S. Nojiri, and S.D. Odintsov, "Introduction to modified gravity and gravitational alternative for dark energy," Int. J. Geom. Methods Mod. Phys. **04**, 115–145 (2007). https://doi.org/10.1142/S0219887807001928

[7] A. Silvestri, and M. Trodden, "Approaches to understanding cosmic acceleration," Rep. Prog. Phys. **72**, 096901 (2009). https://doi.org/10.1088/0034-4885/72/9/096901

[8] R.R. Caldwell, and M. Kamionkowski, "The Physics of Cosmic Acceleration," Annu. Rev. Nucl. Part. Sci. **59**, 397–429 (2009). https://doi.org/10.1146/annurev-nucl-010709-151330
[9] P. Astier, and R. Pain, "Observational evidence of the accelerated expansion of the universe," Comptes Rendus Phys. **13**, 521-538 (2012). https://doi.org/10.1016/j.crhy.2012.04.009
[10] K. Bamba, *et al.*, "Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests," Astrophys. Space Sci. **342**, 155–228 (2012). https://doi.org/10.1007/s10509-012-1181-8
[11] S.M. Carroll, W.H. Press, and E.L. Turner, "The Cosmological Constant," Annu. Rev. Astron. Astrophys. **30**, 499–542 (1992). https://doi.org/10.1146/annurev.aa.30.090192.002435
[12] V. Gorini, A. Kamenshchik, U. Moschella, and V. Pasquier, "The Chaplygin gas as a model for dark energy," arXiv:gr-qc/0403062 (2004). https://doi.org/10.48550/ARXIV.GR-QC/0403062
[13] M.S. Turner, and M. White, "CDM models with a smooth component," Phys. Rev. D, **56**, R4439–R4443 (1997). https://doi.org/10.1103/PhysRevD.56.R4439
[14] Z.-H. Zhu, M.-K. Fujimoto, and D. Tatsumi, "Determining the cosmic equation of state using future gravitational wave detectors," Astron. Astrophys. **372**, 377–380 (2001). https://doi.org/10.1051/0004-6361:20010458
[15] B. Ratra, and P.J.E. Peebles, "Cosmological consequences of a rolling homogeneous scalar field," Phys. Rev. D, **37**, 3406–3427 (1988). https://doi.org/10.1103/PhysRevD.37.3406
[16] V. Sahni, and L. Wang, "New cosmological model of quintessence and dark matter," Phys. Rev. D, **62**, 103517 (2000). https://doi.org/10.1103/PhysRevD.62.103517
[17] S. Weinberg, "The cosmological constant problem," Rev. Mod. Phys. **61**, 1–23 (1989). https://doi.org/10.1103/RevModPhys.61.1
[18] P.J.E. Peebles, and B. Ratra, "The cosmological constant and dark energy," Rev. Mod. Phys. **75**, 559–606 (2003). https://doi.org/10.1103/RevModPhys.75.559
[19] A. De Felice, and S. Tsujikawa, "f(R) Theories," Living Rev. Relativ. **13**, 3 (2010). https://doi.org/10.12942/lrr-2010-3
[20] V. Faraoni, "f(R) gravity: successes and challenges," (2008). https://doi.org/10.48550/ARXIV.0810.2602
[21] K.R. Mule, S.P. Gaikwad, and V.G. Mete, "Bianchi type-III holographic dark energy cosmological model in f (R , T) theory of gravitation," New Astron. **105**, 102087 (2024). https://doi.org/10.1016/j.newast.2023.102087
[22] P.K. Sahoo, B. Mishra, and S.K. Tripathy, "Kaluza–Klein cosmological model in f(R, T) gravity with Λ(T)," Indian J. Phys. **90**, 485–493 (2016). https://doi.org/10.1007/s12648-015-0759-8
[23] M.J.S. Houndjo, "Reconstruction of f(R, T) Gravity Describing Matter Dominated and Accelerated Phases," Int. J. Mod. Phys. D, **21**, 1250003 (2012). https://doi.org/10.1142/S0218271812500034
[24] R. Zaregonbadi, M. Farhoudi, and N. Riazi, "Dark matter from f (R, T) gravity," Phys. Rev. D, **94**, 084052 (2016). https://doi.org/10.1103/PhysRevD.94.084052
[25] S.B. Fisher, and E.D. Carlson, "Reexamining f (R, T) gravity," Phys. Rev. D, **100**, 064059 (2019). https://doi.org/10.1103/PhysRevD.100.064059
[26] H. Shabani, and M. Farhoudi, "f(R,T) cosmological models in phase space," Phys. Rev. D, **88**, 044048 (2013). https://doi.org/10.1103/PhysRevD.88.044048
[27] P. Sarmah, and U.D. Goswami, "Observational constraints on cosmological parameters in the Bianchi type III Universe with f(R,T) gravity theory," Phys. Dark Universe, **48**, 101932 (2025). https://doi.org/10.1016/j.dark.2025.101932
[28] E. Barrientos, *et al.*, "Metric-affine f(R,T) theories of gravity and their applications," Phys. Rev. D, **97**, 104041 (2018). https://doi.org/10.1103/PhysRevD.97.104041
[29] H. Velten, and T.R.P. Caramês, "Cosmological inviability of f(R,T) gravity," Phys. Rev. D, **95**, 123536 (2017). https://doi.org/10.1103/PhysRevD.95.123536
[30] P.J. Steinhardt, and N. Turok, "Cosmic evolution in a cyclic universe," Phys. Rev. D, **65**, (2002). https://doi.org/10.1103/physrevd.65.126003
[31] A. Ijjas, and P.J. Steinhardt, "Fully stable cosmological solutions with a non-singular classical bounce," Phys. Lett. B, **764**, 289-294 (2017). https://doi.org/10.1016/j.physletb.2016.11.047
[32] T. Kobayashi, "Generic instabilities of nonsingular cosmologies in Horndeski theory: A no-go theorem," Phys. Rev. D, **94**, (2016). https://doi.org/10.1103/physrevd.94.043511
[33] D. Battefeld, and P. Peter, "A critical review of classical bouncing cosmologies," Phys. Rep. **571**, 1–66 (2015). https://doi.org/10.1016/j.physrep.2014.12.004
[34] R. Brandenberger, and P. Peter, "Bouncing Cosmologies: Progress and Problems," Found. Phys. **47**, 797–850 (2017). https://doi.org/10.1007/s10701-016-0057-0
[35] J. Martin, and P. Peter, "Parametric amplification of metric fluctuations through a bouncing phase," Phys. Rev. D, **68**, 103517 (2003). https://doi.org/10.1103/PhysRevD.68.103517
[36] T. Singh, R. Chaubey, and A. Singh, "Bouncing cosmologies with viscous fluids," Astrophys. Space Sci. **361**, 106 (2016). https://doi.org/10.1007/s10509-016-2696-1
[37] F.R. Klinkhamer, and Z.L. Wang, "Nonsingular bouncing cosmology from general relativity," Phys. Rev. D, **100**, 083534 (2019). https://doi.org/10.1103/PhysRevD.100.083534
[38] E. Battista, "Nonsingular bouncing cosmology in general relativity: physical analysis of the spacetime defect," Class. Quantum Gravity, **38**, 195007 (2021). https://doi.org/10.1088/1361-6382/ac1900
[39] A.S. Agrawal, *et al.*, "Global phase space analysis for a class of single scalar field bouncing solutions in general relativity," Eur. Phys. J. C, **84**, (2024). https://doi.org/10.1140/epjc/s10052-023-12377-y
[40] S. Chakraborty, S. Mishra, and S. Chakraborty, "A dynamical system analysis of bouncing cosmology with spatial curvature," Gen. Relativ. Gravit. **56**, 83 (2024). https://doi.org/10.1007/s10714-024-03265-1
[41] A.Y. Shaikh, and B. Mishra, "Bouncing scenario of general relativistic hydrodynamics in extended gravity," Commun. Theor. Phys. **73**, 025401 (2021). https://doi.org/10.1088/1572-9494/abcfb2
[42] A.S. Agrawal, L. Pati, S.K. Tripathy, and B. Mishra, "Matter bounce scenario and the dynamical aspects in f(Q,T) gravity," Phys. Dark Universe, **33**, 100863 (2021). https://doi.org/10.1016/j.dark.2021.100863

[43] A. S Agrawal, S.K. Tripathy, S. Pal, and B. Mishra, "Role of extended gravity theory in matter bounce dynamics," Phys. Scr. **97**, 025002 (2022). https://doi.org/10.1088/1402-4896/ac49b2
[44] A.Y. Shaikh, "Exploring the bouncing cosmological models in symmetric teleparallel gravity," Int. J. Geom. Methods Mod. Phys. **20**, 2350220 (2023). https://doi.org/10.1142/S0219887823502201
[45] M. Zubair, *et al.*, "Bouncing universe scenarios in an extended gravitational framework involving curvature-matter coupling," J. High Energy Astrophys. **44**, 243–265 (2024). https://doi.org/10.1016/j.jheap.2024.10.005
[46] L.V. Jaybhaye, R. Solanki, and P.K. Sahoo, "Bouncing cosmological models in f(R,Lm) gravity," Phys. Scr. **99**, 065031 (2024). https://doi.org/10.1088/1402-4896/ad4838
[47] M. Sharif, M.Z. Gul, and I. Hashim, "Theoretical insights and implications of bouncing cosmology in f(R,T^2) theory," Eur. Phys. J. C, **84**, 1094 (2024). https://doi.org/10.1140/epjc/s10052-024-13473-3
[48] M. Koussour, *et al.*, "Bouncing behavior in f(R, Lm) gravity: Phantom crossing and energy conditions," Int. J. Geom. Methods Mod. Phys. **21**, 2450184 (2024). https://doi.org/10.1142/S0219887824501846
[49] Shaily, J.K. Singh, D. Sethi, R. Rani, and K. Bamba, "Bouncing cosmology and the dynamical stability analysis in f(R,Lm) gravity," Nucl. Phys. B, **1013**, 116854 (2025). https://doi.org/10.1016/j.nuclphysb.2025.116854
[50] A.Y. Shaikh, and A.P. Jenekar, "Cosmological implications of dark energy with bouncing scenario in f(R,Lm) gravity," Int. J. Geom. Methods Mod. Phys. S0219887826500222 (2025). https://doi.org/10.1142/S0219887826500222
[51] S.K. Tripathy, R.K. Khuntia, and P. Parida, "Bouncing cosmology in an extended theory of gravity," Eur. Phys. J. Plus, **134**, 504 (2019). https://doi.org/10.1140/epjp/i2019-12879-3
[52] S.K. Tripathy, B. Mishra, S. Ray, and R. Sengupta, "Bouncing universe models in an extended gravity theory," Chin. J. Phys. **71**, 610–622 (2021). https://doi.org/10.1016/j.cjph.2021.03.026
[53] S. Bhattacharjee, and P.K. Sahoo, "Comprehensive analysis of a non-singular bounce in f(R,T) gravitation," Phys. Dark Universe, **28**, 100537 (2020). https://doi.org/10.1016/j.dark.2020.100537
[54] P. Sahoo, S. Bhattacharjee, S.K. Tripathy, and P.K. Sahoo, "Bouncing scenario in f(R,T) gravity," Mod. Phys. Lett. A, **35**, 2050095 (2020). https://doi.org/10.1142/S0217732320500959
[55] A.S. Agrawal, *et al.*, "Bouncing Cosmology in Extended Gravity and Its Reconstruction as Dark Energy Model," Fortschritte Phys. **70**, 2100065 (2022). https://doi.org/10.1002/prop.202100065
[56] A.Y. Shaikh, *et al.*, "Bouncing cosmological model with general relativistic hydrodynamics in extended gravity," Int. J. Geom. Methods Mod. Phys. **19**, 2250193 (2022). https://doi.org/10.1142/S0219887822501936
[57] T. Harko, F.S.N. Lobo, S. Nojiri, and S.D. Odintsov, "f(R,T) gravity," Phys. Rev. D, **84**, 024020 (2011). https://doi.org/10.1103/PhysRevD.84.024020
[58] R.H. Brandenberger, "The Matter Bounce Alternative to Inflationary Cosmology," arXiv:1206.4196 [astro-ph.CO] (2012). https://doi.org/10.48550/arXiv.1206.4196
[59] M. Novello, and S. Bergliaffa, "Bouncing cosmologies," Phys. Rep. **463**, 127–213 (2008). https://doi.org/10.1016/j.physrep.2008.04.006
[60] C. Barragán, G.J. Olmo, and H. Sanchis-Alepuz, "Bouncing cosmologies in Palatini f(R) gravity," Phys. Rev. D, **80**, 024016 (2009). https://doi.org/10.1103/PhysRevD.80.024016
[61] K. Bamba, *et al.*, "Bounce cosmology from $F(R)$ gravity and $F(R)$ bigravity," J. Cosmol. Astropart. Phys. **2014**, 008–008 (2014). https://doi.org/10.1088/1475-7516/2014/01/008
[62] S.K. Tripathy, and B. Mishra, "Phantom cosmology in an extended theory of gravity," Chin. J. Phys. **63**, 448–458 (2020). https://doi.org/10.1016/j.cjph.2019.12.022
[63] B. Mishra, S.K. Tripathy, and S. Tarai, "Accelerating models with a hybrid scale factor in extended gravity," J. Astrophys. Astron. **42**, 2 (2021). https://doi.org/10.1007/s12036-020-09655-6
[64] Yu.L. Bolotin, *et al.*, "Cosmology In Terms Of The Deceleration Parameter. Part I," arXiv:1502.00811 [gr-qc] (2015). https://doi.org/10.48550/ARXIV.1502.00811
[65] G.P. Singh, and B.K. Bishi, "Bulk Viscous Cosmological Model in Brans-Dicke Theory with New Form of Time Varying Deceleration Parameter," Adv. High Energy Phys. **2017**, 1–24 (2017). https://doi.org/10.1155/2017/1390572
[66] H. Matsui, F. Takahashi, and T. Terada, "Non-singular bouncing cosmology with positive spatial curvature and flat scalar potential," Phys. Lett. B, **795**, 152–159 (2019). https://doi.org/10.1016/j.physletb.2019.06.013
[67] M. Koussour, and N. Myrzakulov, "Bouncing cosmologies and stability analysis in symmetric teleparallel f(Q) gravity," Eur. Phys. J. Plus, **139**, 799 (2024). https://doi.org/10.1140/epjp/s13360-024-05574-5
[68] B.P. Schmidt, *et al.*, "The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Curvature of the Universe Using Type Ia Supernovae," Astrophys. J. **507**, 46 (1998). https://doi.org/10.1086/306308
[69] J.L. Tonry, *et al.*, "Cosmological Results from High-z Supernovae," Astrophys. J. **594**, 1 (2003). https://doi.org/10.1086/376865
[70] A.G. Riess, *et al.*, "Type Ia Supernova Discoveries at z > 1 from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution," Astrophys. J. **607**, 665 (2004). https://doi.org/10.1086/383612
[71] Y.-F. Cai, and X. Zhang, "Evolution of metric perturbations in a model of nonsingular inflationary cosmology," J. Cosmol. Astropart. Phys. **2009**, 003 (2009). https://doi.org/10.1088/1475-7516/2009/06/003
[72] Y.-F. Cai, R. Brandenberger, and X. Zhang, "The matter bounce curvaton scenario," J. Cosmol. Astropart. Phys. **2011**, 003 (2011). https://doi.org/10.1088/1475-7516/2011/03/003
[73] V. Sahni, *et al.*, "Statefinder—A new geometrical diagnostic of dark energy," J. Exp. Theor. Phys. Lett. **77**, 201–206 (2003). https://doi.org/10.1134/1.1574831
[74] U. Alam, *et al.*, "Exploring the expanding Universe and dark energy using the statefinder diagnostic," Mon. Not. R. Astron. Soc. **344**, 1057–1074 (2003). https://doi.org/10.1046/j.1365-8711.2003.06871.x
[75] A.Y. Shaikh, and A.P. Jenekar, "Cosmological Dynamics of Barrow Holographic Dark Energy in f(G) gravity: An Analytical and Observational Approach," New Astron. 102497 (2025). https://doi.org/10.1016/j.newast.2025.102497
[76] A.Y. Shaikh, and A.P. Jenekar, "Constraining the Hybrid Model for Investigating Holographic Dark Energy in Modified Gravity," East Eur. J. Phys. (3), 26–40 (2025). https://doi.org/10.26565/2312-4334-2025-3-03

[77] A.Y. Shaikh, A.P. Jenekar, and S.M. Shingne, "Constrained Dynamics of Holographic Dark Energy in Modified f (R) Gravity," East Eur. J. Phys. (3), 4–16 (2025). https://doi.org/10.26565/2312-4334-2025-3-01

КОНЦЕПТУАЛЬНЕ ПОРУШЕННЯ ЕНЕРГЕТИЧНИХ УМОВ У КОСМОЛОГІЇ ВІДСКОКУ

А.Й. Шайх[1], С.М. Шінгне[2], А.П. Дженекар[3]
[1]Кафедра математики, Індіра Ганді Кала Махавідьялая, Ралегаон-445402. (M.S.) Індія
[2]Кафедра математики, Коледж наук, мистецтв та комерції ім. Г.С., Кхамгаон-444312. (M.S.) Індія
[3]Кафедра математики, мистецтв, комерції та науки, Коледж Марегаон-445303. (M.S.) Індія

Ця стаття зосереджена на дослідженні моделі відскоку в плоскому Всесвіті Фрідмана-Робертсона-Вокера (FRW). Параметр рівняння стану (EoS), що стосується розглянутої моделі, визначає особливості функціонування Всесвіту. Кінематику разом із фізичними атрибутами динамічних параметрів моделі всебічно досліджують. У цьому сценарії ми дослідили кілька енергетичних умов (ЕУ). Діагностична пара, що складається з методу визначення стану та параметра ривка, досліджується для ідентифікації суттєво різних космічних фаз. Ми використовували параметр квадрата швидкості звуку , розроблений для задоволення потреб аналізу стійкості нашої моделі. Наш аналіз показав, що результати нашого дослідження відповідають закономірностям, що спостерігаються у сценаріях відскоків, пропонуючи метод пояснення космічного прискорення, а також проблеми сингулярності у нашому Всесвіті.

Ключові слова: *сценарій відскоків; метрика FRW; гравітація; космічне прискорення*