

FUSION OF WEAKLY-BOUND ${}^9\text{Be}$ WITH HEAVY NUCLEI ${}^{208}\text{Pb}$: A MULTI-BODY THREE-STAGE CLASSICAL MOLECULAR DYNAMICS APPROACH

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The fusion of the weakly bound ${}^9\text{Be}$ nucleus with ${}^{208}\text{Pb}$ is investigated using a multi-body three-stage classical molecular dynamics (3S-CMD) approach. This model explicitly treats ${}^9\text{Be}$ as a cluster of ${}^4\text{He}$ and ${}^5\text{He}$, allowing for the relaxation of rigid-body constraints on both projectile fragments and the target. In this paper, the influence of these constraints on the complete fusion (CF) cross-section, considering both central and non-central collisions is studied. Systematic removal of rigidity constraints, particularly on the target and the ${}^9\text{Be}$ fragments, significantly affects the CF cross-section, especially at sub-barrier energies. Calculations show that relaxing these constraints enhances CF, indicating the important role of breakup and internal degrees of freedom. The multibody 3S-CMD model provides a tool for understanding the interplay of breakup and fusion in reactions involving weakly bound nuclei.

Keywords: Weakly-bound nuclei; Heavy-ion collisions; Fusion reactions; Classical Molecular Dynamics

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1. INTRODUCTION

Investigations of nuclear reactions involving weakly bound nuclei have been made possible due to recent developments in experimental techniques and the availability of radioactive beams. Weakly bound nuclei exhibit unique phenomena like increased breakup probabilities, neutron transfer, and complete and incomplete fusion processes. In complete fusion, the entire charge of the projectile is captured by the target, irrespective of the breakup of the projectile. Incomplete fusion refers to the capture of one or more fragments of the projectile taking place, followed by the breakup of the projectile. So, it is essential to understand the effects of breakup on fusion reaction, as it significantly changes the possibilities of various reaction channels.

Fusion reactions near the Coulomb barrier are strongly influenced by the intrinsic degrees of freedom, such as the rotational and vibrational modes of the interacting nuclei [1]. Numerous studies have also explored how breakup processes impact fusion outcomes [2–7]. Significant suppression of complete fusion and an enhancement in incomplete fusion at energy over the fusion barrier have been shown by precision measurements for the ${}^9\text{Be} + {}^{208}\text{Pb}$ reaction [4]. These behaviors are attributed to the breakup of the ${}^9\text{Be}$, which can occur through channels such as ${}^8\text{Be} + n$ ($S_n = 1.67$ MeV) and ${}^5\text{He} + {}^4\text{He}$ ($S_\alpha = 2.42$ MeV).

Reactions involving weakly bound nuclei, such as ${}^6\text{Li} + {}^{209}\text{Bi}$ and ${}^7\text{Li} + {}^{209}\text{Bi}$ have been studied using the Multi-Body Three-Stage Classical Molecular Dynamics (3S-CMD) model [8]. In this paper, we investigate the role of various rigidity constraints and their relative importance in determining the fusion cross-section for the ${}^9\text{Be} + {}^{208}\text{Pb}$ system using the multi-body 3S-CMD model. This study includes calculations of complete fusion (CF) while systematically relaxing the rigid-body constraints on the nuclei involved in the collisions. The details of the model are presented in § 2, while the results for fusion cross-sections for ${}^9\text{Be} + {}^{208}\text{Pb}$ are discussed in § 3. Finally, conclusions are summarized in § 4.

2. MODEL DETAILS

The multibody 3S-CMD model is used to simulate the collision of ${}^9\text{Be} + {}^{208}\text{Pb}$. In this model, ${}^9\text{Be}$ is made up of a cluster of ${}^4\text{He}$ and ${}^5\text{He}$ nuclei held together in a way that matches the observed break-up energy of ${}^9\text{Be}$ into ${}^4\text{He}$ and ${}^5\text{He}$ (2.42 MeV). A potential minimization code using a soft-core Gaussian NN potential given by,

$$V_{ij}(r_{ij}) = -V_0 \left(1 - \frac{C}{r_{ij}} \right) \exp \left(-\frac{r_{ij}^2}{r_0^2} \right) \text{ MeV}, \quad (1)$$

is used to create projectile fragments and targets where the typical form of the Coulomb potential between protons is,

$$V_C(r_{ij}) = \frac{1.44}{r_{ij}} \text{ MeV}. \quad (2)$$

However, a purely phenomenological potential is selected, and its parameters are set to approximately match the ground state properties of the target and projectile fragments. The potential parameter set $V_0 = 710.0$ MeV, $C = 1.88$ fm, and $r_0 = 1.15$ fm is used to produce ground state properties of projectile fragments and target mentioned in Table 1.

Table 1. Ground-state properties.

Nucleus	Calculated		Experimental	
	B.E. (MeV)	R (fm)	B.E. (MeV) [9]	R (fm) [10]
^4He	14.48	1.32	28.29	1.68
^5He	22.34	1.46	27.41	–
^9Be ($^4\text{He} + ^5\text{He}$)	39.23	1.87	58.16	2.51
^{208}Pb	1841.92	6.04	1636.46	5.50

These three stages are involved in the simulation of the multibody 3S-CMD model:

- (1) Rutherford Trajectories: Initially, the target and projectile are brought together along their classical Rutherford trajectories, considering their Coulomb interaction.
- (2) Classical Rigid Body Dynamics (CRBD): The system is dynamically evolved using CRBD to approach a distance close to the fusion barrier, accounting for the collective motion and interactions.
- (3) Classical Molecular Dynamics (CMD): The entire multibody system undergoes CMD evolution, allowing for interactions and dynamic evolution of the system.

In stage 3, one or more projectile fragments are constrained to remain rigid, those nuclei are dynamically evolved as in the CRBD calculation. For ^9Be , the rigidity constraint on the bond between ^4He and ^5He , as well as on the target ^{208}Pb are relaxed when the center of mass distance, R_{cm} becomes less than 14 fm. One of the fragments of projectile ^4He is always kept rigid. Allowing ^5He in the projectile to be non-rigid enables the possibility of its breakup.

3. FUSION CROSS SECTION

The complete fusion cross-section is calculated for weakly bound ^9Be induced collisions to enable a quantitative comparison between model calculations and experimental results. Theoretically, the colliding nuclei are assumed to fuse when they overcome the potential barrier between them and become trapped in a potential pocket. The fusion cross section for a given collision energy E_{cm} is calculated using Wong's formula [11]. Classically, fusion cross sections vanish at energies below the barrier, but here the barrier penetrability is accounted for in the calculation using Wong's formula,

$$\sigma_{\text{fus}}(E_{\text{CM}}) = \frac{R_B^2 \hbar \omega_0}{2E_{\text{CM}}} \ln \left[1 + \exp \left(\frac{2\pi(E_{\text{CM}} - V_B)}{\hbar \omega_0} \right) \right], \quad (3)$$

where, V_B , R_B , and ω_0 represent the barrier parameters. For higher energies, the Wong's formula can be approximated to classical formula,

$$\sigma_{\text{fus}}(E_{\text{c.m.}}) = \pi R_B^2 \left[1 - \frac{V_B}{E_{\text{c.m.}}} \right]. \quad (4)$$

So, at higher energies, the fusion cross sections obtained using the classical formula and Wong's formula are nearly the same; however, below the barrier, Wong's formula accounts for barrier penetrability, making it more suitable for our calculations [12].

The complete fusion cross section, σ_{CF} is defined as the capture of the total projectile charge. This process includes two mechanisms: direct complete fusion, which involves the capture of the projectile charge without breakup, and sequential complete fusion, which involves the capture of the entire projectile charge following the breakup of the projectile.

For head-on collisions ($b = 0$), the orientation-averaged fusion cross section is computed by averaging over a large number of Monte Carlo-sampled initial orientations, spanning collision energies both above and below the barrier. Calculated σ_{CF} for $^9\text{Be} + ^{208}\text{Pb}$ reaction with various assumptions of rigid body constraints for projectile fragments ($^4\text{He} + ^5\text{He}$), the bond between them and target (^{208}Pb), is shown in Figure 1 comparing it with experimental CF cross sections. In the SBPM (Static Barrier Penetration Model) [13] calculations, the nuclei are assumed to be rigid, and all rotational and vibrational degrees of freedom are suppressed, thereby neglecting any dynamical effects. As a result, the calculated complete fusion cross section (σ_{CF}) is suppressed across all energies, as shown in Figure 1. To investigate the influence of rigidity constraints, three cases are considered. In Case (a), the rigidity constraint on ^{208}Pb is removed, while the projectile ^9Be remains rigid. In Case (b), along with the removal of the rigidity constraint on $^9\text{Be} + ^{208}\text{Pb}$, the bond between the projectile fragments ^4He and ^5He (constituents of ^9Be) is removed, while ^4He and ^5He are kept rigid. In Case (c), the rigidity constraint on ^5He is also relaxed.

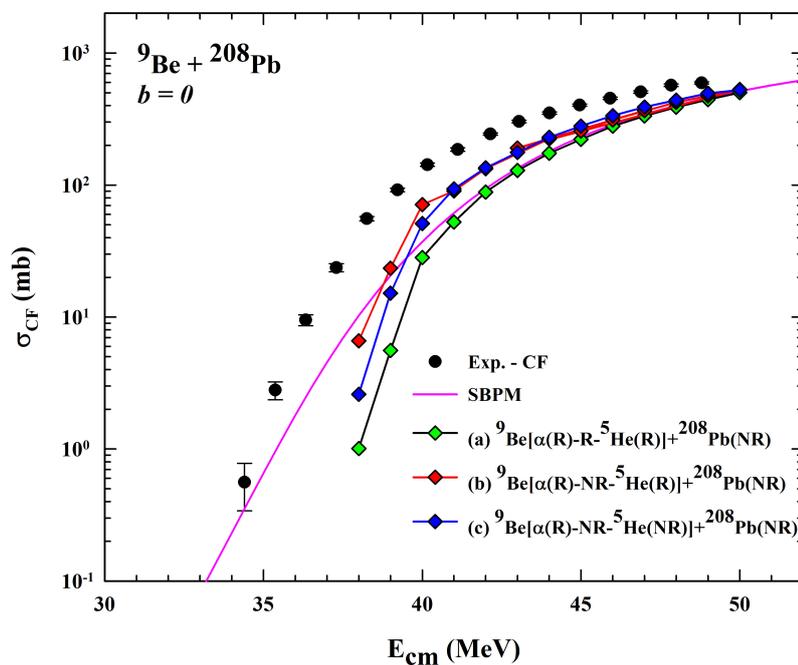


Figure 1. For ($b = 0$), σ_{CF} for ${}^9\text{Be} + {}^{208}\text{Pb}$ reaction with various assumptions of rigid body constraints for a projectile, the bond between them, and the target. Experimental Complete Fusion Cross section (Exp.-CF) [14].

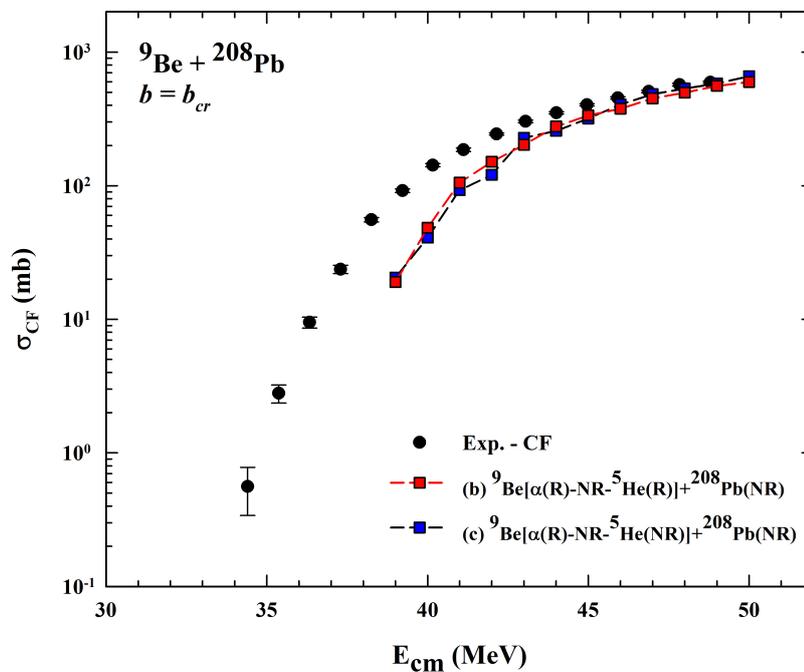


Figure 2. For ($b = b_{\text{cr}}$), σ_{CF} for ${}^9\text{Be} + {}^{208}\text{Pb}$ reaction with various assumptions of rigid body constraints for a projectile, the bond between them, and the target. Experimental Complete Fusion Cross section (Exp.-CF) [14].

As seen in Figure 1, although (σ_{CF}) is suppressed at all energies, the suppression is particularly pronounced at below-barrier energies. Near the barrier energies, (σ_{CF}) improves as the rigidity constraints on the target and projectile are systematically removed, compared to the SBPM calculation. This signifies the dissipation of energy at below-barrier energies as rigidity constraints are relaxed. However, the multibody 3S-CMD calculations appear to follow the trend as the experimental results. For below- and near-barrier energies (σ_{CF}) in case (b) shows improvements compared to cases (a) and (c).

The CF cross-section is also calculated for non-central collisions ($b > 0$). The critical impact parameter for CF, ($b_{cr} - CF$), is determined dynamically during simulation, using the sharp cutoff approximation, where, all trajectories with ($b < b_{cr} - CF$) result in complete fusion, while those with ($b > b_{cr} - CF$) either lead to scattering or incomplete fusion (ICF). The CF cross section determined from ($b_{cr} - CF$) for cases (b) and (c) and shown in Figure 2. As evident from Figure 2, the fusion cross sections for non-central collisions are larger than those for central collisions at higher energies, while at lower energies they nearly coincide. This behavior indicates that as energy increases, non-central trajectories with finite orbital angular momentum can overcome the centrifugal barrier and contribute more to fusion, whereas central collisions correspond only to zero angular momentum $l = 0$. At lower energies, higher angular momentum contributions are strongly suppressed, making the fusion cross sections for central and non-central collisions nearly identical. This energy-dependent behavior is consistent with Wong's formula, where higher angular momentum partial waves become significant only above the Coulomb barrier. Furthermore, for near- and sub-barrier energies, the CF cross section in case (b) shows an improvement compared to case (c), reflecting the influence of different modeling assumptions on fusion probability.

4. CONCLUSIONS

The effects of various degrees of freedom on the ${}^9\text{Be} + {}^{208}\text{Pb}$ reaction are studied using the multi-body Three-Stage Classical Molecular Dynamics (3S-CMD) approach. By appropriately modelling the weakly bound ${}^9\text{Be}$ and systematically relaxing rigidity constraints on the projectile and target, fusion cross sections for both central ($b = 0$) and non-central ($b > 0$) collisions are calculated. The complete fusion (CF) cross-section, σ_{CF} , is significantly influenced by various levels of rigidity constraints both on the target and projectile. For below-barrier energies, the complete fusion cross section is suppressed, while for near-the-barrier energy, it enhances as rigidity constraints are relaxed, indicating the importance of internal degrees of freedom. Mainly for below and near the barrier energies, trends of the multibody 3S-CMD model calculation are consistent with experimental data. Complete fusion cross sections, σ_{CF} , further enhance for non-central collisions. This demonstrates the applicability of this model to study systems with weakly bound nuclei in fusion dynamics.

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ЗЛИТТЯ СЛАБКОЗВ'ЯЗАНИХ ${}^9\text{Be}$ З ВАЖКИМИ ЯДРАМИ ${}^{208}\text{Pb}$: БАГАТОЧАСТИНКОВИЙ ТРЬОХЕТАПНИЙ ПІДХІД КЛАСИЧНОЇ МОЛЕКУЛЯРНОЇ ДИНАМІКИ

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Злиття слабозв'язаного ядра ${}^9\text{Be}$ з ${}^{208}\text{Pb}$ досліджується за допомогою багаточастинкового тристадійного підходу класичної молекулярної динаміки (3S-CMD). Ця модель явно розглядає ${}^9\text{Be}$ як кластер ${}^4\text{He}$ та ${}^5\text{He}$, що дозволяє послабити обмеження твердого тіла як на фрагментах снаряда, так і на мішені. У цій статті досліджується вплив цих обмежень на поперечний переріз повного злиття (CF), враховуючи як центральні, так і нецентральні зіткнення. Систематичне видалення обмежень жорсткості, зокрема на мішені та фрагментах ${}^9\text{Be}$, суттєво впливає на поперечний переріз CF, особливо при підбар'єрних енергіях. Розрахунки показують, що послаблення цих обмежень посилює CF, що вказує на важливу роль розпаду та внутрішніх ступенів свободи. Багаточастинкова 3S-CMD-модель надає інструмент для розуміння взаємодії розпаду та синтезу в реакціях, що включають слабозв'язані ядра.

Ключові слова: слабкозв'язані ядра; зіткнення важких іонів; реакції синтезу; класична молекулярна динаміка