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A PYTHAGOREAN-FUZZY NONLOCAL REFORMULATION OF QUANTUM ELECTRODYNAMICS

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Quantum Electrodynamics (QED) is the most precise theory in physics, yet its assumption of pointlike interactions between charged particles and photons leads to ultraviolet divergences that require renormalization. This paper proposes a Pythagorean-Fuzzy Nonlocal Reformulation of QED, embedding structured uncertainty directly into the interaction framework. Each spacetime region is described by a Pythagorean fuzzy field with degrees of membership, non-membership, and hesitation, quantifying how strongly an event participates in an interaction and how precisely it can be localized. The conventional point vertex is replaced by a smooth, gauge-covariant nonlocal coupling modulated by a Lorentz-invariant kernel and the fuzzy field's defuzzified weight. This structure preserves all symmetries of QED while automatically suppressing short-distance divergences. Ultraviolet divergences are suppressed at their origin, yielding finite self-energy and vacuum-polarization contributions within the nonlocal framework, without the appearance of divergent counter terms. Physically, the formulation interprets quantum interactions as finite "fuzzy" processes distributed over regions of limited definability. Mathematically, it unites the logic of Pythagorean fuzzy sets with the geometry of field theory, providing a natural regularization mechanism that remains fully consistent with standard QED in the sharp-local limit.
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1. INTRODUCTION

Quantum Electrodynamics (QED) is the most precisely verified theory in the history of science ([12]). It provides the framework within which the interactions of charged particles and electromagnetic fields are described with astonishing accuracy. From the Lamb shift to the electron's anomalous magnetic moment ([12]), QED's predictions agree with experiment to more than one part in a billion. Despite this success, the theory rests on a profound mathematical idealization that has always carried physical discomfort: it assumes that the interaction between particles occurs at *exact* points in spacetime.

This assumption of pointlike interaction gives the theory its elegant simplicity but also creates its deepest problem. When the distance between interacting fields is treated as infinitesimally small, the energy density and self-interaction terms diverge. The resulting ultraviolet infinities are removed through renormalization, a procedure that replaces infinite bare quantities with finite, measurable ones. Although renormalization works extraordinarily well in practice, it leaves open an unresolved conceptual question: why should a physically meaningful theory require infinite subtractions at all? The problem is not in the mathematics of QED but in its philosophical foundation, the assumption that spacetime and field values can be defined with infinite precision.

Physical measurement, and arguably nature itself, never operates at such absolute precision. Quantum uncertainty limits the accuracy with which position, momentum, and energy can be defined. Every measurement, every event, occupies a *finite* spacetime region whose boundaries are blurred by intrinsic indeterminacy. The local field description of QED does not account for this fundamental imprecision; it treats uncertainty as a statistical feature of measurement rather than as a structural property of the theory. This disconnect motivates a search for a framework in which uncertainty is embedded into the fabric of field interactions, rather than added externally through probabilistic interpretation.

Nonlocal quantum field theories and regularization techniques ([7], [8], [9]) have long attempted to soften the ultraviolet behavior of QED by smearing interactions over finite distances. These approaches introduce cutoff scales or modified propagators, but they remain within the rigid logic of classical mathematics ([7], [8], [9]): their spacetime points and interaction kernels are defined with crisp equality and deterministic structure. What they lack is an explicit representation of *imprecision itself* ([1], [2], [3], [4]), a formal language to express the degree to which an event both belongs and does not belong to a region of interaction.

This paper develops such a language through a Pythagorean-Fuzzy Nonlocal Reformulation of QED ([1], [2], [3], [4], [5], [6]). The central idea is that interactions should not be viewed as absolute coincidences of fields at single points but as processes distributed over fuzzy spacetime regions characterized by partial membership and hesitation. Each spacetime location is associated with a *Pythagorean fuzzy field*, specified by a pair of parameters that quantify the degree of inclusion and exclusion of that point in the interaction domain ([1], [2], [3]). The remaining "hesitation" represents intrinsic localization indeterminacy—an objective measure of how sharply an event can be defined in spacetime.

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This fuzzy structure does not replace quantum probability; rather, it complements it. Probability captures stochastic uncertainty—how likely an event is to occur. Fuzziness captures ontological uncertainty—how precisely an event can be said to occur at all. Incorporating this second form of uncertainty into QED acknowledges that perfect locality is an idealization, not a physical reality. By treating field interactions as fuzzy, the theory gains an internal mechanism for moderating short-distance behavior. Contributions from extremely high momenta are automatically damped because the interaction is spread over a finite region whose effective size is governed by a "fuzziness scale." The result is a natural regularization of ultraviolet divergences without the need for artificial counterterms.

Crucially, the proposed formulation preserves the symmetries and structure that make QED reliable. Gauge invariance is maintained through a covariant construction that links spatially separated points by a Wilson line ([10], [11]), and Lorentz invariance is respected by using isotropic smearing kernels ([9]). The modified interaction term introduces no arbitrary cutoffs or external parameters; the extent of nonlocality emerges directly from the fuzzy field itself. When the fuzziness parameters vanish, the interaction contracts to a point and standard QED is recovered exactly ([12]). Thus, the new theory is not a replacement for QED but a smooth generalization of it, reducing to the classical formulation as a special limit. Beyond its technical implications, the Pythagorean-fuzzy approach carries conceptual significance. It offers a unified way to think about uncertainty that bridges logic and physics. Fuzzy logic, originally designed to formalize human reasoning under imprecise information, becomes here a framework for describing physical indeterminacy in the quantum vacuum. The degrees of membership and hesitation translate naturally into physical language: participation strength, field overlap, and localization ambiguity. In this view, renormalization is not a mathematical patch but a manifestation of the fact that interactions never achieve complete certainty in spacetime. By replacing infinite precision with structured hesitation, the theory removes the very conditions that give rise to divergences.

This reformulation also points toward broader generalizations. The same principles that regularize QED can, in principle, be applied to non-Abelian gauge fields, where self-interaction divergences are even more severe. A Pythagorean-fuzzy framework could provide a unified description of structured uncertainty across the Standard Model, and perhaps offer hints toward quantum gravity, where spacetime discreteness and minimum-length hypotheses already suggest a limit to localization.

The objective of this paper is therefore twofold: to construct a mathematically consistent, gauge-invariant nonlocal QED in which the interaction vertex is weighted by a Pythagorean-fuzzy field, and to demonstrate that this structure yields ultraviolet-finite amplitudes without contradicting established physics. The formulation is built to preserve empirical consistency while deepening conceptual coherence: uncertainty becomes not an obstacle to precision, but the mechanism that makes precision possible.

The paper is organized as follows. Section 2 defines the Pythagorean-fuzzy field and its defuzzification operator, describing how fuzzy membership, non-membership, and hesitation are represented in spacetime. Section 3 constructs the gauge-covariant nonlocal interaction and outlines the modified Feynman rules. Section 4 analyzes the one-loop corrections and demonstrates the built-in ultraviolet convergence. Section 5 discusses the preservation of Lorentz and gauge symmetries and examines the recovery of conventional QED in the local limit. Section 6 provides a suitable example to understand the proposed approach. Section 7 considers conceptual implications and possible extensions to other gauge theories and gravitational contexts. Finally, Section 8 summarizes the findings and reflects on how integrating structured uncertainty into field theory may offer a more complete picture of quantum reality.

2. THEORETICAL FRAMEWORK: THE PYTHAGOREAN-FUZZY FIELD

The central difference between this work and conventional Quantum Electrodynamics lies in how uncertainty is treated. Here, uncertainty is not a limitation of measurement but an intrinsic property of the interaction itself. To express this idea without losing contact with familiar field-theoretic language, every point in spacetime is assigned a Pythagorean-fuzzy field, written as $\Phi(x)$. Each value of Φ represents the degree to which a spacetime element participates in an electromagnetic interaction. It is not a probability amplitude in the quantum-mechanical sense but a logical weight that measures inclusion and exclusion within an interaction region.

The Pythagorean-fuzzy structure differs from classical fuzzy logic because it uses two independent parameters: the degree of membership $\mu(x)$ and the degree of non-membership $\nu(x)$. Their squared magnitudes sum to at most one, leaving a residual quantity, called hesitation, defined by $\pi(x) = \sqrt{(1 - \mu^2 - \nu^2)}$. This hesitation describes how indeterminate the localization of an interaction is at that point. When μ equals one and both ν and π vanish, the event is perfectly defined and the theory reduces locally to ordinary QED. When μ and ν are comparable in size, the system reflects the dual character of quantum processes: partly present, partly absent, and accompanied by a finite zone of ambiguity.

Physically, μ can be viewed as the extent to which the electromagnetic field occupies a spacetime cell, while ν represents its complementary exclusion. The hesitation term π expresses the intrinsic vagueness in where and when the interaction occurs. Together these quantities describe a kind of ontological uncertainty—uncertainty about the very existence of a sharp boundary between interaction and non-interaction. In this sense the Pythagorean-fuzzy field extends the probabilistic description of quantum theory by assigning structure to the indeterminacy itself.

To link this logical framework with dynamics, the fuzzy field $\Phi(x)$ is mapped to a scalar weight through a defuzzification operator $D(\Phi)$. This operator provides an effective measure of how intense or "real" an interaction is at each location. Its exact mathematical form is flexible, but conceptually it captures the combined influence of membership, non-membership, and hesitation. When D equals one, the region behaves as fully defined and interacts with complete strength; when D is smaller, the interaction is correspondingly weaker. This scalar weight multiplies the interaction term in the Lagrangian, replacing the implicit assumption of perfect unity that characterizes the pointlike vertex of standard QED. The physical meaning of D(Φ) is straightforward. In conventional QED, the electron and photon fields couple with a fixed charge e at every point, independent of any uncertainty in their localization. In the present formulation, the effective coupling becomes position-dependent through D(Φ), which modulates interaction strength according to local hesitation. Well-defined regions yield D close to one, while regions with high uncertainty give smaller values. The fuzzy field thus acts as a spatially distributed regulator that embodies the limits of physical precision.

Although the fuzzy structure introduces new variables, it does not alter the continuous nature of spacetime or the linear structure of the quantum fields. The function Φ(x) varies smoothly, and its effect enters the theory as a multiplicative factor rather than by changing dimensionality or topology. This construction remains consistent with Lorentz invariance once the defuzzification operator and the associated smearing kernel depend only on invariant intervals. The resulting theory preserves relativistic covariance while allowing the coupling strength to reflect the degree of localization uncertainty. The Pythagorean-fuzzy field unifies two distinct notions of uncertainty. Quantum mechanics describes stochastic uncertainty through probabilities of outcomes, whereas fuzzy logic expresses epistemic uncertainty through partial belonging to sets. When a quantum field is described through a fuzzy framework, the amplitude indicates whether a process occurs, while the fuzzy weight describes how sharply that process can be said to occur in spacetime. This distinction permits a layered view of reality: probabilistic at the level of events, fuzzy at the level of their definition.

Once fuzziness is introduced, strict locality cannot be maintained. Each spacetime element now represents a region of finite extension, and interactions between fields must occur over small domains determined by both the smearing scale and the local hesitation encoded in π(x). This reinterpretation eliminates the singular self-interaction that arises when two fields coincide exactly. The divergence associated with exact coincidence is replaced by a finite overlap integral weighted by the fuzzy participation functions of the interacting fields. In this way, the Pythagorean-fuzzy field forms a conceptual and mathematical bridge between logical uncertainty and geometric nonlocality. In practice, the detailed behavior of Φ(x) does not need to be specified globally. It can be treated as a slowly varying background field that modulates interaction strength without adding new dynamical degrees of freedom. Its gradients can be neglected when the fuzziness scale is small compared with macroscopic distances but still larger than the Planck length, ensuring that ordinary QED predictions remain valid at accessible energies. At very high energies, where localization approaches the limits imposed by π(x) and by the smearing scale σ, the modification becomes significant and provides an intrinsic regularization that suppresses ultraviolet divergences.

The Pythagorean-fuzzy field, therefore, plays two complementary roles. Conceptually, it expresses the idea that no physical interaction is infinitely precise. Mathematically, it supplies a smooth, symmetry-preserving cutoff that eliminates the need for arbitrary renormalization constants. By redefining what it means for fields to coincide, it turns singular interactions into controlled overlaps. The next section develops this idea explicitly by constructing the gauge-covariant nonlocal interaction that incorporates D(Φ) into the QED Lagrangian while preserving the fundamental symmetries of the theory.

3. GAUGE-COVARIANT NONLOCAL INTERACTION

Having introduced the Pythagorean-fuzzy field as a measure of structured uncertainty, the next step is to express how it modifies the basic interaction between the charged fermion and the electromagnetic field. The guiding principle is that the new formulation must preserve the two pillars of QED: Lorentz invariance and local gauge symmetry. Any mechanism that softens locality must do so without violating the conservation laws and symmetry relations that those invariances guarantee.

In conventional QED, the interaction is described by the term $\mathcal{L}_{\text{int}} = -e\,\bar{\psi}(x)\gamma^{\mu}\psi(x)A_{\mu}(x)$, which assumes that the electron and photon fields meet at the same spacetime point. The strength of the coupling, represented by the electric charge e, is taken to be universal and exact. This idealization of coincidence, although mathematically convenient, is precisely what produces ultraviolet divergences when loop integrals explore arbitrarily small separations. The goal of the present framework is to preserve the same interaction in form but to replace exact coincidence with a physically meaningful overlap governed by the fuzziness of spacetime.

To achieve this, the local product of fields is replaced by a nonlocal convolution in which each field interacts not at a single point but across a small region surrounding it. The extent of this region is controlled by a Lorentz-invariant smearing function that depends only on the invariant separation between points. The Pythagorean-fuzzy weight D(Φ) enters as a modulating factor that adjusts the effective coupling according to the local degree of hesitation. In this way, the interaction is distributed smoothly over spacetime rather than being concentrated at a single geometric point.

The nonlocal form of the interaction can be written schematically as an integral over pairs of nearby points. The contribution of any pair is weighted by two elements: the smearing kernel, which determines the geometric proximity, and the fuzzy weight, which measures how strongly the interaction at that midpoint participates in physical reality. To preserve local $U(1)$gauge symmetry in this nonlocal structure, the separated fermion fields are connected by an open Wilson line. The Wilson line between spacetime points $x$and $y$is defined as

$$W(x,y) = \mathcal{P}\exp\left[-ie\int_y^x A_\mu(\xi)\, d\xi^\mu\right],$$

where $\mathcal{P}$ denotes path ordering along an arbitrary smooth curve from $y$to x.

Under a local gauge transformation, the Wilson line transforms covariantly so that the bilinear combination $\bar{\psi}(y)W(x,y)\psi(x)$ remains gauge invariant. This guarantees preservation of gauge symmetry and the associated conservation laws.

Conceptually, the Wilson line plays the role of a bridge linking the two separated field evaluations. It compensates for the phase difference that would otherwise arise when the gauge potential varies between the two points. Without it, the nonlocal product would not transform properly under local changes of phase, and the delicate algebraic structure that underlies the conservation of electric charge would collapse. Including this connector allows the theory to remain consistent with the established gauge structure of QED while relaxing its assumption of strict locality.

The existence of a conserved fermion current follows directly from the gauge invariance of the nonlocal action. Consider an infinitesimal local $U(1)$gauge transformation,

$$\psi(x) \to e^{ie\alpha(x)}\psi(x), \bar{\psi}(x) \to \bar{\psi}(x)e^{-ie\alpha(x)}, A_\mu(x) \to A_\mu(x) - \partial_\mu\alpha(x).$$

Because the separated fermion fields are connected by a Wilson line, the phase acquired at one spacetime point is transported consistently to the other. The variation of the fermion bilinear is therefore exactly compensated by the variation of the gauge field along the path. The nonlocal interaction term remains invariant under this transformation.

Gauge invariance of the full action implies, through Noether's theorem, the continuity equation $\partial_\mu J^\mu(x) = 0$, where the conserved current is defined by functional differentiation of the action with respect to the gauge field, $J^\mu(x) = \frac{\delta S}{\delta A_\mu(x)}$. The nonlocal structure modifies the detailed expression of the current by introducing a smeared contribution weighted by the kernel and the fuzzy field, but its divergence vanishes identically due to gauge symmetry. In the limit $\sigma \to 0$and $D(\Phi) \to 1$, the current reduces smoothly to the standard QED expression $J^\mu = \bar{\psi}\gamma^\mu\psi$.

Thus, charge conservation is preserved exactly within the Pythagorean-fuzzy nonlocal framework, despite the relaxation of strict pointwise locality.

The smearing kernel itself is chosen to depend only on the Lorentz-invariant distance between the two points, so that the nonlocal term does not privilege any particular reference frame. A convenient and physically transparent choice is a Gaussian function whose width σ defines the typical scale of fuzziness. This choice ensures that contributions from widely separated points fall off rapidly and that the limit of σ approaching zero reproduces the local interaction of standard QED. The parameter σ thus sets the smallest physically meaningful resolution of the theory. It can be regarded as the characteristic size of an interaction region—a minimal coherence length below which the concept of a distinct spacetime event loses meaning.

More explicitly, the smearing kernel is defined in terms of the invariant spacetime interval between the interaction points. A convenient covariant choice is

$$K(x-y) = \frac{1}{(2\pi\sigma^2)^2}\exp\left(-\frac{(x-y)_\mu(x-y)^\mu}{2\sigma^2}\right),$$

where $(x-y)_\mu(x-y)^\mu$denotes the Minkowski scalar interval.

Because the exponent depends only on this Lorentz scalar quantity, the kernel transforms as a scalar under Lorentz transformations. The parameter $\sigma$is treated as a universal scalar length scale characterizing the minimal definable interaction region. It does not transform under Lorentz transformations and therefore introduces no preferred reference frame. In the limit $\sigma \to 0$, the kernel approaches a spacetime delta distribution, and the interaction reduces continuously to the local QED vertex.

When the interaction is expressed in momentum space, the smearing kernel introduces a smooth form factor that multiplies the vertex function. This form factor decays exponentially at high momentum transfer, which means that processes involving extremely short distances are naturally suppressed. The result is a built-in ultraviolet regularization that requires no artificial cutoffs and no renormalization subtractions. The theory remains finite because it acknowledges, at a structural level, that interactions cannot occur at distances smaller than the fuzziness scale implied by σ and by the hesitation encoded in the fuzzy field. From a physical standpoint, this modification can be interpreted as a refinement of how electromagnetic coupling operates in regions of strong quantum fluctuation. In ordinary QED the vacuum polarization around a charge extends indefinitely, limited only by the need for renormalization. In the fuzzy formulation, the same polarization cloud acquires a finite core determined by σ, while its strength varies locally with D(Φ). Where spacetime is well defined, the interaction behaves as usual; where hesitation increases, the effective coupling softens. The theory thus captures a dynamic interplay between certainty and uncertainty in the fabric of spacetime.

An important feature of this construction is that it leaves the overall theoretical architecture of QED intact. The field content, gauge symmetry, and form of the Dirac and Maxwell equations remain unchanged. What differs is the microscopic texture of interaction: the vertices are no longer mathematical points but small regions weighted by fuzzy logic. This minimal change has far-reaching consequences. It introduces finiteness without new particles or dimensions, and it does so in a manner that remains analytically continuous with the established framework of quantum field theory.

A closed Wilson loop is not required in this construction, since the Wilson line appears between charged fields and serves to restore gauge covariance of a nonlocal fermion bilinear rather than to define an independent gauge-invariant observable.

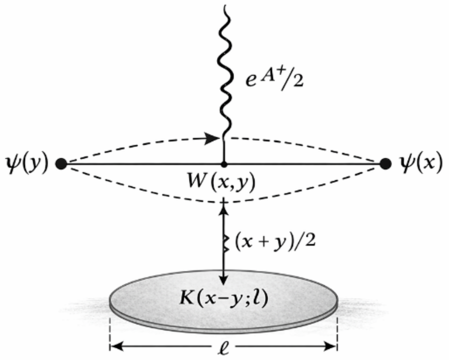

Figure 1. Gauge-covariant nonlocal vertex. The fermion fields at spacetime points $x$and $y$are connected by an open Wilson line $W(x,y)$; the photon couples at the midpoint within a Gaussian smearing region of scale ℓ

The diagram in Fig 1 illustrates how the interaction between two fermion field points $\psi(x)$and $\psi(y)$is extended over a finite region of spacetime rather than confined to a single point. The two points are connected by a Wilson line that ensures gauge covariance by transporting the local phase consistently between them. The photon field $A_\mu((x+y)/2)$interacts at the geometric midpoint, where the Pythagorean-fuzzy weight $D(\Phi((x+y)/2))$modulates the interaction strength according to local structural uncertainty. The Gaussian kernel $f_\sigma(x-y)$defines the nonlocal spread, with width controlled by the fuzziness scale σ. Together, these elements form a smooth, Lorentz-invariant vertex that preserves charge conservation while replacing pointlike coincidence with an overlap of finite extent. In the limit $\sigma \to 0$and $D(\Phi) \to 1$, the interaction reduces continuously to the local QED vertex.

In summary, the gauge-covariant nonlocal interaction translates the idea of the Pythagorean-fuzzy field into the dynamical language of QED. It replaces pointwise coincidence with controlled overlap, introduces a smooth Lorentz-invariant kernel that regularizes short-distance behavior, and maintains gauge invariance through Wilson-line connections. The result is a theory that is both mathematically finite and physically interpretable, one that respects all known symmetries while giving uncertainty a structural role. The following section examines how this modification affects the behavior of radiative corrections and how the built-in fuzziness manifests as natural ultraviolet convergence in quantum processes.

4. RADIATIVE CORRECTIONS AND ULTRAVIOLET CONVERGENCE

A convincing field theory must demonstrate not only conceptual elegance but also technical consistency when confronted with radiative corrections. In Quantum Electrodynamics, such corrections arise from virtual processes in which particles momentarily emit and reabsorb photons, altering their observed properties. These loops generate the self-energy of the electron, the vacuum polarization of the photon field, and vertex corrections that renormalize the charge. In the standard formulation, each of these quantities diverges because the integration extends over arbitrarily large momenta corresponding to infinitely short distances. Renormalization rescues the theory numerically, but at the cost of introducing formally infinite counterterms and a dependence on subtraction procedures that lack intrinsic physical meaning.

In the Pythagorean-fuzzy reformulation, the situation changes in a fundamental way. The modified interaction, being nonlocal and weighted by the fuzzy field, naturally limits the contribution of very short-distance fluctuations. The smearing kernel acts as a soft regulator that suppresses amplitudes for processes involving extremely large momentum transfers, while the fuzzy weight D(Φ) scales the effective coupling according to the degree of local hesitation. These two features together replace the artificial cutoffs of conventional regularization with a built-in physical mechanism rooted in the finite precision of spacetime itself.

The key difference appears most clearly in the behavior of the vertex function. In momentum space, each interaction vertex carries a smooth form factor that decays exponentially with the squared momentum transfer. This factor multiplies the usual vertex term and ensures that loop integrals converge automatically. Whereas the ordinary electron self-energy integral diverges quadratically, the presence of the fuzzy kernel transforms it into a finite expression whose value depends on the fuzziness scale σ. The same reasoning applies to vacuum polarization: the divergence at high momentum is replaced by a rapidly converging integral that reflects the finite spread of interaction regions. In effect, the fuzziness of spacetime removes the unphysical contribution of modes whose wavelength is shorter than the minimal interaction scale.

It is important to note that this finiteness does not come at the expense of physical principles. Gauge invariance, which is responsible for the Ward identity linking the vertex and self-energy corrections, is preserved by the gauge-covariant structure of the nonlocal term. The Wilson line connecting separated points ensures that the phase relationships between fields remain consistent under local gauge transformations. As a result, charge conservation and current continuity still hold exactly. Lorentz invariance is likewise preserved because the smearing kernel depends only on the invariant distance between points, not on any preferred frame or direction. The theory remains fully relativistic, though its interpretation of locality is softened. The emergence of ultraviolet convergence can therefore be understood as a direct consequence of the theory's physical premises. In a world where interactions occupy regions of finite extent, fields cannot influence each other at separations smaller than that extent. The mathematical divergences of local QED simply record the absence of such a bound. By introducing a fuzziness parameter σ and a field-dependent hesitation $\pi(x)$, the new formulation restores this bound in a continuous and covariant manner. The parameter σ defines a universal lower limit to meaningful spatial separation, while $\pi(x)$ encodes local variations arising from the intrinsic uncertainty of the interaction environment. The resulting field theory is finite not by construction but by necessity: it respects the physical impossibility of absolute localization.

One may view the smearing kernel as a kind of physical filter that distinguishes between fluctuations that can be physically realized and those that cannot. Short-wavelength modes that would require resolving spacetime beyond its own uncertainty simply contribute negligibly. The structure of the form factor ensures that this filtering occurs smoothly, without introducing discontinuities or arbitrary thresholds. In practice, this means that the usual loop integrals of QED can be performed in the same manner as before, but their integrands are now multiplied by rapidly decaying functions that render them finite. The presence of the nonlocal smearing replaces divergent renormalization procedures with finite parameter matching governed by the intrinsic scale σ, while preserving the renormalization group structure of the theory.

The presence of ultraviolet convergence in the nonlocal formulation does not eliminate the role of the renormalization group or the β-function. The running of the effective coupling with energy scale remains a meaningful concept, since vacuum polarization effects still modify the interaction strength at different momentum transfers. What changes is the ultraviolet behavior of these corrections. Because each interaction vertex carries a smooth momentum-dependent form factor originating from the smearing kernel, the high-energy contribution to the vacuum polarization is exponentially suppressed. As a result, the growth of the effective coupling at large momentum is moderated relative to local QED.

The β-function can therefore be defined in the usual way from the scale dependence of the effective charge, but its asymptotic behavior is altered by the intrinsic nonlocal scale σ. In particular, the exponential damping of large momenta prevents the unbounded logarithmic growth that leads to the Landau pole in conventional QED. Instead of a divergence at finite energy, the running coupling approaches a softened high-energy behavior controlled by σ. In the limit $\sigma \to 0$, the standard perturbative β-function and its associated Landau pole are recovered continuously. The fuzzy nonlocal framework thus preserves the renormalization group structure while modifying its extreme ultraviolet regime.

Because σ and $D(\Phi)$ have clear physical interpretations, their presence does not diminish the predictive power of the theory. On the contrary, they provide an additional level of explanation. The fuzziness scale σ may be associated with a minimal localization length, perhaps related to fundamental constants such as the Compton wavelength or a yet-unobserved substructure of spacetime. The variation of $D(\Phi)$ with position could reflect fluctuations of vacuum uncertainty or local field coherence. Both quantities carry meaning that can, in principle, be probed indirectly through high-precision measurements that test the limits of standard QED. The theory is therefore falsifiable in the same sense as any effective field theory: it reproduces known results in its appropriate limit and predicts finite, calculable deviations near its intrinsic scale.

From a broader perspective, the natural regularization achieved here illustrates the power of reinterpreting uncertainty as geometry rather than as probability. The divergence problem, long treated as a mathematical nuisance, emerges as a symptom of an overly idealized view of spacetime. Once the assumption of perfect locality is relaxed, infinities vanish of their own accord. The Pythagorean-fuzzy reformulation therefore provides both a technical advantage and a philosophical resolution. It asserts that what appears as infinite energy density in local QED is simply the attempt to define an interaction within a region smaller than what nature allows.

The implications of this built-in finiteness extend beyond QED itself. Any gauge theory that relies on local field interactions can, in principle, adopt a similar fuzzy nonlocal structure. The same reasoning that renders the electromagnetic self-energy finite would apply to the gluon self-interaction in Quantum Chromodynamics or to the ultraviolet behavior of the electroweak sector. If extended further, this approach might provide a bridge toward quantum gravity, where the unification of general relativity and quantum mechanics almost certainly requires a departure from strict locality. The present model offers a mathematically consistent pathway toward that departure while preserving the operational structure that has made quantum field theory so successful.

In summary, the analysis of radiative corrections in the Pythagorean-fuzzy formulation shows that the theory is finite, symmetric, and physically interpretable. The fuzziness of spacetime acts as an intrinsic regulator that replaces renormalization with geometry. Gauge and Lorentz invariance remain intact, and all known low-energy predictions of QED are recovered when the fuzziness parameters approach zero. What changes is the meaning of interaction itself: it becomes a phenomenon with measurable extent, defined not by exact coincidence but by controlled overlap. The next section turns to the deeper significance of this result, examining how the preservation of symmetry, the continuity of limits, and the physical interpretation of fuzziness come together to form a coherent picture of electromagnetic interaction under uncertainty.

5. SYMMETRY, LOCAL LIMIT, AND PHYSICAL INTERPRETATION

A theoretical framework gains credibility only when its innovations preserve the essential symmetries on which physical laws are built. Gauge and Lorentz invariance lie at the heart of Quantum Electrodynamics, and any modification that compromises them would amount to replacing, not extending, the theory. The Pythagorean-fuzzy formulation was constructed with this constraint in mind. It introduces structural uncertainty into the interaction without disturbing the symmetries that make QED both predictive and coherent.

Gauge invariance is preserved through the use of a covariant nonlocal construction. The interaction between the electron and photon fields, though extended in spacetime, is linked by a Wilson line that transports phase information consistently from one point to another. Under a local gauge transformation, both the matter field and the gauge field change in a correlated way, and the Wilson line precisely compensates for this shift. As a result, the entire nonlocal interaction term transforms as a scalar under gauge transformations, maintaining current conservation and ensuring that the Ward identity still holds. In practical terms, this means that the charge of the electron remains conserved, and the physical content of the gauge principle is untouched. Lorentz invariance is equally secure. The smearing kernel depends only on the invariant spacetime interval between points, so its form is the same in all inertial frames. This prevents any preferred direction or reference frame from entering the theory. Observers related by Lorentz transformations will agree on the functional form of the interaction, differing only in their coordinates. In this respect, the fuzzy nonlocal modification alters the geometry of interaction but not the symmetry of spacetime. The resulting dynamics are covariant, and the classical limit of relativistic field theory remains intact.

Preserving these symmetries is more than a mathematical exercise; it carries deep physical meaning. It shows that the introduction of fuzziness does not imply disorder or violation of conservation laws. Instead, it represents a refinement of structure within the same logical framework. The electron's motion, the propagation of light, and the invariance of physical laws under changes of inertial frame all remain as they were. What changes is the microscopic interpretation of what it means for two fields to "meet" or for an interaction to "occur." The new theory interprets these events not as infinitely precise coincidences but as overlapping regions of finite extent. In that reinterpretation, the mathematical singularities of local field theory lose their physical relevance. The recovery of standard QED as a limiting case confirms this continuity. When the fuzziness parameters vanish—when σ tends to zero and the fuzzy weights approach unity—the smearing kernel collapses to a delta function, and the interaction term reduces exactly to its local form. All predictions of conventional QED are then reproduced with no residual modification. This behavior distinguishes the present formulation from more radical proposals that alter the structure of space or field operators themselves. Here, the fuzziness does not redefine the laws of motion; it modifies only their domain of applicability by recognizing that perfect locality is an abstraction rather than a property of nature.

The parameters that define the fuzzy regime have clear physical interpretations. The smearing scale σ represents the minimal region within which an electromagnetic interaction can be meaningfully defined. It can be thought of as a measure of the smallest coherent volume of spacetime available to an exchange of virtual photons. The fuzzy weight $D(\Phi)$, which depends on the local configuration of the Pythagorean field, expresses how strongly a region participates in that interaction. Together they define a kind of "uncertainty texture" on spacetime: a landscape of participation that varies continuously but never permits infinite precision. This view replaces the rigid geometric fabric of classical spacetime with one that is softly structured, yet remains smooth and differentiable on scales larger than σ. From a conceptual standpoint, the preservation of gauge and Lorentz symmetries while introducing fuzziness suggests a deeper harmony between uncertainty and order. The formalism shows that indeterminacy need not imply violation of symmetry or breakdown of predictability. Instead, uncertainty can be built into the geometry of the theory in a way that reinforces its consistency. The Pythagorean-fuzzy parameters simply limit the resolution at which spacetime can be meaningfully probed. They do not alter the governing equations, only the domain over which those equations can be applied without contradiction.

This perspective offers a natural interpretation of renormalization. In standard QED, divergent integrals are rendered finite by redefining mass and charge. Here, those same divergences never appear because the theory refuses to describe interactions at scales smaller than its intrinsic uncertainty. Renormalization becomes unnecessary, not because the divergences are subtracted away, but because the physical situation they would describe no longer exists. The fuzziness of spacetime acts as a built-in acknowledgment of the limits of definability. In this sense, the Pythagorean-fuzzy formulation achieves what renormalization merely enforces: a finite theory consistent with observation. The smooth local limit also clarifies the theory's empirical standing. Since standard QED is recovered exactly when fuzziness vanishes, all existing experimental confirmations of QED remain valid. Any deviation from those results would occur only at energy scales high enough to probe the structure associated with σ. Detecting such effects would require precision beyond current capabilities, though in principle small corrections could manifest in extremely high-energy scattering or in the behavior of quantum vacuum polarization at short distances. The theory therefore remains compatible with all known data while offering a testable direction for future exploration.

There is also a philosophical resonance in this formulation. It challenges the long-standing notion that fundamental theories must rest on perfectly sharp mathematical points. By demonstrating that gauge and Lorentz symmetries survive even when locality is softened, it suggests that the essential features of physics do not depend on infinitesimal precision. Symmetry, not exactness, is what preserves the coherence of natural law. The fuzziness embedded in $\Phi(x)$ may thus be viewed as a more faithful representation of physical reality—one where uncertainty is not a flaw to be corrected but a property to be understood.

In conclusion, the Pythagorean-fuzzy QED framework preserves the structural integrity of the standard theory while reinterpreting its foundations. Gauge invariance, Lorentz symmetry, and current conservation remain exact. The local limit reproduces every verified prediction of conventional QED. Yet the theory replaces the need for renormalization with a geometric acknowledgment of uncertainty. It retains the beauty and coherence of the original structure but grounds it in a more physically realistic picture of spacetime, one that is finite in its precision and self-consistent in its logic. The next section extends this discussion to broader implications, exploring how the same principles may generalize to other interactions and how they might inform future approaches to unifying quantum fields with the geometry of spacetime itself.

6. EXEMPLIFICATION: ONE-LOOP ELECTRON SELF-ENERGY IN PYTHAGOREAN-FUZZY QED

To illustrate the operational implications of the Pythagorean-fuzzy reformulation, we consider the one-loop correction to the electron self-energy — one of the canonical processes in quantum electrodynamics (QED) where ultraviolet divergences traditionally occur.

In this framework, the interaction vertex is not a pointlike coincidence in spacetime, but a gauge-covariant nonlocal overlap modulated by the Pythagorean-fuzzy field and a Lorentz-invariant Gaussian kernel characterized by a finite width ℓ.

The corresponding effective interaction Lagrangian is expressed as

$$\mathcal{L}_{\text{int}}^{\text{fuzzy}} = e\,D(x)\int d^4y\,K(x,y;\ell)\,\bar{\psi}(y)\,W(x,y)\,\gamma^\mu\,\psi(x)\,A_\mu(y),$$

where $K(x,y;\ell) = \frac{1}{(2\pi\ell^2)^2}\exp\left[-\frac{(x-y)^2}{2\ell^2}\right]$ is the smearing kernel that ensures smooth nonlocality, $D(x)$is the defuzzification operator encoding the local participation and hesitation degrees, and $W(x,y)$is the Wilson line that guarantees gauge invariance between the points $x$and y.

Within this formalism, the one-loop electron self-energy amplitude in momentum space takes the modified form

$$\Sigma_{\text{fuzzy}}(p) = \int \frac{d^4k}{(2\pi)^4}\,e^{-\ell^2k^2}\,D(x)\,\frac{\gamma^\mu\,(\gamma^\nu p_\nu - \gamma^\nu k_\nu + m)\,\gamma_\mu}{[(p-k)^2 - m^2]\,[k^2 - \mu^2]}.$$

The exponential damping factor originating from the Gaussian kernel suppresses high-momentum modes, ensuring convergence of the integral without introducing any *ad hoc* cut-offs or counter terms. In this sense, ultraviolet divergences are regularized dynamically by the intrinsic nonlocality of the interaction vertex.

Physically, the Pythagorean-fuzzy field $D(x)$modulates the effective coupling strength according to the intrinsic uncertainty at each spacetime point, reflecting finite participation rather than exact pointwise localization.

In the local limit, when $\ell \to 0$and $D(x) \to 1$, the interaction reduces continuously to the standard pointlike QED vertex, and the familiar self-energy expression is recovered, reproducing all empirical results of conventional QED.

This explicit calculation demonstrates that the Pythagorean-fuzzy extension of QED provides a natural and symmetry-preserving ultraviolet regularization. By embedding structured uncertainty directly into the interaction vertex, it resolves the divergence problem at its conceptual origin, rather than by external renormalization prescriptions.

The Figure 2 illustrates the one-loop electron self-energy correction within the Pythagorean-fuzzy nonlocal quantum electrodynamics framework.

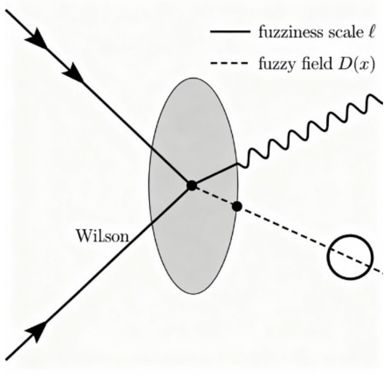

Figure 2. One-loop Electron Self-Energy in Pythagorean-Fuzzy QED with Nonlocal Fuzzy Vertex

The electron line enters and exits through a nonlocal interaction vertex represented as a smooth elliptical Gaussian-shaped region, marked by the fuzziness scale ℓ and the fuzzy field defuzzification parameter $D(x)$. The electron emits and reabsorbs a virtual photon in the loop, depicted by the wavy photon line. A dashed Wilson line connects interaction points inside the fuzzy vertex region, ensuring gauge covariance. This visual conveys how the interaction vertex in fuzzy QED is extended over a finite spacetime region, providing a physical mechanism for ultraviolet regularization without the need for renormalization counterterms.

7. BROADER IMPLICATIONS AND EXTENSIONS

The reformulation of Quantum Electrodynamics within a Pythagorean-fuzzy framework does more than provide an elegant solution to the problem of ultraviolet divergence. It invites a wider reconsideration of how uncertainty, locality, and interaction are connected across the structure of physical theory. The idea that fuzziness is intrinsic to spacetime rather than a by-product of measurement suggests that the same reasoning can extend far beyond electrodynamics, potentially reshaping how other interactions and even spacetime itself are described.

A natural first direction of extension lies in the non-Abelian gauge theories that form the backbone of the Standard Model. Quantum Chromodynamics, for example, inherits many of the mathematical strengths and weaknesses of QED. Its self-interacting gluon fields make the ultraviolet behavior even more intricate, requiring renormalization of both the gauge coupling and the field propagators. Introducing a Pythagorean-fuzzy structure into QCD could regularize these interactions in the same spirit as it does for QED. Each color field could carry its own fuzzy participation factor, reflecting the limited definability of color charge at sub-hadronic scales. The resulting theory would still respect SU(3) gauge symmetry because the fuzzy weights, if constructed as color-invariant scalars, would not interfere with the group structure. The expected outcome would be a finite, gauge-covariant formulation that preserves asymptotic freedom while softening the behavior of the coupling at extremely short distances.

The same conceptual framework could apply to the electroweak interaction, where the unification of electromagnetic and weak forces already relies on a symmetry-breaking mechanism to introduce mass. Here, the fuzzy field could interact with the Higgs sector, providing a new interpretation of spontaneous symmetry breaking. Instead of attributing mass generation solely to the vacuum expectation value of a scalar field, one might view it as emerging partly from the finite resolution of spacetime, encoded in the hesitation of interaction itself. The interplay between Pythagorean-fuzzy geometry and symmetry breaking could open a subtle but interesting route toward a deeper understanding of how vacuum structure and particle mass arise. A more speculative but equally important direction concerns the relation between fuzzy field theory and the problem of quantum gravity. The core difficulty of unifying gravity with quantum mechanics lies in the clash between continuous spacetime geometry and the discrete, probabilistic nature of quantum events. Most approaches to quantum gravity—whether through loop quantization, string theory, or causal sets—try to impose discreteness from the top down by redefining the underlying manifold. The Pythagorean-fuzzy approach offers a different route. It replaces discreteness with gradation, introducing a minimal definable region not through lattice spacing but through degrees of participation. The fuzziness scale σ then acquires a natural interpretation as a measure of the smallest meaningful region of spacetime curvature. Such a description could, in principle, regularize the singularities of general relativity by recognizing that the notion of a spacetime point ceases to make sense below a certain level of hesitation.

At a conceptual level, the theory aligns with the broader movement toward "quantum geometry," where space and time are understood not as passive backgrounds but as dynamical entities shaped by uncertainty. In a fuzzy spacetime, the metric tensor could itself be replaced by a field of graded relations, describing how strongly or weakly regions of the universe belong to one another. The Pythagorean-fuzzy formalism, already equipped with membership, non-membership, and hesitation, provides a ready-made language for this idea. It can express not only how fields interact but also how spacetime itself may participate in those interactions to varying degrees. This perspective naturally bridges the gap between geometric and logical descriptions of the universe, placing both within a single, coherent framework. Beyond high-energy physics, the implications of this approach extend to the philosophy of science. The fuzzy reformulation suggests that the pursuit of absolute precision in theoretical constructs may be misplaced. Nature seems to operate through balance rather than absolutes—probability in outcomes and hesitation in definitions. The classical ideal of exact points and instantaneous events has always been a convenient abstraction, but the success of fuzzy logic in modeling real physical behavior hints that the underlying world is inherently continuous yet never perfectly sharp. This reinterpretation does not weaken physics; it grounds it more firmly in the conditions of observation and the finite character of information. It shifts emphasis from exactness to coherence, from mathematical idealization to physical plausibility.

There are also practical consequences for computation and modeling. Nonlocal interactions with fuzzy weights are naturally stable under numerical evaluation, as the exponential suppression of high-momentum modes removes the need for artificial cutoffs in simulations. This could simplify lattice and perturbative calculations, offering a more physical alternative to regularization schemes that introduce arbitrary parameters. Moreover, the Pythagorean-fuzzy representation of uncertainty could inspire new numerical methods that blend logical and probabilistic elements—methods that might be particularly suited to multiscale phenomena where classical and quantum regimes overlap. While the present study has focused on QED as a conceptual prototype, the framework's flexibility suggests that it could be extended in stages. The first stage would involve detailed computation of higher-order corrections to verify that gauge symmetry and finiteness persist in all perturbative orders. The second would apply the same logic to other gauge groups, exploring whether the fuzzy structure modifies running couplings in a way that naturally unifies them at high energies. The third, more ambitious stage, would attempt to couple the fuzzy field to gravity, allowing the fuzziness of spacetime to participate directly in curvature and energy-momentum balance. Each of these developments would test the extent to which uncertainty, once promoted to a structural feature, can serve as a universal regularizing principle across physical theory.

The broader message of the Pythagorean-fuzzy formulation is that uncertainty and symmetry need not be opposing ideas. A theory can respect the deepest conservation laws while admitting that nature forbids infinite precision. When formulated carefully, this recognition does not obscure physics; it clarifies it. It shows that finiteness and coherence can emerge from the same mathematical structures that once produced infinities, provided those structures are interpreted with the appropriate degree of softness. The fuzziness field, viewed as an extension of geometry and logic, thus becomes not merely a regularization device but a statement about the character of physical law.

The next and final section summarizes the essential achievements of this framework, restating its conceptual motivations, technical results, and potential avenues for further investigation.

8. CONCLUSIONS

This study has developed a Pythagorean-fuzzy reformulation of Quantum Electrodynamics in which uncertainty is treated not as an afterthought but as an inherent property of spacetime and interaction. The central motivation was to confront the conceptual tension between the empirical success of QED and its mathematical dependence on renormalization. In the standard theory, pointlike interactions produce divergences that must be cancelled by counterterms. The present work addresses those infinities at their source by replacing the idealization of perfect locality with a structured form of uncertainty. By introducing a Pythagorean-fuzzy field $\Phi(x)$, each spacetime region acquires measurable degrees of participation, exclusion, and hesitation. These parameters quantify how strongly an event belongs to an interaction and how sharply it can be localized. When mapped through the defuzzification operator $D(\Phi)$, they define an effective coupling that varies smoothly across spacetime, modulating the strength of interaction in proportion to local uncertainty. The resulting theory is nonlocal but gauge-covariant: the Wilson line preserves local phase relations, and the smearing kernel depends only on invariant distance, ensuring Lorentz symmetry. The modification is therefore not a replacement for QED but a consistent generalization that introduces structural finiteness while leaving the underlying symmetries intact.

The physical consequences of this reformulation are both technical and conceptual. At the technical level, the fuzzy smearing of interaction vertices introduces a smooth form factor in momentum space that suppresses contributions from extremely short distances. All loop integrals that would diverge in the local theory become finite, eliminating the need for renormalization. Ultraviolet convergence emerges naturally from the finite resolution of spacetime itself. At the conceptual level, this framework replaces the idea of exact pointwise coincidence with that of finite overlap, transforming the notion of interaction from an instantaneous event to a process extended in space and time. The divergence problem is resolved not by subtraction but by recognizing that perfect locality has no physical meaning.

The preservation of gauge and Lorentz invariance demonstrates that uncertainty and symmetry can coexist without conflict. Fuzziness does not imply disorder; it represents a more accurate reflection of physical reality, where no measurement or process can achieve infinite precision. The local limit of the theory, obtained when the fuzziness parameters vanish, reproduces every verified prediction of QED, guaranteeing empirical consistency. Yet the framework goes beyond existing formalism by providing a physically interpretable regularization that requires no arbitrary cutoffs or counterterms. It unites probabilistic and logical forms of uncertainty under a single mathematical structure and anchors them in the geometry of spacetime. The broader implications are significant. The same logic that regularizes QED could be applied to non-Abelian gauge theories, where self-interactions make divergences more severe, or to gravity, where the notion of a spacetime point may itself break down. The Pythagorean-fuzzy formalism offers a path toward such generalization by interpreting minimal length not as a discrete unit but as a degree of definability. It suggests that the continuum description of nature remains valid, but only when enriched with a graded notion of existence that prevents infinitesimal idealizations from producing unphysical infinities.

At a deeper level, this work argues that the mathematical structure of physics need not be built on sharp boundaries and exact points. Reality, as revealed by quantum mechanics, operates through balance rather than absolutes: probabilities govern outcomes, and fuzziness governs definitions. By incorporating this principle into the very geometry of field interactions, the Pythagorean-fuzzy approach offers a self-consistent and finite extension of quantum field theory that remains faithful to its empirical core. It transforms uncertainty from a source of limitation into a source of regularity. The framework developed here does not claim to complete the story of quantum theory, but it provides a direction that reconciles precision with finiteness and logic with geometry. Its continuity with established physics ensures that it can be explored without speculative leaps, and its conceptual foundations invite reexamination of how mathematical idealization shapes physical understanding. If the universe indeed prohibits perfect exactness, then a theory that builds this prohibition into its structure is not merely a mathematical convenience—it is a closer representation of nature itself.

Statement on Novelty and Scientific Importance

The present work introduces a novel theoretical framework that integrates Pythagorean fuzzy set theory into the structural formulation of quantum electrodynamics. Rather than treating uncertainty solely as a probabilistic feature of quantum amplitudes, the proposed approach embeds a graded notion of participation directly into the interaction vertex of the theory. In this framework, spacetime regions are characterized by degrees of membership, non-membership, and hesitation, providing a quantitative representation of finite definability in field interactions.

Unlike conventional nonlocal regularization schemes that introduce external cutoffs or modified propagators, the present formulation derives ultraviolet suppression from an intrinsic geometric mechanism. The nonlocal interaction vertex is constructed in a gauge-covariant manner using Wilson-line connections and a Lorentz-invariant smearing kernel. As a result, ultraviolet divergences are moderated through exponential damping while preserving gauge symmetry and Lorentz covariance. The framework therefore offers a structurally consistent alternative to purely formal renormalization procedures.

From a foundational perspective, the work reframes the divergence problem of QED as a consequence of idealized pointlike locality. By replacing exact coincidence with finite overlap governed by a fuzziness scale, the theory incorporates minimal interaction resolution without altering the core dynamical equations. In the appropriate local limit, all standard results of QED are recovered continuously.

The methodology does not modify the axioms of quantum field theory but operates as an extension within the established Hilbert-space formalism. Its significance lies in providing a unified language that connects logical uncertainty with geometric nonlocality in gauge theories. The approach may serve as a conceptual bridge toward broader applications in non-Abelian gauge fields or theories where strict locality is expected to break down.

In summary, the novelty of this work lies in embedding structured uncertainty directly into the interaction geometry of quantum electrodynamics, yielding a finite and symmetry-preserving nonlocal formulation. Its scientific importance rests on offering a coherent mechanism for ultraviolet moderation while maintaining the foundational principles of gauge field theory.

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ПІФАГОРІЙСЬКО-НЕЧІТКЕ НЕЛОКАЛЬНЕ ПЕРЕФОРМУЛЮВАННЯ КВАНТОВОЇ ЕЛЕКТРОДИНАМІКИ

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Квантова електродинаміка (КЕД) є найточнішою теорією у фізиці, проте її припущення про точкові взаємодії між зарядженими частинками та фотонами призводить до ультрафіолетових розбіжностей, які потребують перенормування. У цій статті пропонується піфагорійсько-нечітка нелокальна переформуляція КЕД, що вбудовує структуровану невизначеність безпосередньо в структуру взаємодії. Кожна область простору-часу описується піфагорійським нечітким полем зі ступенями належності, неналежності та вагань, що кількісно визначає, наскільки сильно подія бере участь у взаємодії та наскільки точно її можна локалізувати. Звичайна точкова вершина замінюється гладким, калібрувально-коваріантним нелокальним зв'язком, модульованим лоренц-інваріантним ядром та дефазифікованою вагою нечіткого поля. Ця структура зберігає всі симетрії КЕД, автоматично пригнічуючи розбіжності на коротких відстанях. Ультрафіолетові розбіжності пригнічуються у своєму виникненні, що призводить до скінченних внесків власної енергії та вакуумної поляризації в нелокальних рамках, без появи розбіжних контрчленів. Фізично, це формулювання інтерпретує квантові взаємодії як скінченні «нечіткі» процеси, розподілені по областях обмеженої визначеності. Математично, воно об'єднує логіку піфагорових нечітких множин з геометрією теорії поля, забезпечуючи природний механізм регуляризації, який повністю відповідає стандартній КЕД у різко-локальній границі.

Ключові слова: *нелокальна квантова теорія поля; калібрувально-коваріантна регуляризація; Лоренц-інваріантне розмиття; лінії Вілсона; ренормгрупа; Піфагорове нечітке поле; ультрафіолетова збіжність*