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IMPLICIT QUIESCENT OPTICAL SOLITON PERTURBATION HAVING NONLINEAR CHROMATIC DISPERSION AND LINEAR TEMPORAL EVOLUTION WITH KUDRYASHOV'S FORMS OF SELF–PHASE MODULATION STRUCTURE BY LIE SYMMETRY

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The paper retrieves implicit quiescent optical solitons for the nonlinear Schrödinger's equation that is taken up with nonlinear chromatic dispersion and linear temporal evolution. Using a stationary or quiescent approach combined with Lie symmetry analysis, the study systematically examines six distinct self–phase–modulation structures proposed by Kudryashov. The analytical procedure reduces the governing equation to amplitude forms whose solutions are obtained through quadratures, leading to both implicit solitary–wave profiles and one explicit periodic case. The six forms of self–phase modulation structures, as proposed by Kudryashov, yielded solutions in terms of quadratures, periodic solutions as well as in terms of elliptic functions. The existence of each family of solutions is discussed in terms of the admissible parameter relations that ensure physically meaningful solitary profiles. The approach provides a unified framework compared with earlier methods based on direct elliptic–function expansions, highlighting how Lie symmetry facilitates a compact treatment of multiple nonlinear dispersion laws. The results are relevant to understanding stationary optical pulses in nonlinear fibers and photonic crystal fibers, and they establish a foundation for future numerical and experimental studies on nonlinear–dispersion–driven pulse propagation.

Keywords: *Quiescent solitons; Chromatic dispersion; Quadratures; Parameter constraints*

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1. INTRODUCTION

The key ingredients for the propagation of solitons through optical fibers across transcontinetal and transoceanic distances is the the sustainment of the delicate balance between chromatic dispersion (CD) and self–phase modulation (SPM) [1–10]. Any loss of this balance would lead to the solitons getting stalled and/or wave collapse and this would trigger catastrophic consequences. Therefore it is of paramount importance to make sure that the balance is continuously maintained for the smooth propagation of such solitons across intercontinental distances around the planet. Nevertheless, it may so happen that this balance is compromised owing to various factors that are beyond control. One such factor is when the CD is rendered to be nonlinear due to random variations in the fiber diameter or due to the rough handling of fibers during its installation underground or along the ocean bed [2].

The current paper will address this situation. In particular the governing model, namely the nonlinear Schrödinger's equation (NLSE) with nonlinear CD and having six forms of SPM structures that are of non–Kerr type. It must be noted that the concept of quiescent solitons has been addressed for several other models, apart from NLSE, in the past. These include Fokas–Lenells equation, complex Ginzburg–Landau equation and several others [1–9, 12–14]. These six forms of SPM were proposed by Kudryashov during the past few years [3]. In fact, the retrieval of quiescent optical solitons with nonlinear CD and with the six forms of SPM structure have been addressed in the past by the Jacobi's elliptic function approach [3]. The current paper will revisit the model with the same six forms of SPM and the implicit quiescent solitons will be recovered by the aid of Lie symmetry analysis. The details of the mathematical analysis and the derivation mechanism are exhibited in the rest of the paper after a quick introduction to the model.

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1.1. GOVERNING MODEL

The dimensionless form of the governing NLSE in optical fibers and PCF with nonlinear CD and non–Kerr laws of SPM, as proposed by Kudryashov, is structured as:

$$iq_t + a\left(|q|^n q\right)_{xx} + F\left(|q|^2\right) q = i\left[\lambda\left(|q|^{2m} q\right)_x + \theta_1\left(|q|^{2m}\right)_x q + \theta_2 |q|^{2m} q_x\right]. \tag{1}$$

Here in (1) $q(x,t)$ represents the wave amplitude and is a complex–valued function. The independent variables are x and t that represent spatial and temporal variables respectively. The first term is the linear temporal evolution with its coefficient being $i = \sqrt{-1}$. The second term with its coefficient being a real–valued constant, a is the nonlinear CD where the nonlinearity factor is dictated by the exponent n. When $n = 0$, one collapses to linear CD. The third term is from SPM where the functional F accounts for the nonlinear structure of intensity dependent refractive index change. The right hand side comprises of the Hamiltonian perturbation terms with arbitrary intensity. The coefficient of λ is the self–steepening while the coefficients of θ_j for $j = 1, 2$ represents the self–frequency shift effect. The perturbation parameters λ and θ_j are real–valued constants.

2. MATHEMATICAL ANALYSIS

This section will analyze equation (1) with linear temporal evolution for six forms of SPM structures as proposed by Kudryashov. Equation (1) does not support mobile solitons unless $n = 0$ in the CD. Thus, the quiescent optical solitons supported by (1) is taken to be of the form:

$$q(x,t) = \phi(x)\, e^{i\omega t}, \tag{2}$$

where $\phi(x)$ represents the amplitude portion of the soliton while the second factor is the phase component with ω representing the wave number. Substituting (2) into (1) and decomposing into real and imaginary components, the real part yields the ordinary differential equation (ODE) for the amplitude component $\phi(x)$ as

$$a(n+1)\phi(x)^{2m+n}\left[n\left\{\phi'(x)\right\}^2 + \phi(x)\phi''(x)\right] + \phi^2(x)\left[F\left\{\phi^2(x)\right\} - \omega\right] = 0. \tag{3}$$

Then, the imaginary part gives the parameter constraints as

$$(2m+1)\lambda + 2m\theta_1 + \theta_2 = 0. \tag{4}$$

With these parameter constraints, the governing model (1) conveniently reduces to

$$iq_t + a\left(|q|^n q\right)_{xx} + F\left(|q|^2\right) q = i\left[\lambda\left(|q|^{2m} q\right)_x + \theta_1\left(|q|^{2m}\right)_x q - \left\{(2m+1)\lambda + 2m\theta_1\right\}|q|^{2m} q_x\right]. \tag{5}$$

This ODE given by (3) will be addressed in details in the subsequent section with the six forms of SPM structures, as proposed by Kudryashov, corresponding to the functional F, that would yield a variety of solution structures.

2.1. FORM–I

For Form–I, the law of SPM is given as

$$F(s) = \frac{b_1}{s^m} + \frac{b_2}{s^{\frac{m}{2}}} + b_3 s^{\frac{m}{2}} + b_4 s^m. \tag{6}$$

Thus, the corresponding NLSE reduces to

$$iq_t + a\left(|q|^n q\right)_{xx} + \left(\frac{b_1}{|q|^{2m}} + \frac{b_2}{|q|^m} + b_3|q|^m + b_4|q|^{2m}\right) q = i\left[\lambda\left(|q|^{2m} q\right)_x + \theta_1\left(|q|^{2m}\right)_x q + \theta_2|q|^{2m} q_x\right]. \tag{7}$$

Thus, substituting (2) into (7), the imaginary part relation gives (4) which transforms (7) to

$$iq_t + a\left(|q|^n q\right)_{xx} + \left(\frac{b_1}{|q|^{2m}} + \frac{b_2}{|q|^m} + b_3|q|^m + b_4|q|^{2m}\right) q = i\left[\lambda\left(|q|^{2m} q\right)_x + \theta_1\left(|q|^{2m}\right)_x q - \left([2m+1]\lambda + 2m\theta_1\right)|q|^{2m} q_x\right]. \tag{8}$$

For equation (8) to be integrable, one needs to choose $n = 2m$. Therefore, the real part equation modifies (3) to

$$a(n+1)\phi^{4m}(x)\left[n\left\{\phi'(x)\right\}^2 + \phi(x)\phi''(x)\right] + \phi^2(x)\left(\phi^m(x)\left(\phi^m(x)\left(\phi^m(x)\left(b_4\phi^m(x) + b_3\right) - \omega\right) + b_2\right) + b_1\right) = 0. \tag{9}$$

Equation (9) admits the translational Lie point symmetry, namely $\partial/\partial x$ which integrates the ODE (9) to:

$$x = \pm\sqrt{-\frac{A}{B}}\; C, \tag{10}$$

where

$$A = 4a(1+m)(2+m)(1+2m)^2(2+3m)\left(\phi^m - \kappa_1\right)\left(\phi^m - \kappa_2\right)\left(\phi^m - \kappa_3\right)\left(\phi^m - \kappa_4\right), \tag{11}$$

$$\begin{aligned} B &= 12m^5 b_2\kappa_1\kappa_2\phi^m + 26m^4 b_2\kappa_1\kappa_2\phi^m + 18m^3 b_2\kappa_1\kappa_2\phi^m + 4m^2 b_2\kappa_1\kappa_2\phi^m - 12m^5 b_2\kappa_2\kappa_3\phi^m \\ &- 26m^4 b_2\kappa_2\kappa_3\phi^m - 18m^3 b_2\kappa_2\kappa_3\phi^m - 4m^2 b_2\kappa_2\kappa_3\phi^m - 12m^5 b_2\kappa_1\kappa_4\phi^m \\ &- 26m^4 b_2\kappa_1\kappa_4\phi^m - 18m^3 b_2\kappa_1\kappa_4\phi^m - 4m^2 b_2\kappa_1\kappa_4\phi^m + 12m^5 b_2\kappa_3\kappa_4\phi^m \\ &+ 26m^4 b_2\kappa_3\kappa_4\phi^m + 18m^3 b_2\kappa_3\kappa_4\phi^m + 4m^2 b_2\kappa_3\kappa_4\phi^m - 6m^5\omega\kappa_1\kappa_2\phi^{2m} - 19m^4\omega\kappa_1\kappa_2\phi^{2m} \\ &- 16m^3\omega\kappa_1\kappa_2\phi^{2m} - 4m^2\omega\kappa_1\kappa_2\phi^{2m} + 6m^5\omega\kappa_2\kappa_3\phi^{2m} + 19m^4\omega\kappa_2\kappa_3\phi^{2m} + 16m^3\omega\kappa_2\kappa_3\phi^{2m} \\ &+ 4m^2\omega\kappa_2\kappa_3\phi^{2m} + 6m^5\omega\kappa_1\kappa_4\phi^{2m} + 19m^4\omega\kappa_1\kappa_4\phi^{2m} + 16m^3\omega\kappa_1\kappa_4\phi^{2m} + 4m^2\omega\kappa_1\kappa_4\phi^{2m} \\ &- 6m^5\omega\kappa_3\kappa_4\phi^{2m} - 19m^4\omega\kappa_3\kappa_4\phi^{2m} - 16m^3\omega\kappa_3\kappa_4\phi^{2m} - 4m^2\omega\kappa_3\kappa_4\phi^{2m} + 4m^5 b_3\kappa_1\kappa_2\phi^{3m} \\ &+ 14m^4 b_3\kappa_1\kappa_2\phi^{3m} + 14m^3 b_3\kappa_1\kappa_2\phi^{3m} + 4m^2 b_3\kappa_1\kappa_2\phi^{3m} - 4m^5 b_3\kappa_2\kappa_3\phi^{3m} - 14m^4 b_3\kappa_2\kappa_3\phi^{3m} \\ &- 14m^3 b_3\kappa_2\kappa_3\phi^{3m} - 4m^2 b_3\kappa_2\kappa_3\phi^{3m} - 4m^5 b_3\kappa_1\kappa_4\phi^{3m} - 14m^4 b_3\kappa_1\kappa_4\phi^{3m} - 14m^3 b_3\kappa_1\kappa_4\phi^{3m} \\ &- 4m^2 b_3\kappa_1\kappa_4\phi^{3m} + 4m^5 b_3\kappa_3\kappa_4\phi^{3m} + 14m^4 b_3\kappa_3\kappa_4\phi^{3m} + 14m^3 b_3\kappa_3\kappa_4\phi^{3m} + 4m^2 b_3\kappa_3\kappa_4\phi^{3m} \\ &+ 3m^5 b_4\kappa_1\kappa_2\phi^{4m} + 11m^4 b_4\kappa_1\kappa_2\phi^{4m} + 12m^3 b_4\kappa_1\kappa_2\phi^{4m} + 4m^2 b_4\kappa_1\kappa_2\phi^{4m} - 3m^5 b_4\kappa_2\kappa_3\phi^{4m} \\ &- 11m^4 b_4\kappa_2\kappa_3\phi^{4m} - 12m^3 b_4\kappa_2\kappa_3\phi^{4m} - 4m^2 b_4\kappa_2\kappa_3\phi^{4m} - 3m^5 b_4\kappa_1\kappa_4\phi^{4m} - 11m^4 b_4\kappa_1\kappa_4\phi^{4m} \\ &- 12m^3 b_4\kappa_1\kappa_4\phi^{4m} - 4m^2 b_4\kappa_1\kappa_4\phi^{4m} + 3m^5 b_4\kappa_3\kappa_4\phi^{4m} + 11m^4 b_4\kappa_3\kappa_4\phi^{4m} + 12m^3 b_4\kappa_3\kappa_4\phi^{4m} \\ &+ 4m^2 b_4\kappa_3\kappa_4\phi^{4m} + 6m^6 b_1\kappa_1\kappa_2 + 25m^5 b_1\kappa_1\kappa_2 + 35m^4 b_1\kappa_1\kappa_2 + 20m^3 b_1\kappa_1\kappa_2 + 4m^2 b_1\kappa_1\kappa_2 \\ &- 6m^6 b_1\kappa_2\kappa_3 - 25m^5 b_1\kappa_2\kappa_3 - 35m^4 b_1\kappa_2\kappa_3 - 20m^3 b_1\kappa_2\kappa_3 - 4m^2 b_1\kappa_2\kappa_3 - 6m^6 b_1\kappa_1\kappa_4 - 25m^5 b_1\kappa_1\kappa_4 \\ &- 35m^4 b_1\kappa_1\kappa_4 - 20m^3 b_1\kappa_1\kappa_4 - 4m^2 b_1\kappa_1\kappa_4 + 6m^6 b_1\kappa_3\kappa_4 + 25m^5 b_1\kappa_3\kappa_4 + 35m^4 b_1\kappa_3\kappa_4 \\ &+ 20m^3 b_1\kappa_3\kappa_4 + 4m^2 b_1\kappa_3\kappa_4, \end{aligned} \tag{12}$$

and

$$\begin{aligned} C &= \Pi\left(\frac{\kappa_1-\kappa_4}{\kappa_2-\kappa_4}; \sin^{-1}\left(\sqrt{\frac{(\phi^m-\kappa_1)(\kappa_2-\kappa_4)}{(\phi^m-\kappa_2)(\kappa_1-\kappa_4)}}\right) | \frac{(\kappa_2-\kappa_3)(\kappa_1-\kappa_4)}{(\kappa_1-\kappa_3)(\kappa_2-\kappa_4)}\right)(\kappa_1-\kappa_2) \\ &+ F\left(\sin^{-1}\left(\sqrt{\frac{(\phi^m-\kappa_1)(\kappa_2-\kappa_4)}{(\phi^m-\kappa_2)(\kappa_1-\kappa_4)}}\right) | \frac{(\kappa_2-\kappa_3)(\kappa_1-\kappa_4)}{(\kappa_1-\kappa_3)(\kappa_2-\kappa_4)}\right)\kappa_2. \end{aligned} \tag{13}$$

Since the expression under the square root in Eq. (10) must be real, the existence of admissible solutions requires that $AB < 0$. This condition guarantees that the radicand remains non–negative, ensuring the physical realizability of the obtained solitary–wave profiles.

Here, κ_j for $j = 1\cdots 4$ is any solution of the quartic (fourth–degree) equation

$$\begin{aligned} &\left(3b_4m^3 + 11b_4m^2 + 12b_4m + 4b_4\right)\kappa^4 + \left(4b_3m^3 + 14b_3m^2 + 14b_3m + 4b_3\right)\kappa^3 - \left(6m^3\omega + 19m^2\omega + 16m\omega + 4\omega\right)\kappa^2 \\ + &\left(12b_2m^3 + 26b_2m^2 + 18b_2m + 4b_2\right)\kappa + 20b_1m + 4b_1 + 6b_1m^4 + 25b_1m^3 + 35b_1m^2 = 0. \end{aligned} \tag{14}$$

This polynomial is referred to as quartic rather than biquadratic because it contains both odd and even powers of κ. The four roots $\kappa_1, \ldots, \kappa_4$ of this equation parameterize the elliptic–integral representation used in the subsequent analysis.

Here, $\Pi\left(\nu;\psi|m\right)$ the incomplete elliptic integral that is defined as:

$$\Pi\left(\nu;\psi|\mu\right) = \int_0^{\psi} \frac{1}{\left(1-\nu\sin^2\theta\right)\sqrt{1-\mu\sin^2\theta}}d\theta, \tag{15}$$

for

$$-\frac{\pi}{2} < \psi < \frac{\pi}{2}, \tag{16}$$

$$\mu\sin^2(\psi) < 1, \tag{17}$$

and

$$\nu\sin^2(\psi) > 1. \tag{18}$$

Moreover, $F(\psi|\mu)$ is elliptic integral of the first kind which is defined as:

$$F(\psi|\mu) = \int_0^{\psi} \frac{1}{\sqrt{1-\mu\sin^2(\theta)}}\, d\theta, \tag{19}$$

where

$$-\frac{\pi}{2} < \psi < \frac{\pi}{2}, \tag{20}$$

and

$$\mu\sin^2(\psi) < 1. \tag{21}$$

From (10), the solutions will exist provided

$$AB < 0. \tag{22}$$

2.2. FORM–II

For Form–II, the law of SPM is given as

$$F(s) = \frac{b_1}{s^{2m}} + \frac{b_2}{s^{\frac{3m}{2}}} + \frac{b_3}{s^m} + \frac{b_4}{s^{\frac{m}{2}}} + b_5 s^{\frac{m}{2}} + b_6 s^m + b_7 s^{\frac{3m}{2}} + b_8 s^{2m}. \tag{23}$$

Thus the corresponding NLSE reduces to

$$\begin{aligned} & iq_t + a\left(|q|^n q\right)_{xx} + \left(\frac{b_1}{|q|^{4m}} + \frac{b_2}{|q|^{3m}} + \frac{b_3}{|q|^{2m}} + \frac{b_4}{|q|^m} + b_5|q|^m + b_6|q|^{2m} + b_7|q|^{3m} + b_8|q|^{4m}\right)q \\ = \ & i\left[\lambda\left(|q|^{2m}q\right)_x + \theta_1\left(|q|^{2m}\right)_x q + \theta_2|q|^{2m}q_x\right] \end{aligned} \tag{24}$$

The starting hypothesis given by (2) when applied to (24) modifies it to:

$$\begin{aligned} & iq_t + a\left(|q|^n q\right)_{xx} + \left(\frac{b_1}{|q|^{4m}} + \frac{b_2}{|q|^{3m}} + \frac{b_3}{|q|^{2m}} + \frac{b_4}{|q|^m} + b_5|q|^m + b_6|q|^{2m} + b_7|q|^{3m} + b_8|q|^{4m}\right)q \\ = \ & i\left[\lambda\left(|q|^{2m}q\right)_x + \theta_1\left(|q|^{2m}\right)_x q - \{(2m+1)\lambda + 2m\theta_1\}|q|^{2m}q_x\right], \end{aligned} \tag{25}$$

by virtue of the imaginary part. The real part leads to the ODE for $\phi(x)$:

$$\begin{aligned} & a(n+1)\phi(x)^{4m+n}\left[n\{\phi'(x)\}^2 + \phi(x)\phi''(x)\right] + \phi^2(x)\Big\{\phi^m(x)\Big\{\phi^m(x)\Big\{\phi^m(x)\Big\{\phi^m(x)\Big\{\phi^m(x)\Big\{\phi^m(x)\Big\{ \\ \times \ & \phi^m(x)\Big\{b_8\phi^m(x) + b_7\Big\} + b_6\Big\} + b_5\Big\} - \omega\Big\} + b_4\Big\} + b_3\Big\} + b_2\Big\} + b_1\Big\} = 0. \end{aligned} \tag{26}$$

Next, the translational Lie symmetry that is supported by (26), integrates it to the implicit solution in terms of quadratures as:

$$x = \pm\int\sqrt{\frac{A}{B}}\, d\phi, \tag{27}$$

where

$$\begin{aligned} A \ = \ & a(n+1)(n+2)(m-n-2)(2m-n-2)(3m-n-2)(4m-n-2)(m+n+2)\times \\ \times \ & (2m+n+2)(3m+n+2)(4m+n+2)\phi^{4m+n}, \end{aligned} \tag{28}$$

and

$$\begin{aligned} B \ = \ & 2\phi^2\Big\{(4m-n-2)\phi^m\Big\{b_2(n+2)(2m-n-2)(m-n-2)(m+n+2)(2m+n+2)(3m+n+2) \\ \times \ & (4m+n+2) - (3m-n-2)\phi^m\Big\{(2m-n-2)\phi^m\Big\{(m-n-2)\phi^m \end{aligned}$$

$$\begin{aligned}
&\times \quad \Big\{(n+2)\phi^m\Big\{(m+n+2)\phi^m\Big\{(2m+n+2)\phi^m\Big\{b_8(3m+n+2)\phi^m+b_7(4m+n+2)\Big\}\\
&+ \quad b_6(3m+n+2)(4m+n+2)\Big\}+b_5(2m+n+2)(3m+n+2)(4m+n+2)\Big\}\\
&- \quad \omega(m+n+2)(2m+n+2)(3m+n+2)(4m+n+2)\Big\}\\
&- \quad b_4(n+2)(m+n+2)(2m+n+2)(3m+n+2)(4m+n+2)\Big\}\\
&- \quad b_3(n+2)(m-n-2)(m+n+2)(2m+n+2)(3m+n+2)(4m+n+2)\Big\}\Big\}\\
&+ \quad b_1(n+2)(3m-n-2)(2m-n-2)(m-n-2)(m+n+2)(2m+n+2)(3m+n+2)(4m+n+2)\Big\}.
\end{aligned} \tag{29}$$

This solution (29) would remain valid provided

$$AB > 0. \tag{30}$$

The condition $AB > 0$ arises from the requirement that the expression inside the square root in Eq. (29) must be real and positive. This ensures that the radicand of $\sqrt{A/B}$ is non–negative, leading to physically meaningful and admissible solitary–wave solutions. The sign difference from the previous case ($AB < 0$) is due to the absence of the negative sign under the square root in the present formulation.

2.3. FORM–III

The SPM structure here is:

$$F(s) = b_1 s^{\frac{m}{2}} + b_2 s^m + b_3\left(s^{\frac{m}{2}}\right)''. \tag{31}$$

The corresponding NLSE therefore gets structured as:

$$iq_t + a\left(|q|^n q\right)_{xx} + \left[b_1|q|^m + b_2|q|^{2m} + b_3\left(|q|^m\right)_{xx}\right]q = i\left[\lambda\left(|q|^{2m}q\right)_x + \theta_1\left(|q|^{2m}\right)_x q + \theta_2|q|^{2m}q_x\right]. \tag{32}$$

The starting hypthesis to locate for quiescent optical solitons is (2) which when implemented, the imaginary part gives (4). The real part leads to the ODE:

$$\begin{aligned}
&a(n+1)\phi^{2m+n}(x)\left[n\{\phi'(x)\}^2+\phi(x)\phi''(x)\right]+2b_3m\phi^{2m}(x)\left[\phi(x)\phi''(x)+(2m-1)\{\phi'(x)\}^2\right]\\
+&\quad b_1\phi(x)^{m+2}+b_2\phi(x)^{2m+2}-\omega\phi(x)^2=0.
\end{aligned} \tag{33}$$

For (33) to be integrable, one must choose

$$n = -1, \tag{34}$$

which leads to the modification of the governing model to be modified to:

$$\begin{aligned}
&iq_t + a\left(\frac{|q|}{q}\right)_{xx} + \left[b_1|q|^m + b_2|q|^{2m} + b_3\left(|q|^m\right)_{xx}\right]q\\
=&\quad i\left[\lambda\left(|q|^{2m}q\right)_x + \theta_1\left(|q|^{2m}\right)_x q - \{(2m+1)\lambda + 2m\theta_1\}|q|^{2m}q_x\right]
\end{aligned} \tag{35}$$

Also by virtue of (34), the governing ODE for $\phi(x)$ given by (33) condenses to

$$2b_3m\phi^{2m}(x)\left[\phi(x)\phi''(x)+(2m-1)\{\phi'(x)\}^2\right]+b_1\phi(x)^{m+2}+b_2\phi(x)^{2m+2}-\omega\phi(x)^2=0. \tag{36}$$

The translational Lie symmetry supported by (36) leads to the periodic solution given by

$$\phi(x) = \left(\frac{2}{3b_2}\right)^{\frac{1}{m}}\left[\pm\sqrt{2(2b_1^2+9\omega b_2)}\left\{1+\cos\left(\sqrt{\frac{b_2}{b_3}}\,x\right)\right\}\csc^2\left(\sqrt{\frac{b_2}{b_3}}\,x\right)\sin^3\left(\frac{1}{2}\sqrt{\frac{b_2}{b_3}}\,x\right)-b_1\right]^{\frac{1}{m}}. \tag{37}$$

This solution (37) is valid provided

$$b_2b_3 > 0, \tag{38}$$

and

$$2b_1^2 + 9\omega b_2 > 0. \tag{39}$$

2.4. FORM–IV

In this case, the law of SPM is:

$$F(s) = b_1 s^{\frac{m}{2}} + b_2 s^m + b_3 s^{\frac{3m}{2}} + b_4 s^{2m} + b_5 s^{\frac{5m}{2}} + b_6 s^{3m}, \tag{40}$$

which leads to the structure of NLSE to be

$$\begin{aligned} & iq_t + a\left(|q|^n q\right)_{xx} + \left(b_1 |q|^m + b_2 |q|^{2m} + b_3 |q|^{3m} + b_4 |q|^{4m} + b_5 |q|^{5m} + b_6 |q|^{6m}\right) q \\ = \; & i\left[\lambda \left(|q|^{2m} q\right)_x + \theta_1 \left(|q|^{2m}\right)_x q + \theta_2 |q|^{2m} q_x\right] \end{aligned} \tag{41}$$

Substituting (2) into (41), the imaginary part gives (4) which leads to the governing model (41) being modified to:

$$\begin{aligned} & iq_t + a\left(|q|^n q\right)_{xx} + \left(b_1 |q|^m + b_2 |q|^{2m} + b_3 |q|^{3m} + b_4 |q|^{4m} + b_5 |q|^{5m} + b_6 |q|^{6m}\right) q \\ = \; & i\left[\lambda \left(|q|^{2m} q\right)_x + \theta_1 \left(|q|^{2m}\right)_x q - \left\{(2m+1)\lambda + 2m\theta_1\right\} |q|^{2m} q_x\right]. \end{aligned} \tag{42}$$

The real part leads to the ODE for $\phi(x)$ takes the form

$$\begin{aligned} & a(n+1)\phi(x)^{2m+n}\left[n\left\{\phi'(x)\right\}^2 + \phi(x)\phi''(x)\right] \\ + \; & \phi^2(x)\left(\phi^m(x)\left(\phi^m(x)\left(\phi^m(x)\left(\phi^m(x)\left(\phi^m(x)\left(b_6\phi^m(x) + b_5\right) + b_4\right) + b_3\right) + b_2\right) + b_1\right) - \omega\right) = 0. \end{aligned} \tag{43}$$

By the aid of the translational Lie symmetry supported by (43), it integrates in terms of quadratures as given below

$$x = \pm \int \sqrt{-\frac{A}{B}}\, d\phi, \tag{44}$$

where

$$A = a(n+1)\phi^{n-2}, \tag{45}$$

and

$$B = 2\left(\frac{b_2\phi^{2m}}{2m+n+2} + \frac{b_3\phi^{3m}}{3m+n+2} + \frac{b_4\phi^{4m}}{4m+n+2} + \frac{b_5\phi^{5m}}{5m+n+2} + \frac{b_6\phi^{6m}}{6m+n+2} + \frac{b_1\phi^m}{m+n+2} - \frac{\omega}{n+2}\right). \tag{46}$$

The integrability condition given by (22) must be valid in this case too for the solution to exist.

2.5. FORM–V

The SPM structure for this form is:

$$F(s) = b_1 s^{\frac{m}{2}} + b_2 s^m + b_3 s^{\frac{3m}{2}} + b_4 s^{2m} + b_5\left(s^{\frac{m}{2}}\right)'' + b_6\left(s^m\right)''. \tag{47}$$

Therefore, the corresponding NLSE is written as:

$$\begin{aligned} & iq_t + a\left(|q|^n q\right)_{xx} + \left[b_1 |q|^m + b_2 |q|^{2m} + b_3 |q|^{3m} + b_4 |q|^{4m} + b_5\left(|q|^m\right)_{xx} + b_6\left(|q|^{2m}\right)_{xx}\right] q \\ = \; & i\left[\lambda\left(|q|^{2m} q\right)_x + \theta_1\left(|q|^{2m}\right)_x q + \theta_2 |q|^{2m} q_x\right]. \end{aligned} \tag{48}$$

Substituting the hypothesis (2) into (48), the real part leads to the ODE for $\phi(x)$ as:

$$\begin{aligned} & a(n+1)\phi^n(x)\left[n\left\{\phi'(x)\right\}^2 + \phi(x)\phi''(x)\right] + 2b_6 m\phi^{2m}(x)\left[\phi(x)\phi''(x) + (2m-1)\left\{\phi'(x)\right\}^2\right] \\ + \; & b_5 m\phi^m(x)\left[\phi(x)\phi''(x) + (m-1)\left\{\phi'(x)\right\}^2\right] + b_1\phi^{m+2}(x) + b_2\phi^{2m+2}(x) + b_3\phi^{3m+2}(x) + b_4\phi^{4m+2}(x) - \omega\phi^2(x) = 0. \end{aligned} \tag{49}$$

For integrability, one must choose:

$$n = 2, \tag{50}$$

and

$$m = 1. \tag{51}$$

The imaginary part leads to (4) and by virtue of (50) and (51), equation (5) simplifies to:

$$\begin{aligned} & iq_t + a\left(|q|^2 q\right)_{xx} + \left[b_1 |q| + b_2 |q|^2 + b_3 |q|^3 + b_4 |q|^4 + b_5 (|q|)_{xx} + b_6 \left(|q|^2\right)_{xx}\right] q \\ = \ & i\left[\lambda\left(|q|^2 q\right)_x + \theta_1 \left(|q|^2\right)_x q - (3\lambda + 2\theta_1) |q|^2 q_x\right]. \end{aligned} \tag{52}$$

By means of the translational Lie symmetry that is supported by (52), it integrates to

$$x = \pm\sqrt{\frac{A}{B}}\ \ F\left(\sin^{-1}\left(\sqrt{\frac{(3a\phi + b_5 + 2\phi b_6 - \kappa_1)(\kappa_2 - \kappa_4)}{(3a\phi + b_5 + 2\phi b_6 - \kappa_2)(\kappa_1 - \kappa_4)}}\right) | \frac{(\kappa_2 - \kappa_3)(\kappa_1 - \kappa_4)}{(\kappa_1 - \kappa_3)(\kappa_2 - \kappa_4)}\right), \tag{53}$$

where

$$A = 24A_1 A_2, \tag{54}$$

$$A_1 = (2a + b_6)(3a + b_6)(5a + 2b_6)(9a + 4b_6)(21a + 10b_6), \tag{55}$$

$$A_2 = (3a\phi + b_5 + 2\phi b_6 - \kappa_1)(3a\phi + b_5 + 2\phi b_6 - \kappa_2)(3a\phi + b_5 + 2\phi b_6 - \kappa_3)(3a\phi + b_5 + 2\phi b_6 - \kappa_4), \tag{56}$$

$$B = B_1 + B_2 + 2B_3 + 4B_4 + B_5 + B_6, \tag{57}$$

$$\begin{aligned} B_1 \ = \ & 5670a^4\omega - 3240a^4\phi^3 b_3 - 2835a^4\phi^4 b_4 + 540a^3\phi^2 b_3 b_5 + 540a^3\phi^3 b_4 b_5 - 72a^2\phi b_3 b_5^2 - 90a^2\phi^2 b_4 b_5^2 + 6ab_3 b_5^3 \\ + \ & 12a\phi b_4 b_5^3 - b_4 b_5^4, \end{aligned} \tag{58}$$

$$B_2 = \left\{9a^3\left(1147\omega - 604\phi^3 b_3 - 521\phi^4 b_4\right) + 6a^2\phi^2(111b_3 + 106\phi b_4) b_5 - 6a\phi(10b_3 + 11\phi b_4) b_5^2 + (3b_3 + 4\phi b_4) b_5^3\right\} b_6, \tag{59}$$

$$B_3 = \left\{6a^2\left(586\omega - 283\phi^3 b_3 - 241\phi^4 b_4\right) + a\phi^2(135b_3 + 124\phi b_4) b_5 - 6\phi(b_3 + \phi b_4) b_5^2\right\} b_6^2, \tag{60}$$

$$B_4 = \left(531a\omega + 9\phi^2 b_3(-26a\phi + b_5) + \phi^3 b_4(-197a\phi + 8b_5)\right) b_6^3, \tag{61}$$

$$B_5 = 16\left(15\omega - 6\phi^3 b_3 - 5\phi^4 b_4\right) b_6^4 - b_1(2a + b_6)(9a + 4b_6)(21a + 10b_6)(-b_5 + 4\phi(3a + b_6)), \tag{62}$$

(63)

$$B_6 = -b_2(2a + b_6)(21a + 10b_6)\left(b_5^2 - 4\phi b_5(3a + b_6) + 6\phi^2(3a + b_6)(5a + 2b_6)\right) \tag{64}$$

where κ_i $i = 1 \cdots 4$ is any solution of

$$\begin{aligned} & -81a^4\nu_1 - 9a^3 b_6\nu_2 - 6a^2 b_6^2\nu_3 - 368256a^2 b_6^6\omega - 4ab_6^3\nu_4 + b_2\nu_7(2a + b_6)(3a + 2b_6)^2(21a + 10b_6) \\ & -57024ab_6^7\omega - 48b_6^5\nu_6 - 8b_6^4\nu_5 + b_1\nu_8 - 3840b_6^8\omega = 0, \end{aligned} \tag{65}$$

and

$$\nu_1 = 5670a^4\omega - 6ab_3\left(-70b_5\kappa^2 + 84b_5^2\kappa - 35b_5^3 + 20\kappa^3\right) + b_4\left[2b_5\left\{80\kappa^3 - 7b_5\left(-16b_5\kappa + 5b_5^2 + 20\kappa^2\right)\right\} - 35\kappa^4\right] = 0, \tag{66}$$

$$\begin{aligned} \nu_2 \ = \ & 228987a^4\omega + 3ab_3\left(2994b_5\kappa^2 - 3660b_5^2\kappa + 1567b_5^3 - 844\kappa^3\right) \\ + \ & b_4\left(2416b_5\kappa^3 - 4308b_5^2\kappa^2 + 3536b_5^3\kappa - 1147b_5^4 - 521\kappa^4\right), \end{aligned} \tag{67}$$

$$
\begin{aligned}
\nu_3 &= 670680a^4\omega - 9ab_3\left(-1403b_5\kappa^2 + 1750b_5^2\kappa - 773b_5^3 + 390\kappa^3\right)\\
&- 2b_4\left(-1132b_5\kappa^3 + 2055b_5^2\kappa^2 - 1732b_5^3\kappa + 586b_5^4 + 241\kappa^4\right),
\end{aligned} \tag{68}
$$

$$
\begin{aligned}
\nu_4 &= 1116261a^4\omega + 3ab_3\left(2919b_5\kappa^2 - 3720b_5^2\kappa + 1703b_5^3 - 800\kappa^3\right)\\
&+ b_4\left(936b_5\kappa^3 - 1728b_5^2\kappa^2 + 1496b_5^3\kappa - 531b_5^4 - 197\kappa^4\right),
\end{aligned} \tag{69}
$$

$$
\begin{aligned}
\nu_5 &= 384858a^4\omega - 3ab_3\left(15\kappa - 23b_5\right)\left(-13b_5\kappa + 9b_5^2 + 6\kappa^2\right)\\
&- 2b_4\left[b_5\left\{5b_5\left(-8b_5\kappa + 3b_5^2 + 9\kappa^2\right) - 24\kappa^3\right\} + 5\kappa^4\right],
\end{aligned} \tag{70}
$$

$$
\nu_6 = 28143a^3\omega + b_3\left(5b_5\left(2b_5\left(b_5 - 2\kappa\right) + 3\kappa^2\right) - 4\kappa^3\right), \tag{71}
$$

$$
\nu_7 = 6\kappa^2\left(3a + b_6\right)\left(5a + 2b_6\right) - 8b_5\kappa\left(3a + b_6\right)\left(9a + 4b_6\right) + 3b_5^2\left(5a + 2b_6\right)\left(9a + 4b_6\right), \tag{72}
$$

$$
\nu_8 = \left(2a + b_6\right)\left(3a + 2b_6\right)^3\left(9a + 4b_6\right)\left(21a + 10b_6\right)\left\{4\kappa\left(3a + b_6\right) - 3b_5\left(5a + 2b_6\right)\right\}. \tag{73}
$$

Here, $F(\psi|\mu)$ is elliptic integral of the first kind that has been defined in (19). The existence criteria for the solution is given by (30).

2.6. FORM–VI

The SPM structure for this form is:

$$
F(s) = b_1 s^{\frac{m}{2}} + b_2 s^m + b_3 s^{\frac{3m}{2}} + b_4 s^{2m} + b_5 s^{\frac{5m}{2}} + b_6 s^{3m} + b_7\left(s^{\frac{m}{2}}\right)'' + b_8\left(s^m\right)''. \tag{74}
$$

Therefore, the corresponding NLSE shapes up to be:

$$
\begin{aligned}
& iq_t + a\left(|q|^n q\right)_{xx} + \left[b_1|q|^m + b_2|q|^{2m} + b_3|q|^{3m} + b_4|q|^{4m} + b_5|q|^{5m} + b_6|q|^{6m} + b_7\left(|q|^m\right)_{xx} + b_8\left(|q|^{2m}\right)_{xx}\right]q\\
= \;& i\left[\lambda\left(|q|^{2m}q\right)_x + \theta_1\left(|q|^{2m}\right)_x q + \theta_2|q|^{2m}q_x\right].
\end{aligned} \tag{75}
$$

Substituting (2) into (75) and splitting into real and imaginary parts, equation (5) by virtue of (4) simplifies to:

$$
\begin{aligned}
& iq_t + a\left(|q|^n q\right)_{xx} + \left[b_1|q|^m + b_2|q|^{2m} + b_3|q|^{3m} + b_4|q|^{4m} + b_5|q|^{5m} + b_6|q|^{6m} + b_7\left(|q|^m\right)_{xx} + b_8\left(|q|^{2m}\right)_{xx}\right]q\\
= \;& i\left[\lambda\left(|q|^{2m}q\right)_x + \theta_1\left(|q|^{2m}\right)_x q - \left\{(2m+1)\lambda + 2m\theta_1\right\}|q|^{2m}q_x\right].
\end{aligned} \tag{76}
$$

The real part equation (3) in this case, is:

$$
\begin{aligned}
& a(n+1)\phi^n(x)\left[n\left\{\phi'(x)\right\}^2 + \phi(x)\phi''(x)\right] + 2b_8 m\phi^{2m}(x)\left[\phi(x)\phi''(x) + (2m-1)\left\{\phi'(x)\right\}^2\right]\\
+ \;& b_7 m\phi^m(x)\left[\phi(x)\phi''(x) + (m-1)\left\{\phi'(x)\right\}^2\right] + b_1\phi^{m+2}(x) + b_2\phi^{2m+2}(x) + b_3\phi^{3m+2}(x)\\
+ \;& b_4\phi^{4m+2}(x) + b_5\phi^{5m+2}(x) + b_6\phi^{6m+2}(x) - \omega\phi^2(x) = 0.
\end{aligned} \tag{77}
$$

By the translational Lie symmetry, supported by (77), the integral is written in quadratures as:

$$
x = \pm\int\sqrt{\frac{A}{2B}}\,d\phi, \tag{78}
$$

where

$$
A = \exp\left(-2\int^{\phi} -\frac{m\left(2(2m-1)b_8\tau_1^m + (m-1)b_7\right)\tau_1^m + an(n+1)\tau_1^n}{\tau_1\left(m\left(2b_8\tau_1^m + b_7\right)\tau_1^m + a(n+1)\tau_1^n\right)}d\tau_1\right), \tag{79}
$$

and

$$B = \int^{\phi} \frac{\exp\left(-2\int^{\tau_2} -\frac{m\left(2(2m-1)b_8\tau_1^m+(m-1)b_7\right)\tau_1^m+an(n+1)\tau_1^n}{\tau_1\left(m\left(2b_8\tau_1^m+b_7\right)\tau_1^m+a(n+1)\tau_1^n\right)}d\tau_1\right)\Gamma}{m\left(2b_8\tau_2^m+b_7\right)\tau_2^m+a(n+1)\tau_2^n}d\tau_2, \tag{80}$$

with

$$\Gamma = \tau_2\left(\omega-\tau_2^m\left(\tau_2^m\left(\tau_2^m\left(\tau_2^m\left(\tau_2^m\left(b_6\tau_2^m+b_5\right)+b_4\right)+b_3\right)+b_2\right)+b_1\right)\right). \tag{81}$$

The parameter constraint given by (30) must also be satisfied for the solution to exist.

3. CONCLUSIONS

The paper addressed the NLSE with nonlinear CD and linear temporal evolutions but with six forms of SPM structures that were proposed by Kudryashov during the past decade. By applying the Lie symmetry analysis, the study systematically derived implicit quiescent optical solitons and reduced the governing nonlinear partial differential equation to solvable amplitude equations expressed through quadratures. The Lie symmetry analysis is the integration tool that has been implemented to address the models that gave way to the solutions also in terms of quadratures. The results, recovered with the usage of Lie symmetry analysis, were in terms of quadratures and in one particular case, emerged in terms of periodic functions. Among the six Kudryashov SPM structures, five produced implicit solitary–wave profiles while one yielded an explicit periodic solution under well–defined parameter constraints. These constraints delineate the parameter space in which physically admissible stationary solitons can exist, clarifying the interplay among nonlinear chromatic dispersion, self–steepening, and self–frequency–shift effects.

The analytical complexity of the solutions are a far cry from the explicit quiescent soliton solutions that were recovered in 2021 [3]. Compared with the earlier results obtained via extended elliptic–function methods, the Lie symmetry approach offers a unified and transparent framework capable of handling multiple nonlinear dispersion and SPM laws within a single analytical setting. This framework also highlights integrability conditions (such as specific exponent relations) that make the reduced equations analytically tractable.

The results of this work are interesting and thus pave the way for further activities with the model with time. Future investigations will extend the present analysis to include generalized temporal evolution, dissipative terms, and higher–order nonlinearities. Numerical simulations and stability analyses will be performed to validate the analytical predictions and explore the robustness of the stationary profiles under perturbations. Later, these same models are going to be addressed with generalized temporal evolution and the results of those research activities will be presented in a different journal. Overall, this work provides a clear analytical foundation for exploring nonlinear–dispersion–driven pulse dynamics in optical fibers and photonic crystal fibers, serving as a stepping stone toward a comprehensive understanding of nonlinear wave propagation in advanced optical media. This is just a tip of the iceberg!

Disclosure

The authors claim there is no conflict of interest.

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НЕЯВНЕ СПОКІЙНЕ ОПТИЧНЕ СОЛІТОНЕ ЗБУРЕННЯ З НЕЛІНІЙНОЮ ХРОМАТИЧНОЮ ДИСПЕРСІЄЮ ТА ЛІНІЙНОЮ ЧАСОВОЮ ЕВОЛЮЦІЄЮ З ФОРМАМИ САМОФАЗОВОЇ МОДУЛЯЦІЇ КУДРЯШОВА ЗА СИМЕТРІЄЮ ЛІ

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У статті отримано неявні спокійні оптичні солітони для нелінійного рівняння Шредінгера, яке враховує нелінійну хроматичну дисперсію та лінійну часову еволюцію. Використовуючи стаціонарний або спокійний підхід у поєднанні з аналізом симетрії Лі, дослідження систематично розглядає шість різних структур самофазової модуляції, запропонованих Кудряшовим. Аналітична процедура зводить керівне рівняння до амплітудних форм, розв'язки яких отримуються через квадратури, що призводить як до неявних профілів одиночної хвилі, так і до одного явного періодичного випадку. Шість форм структур самофазової модуляції, запропонованих Кудряшовим, дали розв'язки в термінах квадратур, періодичних розв'язків, а також в термінах еліптичних функцій. Існування кожного сімейства розв'язків обговорюється з точки зору допустимих параметричних співвідношень, які забезпечують фізично значущі одиночні профілі. Цей підхід забезпечує єдину структуру порівняно з попередніми методами, заснованими на прямих розкладах еліптичних функцій, підкреслюючи, як симетрія Лі сприяє компактному обробленню кількох нелінійних законів дисперсії. Результати є актуальними для розуміння стаціонарних оптичних імпульсів у нелінійних волокнах та фотоннокристалічних волокнах, і вони закладають основу для майбутніх числових та експериментальних досліджень. нелінійне поширення імпульсів, зумовлене дисперсією.

Ключові слова: *спокійні солітони; хроматична дисперсія; квадратури; параметричні обмеження*