East European Journal of Physics. 4. 240-247 (2025)
DOI:10.26565/2312-4334-2025-4-21 ISSN 2312-4334

THIRD HARMONIC GENERATION OF Q-GAUSSIAN LASER BEAM IN COLLISIONLESS PLASMA

Kulkaran Singh, Keshav Walia*, Taranjot Singh
Department of Physics, DAV University Jalandhar, Punjab-144012, India
**Corresponding Author Email: keshavwalia86@gmail.com*
Received September 13, 2025; revised October 26, 2025; accepted October 27, 2025

In the current study, third-harmonic generation (THG) of a q-Gaussian laser beam propagating through a collisionless plasma is investigated. This nonlinear beam profile induces density gradients in the plasma due to the ponderomotive force. These density gradients excite electron plasma waves (EPW) at twice the pump wave frequency via $\vec{\nabla} \times \vec{B}$ mechanism. The fundamental pump wave and the EPW interact nonlinearly to produce third-harmonic radiation. The nonlinear ODE for the beam waist of the pump beam and the THG conversion efficiency expressions are obtained by employing WKB and paraxial approaches. The influence of key laser-plasma parameters, including plasma density, beam width, intensity, and q-parameter, on the self-focusing of the main beam and the 3rd harmonic efficiency is also analyzed. The results indicate that q-Gaussian beams, due to their higher field amplitudes and broader wings than conventional Gaussian beams, can significantly enhance THG in collisionless plasmas. These findings provide insights into optimizing harmonic generation in structured laser beams for applications in ultrafast optics, particle acceleration, and plasma-based radiation sources.
Keywords: *Collisionless Plasma; Density Gradients; Pump Wave; Electron Plasma Wave; Self-focusing; Third harmonic Generation*
PACS: 52.38.Hb, 52.35.Mw, 52.38.Dx

1. INTRODUCTION

Lasers interaction with plasma medium is an important research topic amongst theoretical/experimental researchers in recent decades, owing to their crucial role in inertial confinement fusion (ICF), charged particle acceleration and radiation sources [1-6]. During laser-plasma interaction, a wide variety of nonlinear processes, including self-focusing, parametric instabilities, self-phase modulation, and harmonic generation, are produced [6-15]. Harmonic generation has received significant attention due to its fundamental significance and technological applications. The propagation dynamics of the laser beam is strongly influenced by harmonic generation. Harmonic generation facilitates beam propagation in overdense regions and supports mode conversion processes. The generation of harmonics has been extensively studied in laser-plasma systems [16-21]. Third harmonics generation becomes an important nonlinear process for ultra-relativistic laser intensities and structured beams. Mostly the work on harmonic generation was explored by considering uniform plane waves. However, actual laser beams have non-uniform spatial irradiance profiles and nonlinear processes are greatly triggered by such spatial profiles. Such non-uniform profiles exhibit self-focusing/Self-defocusing phenomena. So, they modify overall electric field strength and hence enhance efficiency of harmonic production. q-Gaussian beams have higher field amplitude across the wave front in comparison to ordinary Gaussian profiles. So, this motivates us to explore third harmonic radiation from q-Gaussian beam in plasmas. Researchers interest in laser-plasma interaction extends beyond plasma optics. Because, electron plasma wave (EPW) is excited at high frequency during this process. This high frequency plasma wave may result in to production of energetic electrons, which can preheat the fusion target and could cause degradation of implosion performance. Similarly, high frequency plasma wave transfers energy to charged particles through wave-particle interaction. In this process, charged particles acquire extremely high energy thereby causing ultrahigh acceleration gradients. So, in actual practice, nonlinear plasma response that generates harmonic radiation also plays key part in such processes.

Experimental/Theoretical researchers have explored higher harmonic radiations in plasma. Schifano et al. [22] explored generation of harmonics in plasma through filamentation process.

Esarey et al. [23] explored relativistic harmonic radiations in plasma and further the discussed the effect of diffraction in forward harmonic emission. The production of harmonics in forward direction has also been explored in underdense plasma [24-25]. Ganeev et al. [26] explored production of higher harmonics in plasma plumes and Gupta et al. [27] explored third harmonic generation at ultrarelativistic intensities. The use of wiggler magnetic field has been shown to significantly enhance THG efficiency by satisfying phase matching conditions [28-31]. Azad et al. explored that Raman-enhanced nonlinear effects in Hermite-cosh Gaussian beam are found to increase harmonic efficiency in magnetized plasmas [32]. Ganeev et al. explored that higher harmonic generation in boron carbide plasmas are found to show enhancement over carbon and boron plasmas [33]. Laser pulse critically influences harmonic yield, increasing linearly below and decreasing near saturation [34]. R.A. Ganeev explored enhancement of higher harmonics in laser induced plasmas [35]. Furthermore, pulse slippage and density transitions have been identified as critical factors influencing THG

Cite as: K. Singh, K. Walia, T. Singh, East Eur. J. Phys. 4, 240 (2025), https://doi.org/10.26565/2312-4334-2025-4-21

efficiency [37-39]. It is already quite clear from these observations that third harmonic generations in plasmas are sensitive to beam structure, plasma profile, and nonlinear interaction geometry. Recently, a new class of lasers i.e. q-Gaussian lasers has been invented. Such beams have intensity irradiance in the form, $f(r) = f(0)\left(1+\frac{r^2}{qr_0^2}\right)^{-q}$. Setting $q -> \infty$ changes it to conventional Gaussian beam. There is greater flexibility linked with q-Gaussian profiles on account of tunable q-parameter. This q-parameter helps them to change their profiles from Gaussian like to Super-Gaussian like forms. Such beams also exhibit less divergence thereby enabling improved confinement of optical energy. q-Gaussian beam profiles also exhibit reduced power content and broader wings, which makes them suitable candidate in advanced nonlinear plasma studies. The researchers have explored numerous plasma instabilities using q-Gaussian laser beams in laser-plasma interaction studies, due to their strong nonlinear coupling characteristics [40-44]. The aim of present study is to explore THG of q-Gaussian laser beam propagating through collisionless plasma, with particular focus on understanding the roles of plasma density gradients and EPW in enhancing THG efficiency. It analyzes how q-parameter, beam width, intensity and plasma density affect self-focusing and THG efficiency optimization. The plasma electrons get redistributed as a result of ponderomotive force, thereby creating density gradients in plasma. They produce EPW at twice frequency of fundamental wave. The nonlinear interaction of EPW excited at 2ω with fundamental wave having frequency ω produces third harmonic generation. The paper is organized as follows: In Section 2, wave equation for q-Gaussian beam in collisionless has been derived using paraxial theory. In Section 3, we derived the amplitude of electron plasma wave at 2ω starting from the fluid equations and further using perturbation analysis and process of linearization. In Section 4, we derived the efficiency of 3rd harmonics. Sections 5 & 6 are devoted to discussion of results and conclusion respectively.

2. SOLUTION OF WAVE EQUATION FOR PUMP BEAM

The current study explores transition of q-Gaussian beam in plasma in z-direction. The beam's intensity at plane $z = 0$ is represented as

$$E \cdot E^*|_{z=0} = E_0^2\left(1+\frac{r^2}{qr_0^2}\right)^{-q}. \tag{1}$$

In Eq. (1), E_0 and r_0 denote field amplitude and beam radius at $z = 0$. q-Gaussian beam profiles form more generalized concept of normal Gaussian profile through q value. Increasing q values make them closer to Gaussian beam profiles. If we keep the values of q very low, then we are in a position to produce broader/flatter profiles. For $z > 0$, the beam irradiance takes the form

$$E \cdot E^* = \frac{E_0^2}{f^2}\left(1+\frac{r^2}{qr_0^2 f^2}\right)^{-q} \tag{2}$$

In Eq. (2), f is a parameter telling us about the contraction/divergence of the beam as it transits through plasma. This parameter, known as the beam waist, is related to the equilibrium beam radius. The Wave equation for the pump beam is expressed as

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E = 0 \tag{3}$$

In writing Eq. (3), we just ignored $\nabla(\nabla \cdot E)$ term. When we are having transverse waves then, $\nabla \cdot E = \mathrm{k} \cdot E = 0$, where k is wave vector. Then term $\nabla(\nabla \cdot E)$ has also been neglected in present problem considering $\frac{c^2}{\omega^2}\left|\frac{1}{\varepsilon}\nabla^2 \ln \varepsilon\right| \leq 1$. The alteration in plasma's dielectric response is observed in course of laser propagation within plasma. The Plasma's total dielectric function is comprising of two parts, i.e.

$$\varepsilon = \varepsilon_0 + \Phi(E \cdot E^*) \tag{4}$$

In Eq. (4), there are two contributions on RHS, $'\varepsilon_0'$ being linear contribution while $'\Phi(E \cdot E^*)'$ being nonlinear contribution. Both these terms are expressed mathematically as

$$\varepsilon_0 = 1 - \frac{\omega_p^2}{\omega^2} \tag{5}$$

$$\Phi(E \cdot E^*) = \frac{\omega_p^2}{\omega^2}\left[1 - \frac{N_{0e}}{N_0}\right] \tag{6}$$

In the above Eqs. (5) and (6), $\omega_p = \sqrt{\frac{4\pi n e^2}{m}}$ is plasma frequency. $'N_{0e}'$ and $'N_0'$ denote the concentrations of plasma electrons in presence/absence of beams. Suppose a beam profile with angular frequency ω and wave vector k is transiting along the z-axis in a hot collisionless plasma. During the transition of the laser beam through plasma, the nonlinear ponderomotive force induces density variations in plasma. This further changes the plasma density profile in the transverse direction as

$$\frac{N_{0e}}{N_0} = exp\left(-\frac{3}{4}\beta\frac{m}{M}E \cdot E^*\right) \tag{7}$$

In Eq. (7), $\beta = \frac{e^2 M}{6K_B T_0 \gamma \omega^2 m^2}$; Where, e, m, M, T_0 and K_B correspond to electronic charge, electronic mass, ionic mass, plasma temperature and Boltzmann constant respectively. So, on using Eq. (7), we can write the nonlinear portion for collisionless plasma as

$$\Phi(E \cdot E^*) = \frac{\omega_p^2}{\omega^2}\left[1 - exp\left(-\frac{3}{4}\beta\frac{m}{M}E \cdot E^*\right)\right] \tag{8}$$

Now, we will make use of the approach [45-46] to obtain a solution of Eq. (3)

$$E = E_0(r,z)\exp[i(\omega t - k(S+z))] \tag{9}$$

$$E_0 \cdot E_0^* = \frac{E_{00}^2}{f^2}\left(1 + \frac{r^2}{qr_0^2 f^2}\right)^{-q} \tag{10}$$

$$S = \frac{1}{2}r^2\frac{1}{f}\frac{df}{dz} + \Phi_0(z) \tag{11}$$

$$k = \frac{\omega}{c}\sqrt{\varepsilon_0} \tag{12}$$

Here, f is a parameter describing beam waist of q-Gaussian beam and it satisfies 2nd order differential equation as

$$\frac{d^2 f}{d\eta^2} = \frac{q+4}{qf^3} - \left(\frac{\omega_p r_0}{c}\right)^2\left(\frac{3}{4}\beta\frac{m}{M}E_{00}^2\right) exp\left[-\frac{3}{4}\beta\frac{m}{M}\frac{E_{00}^2}{f^2}\right]\frac{1}{f^3} \tag{13}$$

Eq. (13) denotes variation in beam waist with normalized propagation distance as laser beam transition in hot collisionless plasma take place. The boundary conditions are $f = 1$ & $\frac{df}{d\eta} = 0$ at $\eta = 0$.

3. EXCITATION OF ELECTRON PLASMA WAVE

We have considered the dynamics of plasma electrons in exploring excitation mechanism of EPW. Since, mass of ions is almost 2000 times mass of electrons. So, they will not react to large frequency field and will be treated as fixed in their respective position. Moreover, ions don't play any role in excitation mechanism of EPW. The nonlinear ponderomotive force produces density gradients in collisionless plasma, which act as source for EPW. We will start from the following Fluid equations in order to derive source term for THZ.

$$\frac{\partial N_e}{\partial t} + \nabla \cdot (N_e V) = 0 \tag{14}$$

$$\nabla \cdot E = 4\pi(ZN_{oi} - N_e)e \tag{15}$$

$$\frac{P}{N_e^{\gamma}} = Constant \tag{16}$$

$$m\left[\frac{\partial V}{\partial t} + (V \cdot \nabla)\mathrm{V}\right] = -e\left[E + \frac{1}{c}V \times B\right] - 2\Gamma mV - \frac{\gamma}{N_e}\nabla P_e \tag{17}$$

Using perturbation analysis and further linearizing the equations, amplitude term in case of EPW is obtained as

$$n_{2\omega}(r) = \frac{en_0}{m}\frac{E_{00}}{f}\left(1 + \frac{r^2}{qr_0^2 f^2}\right)^{\frac{-q}{2}}\left\{\frac{r}{r_0^2 f^2}\right\}\frac{1}{\left\{4\omega^2 - k^2 v_{th}^2 - \omega_p^2\left(exp\left[-\frac{3}{4}\beta\frac{mE_{00}^2}{M f^2}\right]\right)^2\right\}} \tag{18}$$

4. EFFICIENCY OF THIRD HARMONICS

Third harmonic radiation is produced through coupling of interaction of amplitude term for EPW $n_{2\omega}$ with fundamental beam.

$$J_3^{NL} = -en_{2\omega}v_0 \tag{19}$$

In presence of input field, an electron oscillatory velocity is

$$v_0 = -\frac{eE_0}{mi\omega} \tag{20}$$

$$\text{Hence, } J_3^{NL} = \frac{e^2}{mi\omega}n_{2\omega}E_0 \tag{21}$$

The field vector E_3 of THG obeys wave equation

$$\nabla^2 E_3 + \frac{\omega_3^2}{c^2}\varepsilon_3(\omega_3)E_3 = -\frac{8\pi i\omega_3}{c^2}J_3^{NL} \tag{22}$$

On substituting value of J_3^{NL} from Eq. (21) in Eq. (22), one can obtain

$$\nabla^2 E_3 + \frac{\omega_3^2}{c^2}\varepsilon_3(\omega_3)E_3 = \frac{\omega_p^2}{c^2}\frac{n_{2\omega}}{n_0}E_0 \tag{23}$$

In Eq. (23), $\omega_3 = 3\omega$ and ε_3 are for 3rd harmonic frequency and 3rd harmonic dielectric function respectively. We can easily solve Eq. (23) to obtain

$$E_3 = \frac{\omega_p^2}{c^2}\frac{n_{2\omega}}{n_0}\frac{E_{00}}{\mathrm{f}}\left(1+\frac{r^2}{qr_0^2f^2}\right)^{\frac{-q}{2}}\frac{1}{(k_3^2-9k^2)} \tag{24}$$

Now, the power of 3rd harmonics is

$$P_3 = \iint |E_3|^2\,dxdy \tag{25}$$

The power of fundamental beam is

$$P_0 = \iint |E_0|^2\,dxdy \tag{26}$$

The efficiency of 3rd harmonics is expressed as

$$Y_3 = \frac{\iint\left(\frac{\omega_p^2}{c^2}\frac{e}{m}\left(\frac{E_{00}}{f}\left(1+\frac{r^2}{qr_0^2f^2}\right)^{\frac{-q}{2}}\right)^2\left\{\frac{r}{r_0^2f^2}\right\}\frac{1}{\left\{4\omega^2-k^2v_{th}^2-\omega_p^2\left(exp\left[-\frac{3}{4}\beta\frac{m}{M}\frac{E_{00}^2}{f^2}\right]\right)^2\right\}}\frac{1}{(k_3^2-9k^2)}\right)^2 rdrd\theta}{\iint\left(\frac{E_{00}}{f}\left(1+\frac{r^2}{qr_0^2f^2}\right)^{\frac{-q}{2}}\right)^2 rdrd\theta} \tag{27}$$

5. DISCUSSION

The Eqs. (13) and (27) don't have analytical solution on account of their nonlinear and coupled character. So, numerical simulations of these equations are performed by taking established laser and plasma parameters;

$$\alpha\left(=\frac{3}{4}\beta\frac{m}{M}\right)E_{00}^2 = 3.0, 4.0, 5.0\,;\ \frac{\omega_p^2}{\omega^2} = 0.3, 0.5, 0.7\,;\ r_0 = 15\ \mu\mathrm{m}, 20\ \mu\mathrm{m}, 25\ \mu\mathrm{m},\ q = 1, 2, 3$$

Eq. (13) contains two contributions on RHS with each contribution has fundamental physical effect. The first term causes spreading of beam as it transits through medium. The second term causes nonlinear self-focusing of beam. So, the overall beam response in plasma is decided through interplay between these two opposite mechanisms. If first term is stronger, then beam divergence takes place. If second term is stronger, then contraction of beam takes place. When these two opposing mechanisms exactly balance each other in magnitude, then beam neither contracts nor diverges. On the other hand, beam's radius remains almost fixed during its transition through plasma medium. This equilibrium state so obtained is known as self-trapping, where beam transits through plasma medium without spreading or collapsing.

Figure 1 illustrates variation of f with η for three different beam intensities i.e. $\alpha E_{00}^2 = 3.0, 4.0, 5.0$. The curves corresponding to $\alpha E_{00}^2 = 3.0$, $\alpha E_{00}^2 = 4.0$, and $\alpha E_{00}^2 = 5.0$ are shown in blue, green and red respectively. It is clear from results that increment in αE_{00}^2 parameter reduces tendency of beam to focus. This behavior arises due to reason that divergence term becomes more dominant than convergence term at higher αE_{00}^2 parameter. So, there is weakening of self-focusing at higher αE_{00}^2.

Figure 2 illustrates variation of f with η for three different plasma densities i.e. $\frac{\omega_p^2}{\omega^2} = 0.3, 0.5, 0.7$. The curves corresponding to $\frac{\omega_p^2}{\omega^2} = 0.3$, $\frac{\omega_p^2}{\omega^2} = 0.5$, and $\frac{\omega_p^2}{\omega^2} = 0.7$ are shown in blue, green and red respectively. It is clear from results that increment in $\frac{\omega_p^2}{\omega^2}$ parameter enhances tendency of beam to focus. This behavior arises due to reason that refractive term becomes more dominant than diffraction term at higher $\frac{\omega_p^2}{\omega^2}$ values. So, there is strengthening of self-focusing at higher $\frac{\omega_p^2}{\omega^2}$ values.

Figure 3 illustrates variation of f with η for three different beam radii i.e. $r_0 = 15\mu m, 20\mu m, 25\mu m$. The curves corresponding to $r_0 = 15\mu m$, $r_0 = 20\ \mu m$, and $r_0 = 25\ \mu m$ are shown in Blue, Green and Red respectively. It is clear from results that increment in r_0 parameter enhances tendency of beam to focus. This behavior arises due to reason that refractive term becomes more dominant than diffraction term at higher r_0 value. So, there is strengthening of self-focusing at higher r_0.

Figure 4 illustrates variation of f with η for three different q-values i.e. $q = 1,2,3,$ and ∞. The curves corresponding to $q = 1$, $q = 2, q = 3,$ and $q = \infty$ shown in Blue, Green, Red and Black respectively. It is clear from figures that increment in q parameter enhances tendency of beam to focus. This is due to increase in intensity localization along axis of beam with rapid increase in q values. So, axial rays start undergoing focusing earlier than off-axial rays, thereby improving focusing characteristics of beam. For $q \to \infty$, the beam profile changes to ordinary Gaussian beam, and has less sharply localized intensity thereby reducing nonlinear focusing effect, hence decreasing self-focusing compared to finite q values.

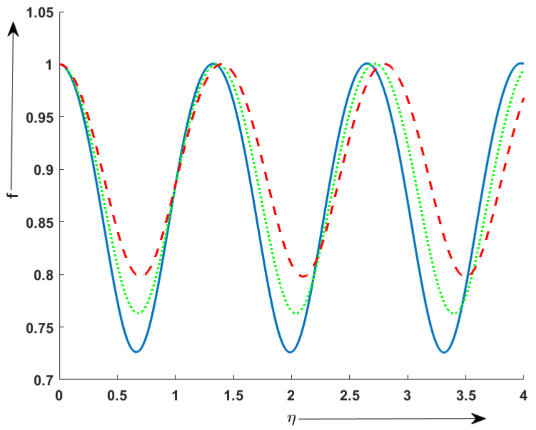

Figure 1. Variation of f with η for three different beam intensities i.e. $\alpha E_{00}^2 = 3.0, 4.0, 5.0$. The curves corresponding to $\alpha E_{00}^2 = 3.0$, $\alpha E_{00}^2 = 4.0$, and $\alpha E_{00}^2 = 5.0$ are shown in blue, green and red respectively

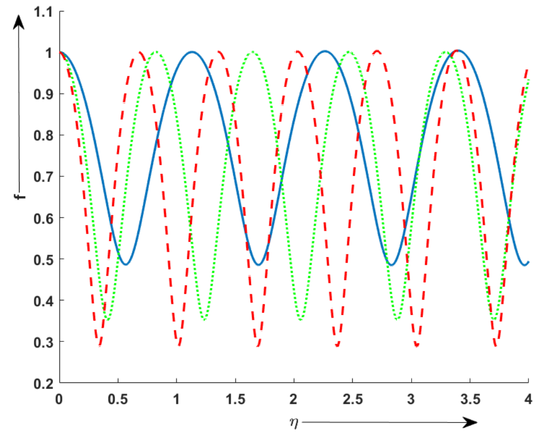

Figure 2. Variation of f with η for three different plasma densities i.e. $\frac{\omega_p^2}{\omega^2} = 0.3, 0.5, 0.7$. The curves corresponding to $\frac{\omega_p^2}{\omega^2} = 0.3$, $\frac{\omega_p^2}{\omega^2} = 0.5$, and $\frac{\omega_p^2}{\omega^2} = 0.7$ are shown in blue, green and red respectively

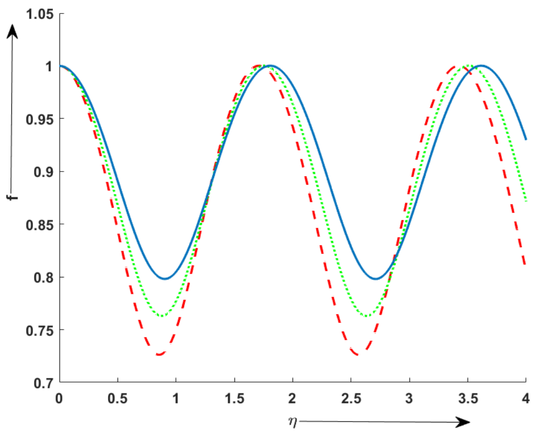

Figure 3. Variation of f with η for three different beam radii i.e. $r_0 = 15\mu m, 20\mu m, 25\mu m$. The curves corresponding to $r_0 = 15\mu m$, $r_0 = 20\ \mu m$, and $r_0 = 25\ \mu m$ are shown in Blue, Green and Red respectively

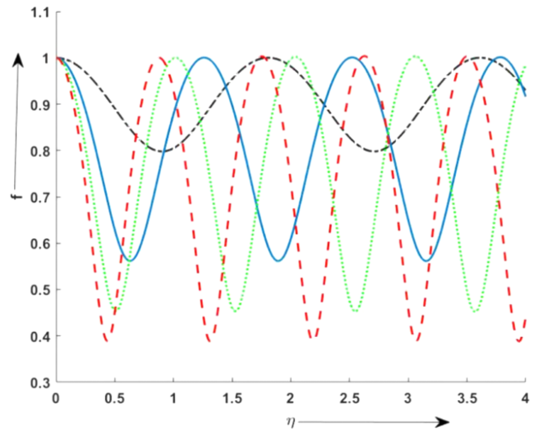

Figure 4. Variation of f with η for three different q-values i.e. $q = 1,2,3,\infty$. The curves corresponding to $q = 1$, $q = 2$, $q = 3$, and $q = \infty$ shown in Blue, Green, Red and Black respectively

Figure 5 illustrates variation of Y_3 with η for two different beam intensities i.e. $\alpha E_{00}^2 = 4.0, 5.0$. The curves corresponding to $\alpha E_{00}^2 = 4.0$, and $\alpha E_{00}^2 = 5.0$ are shown in Blue, and Red respectively. It is clear from results that there is decrease in magnitude of Y_3 with enhancement in αE_{00}^2 parameter. This reduction is directly connected with focusing characteristics of pump beam. Since, focusing ability of pump beam is found to get weakened by increasing αE_{00}^2 parameter. So, amplitude of EPW and hence yield of 3rd harmonics are decreased accordingly.

Figure 6 illustrates variation of Y_3 with η for two different plasma densities i.e. $\frac{\omega_p^2}{\omega^2} = 0.5, 0.7$. The curves corresponding to $\frac{\omega_p^2}{\omega^2} = 0.5$, and $\frac{\omega_p^2}{\omega^2} = 0.7$ are shown in Blue, and Red respectively. It is clear from the results that there is increase in magnitude of Y_3 with enhancement in $\frac{\omega_p^2}{\omega^2}$ parameter. This amplification is directly connected with focusing characteristics of pump beam. Since, focusing ability of pump beam is found to get strengthened by increasing $\frac{\omega_p^2}{\omega^2}$ parameter. So, amplitude of EPW and hence yield of 3rd harmonics are enhanced accordingly.

Figure 7 illustrates variation of Y_3 with η for two different beam radii i.e. $r_0 = 20\mu m, 25\mu m$. The curves corresponding to $r_0 = 20\mu m$, and $r_0 = 25\mu m$ are shown in Blue, and Red respectively. It is clear from the results that there is increase in magnitude of Y_3 with enhancement in r_0 parameter. This amplification is directly connected with focusing characteristics of pump beam. Since, focusing ability of pump beam is found to get strengthened by increasing r_0 parameter. So, amplitude of EPW and hence yield of 3rd harmonics are enhanced accordingly.

Figure 8 illustrates variation of Y_3 with η for two different q-values i.e. $q = 1, 2$ and ∞. The curves corresponding to $q = 1$, $q = 2$ and $q = \infty$ are shown in Blue, Red and Black respectively. It is clear from results that magnitude of Y_3 is amplified with enhancement in q values. This is on account of reason that increase in q value enhances axial self-focusing of pump beam thereby amplifying nonlinear interaction that produces third harmonics. So, dominant role is played by q-parameter in optimizing THG output. For $q \to \infty$, the beam profile changes to ordinary Gaussian beams and has less localized intensity, reducing axial self-focusing and thus lowering the efficiency of 3rd harmonics.

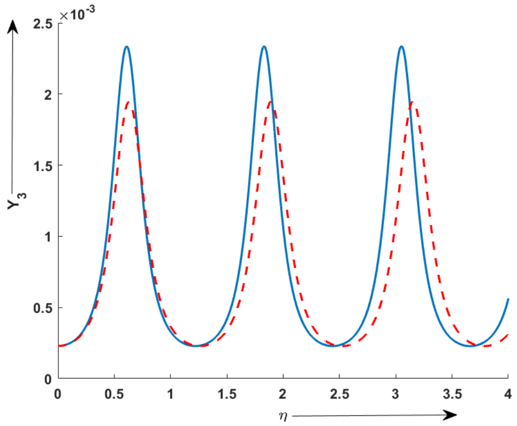

Figure 5. Variation of Y_3 with η for two different beam intensities i.e. $\alpha E_{00}^2 = 4.0, 5.0$. The curves corresponding to $\alpha E_{00}^2 = 4.0$, and $\alpha E_{00}^2 = 5.0$ are shown in Blue, and Red respectively

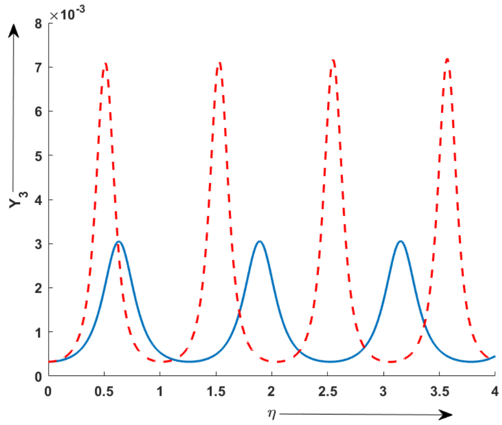

Figure 6. Variation of Y_3 with η for two different plasma densities i.e. $\frac{\omega_p^2}{\omega^2} = 0.5, 0.7$. The curves corresponding to $\frac{\omega_p^2}{\omega^2} = 0.5$, and $\frac{\omega_p^2}{\omega^2} = 0.7$ are shown in Blue, and Red respectively

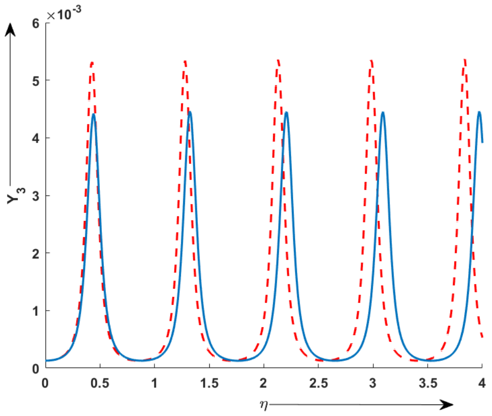

Figure 7. Variation of Y_3 with η for two different beam radii i.e. $r_0 = 20\mu m, 25\mu m$. The curves corresponding to $r_0 = 20\mu m$, and $r_0 = 25\mu m$ are shown in Blue, and Red respectively

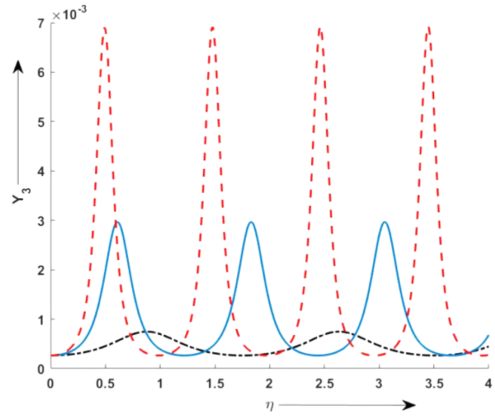

Figure 8. Variation of Y_3 with η for two different q-values i.e. $q = 1, 2$ and ∞. The curves corresponding to $q = 1$, $q = 2$ and $q = \infty$ are shown in Blue, Red and Black respectively

6. CONCLUSIONS

The present study explores third-harmonic generation of a q-Gaussian beam in a Collisionless plasma using the paraxial approach. The analysis reveals two key outcomes:

1. Beam Focusing: The beam's self-focusing capability is strengthened with enhancement in plasma density, beam radius, q-parameter, and with a decrease in beam intensity. To optimize self-focusing, we need careful control of beam and plasma parameters.
2. THG Efficiency: The efficiency of 3rd harmonics is amplified with enhancement in plasma density, beam radius, q-parameter, and with reduction in beam intensity. So, to optimize THG efficiency, we need to have careful control of beam and plasma parameters

These results are highly crucial for applications in laser-driven fusion and other laser-plasma interaction schemes, where the production of 3rd harmonics plays an important role.

ORCID

Keshav Walia, https://orcid.org/0000-0001-9547-3027

REFERENCES

[1] H. Hora, "New aspects for fusion energy using inertial confinement," Laser Part. Beams, **25**, 37–45 (2007). https://doi.org/10.1017/s0263034607070073

[2] T. Feder, "Petawatt laser probes nature at Texas University," Phys. Today, **61**, 27 (2008). https://doi.org/10.1063/1.3001859
[3] R. Singh, A. Sharma, and V. Tripathi, "Relativistic self-distortion of a laser pulse and ponderomotive acceleration of electrons in an axially inhomogeneous plasma," Laser Part. Beams, **28**, 299 (2010). https://doi.org/10.1017/s0263034610000200
[4] E. Esarey, C.B. Schroeder, and W.P. Leemans, "Physics of laser-driven plasma-based electron accelerators," Rev. Mod. Phys., **81**, 1229 (2009). https://doi.org/10.1103/revmodphys.81.1229
[5] R. Bingham, J.T. Mendonca, and P.K. Shukla, "Topical review: plasma based charged-particle accelerators," Plasma Phys. Controlled Fusion, **46**, R1 (2004). https://doi.org/10.1088/0741-3335/46/1/r01
[6] Z. Zeng, Y. Cheng, X. Song, R. Li, and Z. Xu, "Generation of an extreme ultraviolet supercontinuum in a two color laser field," Phys. Rev. Lett. **98**, 203901 (2007). https://doi.org/10.1103/physrevlett.98.203901
[7] K. Walia, "Propagation characteristics of a high-power beam in weakly relativistic-ponderomotive thermal quantum plasma," Commun. Theor. Phys., **75**, 095501 (2023). https://doi.org/10.1088/1572-9494/accf82
[8] K. Walia, "Nonlinear interaction of high power beam in weakly relativistic and ponderomotive cold quantum plasma," Optik, **219**, 165040 (2020). https://doi.org/10.1016/j.ijleo.2020.165040
[9] K. Walia, "Nonlinear Interaction of High Power Elliptical Laser Beam with Cold Collisionless Plasma," J. Fusion Energ. **35**, 446 (2016). https://doi.org/10.1007/s10894-016-0059-0
[10] T. Singh, and K. Walia, "Impact of High-Power Cosh-Gaussian Beam on Second Harmonic Generation in Collisionless Magnetoplasma," J. Contemp. Phys. **59**, 254 (2024). https://doi.org/10.1134/s106833722470049x
[11] K. Walia, "Self-focusing of high power beam in unmagnetized plasma and its effect on Stimulated Raman scattering process," Optik, **225**, 165592 (2021). https://doi.org/10.1016/j.ijleo.2020.165592
[12] K. Walia, "Stimulated Brillouin Scattering of high power beam in unmagnetized plasma: Effect of relativistic and ponderomotive nonlinearities," Optik, **221**, 165365 (2020). https://doi.org/10.1016/j.ijleo.2020.165365
[13] K.M. Gavade, T.U. Urunkar, B.D. Vhanmore, A.T. Valkunde, M.V. Takale, and S.D. Patil, "Self-focusing of Hermite cosh Gaussian laser beams in a plasma under a weakly relativistic and ponderomotive regime," J. Appl. Spectrosc. **87**, 499–504 (2020). https://doi.org/10.1007/s10812-020-01030-1
[14] S. Chib, L.D. Essakali, and A. Belafhal, "Propagation properties of a novel generalized flattened Hermite-cosh-Gaussian light beam," Opt. Quantum Electron. **52**, 277 (2020). https://doi.org/10.1007/s11082-020-02406-5
[15] M.A. Wani, and N. Kant, "Investigation of relativistic self-focusing of Hermite-cosine-Gaussian laser beam in collisionless plasma," Optik, **127**, 4705 (2016). https://doi.org/10.1016/j.ijleo.2016.02.026
[16] K. Singh, and K. Walia, "Second Harmonic Generation of High Power Cosh-Gaussian Beam in Thermal Quantum Plasma: Effect of Relativistic and Ponderomotive Nonlinearity," J. Contemp. Phys. **59**, 244 (2024). https://doi.org/10.1134/s1068337224700488
[17] K. Singh, and K. Walia, "Influence of Self-Focused Elliptical Laser Beam on Second Harmonic Generation in Cold Quantum Plasma," J. Contemp. Phys. **59**, 154 (2024). https://doi.org/10.1134/s1068337224700300
[18] J. Wadhwa, and A. Singh, "Generation of second harmonics by a self-focused Hermite-Gaussian laser beam in collisionless plasma," Phys. Plasmas, **26**, 062118 (2019). https://doi.org/10.1063/1.5086858
[19] R. Singh, and V.K. Tripathi, "Brillouin shifted third harmonic generation of a laser in a plasma," J. Appl. Phys. **107**, 113308 (2010). https://doi.org/10.1063/1.3431402
[20] H.R. Askari, and M. Noroozi, "Effect of a wiggler magnetic field and the ponderomotive force on the second harmonic generation in laser-plasma interactions," Turkish J. Phys. **33**, 299 (2009). https://doi.org/10.3906/fiz-0810-14
[21] M. Abedi-Varaki, and S. Jafari, "Second-harmonic generation of a linearly polarized laser pulse propagating through magnetized plasma in the presence of a planar magnetostatic wiggler," Eur. Phys. J. Plus, 133, 1 (2018). https://doi.org/10.1140/epjp/i2018-11975-2
[22] E. Schifano, S.D. Baton, C. Labaune, N. Renard, V. Biancalana, A. Giulietti, and D. Giulietti, "Second harmonic emission from laser-preformed plasmas as a diagnostic for filamentation in various interaction conditions," Laser Part. Beams, **12**, 435 (1994). https://doi.org/10.1017/s0263034600008296
[23] E. Esarey, A. Ting, P. Sprangle, D. Umstadter, and X. Liu, "Nonlinear analysis of relativistic harmonic generation by intense lasers in plasmas," IEEE Trans. Plasma Sci. 21, 95 (1993). https://doi.org/10.1109/27.221107
[24] S.D. Baton, H.A. Baldis, T. Jalinaud, and C. Labaune, "Fine-Scale Spatial and Temporal Structures of Second-Harmonic Emission from an Underdense Plasma," Europhys. Lett. **23**, 191 (1993). https://doi.org/10.1209/0295-5075/23/3/006
[25] F. Brandi, F. Giammanco, and W. Ubachs, "Spectral red shift in harmonic generation from plasma dynamics in laser focus," Phys. Rev. Lett. **96**, 123904 (2006). https://doi.org/10.1103/physrevlett.96.123904
[26] R.A. Ganeev, M. Suzuki, M. Baba, and H. Kuroda, "Higher order harmonic generation from laser plasma produced by pulses of different duration," Phys. Rev. A, **76**, 023805 (2007). https://doi.org/10.1103/physreva.76.023805
[27] M.K. Gupta, R.P. Sharma, and S.T. Mahmoud, "Generation of plasma wave and third harmonic generation at ultra-relativistic laser power," Laser Part. Beams, **25**, 211 (2007). https://doi.org/10.1017/s0263034607000031
[28] H. Sharma, H. Jaloree, and J. Prashar, "Magnetic field wiggler assisted third harmonic generation of a Gaussian laser pulse in plasma," Turk. J. Phys. **37**, 368–374 (2013). https://doi.org/10.3906/fiz-1212-12
[29] V. Sharma, V. Thakur, and N. Kant, "Third harmonic generation of a relativistic self-focusing laser in plasma in the presence of wiggler magnetic field," High Energy Dens. Phys. **32**, 51–55 (2019). https://doi.org/10.1016/j.hedp.2019.04.007
[30] M. Abedi-Varaki, "Relativistic laser third-harmonic generation from magnetized plasmas under a tapered magnetostatic wiggler," Phys. Plasmas, **30**, 083102 (2023). https://doi.org/10.1063/5.0155016
[31] M. Singh, and D.N. Gupta, "Relativistic third-harmonic generation of a laser in a self-sustained magnetized plasma channel," IEEE J. Quantum Electron. **50**, 491-496 (2014). https://doi.org/10.1109/jqe.2014.2320763
[32] T. Azad, N. Kant, A. Paknezhad, *et al.* "Raman-enhanced nonlinear effects on third harmonic generation in plasma using Hermite cosh Gaussian laser beam," Opt. Quant. Electron. **57**, 239 (2025). https://doi.org/10.1007/s11082-025-08143-x
[33] R.A. Ganeev, "High-order harmonic generation in boron carbide laser-induced plasma: comparison with carbon and boron plasmas, two-color-pumping-induced enhancement, quasi-phase-matching, and tuning of harmonics," Optics Continuum, **3**(12), 2272–2285 (2024). https://doi.org/10.1364/optcon.540708

[34] S. Westerberg, M. Redon, A.-K. Raab, G. Beaufort, *et al*. "Influence of the laser pulse duration in high-order harmonic generation," APL Photonics, **10**, 096103 (2025). https://doi.org/10.1063/5.0272968
[35] R.A. Ganeev, "High-order harmonics enhancement in laser-induced plasma," Sci. Rep. **13**, 13951 (2023). https://doi.org/10.1038/s41598-023-41239-6
[36] V. Thakur, and N. Kant, "Effect of pulse slippage on density transition-based resonant third harmonic generation of short pulse laser in plasma," Front. Phys. **11**, 115202 (2016). https://doi.org/10.1007/s11467-016-0563-8
[37] V. Sharma, V. Thakur, A. Singh, and N. Kant, "Third harmonic generation of a relativistic self-focusing laser in plasma under exponential density ramp," Z. Naturforsch. A, **77**, 323–328 (2022). https://doi.org/10.1515/zna-2021-0266
[38] M.J. Basiry, M. Sharifian, M. Hashemzadeh, M. Borhani, and H. Alirezaie, "Influence of rippled density and laser profile on third harmonic generation using cosh–Gaussian laser pulses in inhomogeneous magnetized plasmas," Contrib. Plasma Phys. **63**, 202200196 (2023). https://doi.org/10.1002/ctpp.202200196
[39] K.Y. Khandale, P.T. Takale, T.U. Urunkar, S.S. Patil, P.P. Nikam, M.B. Mane, V.S. Pawar, *et al*. "On the exploration of q parameter in propagation dynamics of q-Gaussian laser beam in underdense collisional plasma," Bulg. J. Phys. **49**, 375–385 (2022). https://doi.org/10.55318/bgjp.2022.49.4.375
[40] D.N. Yadav, and S.C. Sharma, "Electron plasma wave excitation by a q-Gaussian laser beam and subsequent electron acceleration," Phys. Plasmas, **27**, 093106 (2020). https://doi.org/10.1063/5.0007998
[41] A.S. Ahmed, H.A. Salih, and K.I. Hassoon, "Relativistic self-focusing of a q-Gaussian laser beam in plasma by the influence of magnetic field," Braz. J. Phys. **53**, 112 (2023). https://doi.org/10.1007/s13538-023-01324-7
[42] D. Tripathi, S. Kaur, A. Vijay, and K. Walia, "Nonlinear Dynamics of q-Gaussian Laser Beam in Collisional Plasma: Effect of Linear Absorption," J. Contemp. Phys. **60**, 16 (2025). https://doi.org/10.1134/s1068337225700409
[43] K. Walia, N. Mehra, and S. Pandit, "Propagation Characteristics of q-Gaussian Laser Beam in Cold Collisionless Plasma," J. Contemp. Phys. 59, 378 (2024). https://doi.org/10.1134/s1068337225700203
[44] K. Walia, "Self-focusing of q-Gaussian beam in unmagnetized plasma and its impact on second harmonic generation," Optik, **277**, 170681 (2023). https://doi.org/10.1016/j.ijleo.2023.170681
[45] S.A. Akhmanov, A.P. Sukhorokov, and R.V. Kokhlov, "Self-focusing and diffraction of light in nonlinear medium," Sov. Phys. Uspekhi, 10, 609 (1968).
[46] M.S. Sodha, A.K. Ghatak, and V.K. Tripathi, "Self focusing of laser beams in plasmas and semiconductors," Prog. Opt. **13**, 169 (1976). https://doi.org/10.1016/s0079-6638(08)70021-0

ГЕНЕРАЦІЯ ТРЕТЬОЇ ГАРМОНІКИ Q-ГАУСІВСЬКОГО ЛАЗЕРНОГО ПРОМЕНЯ У ПЛАЗМІ БЕЗ ЗІТКНЕНЬ

Кулкаран Сінгх, Кешав Валья, Таранджот Сінгх
Кафедра фізики, Університет DAV, Джаландхар, Пенджаб-144012, Індія

У цьому дослідженні досліджується генерація третьої гармоніки (ГТГ) q-гауссового лазерного променя, що поширюється через беззіткнювальну плазму. Цей нелінійний профіль променя індукує градієнти густини в плазмі через пондеромоторну силу. Ці градієнти густини збуджують електронні плазмові хвилі (ЕПХ) з подвоєною частотою хвилі накачування за допомогою механізму $\vec{\mathbf{V}} \times \vec{\mathbf{B}}$. Фундаментальна хвиля накачування та ЕПХ взаємодіють нелінійно, створюючи випромінювання третьої гармоніки. Нелінійне ЗДР для перетяжки променя накачування та вирази ефективності перетворення ГТГ отримані за допомогою методу ВКБ та параксіального підходів. Також проаналізовано вплив ключових параметрів лазерної плазми, включаючи густину плазми, ширину променя, інтенсивність та q-параметр, на самофокусування основного променя та ефективність третьої гармоніки. Результати показують, що q-гаусові пучки, завдяки своїм вищим амплітудам поля та ширшим крилам, ніж у звичайних гаусових пучках, можуть значно покращити генерацію гармонік у беззіткнювальній плазмі. Ці результати дають уявлення про оптимізацію генерації гармонік у структурованих лазерних променях для застосувань в надшвидкій оптиці, прискоренні частинок та джерелах випромінювання на основі плазми.

Ключові слова: *беззіткнювальна плазма; градієнти густини; хвиля накачування; електронна плазмова хвиля; самофокусування; генерація третьої гармоніки*