EAST EUROPEAN JOURNAL OF PHYSICS. **4**. 75–86 (2025)
DOI: 10.26565/2312-4334-2025-4-07 ISSN 2312-4334

QUANTUM-CORRECTED THERMODYNAMICS OF AdS-RINDLER BLACK HOLES

Aram Bahroz Brzo[1,2*], **Peshwaz Abdulkareem Abdoul**[3**], **Behnam Pourhassan**[4,5,6***]
[1]*Physics Department, College of Education, University of Sulaimani, Sulaimani 46001, Kurdistan Region, Iraq*
[2]*Research and Development Center, University of Sulaimani, Sulaimani 46001, Kurdistan Region, Iraq*
[3]*Physics Department, College of Science, Charmo University: Chamchamal, Sulaimani, Kurdistan Region, Iraq*
[4]*School of Physics, Damghan University, Damghan 3671645667, Iran*
[5]*Center for Theoretical Physics, Khazar University, 41 Mehseti Street, Baku, AZ1096, Azerbaijan*
[6]*Centre of Research Impact and Outcome, Chitkara University, Punjab, Rajpura, 140417, India*
[*]*Corresponding Author e-mail: aram.brzo@univsul.edu.iq*
[**]*peshwaz.abdoul@chu.edu.iq*; [***]*b.pourhassan@du.ac.ir*
Received September 2, 2025; revised October 29, 2025; accepted November 5, 2025

We investigate the thermodynamic properties and stability of hyperbolic (AdS–Rindler) black holes, emphasizing the effects of non perturbative quantum correction. Using standard thermodynamic formulations alongside the Poincaré disk method, we compute key quantities including mass, Hawking temperature, entropy, and heat capacity. To account for quantum gravitational effects, we introduce an exponential correction to the Bekenstein–Hawking entropy and systematically derive the modified thermodynamic parameters. While the corrected entropy yields consistent adjustments, the heat capacity exhibits nontrivial behavior, leading to narrower and more gradual stable regions $\left(\Delta r_{(d)}\right)$ for each dimension d. Moreover, the smoothing of sharp entropy variations near $r_h = 1$ emphasizes how horizon geometry governs the impact of quantum corrections. This study provides the novel systematic identification of stable regions before and after exponential corrections of (AdS–Rindler) black holes, offering new insights into the interplay of geometry, dimensionality, and quantum effects in black hole thermodynamics.

Keywords: *Hyperbolic black holes; Quantum entropy correction; Stability analysis*

PACS: 04.60.-m, 04.70.Dy, 04.70.-s, 04.20.-q

1. INTRODUCTION

The study of black hole (BH) thermodynamics has been a cornerstone in understanding the intricate relationship between gravity, quantum mechanics, and statistical physics. Central to this field is the exploration of how black holes (BHs), as thermodynamic systems, adhere to laws analogous to the classical laws of thermodynamics. The seminal works of Bekenstein and Hawking established that BHs possess an entropy proportional to their horizon area [1, 2]. The formulation of BH mechanics as analogous to the laws of thermodynamics was first systematically established by Bardeen, Carter, and Hawking [3], who demonstrated that the surface gravity of a BH remains constant over the horizon, analogous to temperature in conventional thermodynamic systems. This foundational framework later faced challenges with the information paradox, which found a potential resolution pathway in Page's work [4], suggesting that information might be preserved in BH evaporation through subtle correlations in the radiation, with information beginning to emerge after approximately half the entropy has been radiated away—a concept now known as the 'Page curve.' A complementary breakthrough came from string theory when Strominger and Vafa [5] provided the first precise microstate counting for certain supersymmetric BHs, demonstrating that the Bekenstein-Hawking entropy formula could be derived from first principles by counting the degeneracy of D-brane configurations, thus establishing a crucial link between gravity and quantum theory that continues to influence the development of quantum corrections to BH thermodynamics. These discoveries have spurred extensive research into the microscopic origins of BH entropy and the potential quantum corrections that arise in various gravitational settings. In particular, the Anti-de Sitter (AdS) spacetime has garnered significant attention due to its role in the AdS/CFT correspondence, which posits a duality between a gravitational theory in AdS space and a conformal field theory on its boundary. Within this framework, Rindler-AdS spacetime emerge as a fascinating subclass, characterized by their hyperbolic horizons and constant acceleration analogous to Rindler coordinates in flat spacetime. These spacetime provide fertile ground for investigating the thermodynamic properties of BHs, especially when considering the implications of quantum corrections to classical entropy formulations [6, 7]. The importance of AdS black holes has been widely recognized, leading to extensive research in various directions. For instance, the reconciliation of the Weak Gravity Conjecture (WGC) and the Weak Cosmic Censorship Conjecture (WCCC) by examining Einstein-Euler-Heisenberg-AdS BHs in four-dimensional spacetime has been investigated in [8] by applying certain conditions to the metric parameters, they show that the (WGC) and the (WCCC) can be simultaneously satisfied. Furthermore, the investigation of the thermodynamic topology of AdS Einstein–power–Yang–Mills BHs using both bulk-boundary and restricted phase space (RPS) approaches, considering different non-extensive entropy models, has been studied [9]. Also,

Cite as: A.B. Brzo, P.A. Abdoul, B. Pourhassan, East Eur. J. Phys. 4, 75 (2025), https://doi.org/10.26565/2312-4334-2025-4-07

the thermodynamic topology of Schwarzschild-AdS BHs under the frameworks of non-commutative geometry and Barrow entropy, providing an overview of the black hole's properties and the relevance of Barrow entropy studied in [10].

The Bekenstein-Hawking entropy formula, which equates the entropy of a BH to a quarter of its horizon area, serves as the classical foundation for BH thermodynamics. However, various approaches in quantum gravity suggest that this entropy receives corrections, often manifesting as logarithmic terms including thermal fluctuations [11], higher-order entropy corrections [12], non-perturbative exponential corrections [13]. These corrections are pivotal in understanding the microstates contributing to BH entropy and have profound implications for the stability and phase structure of BHs [1, 2].

Recently the thermodynamic properties of AdS–Schwarzschild-type BHs through the framework of loop quantum gravity has been tested, showing that quantum corrections play a crucial role in reshaping their critical behavior and phase transition characteristics [14, 15, 16]. These corrections modify key aspects such as the equation of state, critical points, and heat capacity, ultimately affecting the stability conditions of BHs. Such insights highlight the importance of accounting for quantum effects to achieve a deeper and more complete understanding of black-hole thermodynamics [17, 18, 19].

In the realm of Rindler-AdS spacetimes, the entanglement entropy of holographic quantum fields has been the subject of extensive investigation. Emparan and Magán [20] demonstrated how quantum disentanglement modifies the entanglement structure of holographic quantum fields in Rindler–AdS geometry, while in [21] extending the Rindler method to compute timelike entanglement entropy in AdS_3/CFT_2 has been studied. Further developments in [22, 23] analyzed generalized Rindler wedges and inner-horizon entanglement, respectively, highlighting the geometric richness of these setups. Complementarily, Miao [24] investigated the Casimir and holographic dual aspects of AdS wedges, providing deeper insight into quantum correlations across Rindler horizons.

Interestingly, holographic analyses indicate that the entanglement entropy remains finite even in the zero-temperature limit, creating a puzzling scenario that has motivated extensive investigations into the quantum properties of Rindler–AdS BHs [20, 21, 22, 23].

In recent years, the study of quantum corrections to BH thermodynamics has gained significant attention [25, 26, 27, 28, 29], as these corrections might provide insight into the quantum nature of gravity and potentially resolve longstanding paradoxes such as the information loss problem [30]. Prior attempts to consider corrections to the BH entropy are due to the thermal fluctuations which at leading order add an alogarithmic term to the BH entropy [31, 32, 33, 34, 35, 36]. Higher-order entropy corrections represent another important approach to quantum BH thermodynamics. Upadhyay et al. [12] studied the P-V criticality of AdS BHs corrected for first-order entropy in massive gravity, finding that entropy corrections can substantially alter phase transition behavior. The application of quantum gravity approaches to BH thermodynamics represents the frontier of this field. Pourhassan et al. [37] investigated quantum gravitational corrections to the geometry of charged AdS BHs, while Lone et al. [38] applied topos theory to derive quantum gravitational corrections to a Kerr BH. These studies suggest that a full quantum theory of gravity might resolve longstanding issues in BH thermodynamics, including the information loss paradox. The classical Bekenstein-Hawking entropy formula ($\overset{(C)}{S}_{BH}$) is given by:

$$\overset{(C)}{S}_{BH} = \frac{k_B A}{4\pi\hbar G}, \tag{1}$$

where A is the area of the horizon given by:

$$A = n\epsilon\, l_p^2 \tag{2}$$

where ϵ is constant and n is the quantum number. In addition, the surface gravity κ is computed as:

$$\kappa = \frac{1}{2}\left.\frac{df}{dr}\right|_{r=r_h} \tag{3}$$

where $f(r)$ is the blackening factor. It is important to note that the expression for surface gravity presented in Eq. (3) is applicable only to static black hole configurations, such as the Schwarzschild and Reissner–Nordström solutions, and fails to describe rotating (Kerr) geometries (see [3, 39, 40]). Furthermore, the temperature (T) of a BH is related to its surface gravity (κ) by the formula:

$$T_H = \frac{\hbar\kappa}{2\pi}, \tag{4}$$

where, $\hbar$ is the reduced Planck constant. This relation arises from the Hawking temperature formula, which states that the temperature of a BH is proportional to its surface gravity. This result is fundamental in BH thermodynamics and is derived from quantum field theory in curved spacetime.

Quantum gravitational effects are expected to introduce corrections to this entropy. Various approaches, including string theory and loop quantum gravity, suggest modifications that often take the form of logarithmic or exponential terms.

For example, Calmet and Kuipers [41] calculated quantum gravitational corrections to the entropy of a Schwarzschild BH using the Wald entropy formula within an effective field theory approach.

BH entropy is generally expressed in the following form [42]:

$$S = \frac{A}{4\ell_P^2} + \alpha \ln \frac{A}{4\ell_P^2} + \beta \frac{4\ell_P^2}{A} + \cdots + \exp\left(-\delta \frac{A}{4\ell_P^2}\right) + \cdots \tag{5}$$

where the constants $(\alpha, \beta, \delta, \eta)$, and similar terms, are universal. For black holes with small horizon areas of order $(O(l_p^2))$, a complete quantum gravity framework is required for a proper description. In such cases, the logarithmic and higher-order correction terms involving $((l_p^2/A))$ may be modified or even absent. Pourhassan et al. [13] applied non-perturbative quantum corrections to a Born-Infeld BH and analyzed its information geometry, revealing new aspects of BH microstructure. More recently, Pourhassan et al. [43] investigated non-perturbative corrections to BH geometry, showing how quantum effects fundamentally alter spacetime structure near BH horizons. The study of quantum corrections in various BH backgrounds has also extended to higher-dimensional and exotic BH solutions. Han et al. [44] examined the the impact of fluctuations on AdS BHs in arbitrary dimensions, deriving how horizon perturbations change Hawking temperature and Bekenstein–Hawking entropy and discussing resulting thermodynamic consequences. While Pourhassan et al. [45] investigated the quantum thermodynamics of an M2-M5 brane system. These studies highlight the universal nature of quantum corrections in different theories and dimensions of gravitation. Recent work has also connected quantum-corrected BH thermodynamics to holographic concepts such as the AdS/CFT correspondence. Kumar et al. [46] studied the stabilizing effects of higher-order quantum corrections on charged BTZ BH thermodynamics, demonstrating how quantum corrections affect the stability of BHs in AdS space. Moreover, Pourhassan et al. [47] examined thermal fluctuation effects on the shear viscosity to entropy ratio in five-dimensional Kerr-Newman BHs, providing important connections to hydrodynamic properties of dual field theories.

In the context of Rindler-AdS hyperbolic BHs, understanding these quantum corrections is crucial, as they can significantly influence the thermodynamic stability and phase structure of the system. Studies have shown that such corrections can alter the heat capacity and, consequently, the stability criteria of BHs [48].

Recent studies continue to explore quantum gravity's impact on BH thermodynamics, from shadow imprints and singularity resolution in regular BHs [15, 49] to phase transitions in hyperscaling-violating spacetimes [50]. Investigations into AdS BHs further show how corrections from the generalized uncertainty principle alter their evaporation and stability [51], with thermodynamic geometry providing key insights into the resulting phase structures [52]. Our work builds upon this foundation by applying an exponential entropy correction to the distinctive AdS-Rindler geometry, revealing its unique stability signatures.

This article is organized as follows. Section 2 presents the theoretical model, establishing the foundational framework for our analysis of AdS-Rindler BHs. Section 3 explores the mathematical aspects of computing the area of AdS-Rindler BHs. In Sec. 4, we develop the thermodynamic framework before adding quantum correction, introducing key concepts and methodologies for analyzing BH thermodynamics from a thermal point of view. Section 5 provides a detailed analysis non-perturbative quantum corrections of entropy and other thermodynamic function model. Section 6 explores the comparison between the thermodynamics before and after adding quantum corrections. Finally, section 7 summarizes our findings and discusses their implications, along with future research directions in this field.

2. GENERAL FORM OF HYPERBOLIC BLACK HOLES

The action for a hyperbolic BH in the Rindler-AdS spacetime is typically derived from the Einstein-Hilbert action with a negative cosmological constant. The general form of the action in $(d+1)-$dimensional space-time is given by:

$$S = \frac{1}{16\pi G} \int d^{d+1}x\sqrt{-g}\,(R - 2\Lambda) \tag{6}$$

where, R is the Ricci scalar, and $\Lambda = -\frac{d(d-1)}{2l^2}$ is the cosmological constant for AdS spacetime with curvature scale l. Unlike their spherical counterparts, these BHs exhibit distinct thermodynamic behaviors because of their horizon geometry. The variation of this action with respect to the metric $g_{\mu\nu}$ gives Einstein's field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = 0. \tag{7}$$

Taking the trace by contracting with $g^{\mu\nu}$, leads to:

$$R = -\frac{d(d+1)}{l^2}, \tag{8}$$

Substituting this into Eq.(7), we obtain the Einstein tensor $(G_{\mu\nu})$ as the following.

$$G_{\mu\nu} = R_{\mu\nu} + \left(\frac{d}{\ell^2}\right) g_{\mu\nu} = 0. \tag{9}$$

This describes an AdS spacetime with constant curvature. The general form of the metric for a $(d+1)$-dimensional hyperbolic BH can be expressed as:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dH^2_{d+1}, \tag{10}$$

where dH^2_{d+1} denotes the unit metric of the hyperbolic static BH in $d+1$ dimensional space [48]. Although the metric considered here, Eq. (10), does not explicitly use Rindler coordinates, it corresponds to the hyperbolic patch of anti–de Sitter (AdS) spacetime that is locally equivalent to the Rindler wedge of AdS under an appropriate coordinate transformation (see Refs. [48, 53]). Hence, following common usage in the literature, we refer to this background as a Rindler–AdS spacetime. This designation emphasizes the constant–acceleration interpretation of observers in the hyperbolic AdS region and justifies the thermodynamic analysis in analogy with Rindler horizons. Or, in the matrix form the metric tensor $g_{\mu\nu}$ is a $(d+1)\times(d+1)$ symmetric matrix:

$$g_{\mu\nu} = \begin{bmatrix} -f(r) & 0 & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{f(r)} & 0 & 0 & \cdots & 0 \\ 0 & 0 & r^2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & r^2\sinh^2\chi & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & r^2\sinh^2\chi\cdots\sinh^2\theta_{d-3} \end{bmatrix}. \tag{11}$$

The Ricci tensor components presented in Eq. (9) can be calculated as:

$$R_{\mu\nu} = \partial_\alpha\Gamma^\alpha_{\mu\nu} - \partial_\nu\Gamma^\alpha_{\mu\alpha} + \Gamma^\alpha_{\alpha\beta}\Gamma^\beta_{\mu\nu} - \Gamma^\alpha_{\nu\beta}\Gamma^\beta_{\mu\alpha} \tag{12}$$

where $\Gamma^\alpha_{\mu\nu}$ is the Christoffel symbols, can be found:

$$\Gamma^\mu_{\nu\alpha} = \frac{1}{2}g^{\mu\beta}\Big(\partial_\nu(g_{\alpha\beta}) + \partial_\alpha(g_{\nu\beta}) - \partial_\beta(g_{\nu\alpha})\Big) \tag{13}$$

Substituting Eq. (11) and Eq. (12) into Eq. (7) the blackening factor $(f(r))$ can be found as:

$$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{M}{r^{d-2}}, \tag{14}$$

The event horizon is located at $r = r_h$, where $f(r_0) = 0$ and M is the mass of the BH. Then solving $(f(r_h) = 0)$ yields the following:

$$M = r_h^{d-2}\left(\frac{r_h^2}{\ell^2} - 1\right). \tag{15}$$

Thus, substitution of Eq. (15) Eq. (14) reduces to,

$$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{r_h^{d-2}}{r^{d-2}}\left(\frac{r_h^2}{\ell^2} - 1\right). \tag{16}$$

This metric function is the key factor for calculating surface gravity, Hawking temperature, and other thermodynamic parameters, as we will see in the upcoming sections.

3. AREA OF HYPERBOLIC SPACE

To find the area of a hyperbolic space, we consider the metric of hyperbolic space H_{d-1} and compute the surface integral over a given region. The hyperbolic space (H_{d-1}) can be represented using the Poincaré disk model, where the metric in polar coordinates is given by:

$$ds^2 = \frac{dr^2 + r^2 d\Omega^2_{d-2}}{(1-r^2)^2} \tag{17}$$

where (r) is the radial coordinate on the Poincaré disk $(0 \le r < 1)$ and $(d\Omega^2_{d-2})$ is the metric on the unit $(d-2)-$dimensional sphere. The area element in H_{d-1} (with the dimension of length l normalized to unity) is given by [53]:

$$dA = \frac{r^{d-2}\,dr\,d\Omega_{d-2}}{(1-r^2)^{d-1}} \tag{18}$$

where $\frac{r^{d-2}}{(1-\frac{r^2}{l^2})^{d-1}} = \sqrt{g}$ is the metric determinant. The total area of a hyperbolic space up to a radius (r_h) is calculated as (for simplicity we assume $l = 1$):

$$\begin{aligned} A(r_h) &= \int_0^{r_h} \int_{\Omega_{d-2}} \frac{r^{d-2}\, dr\, d\Omega_{d-2}}{(1-r^2)^{d-1}} \\ &= \frac{2\pi^{(d-1)/2}}{\Gamma((d-1)/2)} \int_0^{r_h} \frac{r^{d-2}\, dr}{(1-r^2)^{d-1}} \end{aligned} \tag{19}$$

Since the factor $(\Omega_{d-2} = \frac{2\pi^{(d-1)/2}}{\Gamma((d-1)/2)})$ is the total solid angle for a unit $(d-2)-$sphere. Making the substitution $(x = r^2,\ dx = 2rdr)$, Eq.(19) reduces to:

$$A(r_h) = \frac{2\pi^{(d-1)/2}}{\Gamma((d-1)/2)} \int_0^{r_h^2} \frac{1}{2} x^{(d-3)/2} (1-x)^{-(d-1)}\, dx \tag{20}$$

Thus, the total area of hyperbolic space obtained as [54]:

$$A(r_h) = \frac{2\pi^{(d-1)/2}}{\Gamma\left(\frac{d-1}{2}\right)} \cdot \frac{r_h^{d-1}}{d-1}\ {}_2F_1\left(\frac{d-1}{2},\ d-1;\ \frac{d+1}{2};\ r_h^2\right) \tag{21}$$

where, ${}_2F_1\left(\frac{d-1}{2},\ d-1;\ \frac{d+1}{2};\ r_h^2\right)$ is the Gauss hypergeometric function. Thus, Eq.(21) gives the area enclosed within the radius r_h in the hyperbolic space.

4. THERMODYNAMICS OF ADS-RINDLER BHS

In this section, we derive the fundamental thermodynamic properties of BHs using established formalisms in BH thermodynamics. We begin by computing the temperature and entropy, followed by the heat capacity, which plays a key role in determining the stability of the BH. The upcoming calculations provide a comprehensive understanding of the black hole's thermodynamic structure and its response to quantum corrections or modifications in the entropy function. Substituting the area presented in Eq. (21) into Eq. (1) (assuming $k_B = \hbar = G = 1$) we obtain the following.

$$\overset{(C)}{S} = B(d)\ r_h^{d-1}\ {}_2F_1\left(\frac{d-1}{2},\ d-1;\ \frac{d+1}{2};\ r_h^2\right), \tag{22}$$

where,

$$B(d) = \frac{\pi^{(d-1)/2}}{2(d-1)\,\Gamma\left(\frac{d-1}{2}\right)}$$

Similarly, substituting Eq. (16) into Eq. (3) leads to the following $(\ell = 1)$:

$$\kappa = \frac{d}{2}\left[r_h - \frac{1}{r_h} + \frac{2}{d\ r_h}\right] \tag{23}$$

Thus, Hawking temperature can easily be found from Eq. (23) and Eq. (4) as:

$$T = \frac{d}{4\pi}\left[r_h - \frac{1}{r_h} + \frac{2}{d\ r_h}\right] \tag{24}$$

where r_h denotes the black hole's horizon's radius. Furthermore, the heat capacity of the BH is an essential thermodynamic quantity that determines its stability. From the first law of BH thermodynamics, it is found to have following relation [55, 56]:

$$\begin{aligned} C &= \frac{\partial M}{\partial T} \\ &= \left(\frac{\partial M}{\partial r_h}\right) \Big/ \left(\frac{\partial T}{\partial r_h}\right), \end{aligned} \tag{25}$$

Using Eq.(15) and Eq.(24), the partial derivatives of mass and Hawking temperature with respect to the horizon radius r_h are expressed by

$$\frac{\partial M}{\partial r_h} = r_h^{d-3}\left[d\ r_h^2 - (d-2)\right], \tag{26}$$

and

$$\frac{\partial T}{\partial r_h} = \frac{1}{4\pi}\left[d + \frac{d-2}{r_h^2}\right], \tag{27}$$

respectively. Finally, by substituting (26) and (27) into (25), the uncorrected (or classical) heat capacity is given by:

$$\overset{(C)}{C} = 4\pi\left[\frac{d\,r_h^2 - (d-2)}{d\,r_h^2 + (d-2)}\right] r_h^{d-1}, \tag{28}$$

The pressure associated with the BH can be expressed as [55]:

$$P = \frac{1}{2}TS. \tag{29}$$

This relation establishes a direct connection between the temperature, entropy, and thermodynamic pressure of the BH. Thus, putting (22) and (24) into (29) the pressure of the AdS-Rindler black hole obtained as,

$$\overset{(C)}{P} = \frac{B(d)}{8\,\pi^{(1/2)}}\, r_h^{d-2}\left(dr_h^2 - d + 2\right) {}_2F_1\left(\frac{d-1}{2},\, d-1;\, \frac{d+1}{2};\, r_h^2\right). \tag{30}$$

where we have used the derivatives of the the hypergeometric function formula:

$$\frac{d}{dz}\,{}_2F_1\,(a,\, b;\, c;\, z) = \frac{ab}{c}\,{}_2F_1\,(a+1,\, b+1;\, c+1;\, z)$$

The thermodynamic volume, which is conjugate to the pressure, is obtained using the following definition [13].

$$\begin{aligned} V &= \frac{\partial M}{\partial P} \\ &= \left(\frac{\partial M}{\partial r_h}\right)\Big/\left(\frac{\partial P}{\partial r_h}\right). \end{aligned} \tag{31}$$

This expression describes how the mass M of the BH varies with respect to pressure, providing insight into its extended phase-space thermodynamics. In so doing, applying Eq. (31) to Eqs. (15) and (30) the BH volume can be calculated as:

$$\overset{(C)}{V} = \frac{8\,\pi^{(1/2)}}{B(d)}\cdot\frac{d\,r_h^2 - (d-2)}{\left(d^2\,r_h^2 - (d-2)^2\right){}_2F_1\left(\frac{d-1}{2},\, d-1;\, \frac{d+1}{2};\, r_h^2\right) + \frac{2(d-1)^2}{d+1} r_h^2 (d\,r_h^2 - d + 2)\,{}_2F_1\left(\frac{d+1}{2},\, d;\, \frac{d+3}{2};\, r_h^2\right)} \tag{32}$$

Finally, the Gibbs free energy, which characterizes the thermodynamic stability and phase behavior of the BH, is given by [13]:

$$G = M - TS. \tag{33}$$

This quantity plays a crucial role in understanding phase transitions and critical phenomena in BH thermodynamics. Doing so, substitution of Eqs.(15), (22) and (24) into Eq.(33) leads to find the Gibbs free energy as:

$$\overset{(C)}{G} = r_h^{d-2}\left[r_h^2 - 1 - \frac{B(d)}{4\,\pi^{(1/2)}}\left(d\,r_h^2 - d + 2\right) {}_2F_1\left(\frac{d-1}{2},\, d-1;\, \frac{d+1}{2};\, r_h^2\right)\right] \tag{34}$$

Ultimately, we can say these functions provide the complete semiclassical thermodynamic description of AdS-Rindler hyperbolic BHs.

5. THERMODYNAMICS OF ADS-RINDLER BHS WITH QUANTUM CORRECTION

In this section, we systematically incorporate quantum gravitational effects into our thermodynamic analysis of hyperbolic BHs. Following the approach outlined in section (1), we employ an exponential correction term to the Bekenstein-Hawking entropy, which captures non-perturbative quantum effects near the Planck scale. This correction provides a more complete description of BH microstate structure when quantum gravitational fluctuations become significant. We begin by modifying the classical entropy with an exponential correction term,

$$\overset{(Q)}{S} = \overset{(C)}{S} + \eta\, e^{-\delta\, \overset{(C)}{S}}$$

$$= B(d)\, r_h^{d-1}\, {}_2F_1\left(\frac{d-1}{2},\, d-1;\, \frac{d+1}{2};\, r_h^2\right)$$
$$+ \eta\ \exp\left[-\delta\, B(d)\, r_h^{d-1}\, {}_2F_1\left(\frac{d-1}{2},\, d-1;\, \frac{d+1}{2};\, r_h^2\right)\right], \tag{35}$$

where δ is the quantum correction parameter that controls the strength of quantum gravitational effects. This parameter has dimensions of area and is expected to be on the order of the Planck area. In the limit $\delta \to 0$, we recover the classical entropy formula, and as δ increases, quantum effects become more pronounced.

With the corrected entropy in hand, we now derive the corresponding modifications to the other thermodynamic quantities. The non perturbative quantum corrected temperature can be found from the first law of BH thermodynamics ($dM = T\, dS$) using Eq,(15) and (35) as:

$$\overset{(Q)}{T} = \frac{\partial M}{\partial S} = \left(\frac{\partial M}{\partial r_h}\right) \Big/ \left(\frac{\partial S}{\partial r_h}\right) = \frac{d\, r_h^2 - (d-2)}{(d-1)\, B(d)\, r_h \left[(d-1)W_1 + \frac{2(d-1)^2}{d+1} r_h^2\, W_2\right] \left[1 - \eta\, \delta\, e^{-\delta\, B(d)\, r_h^{d-1} W_1}\right]} \tag{36}$$

where,

$$W_1 = {}_2F_1\left(\frac{d-1}{2},\, d-1;\, \frac{d+1}{2};\, r_h^2\right), \tag{37}$$

$$W_2 = {}_2F_1\left(\frac{d+1}{2},\, d;\, \frac{d+3}{2};\, r_h^2\right). \tag{38}$$

Significant changes appear in the non-perturbative quantum corrected heat capacity $\overset{(Q)}{C}$, which is defined through the mass-temperature variation. The expression is derived using Eq. (25) for heat capacity, Eq. (26) for the mass derivative, and Eq. (36) for the quantum temperature derivative:

$$\overset{(Q)}{C} = \left(\frac{\partial M}{\partial r_h}\right) \Big/ \left(\frac{\partial \overset{(Q)}{T}}{\partial r_h}\right)$$

$$= \frac{r_h^{d-1}(d-1)^2[B(d)]^2[D_1(r_h)]^2[D_2(r_h)]^2\left[dr_h^2 - (d-2)\right]}{2d(d-1)B(d)r_h^2 D_1(r_h)D_2(r_h) - \left[dr_h^2 - (d-2)\right] D_3} \tag{39}$$

where,

$$D_1(r_h) = (d-1)W_1 + \frac{2(d-1)^2}{d+1} r_h^2 W_2$$
$$D_2(r_h) = 1 - \eta\ \delta\ e^{-\delta B(d) r_h^{d-1} W_1}$$
$$D_3(r_h) = (d-1)B(d)\left[D_1(r_h)D_2(r_h) + r_h D_1'(r_h)D_2(r_h) + r_h D_1(r_h)D_2'(r_h)\right]$$
$$D_1'(r_h) = 2(d-1)^2 r_h W_2 + \frac{4d(d-1)^2}{d+3} r_h^3 W_3$$
$$D_2'(r_h) = \eta\delta^2 B(d)\, e^{-\delta\, B(d)\, r_h^{d-1} W_1}\left[(d-1)r_h^{d-2}W_1 + \frac{2(d-1)^2}{d+1} r_h^d W_2\right]$$
$$W_3 = {}_2F_1\left(\frac{d+3}{2},\, d+1;\, \frac{d+5}{2};\, r_h^2\right).$$

The quantum-corrected pressure is calculated by applying Eq.(29) to the modified entropy Eq.(35) and modified temperature Eq.(36). Thus, we obtain:

$$\overset{(Q)}{P} = \frac{\left[dr_h^2 - (d-2)\right]\left[B(d)\, r_h^{d-1} W_1 + \eta\, e^{-\delta\, B(d) r_h^{d-1} W_1}\right]}{2(d-1)B(d)\, r_h \left[(d-1)W_1 + \frac{2(d-1)^2}{d+1} r_h^2 W_2\right]\left[1 - \eta\, \delta\, e^{-\delta\, B(d)\, r_h^{d-1} W_1}\right]}. \tag{40}$$

The corrected thermodynamic volume $\overset{(Q)}{V}$, can be calculated from Eq.(31) as:

$$\overset{(Q)}{V} = \frac{\partial M}{\partial \overset{(Q)}{P}} \tag{41}$$

where the numerator of the above equation are presented in Eq.(26). While, one can find $\frac{\partial \overset{(Q)}{P}}{\partial r_h}$ in the denominator from Eq.(40). This yield extremely lengthy expressions due to the complexity of the quantum-corrected pressure function Eq.(40), which involve products of hypergeometric functions and exponential correction terms. For this reason, the fully expanded form is not written here, which would span multiple lines and obscure the physical interpretation. Finally, the corrected Gibbs free energy can be calculated as,

$$\overset{(Q)}{G} = r_h^{d-2}\left(\frac{r_h^2}{\ell^2} - 1\right) - \frac{\left[d\ r_h^2 - (d-2)\right]\left[B(d)\ r_h^{d-1} W_1 + \eta\ e^{-\delta\, B(d)\, r_h^{d-1} W_1}\right]}{(d-1)^2 r_h W_1 + \frac{2(d-1)^3}{d+1} r_h^3\ W_2 - \eta\,\delta\left((d-1)^2 r_h W_1 + \frac{2(d-1)^3}{d+1} r_h^3 W_2\right) e^{-\delta\, B(d)\, r_h^{d-1} W_1}}. \tag{42}$$

These quantum-corrected formulations yield several key physical insights. The exponential correction term introduces a regularization effect that becomes significant as r_h gets small, smoothing out the divergent behavior observed in the classical case. Unlike logarithmic corrections that dominate at large horizon areas, our exponential correction becomes most relevant at intermediate scales, particularly in the small scale. The quantum corrections preserve the overall thermodynamic structure, particularly affecting the stability conditions indicated by the heat capacity.
In the following section, we will analyze these results graphically to better visualize the impact of quantum corrections on the thermodynamic landscape of hyperbolic BHs.

6. RESULTS AND DISCUSSION

This section conducts a stability analysis in hyperbolic BH solutions. The impacts of quantum corrections on thermodynamic properties are visualized through graphical representations. First, the heat capacity in Eq.(28) serves as a key indicator of stability, with sign changes corresponding to transitions between stable and unstable regimes. As shown in Figs. (1) and (2), the heat capacity exhibits a singularity at $r_h = 1$, which indicates a critical point in the thermodynamic behavior of the BH. In addition, according to Eq. (25), a divergence in the uncorrected (or classical) heat capacity occurs when the temperature reaches an extremum ($\partial T/\partial r_h = 0$). Thermodynamic stability changes whenever the heat capacity changes sign—occurring at $T = 0$ or $\partial M/\partial r_h = 0$. A positive heat capacity corresponds to stability, while a negative value indicates instability. Thus, analysing zeros and singularities in the heat capacity is essential for determining BH stability and identifying phase transitions. From Eq.(27), assuming $(\partial T/\partial r_h) = 0$, we shall determine the range of d values for which the temperature profiles exhibit extremal points (heat capacity diverges). Thus, we obtain:

$$r_h = r_\infty^\pm(d) = \pm\sqrt{\frac{2-d}{d}} \tag{43}$$

The negative solution is physically unacceptable. Moreover, in BH thermodynamics, only dimensions with $d \geq 2$ are admissible. Considering this, the preceding equation reveals that the only real solution occurs at $d = 2$, giving $r_h = r_\infty^+(d = 2) = 0$. Substituting $d = 2$ into Eq. (28) yields $\overset{(C)}{C} = 4\pi r_h$, showing that the heat capacity remains finite for all non-negative values of r_h and never diverges. Therefore, the obtained result $r_h = r_\infty^+(d = 2) = 0$ should be regarded as an apparent solution. On the other hand, a change in the sign of the heat capacity indicates a transition between stable and unstable BH states, with the transition occurring at points where the heat capacity reaches zero. From Eq.(26), assuming $C = 0$ (corresponding to $T_h = 0$ or $\partial M/\partial r_h = 0$), the solution becomes:

$$r_h = r_0^\pm(d) = \pm\sqrt{\frac{d-2}{d}}. \tag{44}$$

Ignoring the nonphysical negative root, we see that for $d \geq 2$, the regions of instability and stability correspond to $r_h < r_0^+(d)$ and $r_h > r_0^+(d)$, respectively. Specifically, for $d = 2, 3, 4, 5$ we have $r_0^+(d) = 0, 1/\sqrt{3} = 0.577350, 1/\sqrt{2} = 0.707107, \sqrt{3/5} = 0.774597$ (indicated by stars in Fig. (2) (panel (a)). Moreover, Fig. (1) presents both classical entropy (panel (a)) and quantum-corrected entropy (panel (b)) as a function of the horizon radius r_h. As can be seen, there is a sharp broadening around r_h =1, which suggests a rapid entropy change near the critical horizon radius. This could be an indicator of a phase transition or a region in which the thermodynamic behavior of the BH changes drastically. After adding quantum correction, the most notable effect is that the sharp broadening around $r_h = 1$ is smoothed out, leading to a more gradual change in entropy, especially as the negative part becomes positive.

Furthermore, to analyze BH instabilities in the quantum-corrected case, we shall use Eq. (39) and determine the critical values of the horizon radii, namely $r_0^+(d)$ and $r_\infty^+(d)$. Comparing Eq. (39) with Eq. (25), we see that both share the

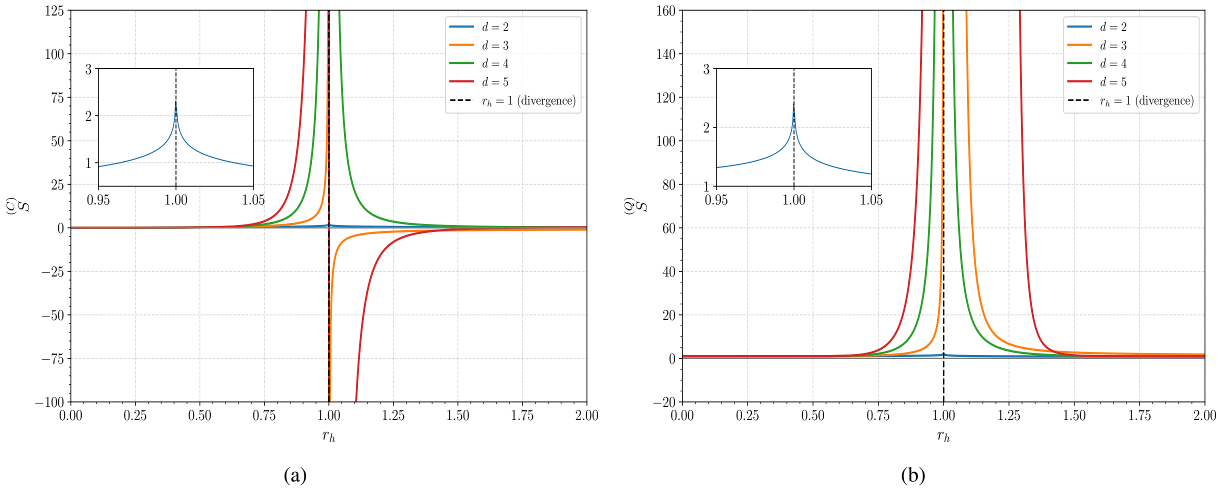

Figure 1. The plots show the classical behavior of entropy before correction (panel a) and the quantum corrected entropy (panel b) of the AdS-Rindler BHs for $\eta = \delta = 1$.

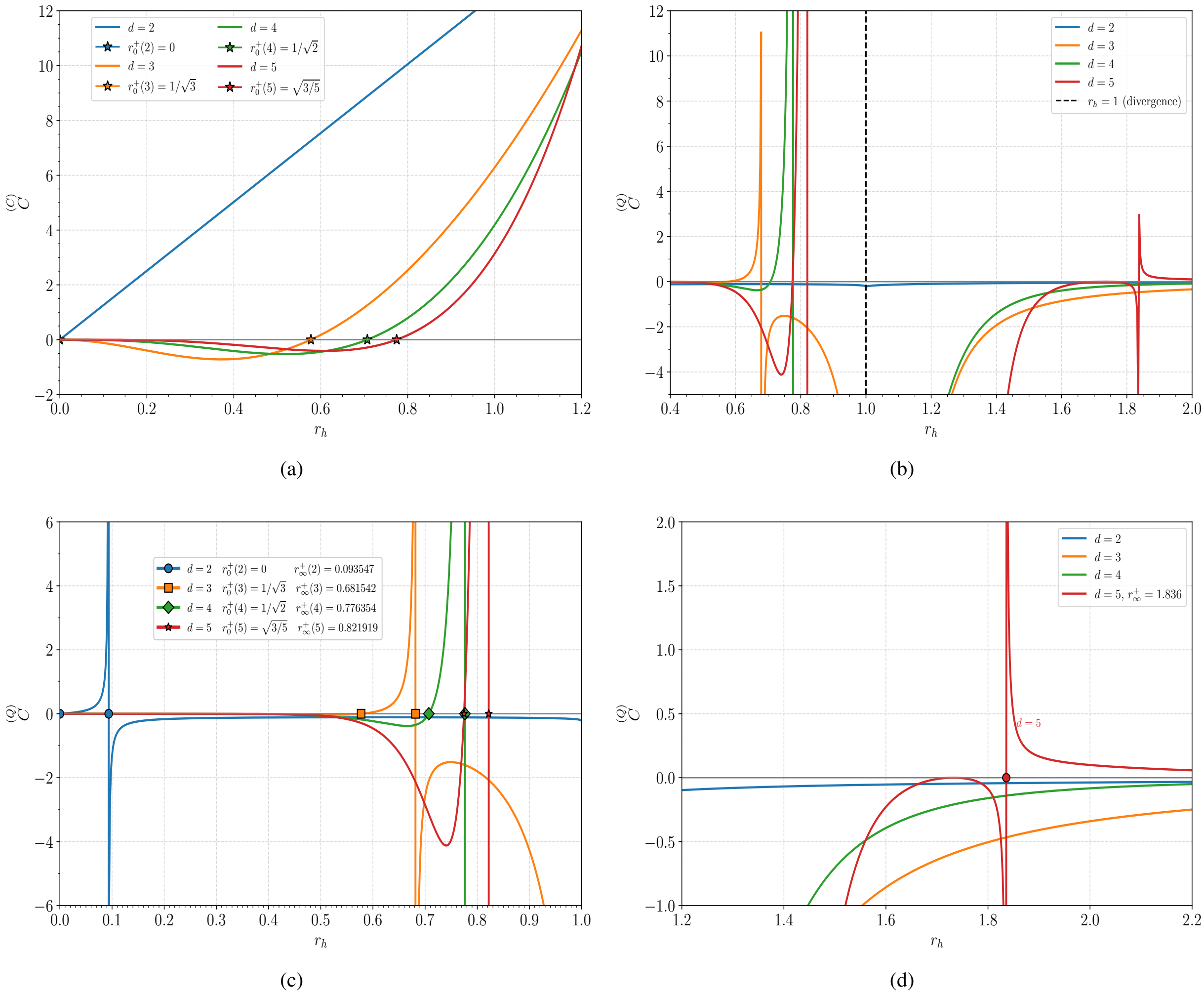

Figure 2. The plots show the semiclassical behavior of heat capacity before correction (panel a) and the quantum corrected heat capacity (panel b, c and d) of the AdS-Rindler BHs for $\eta = \delta = 1$.

common factor $(\partial M/\partial r_h)$ in the numerator. Therefore, the zeros $(r_0^+(d))$ of the quantum-corrected heat capacity $(\overset{(Q)}{C})$ are identical to those of the classical heat capacity $(\overset{(C)}{C})$, which were analytically obtained in Eq. (44).

However, due to the complexity and nonlinearity of the denominator $(\partial \overset{(Q)}{T} /\partial r_h)$ of Eq. (39), which involves transcendental functions, obtaining analytical solutions for $r_\infty^+(d)$ is impractical. Therefore, the roots can be computed numerically. For each value of d, the stable region for the BHs, corresponding to $\overset{(Q)}{C} > 0$, is indicated by the interval between these critical values, i.e., $\Delta r_{(d)} = \left|r_\infty^+(d) - r_0^+(d)\right|$. Specifically, for the values of $d = 2, 3, 4, 5$, the stable regions are defined as follows: $\Delta r_{(2)} = |0.093547 - 0| = 0.093547$, $\Delta r_{(3)} = |0.681542 - 0.577350| = 0.104192$, $\Delta r_{(4)} = |0.776354 - 0.707107| = 0.069247$, $\Delta r_{(5)} = |0.821919 - 0.774597| = 0.047322$, respectively. These points are highlighted with different markers in Fig. (2) (panel c). In addition, in the case where $d = 5$, we note that the heat capacity $\left(\overset{(Q)}{C}\right)$ exhibits an additional divergent point at the vertical asymptote $(r_h = r_\infty^+(5) = 1.836)$. Beyond this point, the black hole re-enters a stable region, as illustrated in Fig. (2) (panel d).

7. CONCLUSIONS

In this study, we conducted a comprehensive thermodynamic and stability analysis of AdS–Rindler (hyperbolic) BHs, a theoretically significant yet relatively unexplored class of BH geometries. Using the framework of Hawking entropy and quantum-corrected thermodynamics, we examined how quantum effects influence the stability of these BHs.

Our results reveal that the heat capacity plays a decisive role in determining stability regimes. For the uncorrected heat capacity case, we have found the critical radii $\left(r_0^+(d)\right)$ (as in Eq.(44)) in which C becomes positive and enters a stable region for all $r_h > r_0^+(d)$. While, upon introducing exponential quantum corrections to the entropy, we observed the drastically change in the stability regions, leading to narrower but smoother stable regions where calculated by $\Delta r_{(d)}$ in Sec.(6). Quantitatively, the size of these intervals for small black holes varies with the spacetime dimension d, reflecting their measurable sensitivity to quantum fluctuations across different dimensions.

The quantum-corrected heat capacity generally destabilized large Rindler–AdS (hyperbolic) black holes across all studied values of d. However, in the special case $d = 5$, the system regains stability for horizon radii satisfying $r_h > 1.836$, where the heat capacity C becomes positive.

Overall, our findings demonstrate that quantum corrections drastically alter the qualitative stability of hyperbolic BHs and do refine their thermodynamic structure, particularly by smoothing sharp entropy variations and modifying the stability of such BHs due to studying heat capacity .

In summary, the interplay between geometry, quantum corrections, and thermodynamic behavior in AdS–Rindler BHs provides valuable insights into nonclassical BH thermodynamics. In future work, the calculation of additional thermodynamic quantities such as pressure, volume, and the Gibbs free energy will enable the study of phase transitions, equations of state, and thermodynamic topology.

ORCID

Aram Bahroz Brzo, https://orcid.org/0000-0002-1257-9377; **Peshwaz Abdulkareem Abdoul**, https://orcid.org/0000-0002-2144-8336; **Behnam Pourhassan**, https://orcid.org/0000-0003-1338-7083

REFERENCES

[1] J. D. Bekenstein, "Black holes and entropy," Phys. Rev. D, **7**, 2333 (1973). https://doi.org/10.1103/physrevd.7.2333

[2] S. W. Hawking, "Particle Creation by Black Holes," Commun. Math. Phys. **43**, 199 (1975). https://doi.org/10.1007/bf02345020

[3] J.M. Bardeen, B. Carter, and S.W. Hawking, "The four laws of black hole mechanics," Commun. Math. Phys. **31**, 161 (1973). https://doi.org/10.1007/bf01645742

[4] D.N. Page, "Information in black hole radiation," Phys. Rev. Lett. **71**, 3743 (1993). https://doi.org/10.1103/physrevlett.71.3743

[5] A. Strominger, and C. Vafa, "Microscopic origin of the Bekenstein-Hawking entropy," Phys. Lett. B, **379**, 99 (1996). https://doi.org/10.1016/0370-2693(96)00345-0

[6] J.M. Maldacena, "The Large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998). https://doi.org/10.4310/atmp.1998.v2.n2.a1

[7] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998). https://doi.org/10.4310/atmp.1998.v2.n2.a2

[8] S.N. Gashti, M.A.S. Afshar, M.R. Alipour, I. Sakallı, B. Pourhassan, and J. Sadeghi, "Assessing WGC Compatibility in ModMax Black Holes via Photon Spheres Analysis and WCCC Validation," arXiv preprint arXiv, **2504**, 11939 (2025). https://doi.org/10.48550/arXiv.2504.11939

[9] S.N. Gashti, B. Pourhassan, and I. Sakallı, "Thermodynamic topology and phase space analysis of AdS black holes through non-extensive entropy perspectives," The European Physical Journal C, **85**(3), 305 (2025). https://doi.org/10.1140/epjc/s10052-025-14035-x

[10] A.B. Brzo, S.N. Gashti, B. Pourhassan, and S. Beikpour, "Thermodynamic Topology of AdS Black Holes within Non-Commutative Geometry and Barrow Entropy," Nuclear Physics B, **1012**, 116840 (2025). https://doi.org/10.1016/j.nuclphysb.2025.116840

[11] B. Pourhassan, M. Faizal, S. Upadhyay, and L. Al Asfar, "Thermal Fluctuations in a Hyperscaling Violation Background," Eur. Phys. J. C, **77**, 555 (2017). https://doi.org/10.1140/epjc/s10052-017-5125-x

[12] S. Upadhyay, B. Pourhassan, and H. Farahani, "P-V criticality of first-order entropy corrected AdS black holes in massive gravity," Phys. Rev. D, **95**, 106014 (2017). https://doi.org/10.1103/PhysRevD.95.106014

[13] B. Pourhassan, "Exponential corrected thermodynamics of black holes," J. Stat. Mech. **2107**, 073102 (2021). https://doi.org/10.1088/1742-5468/ac0f6a

[14] R.B. Wang, S.J. Ma, L. You, Y.C. Tang, Y.H. Feng, X.R. Hu, and J.B. Deng, "Thermodynamics of AdS-Schwarzschild-like black hole in loop quantum gravity," The European Physical Journal C, **84**(11), 1161 (2024). https://doi.org/10.1140/epjc/s10052-024-13505-y

[15] J.-P. Ye, Z.-Q. He, A.-X. Zhou, Z.-Y. Huang, and J.-H. Huang, "Shadows and photon rings of a quantum black hole," Physics Letters B, **851**, 138566 (2024). https://doi.org/10.1016/j.physletb.2024.138566

[16] F.G. Menezes, H.A. Borges, I.P.R. Baranov, and S. Carneiro, "Thermodynamics of effective loop quantum black holes," arXiv preprint arXiv, **2504**, 06964 (2025). https://doi.org/10.48550/arXiv.2504.06964

[17] L. You, R.-B. Wang, Y.-C. Tang, J.-B. Deng, and X.-R. Hu, "Thermal chaos of quantum-corrected-AdS black hole in the extended phase space," The European Physical Journal C, **84**, 11 (2024). https://doi.org/10.1140/epjc/s10052-024-13417-x

[18] B. Hamil, B. C. Lütfüoğlu, and L. Dahbi, "Quantum-corrected Schwarzschild AdS black hole surrounded by quintessence: Thermodynamics and shadows," Modern Physics Letters A, **39**(33n34), 2450161 (2024). https://doi.org/10.1142/s021773232450161x

[19] B. Tan, "Thermodynamics of high order correction for Schwarzschild-AdS black hole in non-commutative geometry," Nuclear Physics B, **1014**, 116868 (2025). https://doi.org/10.1016/j.nuclphysb.2025.116868

[20] R. Emparan, and J.M. Magán, "Tearing down spacetime with quantum disentanglement," Journal of High Energy Physics, **(3)**, 078 (2024). https://doi.org/10.1007/jhep03(2024)078

[21] P.Z. He, and H.Q. Zhang, "Holographic timelike entanglement entropy from Rindler method," Chinese Physics C, **48**(11), 115113 (2024). https://doi.org/10.1088/1674-1137/ad57a8

[22] X.X. Ju, B.H. Liu, W.B. Pan, Y.W. Sun, and Y.T. Wang, "Squashed entanglement from generalized Rindler wedge," Journal of High Energy Physics, **9**, 1-48 (2025). https://doi.org/10.1007/jhep09(2025)006

[23] Q. Wen, M. Xu, and H. Zhong, "Timelike and gravitational anomalous entanglement from the inner horizon," SciPost Physics, **18**(6), 204 (2025). https://doi.org/10.21468/scipostphys.18.6.204

[24] R.X. Miao, "Casimir effect and holographic dual of wedges," Journal of High Energy Physics, **6**, 1-35 (2024). https://doi.org/10.1007/jhep06(2024)084

[25] R. Campos Delgado, "Quantum gravitational corrections to the entropy of a Reissner–Nordström black hole," The European Physical Journal C, **82**(3), 272 (2022). https://doi.org/10.1140/epjc/s10052-022-10232-0

[26] S. Wu, and C. Liu, "The Quantum Corrections on Kerr-Newman Black Hole Thermodynamics by the Generalized Uncertainty Principle," International Journal of Theoretical Physics, **59**, 2681-2693 (2020). https://doi.org/10.1007/s10773-020-04468-3

[27] H.L. Li, D.W. Song, and W. Li, "Phase transition and entropy correction of a quantum correction black hole close to planck scale," General Relativity and Gravitation, **51**, 1-14 (2019). https://doi.org/10.1007/s10714-019-2504-7

[28] D. Ma, T. Huo, and C. Liu, "Thermodynamics and its Quantum Correction of Vacuum Nonsingular Black Hole," Astrophysics, **67**(4), 556-570 (2025). https://doi.org/10.1007/s10511-025-09851-8

[29] J.J. Song, and C.Z. Liu, "Thermodynamics in a quantum corrected Reissner–Nordström–AdS black hole and its GUP-corrections," Chinese Physics B, **33**(4), 040402 (2024). https://doi.org/10.1088/1674-1056/ad1a8a

[30] B. Pourhassan, "Resolving the information loss paradox from the five-dimensional supergravity black hole," Nuclear Physics B, **976**, 115713 (2022). https://doi.org/10.1016/j.nuclphysb.2022.115713

[31] B. Pourhassan, and M. Faizal, "Thermal Fluctuations in a Charged AdS Black Hole," Europhysics Letters, **111**, 40006 (2015). https://doi.org/10.1209/0295-5075/111/40006

[32] S. Upadhyay, N. ul Islam, and P.A. Ganai, "A modified thermodynamics of rotating and charged BTZ black hole," Journal of Holography Applications in Physics, **2**(1), 25-48 (2022). https://doi.org/10.1209/0295-5075/111/40006

[33] A. Sen, "Logarithmic Corrections to $N = 2$ Black Hole Entropy: An Infrared Window into the Microstates," General Relativity and Graviton, **44**(5), 1207-1266 (2012). https://doi.org/10.1007/s10714-012-1336-5

[34] B. Pourhassan, M. Faizal, and U. Debnath, "Effects of Thermal Fluctuations on the Thermodynamics of Modified Hayward Black Hole," Eur. Phys. J. C, **76**, 145 (2016). https://doi.org/10.1140/epjc/s10052-016-3998-8

[35] R. Ali, R. Babar, Z. Akhtar, and A. Övgün, "Thermodynamics and logarithmic corrections of symmergent black holes," Results Phys. **46**, 106300 (2023). https://doi.org/10.1016/j.rinp.2023.106300

[36] F.M. Mele, J. Münch, and S. Pateloudis, "Quantum corrected polymer black hole thermodynamics: mass relations and logarithmic entropy correction," Journal of Cosmology and Astroparticle Physics, **02**, 011 (2022). https://doi.org/10.1088/1475-7516/2022/02/011

[37] B. Pourhassan, R.C. Delgado, S. Upadhyay, H. Farahani, and H. Kumar, "Quantum gravitational corrections to the geometry of charged AdS black holes," Nucl. Phys. B, **1012**, 116830 (2025). https://doi.org/10.1016/j.nuclphysb.2025.116830

[38] B. Pourhassan, X. Shi, S.S. Wani, S. Al-Kuwari, İ. Sakallı, N.A. Shah, M. Faizal, and A. Shabir, "Quantum gravitational corrections to a Kerr black hole using Topos theory," Annals of Physics, **477**, 169983 (2025). https://doi.org/10.1016/j.aop.2025.169983

[39] R.M. Wald, *General Relativity*, (University of Chicago Press, 1984).

[40] S.M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity*, (Addison Wesley, 2004).

[41] X. Calmet, and F. Kuipers, "Quantum Gravitational Corrections to the Entropy of a Schwarzschild Black Hole," Phys. Rev. D, **104**(6), 066012 (2021). https://doi.org/10.1103/physrevd.104.066012

[42] A. Chatterjee, and A. Ghosh, "Exponential corrections to black hole entropy," Phys. Rev. Lett. **125**(4), 041302 (2020). https://doi.org/10.1103/physrevlett.125.041302

[43] B. Pourhassan, H. Farahani, F. Kazemian, I. Sakalli, S. Upadhyay, and D.V. Singh, "Non-perturbative correction on the black hole geometry," Physics of the Dark Universe, **44**, 101444 (2024). https://doi.org/10.1016/j.dark.2024.101444

[44] H. Han, and B. Gwak, "Effects of fluctuations in higher-dimensional AdS black holes," Physical Review D, **110**, 066013 (2024). https://doi.org/10.1103/physrevd.110.066013

[45] B. Pourhassan, H. Aounallah, M. Faizal, S. Upadhyay, S. Soroushfar, Y.O. Aitenov, and S.S. Wani, "Quantum Thermodynamics of an M2-M5 Brane System," Journal of High Energy Physics, **05**, 030 (2022). https://doi.org/10.1007/jhep05(2022)030

[46] H. Kumar, B. Pourhassan, and I. Sakalli, "Stabilizing Effects of Higher-Order Quantum Corrections on Charged BTZ Black Hole Thermodynamics," Nucl. Phys. B, **1007**, 116672 (2024). https://doi.org/10.1016/j.nuclphysb.2024.116672

[47] B. Pourhassan, I. Sakalli, and A.B. Brzo, "Thermal Fluctuation Effects on Shear Viscosity to Entropy Ratio in Five-Dimensional Kerr-Newman Black Holes," Eur. Phys. J. C, **85**, 206 (2025). https://doi.org/10.1140/epjc/s10052-025-13893-9

[48] R. Emparan, "AdS/CFT duals of topological black holes and the entropy of zero-energy states," Journal of High Energy Physics, **06**, 036 (1999). https://doi.org/10.1088/1126-6708/1999/06/036

[49] D. Ma, T. Huo, and C. Liu, "Thermodynamics and its Quantum Correction of Vacuum Nonsingular Black Hole," Astrophysics, **67**(4), 556-570 (2024). https://doi.org/10.1007/s10511-025-09851-8

[50] Ali Övgün, Reggie C. Pantig, and Ángel Rincón, "Shadow and greybody bounding of a regular scale-dependent black hole solution," Annals of Physics, **463**, 169625 (2024). https://doi.org/10.1016/j.aop.2024.169625

[51] J.J. Song, and C.Z. Liu, "Thermodynamics in a quantum corrected Reissner-Nordström-AdS black hole and its GUP-corrections," Chinese Physics B, **33**(4), 040402 (2024). https://doi.org/10.1088/1674-1056/ad1a8a

[52] Q.Q. Li, Y. Zhang, Q. Sun, C.H. Xie, and Y.L. Lou, "Phase structure of quantum corrected charged AdS black hole surrounded by perfect fluid dark matter," Chinese Journal of Physics, **92**, 1-9 (2024). https://doi.org/10.1016/j.cjph.2024.09.001

[53] H. Casini, M. Huerta, and R.C. Myers, "Towards a derivation of holographic entanglement entropy," Journal of High Energy Physics, **5**, 1-41 (2011). https://doi.org/10.1007/jhep05(2011)036

[54] I.S. Gradshteyn, and I.M. Ryzhik, *Table of integrals, series, and products*, Academic press, 2014).

[55] B.P. Dolan, "The cosmological constant and black hole thermodynamic potentials," Class. Quantum Gravity, **28**, 125020 (2011). https://doi.org/10.1088/0264-9381/28/12/125020

[56] J. Tarrio, and S. Vandoren, "Black holes and black branes in Lifshitz spacetimes," J. High Energy Phys. **1109**, 017 (2011). https://doi.org/10.1007/JHEP09(2011)017

КВАНТОВО СКОРИГОВАНА ТЕРМОДИНАМІКА AdS-RINDLER ЧОРНИХ ДІР

Арам Бахроз Брзо[1,2], Пешваз Абдулкарім Абдул[3], Бенам Пурхасан[4,5,6]

[1] *Фізичний факультет, Педагогічний коледж, Університет Сулеймані, Сулеймані 46001, регіон Курдистан, Ірак*
[2] *Центр досліджень і розвитку, Університет Сулеймані, Сулеймані 46001, Курдистан, Ірак*
[3] *Фізичний факультет, Науковий коледж, Університет Чармо: Чамчамал, Сулеймані, регіон Курдистан, Ірак*
[4] *Центр теоретичної фізики, Хазарський університет, вул. Мехсеті, 41, Баку, AZ1096, Азербайджан*
[5] *Школа фізики Дамганського університету, Дамган 3671645667, Іран*
[6] *Центр впливу досліджень & Результатів, Інститут інженерії та технологій Університету Чіткара, Університет Чіткара, Раджпура, 140401, Пенджаб, Індія*

Ми досліджуємо термодинамічні властивості та стабільність гіперболічних AdS-Rindler чорних дір, підкреслюючи вплив непертурбативної квантової корекції. Використовуючи стандартні термодинамічні формулювання разом з методом диска Пуанкаре, ми обчислюємо ключові величини, включаючи масу, температуру Хокінга, ентропію та теплоємність. Щоб врахувати квантові гравітаційні ефекти, ми вводимо експоненціальну корекцію до ентропії Бекенштейна-Хокінга та систематично виводимо модифіковані термодинамічні параметри. Хоча скоригована ентропія призводить до рівномірного зсуву в багатьох величинах, теплоємність зазнає нетривіальних змін, що призводить до вужчих та гладкіших стабільних областей $(\Delta r_{(d)})$ для кожного виміру d. Більше того, згладжування різких варіацій ентропії поблизу $r_h = 1$ підкреслює, як геометрія горизонту керує впливом квантових корекцій. Це дослідження пропонує нову систематичну ідентифікацію стабільних областей до та після експоненціальних корекцій (AdS-Ріндлерових) чорних дір, пропонуючи нове розуміння взаємодії геометрії, розмірності та квантових ефектів у термодинаміці чорних дір.

Ключові слова: *гіперболічні чорні діри; квантова корекція ентропії; аналіз стабільності*