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EFFECT OF ION TEMPERATURE ON THE DYNAMICS OF ANALYTICAL SOLITARY WAVE SOLUTION OF THE DUST ION ACOUSTIC WAVES FOR THE DAMPED FORCED KdV EQUATION IN q-NONEXTENSIVE PLASMAS

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This paper examines the dynamical properties of the analytical solitary wave solution of dust ion acoustic (DIA) solitary waves induced by the damped forced Korteweg-de Vries (DFKdV) equation in an unmagnetized collisional dusty plasma that contains neutral particles, q-nonextensive electrons, positively charged ions, and negatively charged dust grains in the presence of an external periodic force. To obtain the damped forced Korteweg-de Vries (DFKdV) equation, the reductive perturbation approach was developed. It is observed that both the compressive and rarefactive dust-ion acoustic (DIA) solitary-wave solutions are possible for this plasma model. The effects of a number of physical parameters are taken into account: the entropic index (q), dust ion collisional frequency (ν_{id0}), traveling wave speed (M), periodic force frequency (ω), ion-to-electron temperature ratio (σ), the parameter that is the ratio between the unperturbed densities of the dust ions and electrons (μ), the strength and frequency of the external periodic force (f_0). It is observed that those parameters have significant effects on the structures of the damped forced dust-ion acoustic solitary waves. The implication of the outcomes of this investigation may be relevant for understanding the dynamics of dust-ionacoustic (DIA) solitary waves in laboratory plasma as well as in space plasma environment.

Keywords: *Ion-acoustic soliton, Solitary wave, Dusty plasma, Reductive perturbation method, nonextensive electron, Damped forced KdV equation*

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1. INTRODUCTION

In plasmas, ions are able to exchange vibrations with one another due to their charge despite the absence of collisions. As the movement of large numbers of ions is included, these vibrations take the form of low-frequency oscillations, referred to as Ion-Acoustic (IA) waves [1]. One of the most exciting features of IA waves is that they can occur as types of waves similar to electron solitary waves [2, 3, 4, 5, 6, 7]. The solitary waves are localised waves which are seen from a mutual balance between the effects of dispersion and nonlinearity in a medium. A solitary wave is referred to as a soliton if it has two characteristics: it moves for a long period without changing its shape or speed, and it undergoes further phase shifts when it interacts with other waves while retaining its identities and stabilities [8]. The word "soliton" was first used by Zabusky and Kruskal [9] to describe the numerical solution of the Korteweg-de Vries (KdV) problem. Solitary waves have been thoroughly investigated both theoretically and experimentally as a significant non-linear topic (Shukla [10]; Verheest [12]; Tian and Gao [13]; Misra and Bhowmik [11]). Many writers have examined ion-acoustic solitons using the reductive perturbation approach (Bharuthram and Shukla[14]; Yadav and Sharma[15]). It has been noted that nonlinear waves in plasmas with extra components, such as positrons, behave differently from those in plasmas that are composed of electrons and positive ions (Rizzato[16]). Propagation of ion-acoustic solitons in magnetised and unmagnetised plasmas has been studied theoretically (Washimi and Taniuti[20]; Ostrovoski et al.[23]; Boldyrev et al.[24]; Kadomstev and Karpman[21]; Gresillon and Doveil[22]) and experimentally by Ludwig et al.[28]; Cooney et al.[29]; Okutsu et al.[27]; Ikezi et al.[25]; Ikezi[26] in the past, and interesting results have been obtained.

Dusty plasma is an ionised gas that contains huge solid charged dust grains that are micrometres in size, as well as electrons, positive ions, and neutral atoms. The dusty plasma has become one of the prominent matters of research during the last five decades because of its great presence in the universe. This is the most astounding and fascinating matter of fact: over ninety-nine percent of the known universe is in the form of plasma, and the remaining one percent or less which includes our earth, is not in the form of plasma. It may be found in a wide range of astrophysical objects, such as pulsar magnetospheres [18, 19], planetary rings, comet tails, the interstellar medium, noctilucent clouds, the solar atmosphere [30, 47], active galactic nuclei [17], and more [32, 33, 34]. Besides its application in astrophysical studies, nowadays investigation of the plasma becomes a promising topic because of its important role in the semiconductor processing industry, nanoparticle production, and film deposition reactors, etc.,[35, 36].

During the last three decades, physicists have become quite interested in observing DIAW in a dusty plasma system. Tonks et al. made the first theoretical prediction that ion-acoustic waves (IAW) would be present in ionised gas [37], while

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Rewans made the first practical observation of IAW in gas-discharge plasma in [38]. Sazdeev [39] studied theoretically these types of waves in plasma systems, and Ikezi et.al. [40] observed the same experimentally. Subsequently, numerous experimental and theoretical studies had been conducted in various plasma fields, and it was found that dust grains generate a variety of new wave features, viz., dust ion acoustic mode , dust acoustic mode [41, 42], dust drift mode [43], dust lattice mode [44], Shukla–Varma mode [45], dust cyclotron mode [46], dust Berstain–Green–Krushkal mode [47], etc. Shukla and Silin[49] predicted theoretically DIAWs in dusty plasma consisting of negatively charged static dust grains for the first time, and Barken et al.[50] observed the existence of DIAWs experimentally. Linear and nonlinear DIAWs had also been experimentally investigated by Nakamura et al.[51] in a homogeneous unmagnetized dusty plasma. They observed that the phase velocity of the wave increases and the wave endures heavy damping with increasing dust density in the linear regime. Recently, Jharna et al.[48] studied the DIAWs in an unmagnetized collisional nonextensive dusty plasma. They showed that characteristic of the wave affected by the nonextensive parameter and dust ion collisional frequency. Renyi [52] first introduced the nonextensive generalisation of BGS entropy for statistical equilibrium. Liyan and Du [53] studied ion acoustic solitary waves (IASWs) in the plasma with power-law q-distribution in non-extensive statistics, and they suggested that Tsallis [54] statistics are suitable for the system being the non-equilibrium stationary state with inhomogeneous temperature and containing a huge supply of the superthermal low-velocity particles. Saha et al. [55, 56, 57] studied the dynamical behaviour of DIAWs in the presence of an external periodic force. L. Mandi, K.K. Mondal and P. Chatterjee studied the Analytical solitary wave solution of the dust ion acoustic waves for the damped forced modified Korteweg-de Vries equation in q-nonextensive plasmas [56]. Considering the external periodic perturbation, Saha et al.[57] observed the quasiperiodic, periodic, and chaotic structures of DIAWs. Shalini and Saini [58] studied the properties of DIA rogue waves in an unmagnetised collisionless plasma system composed of charged dust grains, superthermal electrons, and warm ions. It is noteworthy that the external periodic perturbation has a significant effect in many real physical situations [59, 60]. P. Chatterjee, R. Ali, A. Saha [61] studied the analytical solitary wave solution of the dust ion acoustic (DIA) waves in the framework of the damped forced Korteweg–de Vries (DFKdV) equation in superthermal collisional dusty plasmas, but they did not take the ion temperature effect and $q-$ nonextensive electron into account in their investigation. This inspired us to investigate how solitary wave structure forms in a dusty plasma system with warm fluid ions and $q-$ nonextensive electrons. As far as we are aware, there has never been an attempt to investigate the characteristics of DIA solitary waves in this kind of plasma model.

Our objective in this work is to derive the analytical DIA solitary wave of the damped force KdV (DFKdV) equation in an unmagnetized collisional dusty plasma with $q-$ nonextensive electrons, neutral particles, positively charged ions, and negatively charged dust grains in the presence of an external periodic force. Furthermore, the effect of the entropic index q, dust ion collisional frequency ν_{id0}, the speed of the travelling wave M, ion-to-electron temperature ratio σ, strength f_0, and frequency ω, of the periodic force and the parameter μ, which is the ratio between the unperturbed densities of the dust ions and electrons, are studied on the analytical solution of DIA solitary waves. The rest of the paper is organized as follows. The basic governing equations are provided in Section 2. In sections 3 and 4, we have derived the damped forced Korteweg-de Vries equation (DFKdV) and the solitary wave solution for non-linear propagation of dust ion acoustic solitary waves. Section 5 presents the result and discusses the effect of the different parameters on the analytical solitary wave solution of DFKdV. Section 6 states the conclusions. At the end, the 'references' are included.

2. BASIC PLASMA MODEL EQUATIONS

In this article, we consider an unmagnetized collisional dusty plasma which is composed of q-nonextensive electrons, stationary dust with negative charge, and hot inertial ions. The normalized ion fluid equations, which comprise the equation of continuity, equation of momentum balance and Poisson equation, governing the dynamics of DIA waves in such a plasma system are as follows:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\sigma}{n}\frac{\partial n}{\partial x} = -\frac{\partial \phi}{\partial x} - \nu_{id} u \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1-\mu) n_e - n + \mu \tag{3}$$

Where n, u and ϕ indicate the number density of ions, ion fluid velocity, electrostatic wave potential respectively normalised to its equilibrium value n_0, ion acoustic speed $C_s = \sqrt{(\frac{K_B T_e}{m_i})}$, with T_e as electron temperature, K_B as Boltzmann constant and m_i as mass of ions and $\frac{K_B T_e}{e}$, with e as magnitude of electron charge. Also, the space variable x and time t are normalized to the Debye length $\lambda_D = \sqrt{(\frac{T_e}{4\pi n_{e0} e^2})}$ and $\omega_{pi}^{-1} = \sqrt{\frac{m_i}{4\pi n_{e0} e^2}}$, with ω_{pi} as ion-plasma frequency. Here ν_{id} is the dust-ion collisional frequency and $\mu = \frac{Z_d n_{d0}}{n_0}$ (dust-to-ion number density ratio). Here z_d are the charge number of dust particle. Also we define $\sigma = \frac{T_i}{T_e}$ (ion-to-electron temperature ratio)

To explain the q-nonextensive electron, we take into account the distribution function[62] that follows.

$$f_e(v) = C_q\{1 + (q-1)\left[\frac{m_e v^2}{2K_B T_e} - \frac{e\phi}{K_B T_e}\right]\}^{\frac{1}{q-1}} \tag{4}$$

Where ϕ indicates the electrostatic potential and other variables or parameter have their typical meaning. It is important to note that this particular distribution function $f_e(v)$ maximizes the Tsallis entropy and hence complies the principles of thermodynamics. Then, the constant of normalization is given by

$$C_q = n_{e0}\frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q}-\frac{1}{2}\right)}\sqrt{\frac{m_e(1-q)}{2\pi k_B T_e}}, \quad \text{for} \quad -1 < q < 1 \tag{5}$$

$$C_q = n_{e0}\frac{1 + q\Gamma\left(\frac{1}{1-q}+\frac{1}{2}\right)}{2\Gamma\left(\frac{1}{1-q}\right)}\sqrt{\frac{m_e\,(1-q))}{2\pi k_B T_e}}, \quad \text{for} \quad q > 1‘ \tag{6}$$

Integrating the distribution function $f_e(v)$ after normalization across the velocity space, the q-nonextensive electron number density may be found as

$$n_e = n_{e0}\{1 + (q-1)\frac{e\phi}{k_B T_e}\}^{\frac{q+1}{2(q-1)}} \tag{7}$$

As a result, the normalized q-nonextensive electron number density is expressed as

$$n_e = \{1 + (q-1)\phi\}^{\frac{q+1}{2(q-1)}} \tag{8}$$

3. DERIVATION OF DAMPED FORCED KORTEWEG-de VRIES (DFKdV) EQUATION

In order to examine the nonlinear wave propagation of DIA solitary wave in unmagnetised collisional dusty plasma, we proceed to derive the damped forced KdV (DFKdV) equation from the set of governing equation by utilizing standard reductive perturbation technique (RPT). According to the RPT, the independent variables are stretched as

$$\xi = \epsilon^{\frac{1}{2}}(x - Ut), \;\; \tau = \epsilon^{\frac{3}{2}}t \tag{9}$$

Where ϵ is a dimensionless parameter indicating the amplitude of the perturbation and U indicate the phase velocity of the DIA solitary wave to be determined from the lowest order of ϵ. Now, the expression of the dependent variables n, u, ϕ, v, are as follows:

$$\left.\begin{array}{l} n = 1 + \epsilon n_1 + \epsilon^2 n_2 + \epsilon^3 n_3 + \dots \\ u = 0 + \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \dots \\ \phi = 0 + \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \dots \\ V_{id} \sim \epsilon^{\frac{3}{2}} v_{id_0} \end{array}\right\} \tag{10}$$

Substituting the above transformation (10) along with stretching coordinate (9) into the set of governing equations (1)-(3) then collecting the coefficient of lowest order terms in ϵ, and after integration with the boundary conditions: $n_1 = 0, u_1 = 0, \phi_1 = 0$ at $|\xi| \to \infty$, we obtain the first order perturbed terms as

$$\left.\begin{array}{l} n_1 = \frac{\phi_1}{E} \\ u_1 = \frac{\phi_1 U}{E} \\ n_1 = a(1-\mu)\phi_1 \end{array}\right\} \tag{11}$$

Where $a = \frac{q+1}{2}$ and $E = U^2 - \sigma$ also found the dispersion relation is obtained as

$$U = \sqrt{\frac{1}{a(1-\mu)} + \sigma} \tag{12}$$

Again, taking the coefficient of next higher order of ϵ (i.e the coefficient of $\epsilon^{\frac{5}{2}}$ from equation (1) and (2) and coefficient of ϵ^2 from equation (3)), we obtain the following equations

$$\frac{\partial n_1}{\partial\tau} - U\frac{\partial n_2}{\partial\xi} + \frac{\partial u_2}{\partial\xi} + \frac{\partial(n_1 u_1)}{\partial\xi} = 0 \tag{13}$$

$$\frac{\partial u_1}{\partial \tau} - U\frac{\partial u_2}{\partial \xi} - n_1 U\frac{\partial u_1}{\partial \xi} + u_1\frac{\partial u_1}{\partial \xi} + \sigma\frac{\partial n_2}{\partial \xi} + \frac{\partial \phi_2}{\partial \xi} + n_1\frac{\partial \phi_1}{\partial \xi} + V_{id_0}u_1 = 0 \tag{14}$$

$$a(1-\mu)\phi_1 - n_1 = 0 \tag{15}$$

Now eliminating n_2,u_2 from equation (13)-(15), then using equation(11) and equation (12), we get the DKdV equation

$$\frac{\partial \phi_1}{\partial \tau} + P\phi_1\frac{\partial \phi_1}{\partial \xi} + Q\frac{\partial^3 \phi_1}{\partial \xi^3} + R\phi_1 = 0 \tag{16}$$

Where $P = \frac{U}{E} + \frac{1-2E^2b(1-\mu)}{2U}$, $Q = \frac{E^2}{2U}$ and $R = \frac{V_{id_0}}{2}$ with $b = \frac{(q+1)(3-q)}{8}$.

The damped KdV equation (16) is a nonlinear partial differential equation. Here P is the nonlinear coefficient, which determines the steepness/sharpness of the solitary excitation, Q is the dispersion coefficient, which measures the broadening of the solitary waves, and R is the dissipation/damping coefficient, which measures the decay of the solitary wave over time while propagating. It has been shown that the existence of an external periodic force significantly changes the behaviour of nonlinear waves. In the presence of an external magnetic force, resistive wall modes of the plasma have been discussed and it has been demonstrated that a flexible, high-speed waveform generator may generate such a force. When an external periodic force $f_0\cos(\omega\tau)$ is taken into account, the damped KdV has the following form:

$$\frac{\partial \phi_1}{\partial \tau} + P\phi_1\frac{\partial \phi_1}{\partial \xi} + Q\frac{\partial^3 \phi_1}{\partial \xi^3} + R\phi_1 = f_0\cos(\omega\tau) \tag{17}$$

Which is termed as the damped forced KdV equation.

4. SOLITARY WAVE SOLUTION OF DAMPED FORCED KORTEWEG-de VRIES (DFKdV) EQUATION

In the absence of R and f_0, that is, $R = 0$ and $f_0 = 0$, of the damped forced KdV equation (17) takes the form of the well-known KdV equation

$$\frac{\partial \phi_1}{\partial \tau} + P\phi_1\frac{\partial \phi_1}{\partial \xi} + Q\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{18}$$

with the solitary wave solution

$$\phi_1 = \phi_m \operatorname{sech}^2\left(\frac{\xi - M\tau}{W}\right) \tag{19}$$

Where $\phi_m = \frac{3M}{P}$ is the amplitude and $W = 2\sqrt{\frac{Q}{M}}$ is the width of the DIA solitary wave with M consist the speed of the DIA solitary wave. In this case, it is well established that

$$\int_{-\infty}^{\infty} \phi_1^2 d\xi \tag{20}$$

is a conserved quantity.

For small values of R and f_0, let us assume that amplitude, width and velocity of the DIA solitary waves are dependent on τ and the approximate solution of equation (17) is of the form

$$\phi_1 = \phi_m(\tau)\operatorname{sech}^2\left(\frac{x - M(\tau)\tau}{W(\tau)}\right) \tag{21}$$

Where $M(\tau)$ is an unknown function of τ and $\phi_m(\tau) = \frac{3M(\tau)}{P}$ and $W(\tau) = 2\sqrt{\frac{Q}{M(\tau)}}$.

Differentiating (19) with respect to τ and using (17), one can obtain

$$\frac{dI}{d\tau} + 2CI = 2f_0\cos(\omega\tau)\int_{-\infty}^{\infty}\phi_1 d\xi, \tag{22}$$

$$\frac{dI}{d\tau} + 2CI = \frac{24f_0\sqrt{Q}}{P}\sqrt{M(\tau)}\cos(\omega\tau). \tag{23}$$

Again

$$I = \int_{-\infty}^{\infty}\phi_1^2 d\xi, \tag{24}$$

$$I = \int_{-\infty}^{\infty}\phi_m^2(\tau)\operatorname{sech}^4\left(\frac{\xi - M(\tau)\tau}{W(\tau)}\right)d\xi, \tag{25}$$

$$I = \frac{24\sqrt{Q}}{P^2} M^{\frac{3}{2}}(\tau) \tag{26}$$

From (21) and (25), the expression of $M(\tau)$ is obtained as

$$M(\tau) = \left(M - \frac{8PCf_0}{16C^2 + 9\omega^2} \right) e^{\frac{4}{3}c\tau} + \frac{6Pf_0}{16C^2 + 9\omega^2} \left(\frac{4}{3} C\cos(\omega\tau) + \omega \sin(\omega\tau) \right) \tag{27}$$

Therefore, the analytical solitary wave solution of the DIA solitary waves for the damped forced KdV (17) is

$$\phi_1 = \phi_m(\tau)\,\mathrm{sech}^2\left(\frac{\xi - M(\tau)\tau}{W(\tau)} \right), \tag{28}$$

Where $M(\tau)$ is given by equation (27), and the amplitude and width are as follows:

$$\phi_m(\tau) = \frac{3\left(M - \frac{8PCf_0}{16C^2+9\omega^2} \right) e^{\frac{4}{3}C\tau} + \frac{18Pf_0}{16C^2+9\omega^2}\left(\frac{4}{3}C\cos(\omega\tau) + \omega\sin(\omega\tau) \right)}{P} \tag{29}$$

$$W(\tau) = \frac{2\sqrt{Q}}{\sqrt{\left(M - \frac{8PCf_0}{16C^2+9\omega^2} \right) e^{\frac{4}{3}C\tau} + \frac{6Pf_0}{16C^2+9\omega^2}\left(\frac{4}{3}C\cos(\omega\tau) + \omega\sin(\omega\tau) \right)}} \tag{30}$$

5. RESULTS AND DISCUSSION

In this manuscript, we have explored the impact of various physical parameters, such as the dust-ion collisional frequency (ν_{id0}), frequency (ω), strength of the external periodic force (f_0), ion-to-electron temperature ratio (σ) and entropic index (q) on the formation and existence of DIA solitary wave by the DIA solitary wave solution of the damped forced KdV equation (17) using numerical computations.

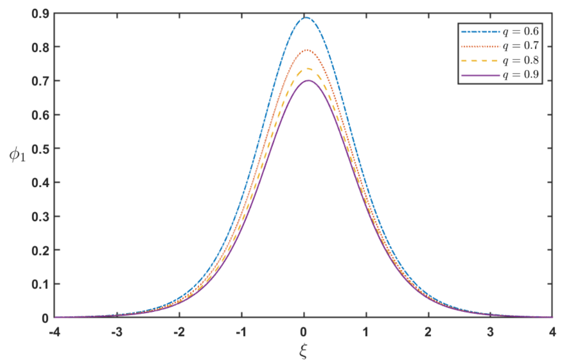


Figure 1. Profile of solitary wave potential ϕ_1 with ξ for different values of q and fixed values of $\mu = 0.2$, $\sigma = 0.1$, $\nu_{id0} = 0.01$, $\omega = 0.5$, $f_0 = 0.05$, $M = 0.1$, and $\tau = 2$.

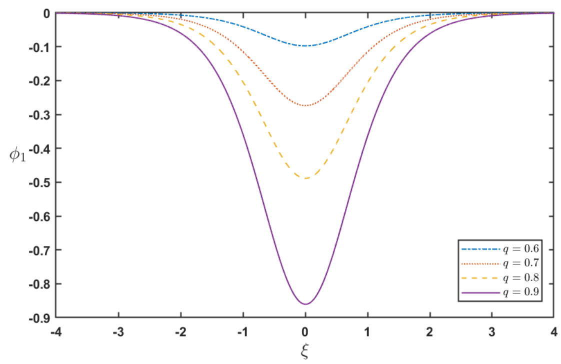


Figure 2. Profile of solitary wave potential ϕ_1 with ξ for different values of q and fixed values of $\mu = 0.5$, $\sigma = 0.1$, $\nu_{id0} = 0.01$, $\omega = 0.5$, $f_0 = 0.05$, $M = 0.1$, and $\tau = 2$.

Figure (1) depict the graph of the variation of compressive solitary wave profile ϕ_1 versus ξ of the damped forced KdV equation (17) for the different values of the entropic index $q = 0.6, 0.7, 0.8, 0.9$ where q is taken from the interval $(0.6, 0.9)$ with special values of the other fixed parameter $M = 0.1, \sigma = 0.1, \mu = 0.2, \nu_{id0} = 0.01, \omega = 0.5, f_0 = 0.05$ and $\tau = 2$. It is observed that the amplitude of the compressive solitary wave decreases and width also decreases as the value of entropic index q increases. A right hand shifting of the DIA solitary wave solution is also observed as q increases.

Figure (2) represent the variation of the rarefactive solitary wave profile for the DIA solitary wave corresponding to the damped forced KdV equation (17) for different entropic index $q = 0.6, 0.7, 0.8, 0.9$ with dust-to-ion number density ratio $\mu = 0.5$, and the other parameters are the same as in Figure (1). In this figure we observed that the amplitude and width of the rarefactive solitary wave increases as the value of entropic index q increases. Thus the DIA rarefactive solitary wave flourishes as the entropic index q grows rapidly. Here we also observed from Figure (1) and Figure (2), and it is interesting to note that the reverse effect happened after changing the value of dust-to-ion number density ratio (μ) from 0.2 to 0.5. that means the value of dust-to-ion number density ratio μ has a significant effect on the dynamics of the DIA solitary wave of the damped forced KdV equation.

Figure (3) depict the variation of the compressive solitary wave profile ϕ_1 versus ξ of the damped forced KdV equation (17) for varying values of the strength of the external periodic force $f_0 = 0.01, 0.03, 0.06, 0.09$, here f_0 is taken from the interval (0.01, 0.09) with special fixed values of the other parameters $\sigma = 0.1, \mu = 0.2, M = 0.1, \nu_{id0} =$

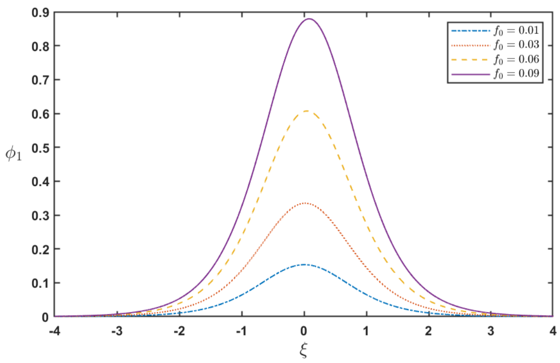


Figure 3. Profile of solitary wave potential ϕ_1 with ξ for different values of f_0 and fixed values of $\mu = 0.2$, $\sigma = 0.1$, $\nu_{id0} = 0.01$, $\omega = 0.5$, $q = 0.8$, $M = 0.1$, and $\tau = 2$.

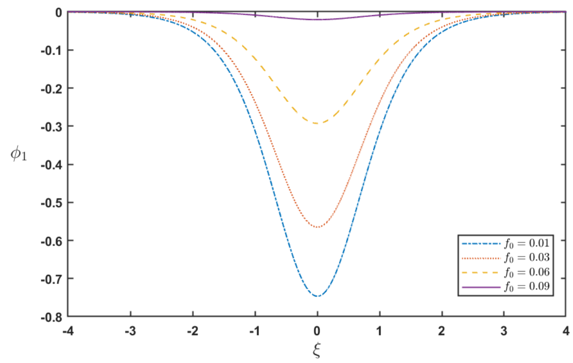


Figure 4. Profile of solitary wave potential ϕ_1 with ξ for different values of f_0 and fixed values of $\mu = 0.5$, $\sigma = 0.1$, $\nu_{id0} = 0.01$, $\omega = 0.5$, $q = 0.8$, $M = 0.1$, and $\tau = 2$.

$0.01, \omega = 0.5, q = 0.8$ and $\tau = 2$. We observed that as the strength of the external periodic force f_0 rises, it is found that the compressive DIA solitary wave's amplitude and breadth also increase.

Figure (4) shows a graph of the variation of the rarefactive solitary wave profile ϕ_1 versus ξ of the damped forced KdV equation (17) for varying values of the strength of the external periodic force $f_0 = 0.01, 0.03, 0.06, 0.09$ with dust-to-ion number density ratio $\mu = 0.5$. All other parameters are the same as in Figure (3). Here, we found that when the external periodic force f_0 becomes stronger, the rarefactive DIA solitary wave's amplitude and width decrease. Figure (3) and Figure (4) also showed us that the behaviour of the DIA solitary wave completely changes when the dust-to-ion number density ratio (μ) is changed from 0.2 to 0.5. It is interesting to node that Figure (3) shows compressive and right hand shifting of the DIA solitary wave and Figure (4) shows rarefactive and not right hand shifting.

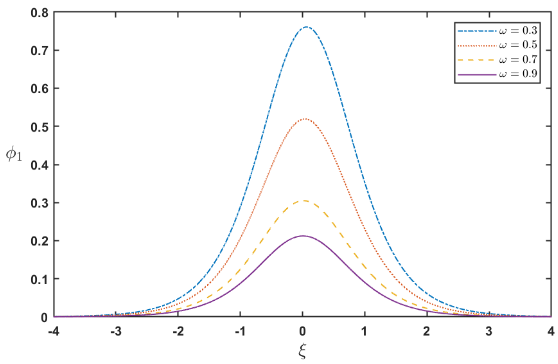


Figure 5. Profile of solitary wave potential ϕ_1 with ξ for different values of ω and fixed values of $\mu = 0.2$, $\sigma = 0.1$, $\nu_{id0} = 0.01$, $f_0 = 0.05$, $q = 0.8$, $M = 0.05$, and $\tau = 2$.

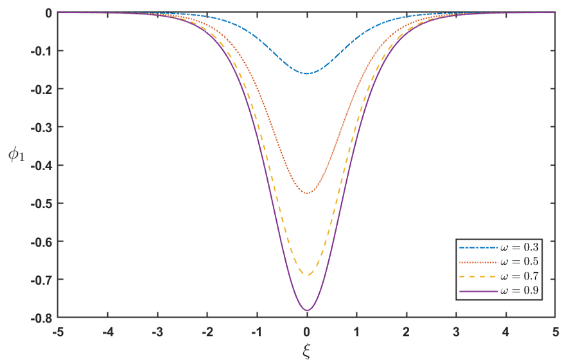


Figure 6. Profile of solitary wave potential ϕ_1 with ξ for different values of ω and fixed values of $\mu = 0.5$, $\sigma = 0.1$, $\nu_{id0} = 0.01$, $f_0 = 0.05$, $q = 0.8$, $M = 0.05$, and $\tau = 2$.

Figure (5) also shows the variation of the compressive DIA solitary wave profile ϕ_1 versus ξ of the damped forced KdV equation (17) for different values of the frequencies $\omega = 0.3, 0.5, 0.7, 0.9$, here ω is taken from the interval $(0.3, 0.9)$ with special fixed values of the other parameters $\sigma = 0.1, \mu = 0.2, M = 0.05, \nu_{id0} = 0.01, f_0 = 0.05, q = 0.8$ and $\tau = 2$. It is observed from the graph that the amplitude and width of the compressive DIA solitary wave decreases with the value of the frequencies ω increases.

Figure (6) depict the graph of the variation of the rarefactive solitary wave profile ϕ_1 versus ξ of the damped forced KdV equation (17) for different values of the frequencies $\omega = 0.3, 0.5, 0.7, 0.9$, here ω is taken from the interval $(0.3, 0.9)$ with dust-to-ion number density ratio $\mu = 0.5$ and other parameter are the same as in Figure (5). Figure (5) and Figure (6) are shown different behaviour compressive right hand shifting and rarefactive not right hand shifting after changing the parameter dust-to-ion number density ratio μ value from 0.2 to 0.5.

Figure (7) reflects the variation of the compressive solitary wave profile ϕ_1 versus ξ of the damped forced KdV equation (17) for the different values of dust ion collisional frequencies $\nu_{id0} = 0.01, 0.05, 0.09, 0.13$, with special values of the other parameters $\sigma = 0.1, \mu = 0.2, M = 0.05, \omega = 0.4, f_0 = 0.05, q = 0.8, \tau = 2$ and Figure (8) reflects the variation of the rarefactive solitary wave profile ϕ_1 versus ξ of the damped forced KdV equation (17) for the different values of dust ion collisional frequencies $\nu_{id0} = 0.01, 0.05, 0.09, 0.13$, $\mu = 0.5$ and other parameter are the same as in Figure (7) and ν_{id0} is taken from the interval $(0.01, 0.13)$ and to discuss the changes, we observed from Figure (7) that the amplitude and width decreases as the value of dust ion collisional frequencies ν_{id0} increases and from Figure (8) we observed that

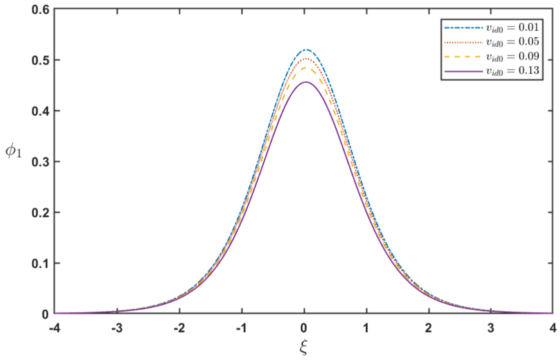


Figure 7. Profile of solitary wave potential ϕ_1 with ξ for different values of v_{ido} and fixed values of $\mu = 0.2$, $\sigma = 0.1$, $\omega = 0.5$, $f_0 = 0.05$, $q = 0.8$, $M = 0.05$, and $\tau = 2$.

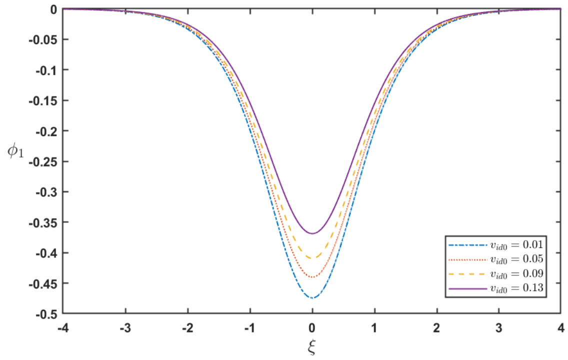
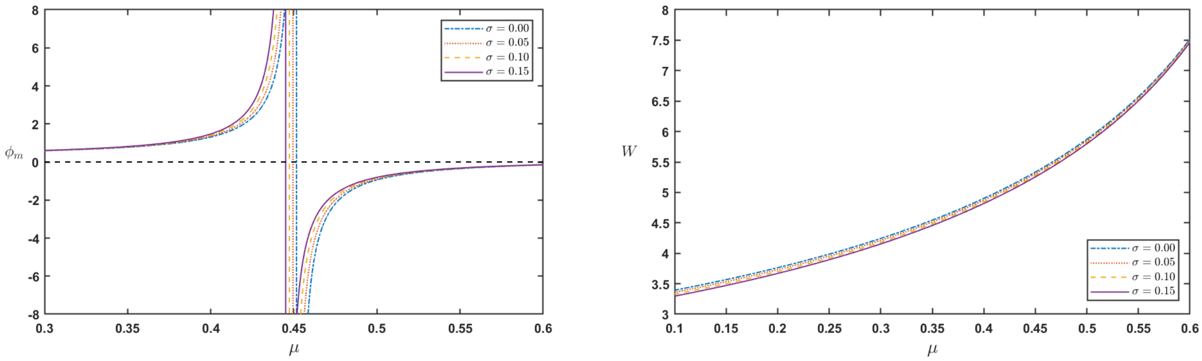


Figure 8. Profile of solitary wave potential ϕ_1 with ξ for different values of v_{ido} and fixed values of $\mu = 0.5$, $\sigma = 0.1$, $\omega = 0.5$, $f_0 = 0.05$, $q = 0.8$, $M = 0.05$ and $\tau = 2$.

the amplitude and width of the rarefactive solitary wave decreases as the values of dust ion collisional frequencies v_{id0} increases. Here from both the graph we also observed that Figure (7) and Figure (8) shows the compressive and rarefactive DIA solitary wave behaviour after changing the only one parameter value μ from 0.2 to 0.5.



(a) (b)

Figure 9. (a) Soliton amplitude ϕ_m and (b) Soliton width W with μ for different values of σ and fixed values of $\omega = 0.5$, $f_0 = 0.05$, $q = 0.8$, $M = 0.1$ and $\tau = 2$.

Figure (9a) shows the graph amplitude ϕ_m versus μ of compressive and rarefactive damped forced KdV solitons for different values of $\sigma = 0.00, 0.05, 0.10, 0.15$ and fixed values of $\omega = 0.5$, $f_0 = 0.05$, $q = 0.8$, $M = 0.1$ and $\tau = 2$. Here we observed that an increase of the amplitudes of compressive DIA KdV solitons suddenly shows a rapid and convex increase within a very small interval of μ, but their rarefactive counter parts sharply decrease concavely for each increasing value of σ in a similar but opposite pattern. Figure (9b) shows the graph the variation of width W of the rarefactive solitary wave solution of the damped forced KdV with respect to μ for different values of $\sigma = 0.00, 0.05, 0.10, 0.15$ with fixed values of $\omega = 0.5, f_0 = 0.05, q = 0.8, M = 0.1$ and $\tau = 2$. The width of DIA solitary waves is seen to increase as the dust-to-ion number density ratio μ increases. Also, at the same time, it is observed that the width of the solitary wave increases rapidly with the different increase in values of the ion-to-electron temperature ratio σ.

6. CONCLUSIONS

In this study, we examined DIA solitary waves in a dusty plasma containing immobile dust particles, non-extensive electrons, and negatively charged ions. We derive the damped forced KdV equation using the reductive perturbation approach. An analytical solitary wave solution has been derived for the damped forced KdV equation in the presence of a small damped and externally applied periodic force. The effect of parameters q, f_0, M, σ and ω on the acoustic solitary wave solution of dust ions with fixed values of the other physical parameters μ, τ has been presented. The parameters $q, f_0, M, v_{id0}, \sigma$ and ω have played an important role in the nonlinear structure of the DIA solitary wave in a collisional dusty plasma.

The present theoretical finding should be crucial for understanding how nonlinear DIA solitary waves behave in a variety of plasma environments, such as the Earth's magnetosphere in space plasma, the pulsar magnetosphere of astrophysical plasmas, and laboratory plasmas that contain the plasma under consideration.

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REFERENCES

[1] F.F. Chen, *Introduction to Plasma Physics and Controlled Fusion*, 3rd edition, (Springer, 2016).

[2] H. Schamel, and V.I. Maslov, "Adiabatic growth of electron holes in current-carrying plasmas", Physica Scripta, **T50**, 42 (1994). https://doi.org/10.1088/0031-8949/1994/t50/006

[3] V.I. Maslov, and H.Schamel, "Growing electron holes in drifting plasmas", Physics Letters A, **178**, 171 (1993). https://doi.org/10.1088/0031-8949/1994/t50/006

[4] H. Schamel, and V. Maslov, "Langmuir Wave Contraction Caused by Electron Holes," Physica Scripta, **82**, 122 (1999). https://doi.org/10.1238/physica.topical.082a00122

[5] D. Batani, V. Maslov, and D. Bondar, *et al*. "Smoothing of transverse nonuniformities at the critical density in laser interaction with nonuniform plasmas," Laser and Particle Beams, **43**, (e3) (2025). https://doi.org/10.1017/lpb.2025.10003

[6] V.I. Maslov, "Electron beam excitation of a potential well in a magnetized plasma waveguide," Physics Letters A, **165**(1), 63–68 (1992). https://doi.org/10.1016/0375-9601(92)91055-v

[7] V.I. Maslov, I.N.Onishchenko, I.P.Yarovaya, *et al*. "Excitation and properties of large amplitude soliton near foil at laser pulse interaction with it," Problems of Atomic Science and Technology, (1), 324–327 (2012).

[8] A. Scott, *Encyclopedia of nonlinear science*, 1st edition, (Routledge, 2006).

[9] N.J. Zabusky, and M.D. Kruskal, "Interaction of solitons in a collisionless plasma and the recurrence of initial states," Physical review letters, **15**(6), 240 (1965). https://doi.org/10.1103/physrevlett.15.240

[10] P.K. Shukla, "Nonlinear waves and structures in dusty plasmas," Physics of Plasmas, **10**(5), 1619-1627 (2003). https://doi.org/10.1063/1.1557071

[11] A.P. Misra, and C. Bhowmik, "Nonplanar ion-acoustic waves in a quantum plasma," Physics Letters A, **369**(1-2), 90-97 (2007). https://doi.org/10.1016/j.physleta.2007.04.066

[12] F. Verheest, "Waves and instabilities in dusty space plasmas," Space Science Reviews, **77**(3), 267-302 (1996). https://doi.org/10.1007/bf00226225

[13] B. Tian, and Y.T. Gao, "Symbolic computation on cylindrical-modified dust-ion-acoustic nebulons in dusty plasmas," Physics Letters A, **362**(4), 283-288 (2007). https://doi.org/10.1016/j.physleta.2006.10.094

[14] R. Bharuthram, and P.K. Shukla, "Large amplitude ion-acoustic double layers in a double Maxwellian electron plasma," The Physics of fluids, **29**(10), 3214-3218 (1986). https://doi.org/10.1063/1.865839

[15] L.L. Yadav, and S.R. Sharma, "Obliquely propagating ion-acoustic double layers in a multicomponent magnetized plasma," Physica scripta, **43**(1), 106 (1991). https://doi.org/10.1088/0031-8949/43/1/018

[16] F.B. Rizzato, "Weak nonlinear electromagnetic waves and low-frequency magnetic-field generation in electron-positron-ion plasmas," Journal of plasma physics, **40**(2), 289-298 (1988). https://doi.org/10.1017/s0022377800013283

[17] H.R. Miller, and P.J. Wiita, *Active galactic nuclei*, vol. 30, (1988).

[18] V.S. Beskin, A.V. Gurevich, and Y.N. Istomin, *Physics of the pulsar magnetosphere*, (Cambridge university press, 1993).

[19] F.C. Michel, "Theory of pulsar magnetospheres," Reviews of Modern Physics, **54**(1), 1 (1982). https://doi.org/10.1103/revmodphys.54.1

[20] H. Washimi, and T. Taniuti, "Propagation of ion-acoustic solitary waves of small amplitude," Physical Review Letters, **17**(19), 996 (1966). https://doi.org/10.1103/physrevlett.17.996

[21] B.B. Kadomtsev, and V.I. Karpman, "Nonlinear waves," Soviet Physics Uspekhi, **14**(1), 40 (1971).

[22] D. Gresillon, and F. Doveil, "Normal modes in the ion-beam-plasma system," Physical Review Letters, **34**(2), 77 (1975). https://doi.org/10.1103/physrevlett.34.77

[23] B.C. Kalita, and S.N. Barman, "Fully nonlinear ion-acoustic solitary waves in a warm magnetized plasma with electron inertia," Astrophysics and Space Science, **330**, 311-316 (2010). https://doi.org/10.1007/s10509-010-0394-y

[24] S.A. Boldyrev, S.V. Vladimirov, and V.N. Tsytovich, "Coupled Langmuir and ion-acoustic solitons," Soviet Journal of Plasma Physics, **18**(11), 727-732 (1992).

[25] H. Ikezi, R.J. Taylor, and D.R. Baker, "Formation and interaction of ion-acoustic solitions," Physical Review Letters, **25**(1), 11 (1970). https://doi.org/10.1103/physrevlett.25.11

[26] H. Ikezi, Experiments on ion-acoustic solitary waves. Research Report: (IPPJ, 1973).

[27] Y. Nakamura, and T. Ogino, "Numerical and laboratory experiments on spherical ion-acoustic solitons," Plasma Physics, **24**(10), 1295 (1982). https://doi.org/10.1088/0032-1028/24/10/007

[28] G.O. Ludwig, J.L. Ferreira, and Y. Nakamura, "Observation of ion-acoustic rarefaction solitons in a multicomponent plasma with negative ions," Physical review letters, **52**(4), 275 (1984). https://doi.org/10.1103/physrevlett.52.275

[29] J.L. Cooney, M.T. Gavin, and K.E. Lonngren, "Experiments on Korteweg–de Vries solitons in a positive ion–negative ion plasma," Physics of Fluids B: Plasma Physics, **3**(10), 2758-2766 (1991). https://doi.org/10.1063/1.859912

[30] W.H. Julian, and P. Goldreich, "Pulsar electrodynamics," Astrophys. J, **157**, 869 (1969). https://doi.org/10.1086/150119

[31] E. Tandberg-Hanssen, and A.G. Emslie, *The physics of solar flares*, vol. 14, (Cambridge University Press, 1988).

[32] C.K. Goertz, "Dusty plasmas in the solar system. Reviews of Geophysics," **27**(2), 271-292 (1989). https://doi.org/10.1029/rg027i002p00271

[33] O. Havnes, T. Aslaksen, F. Melandsø, and T. Nitter, "Collisionless braking of dust particles in the electrostatic field of planetary dust rings," Physica Scripta, **45**(5), 491 (1992). https://doi.org/10.1088/0031-8949/45/5/012

[34] E.C. Whipple, T.G. Northrop, and D.A. Mendis, "The electrostatics of a dusty plasma," Journal of Geophysical Research: Space Physics, **90**(A8), 7405-7413 (1985). https://doi.org/10.1029/ja090ia08p07405

[35] P.K. Shukla, and A.A. Mamun, *Introduction to dusty plasma physics*, 1st edn. (IOP, London, 2002).

[36] F. Verheest, "Waves and instabilities in dusty space plasmas," Space Science Reviews, **77**(3), 267-302 (1996). https://doi.org/10.1007/bf00226225

[37] L. Tonks, and I. Langmuir, "Oscillations in ionized gases," Physical Review, **33**(2), 195 (1929). https://doi.org/10.1103/physrev.33.195

[38] R.W. Revans, "The transmission of waves through an ionized gas," Physical Review, **44**(10), 798 (1933). https://doi.org/10.1103/physrev.44.798

[39] M.A. Leontovich, editor, *Reviews of Plasma Physics*, (Springer Science & Business Media, (2012).

[40] H. Ikezi, R.J. Taylor, and D.R. Baker, "Formation and interaction of ion-acoustic solitions," Physical Review Letters, **25**(1), 11 (1970). https://doi.org/10.1103/physrevlett.25.11

[41] A.A. Mamun, "Arbitrary amplitude dust-acoustic solitary structures in athree-component dusty plasma," Astrophysics and Space Science, **268**, 443-454 (1999). https://doi.org/10.1023/a:1002031022895

[42] N.N. Rao, P.K. Shukla, and M.Y. Yu, "Dust-acoustic waves in dusty plasmas," Planetary and space science, **38**(4), 543-546 (1990). https://doi.org/10.1016/0032-0633(90)90147-i

[43] P.K. Shukla, M.Y. Yu, and R. Bharuthram, "Linear and nonlinear dust drift waves," Journal of Geophysical Research: Space Physics, **96**(A12), 21343-21346 (1991). https://doi.org/10.1029/91ja02331

[44] F. Melandso/, "Lattice waves in dust plasma crystals," Physics of Plasmas, **3**(11), 3890-3901 (1996). https://doi.org/10.1063/1.871577

[45] P.K. Shukla, and R.K. Varma, "Convective cells in nonuniform dusty plasmas," Physics of Fluids B: Plasma Physics, **5**(1), 236-237 (1993). https://doi.org/10.1063/1.860864

[46] R.L. Merlino, A. Barkan, C. Thompson, and N. D'angelo, "Laboratory studies of waves and instabilities in dusty plasmas," Physics of Plasmas, **5**(5), 1607-1614 (1998). https://doi.org/10.1063/1.872828

[47] M. Tribeche, and T.H. Zerguini, "Small amplitude Bernstein–Greene–Kruskal solitary waves in a thermal charge-varying dusty plasma," Physics of Plasmas, **11**(8), 4115-4122 (2004). https://doi.org/10.1063/1.1768957

[48] J. Tamang, K. Sarkar, and A. Saha, "Solitary wave solution and dynamic transition of dust ion acoustic waves in a collisional nonextensive dusty plasma with ionization effect," Physica A: Statistical Mechanics and its Applications, **505**, 18-34 (2018). https://doi.org/10.1016/j.physa.2018.02.213

[49] P.K. Shukla, and V.P. Silin, "Dust ion-acoustic wave," Physica Scripta, **45**(5), 508 (1992). https://doi.org/10.1088/0031-8949/45/5/015

[50] A. Barkan, N. D'angelo, and R.L. Merlino, "Experiments on ion-acoustic waves in dusty plasmas," Planetary and Space Science, **44**(3), 239-242 (1996). https://doi.org/10.1016/0032-0633(95)00109-3

[51] Y. Nakamura, H. Bailung, and P.K. Shukla, "Observation of ion-acoustic shocks in a dusty plasma," Physical review letters, **83**(8), 1602 (1999). https://doi.org/10.1103/physrevlett.83.1602

[52] A. Rényi, "On a new axiomatic theory of probability," Acta Mathematica Academiae Scientiarum Hungarica, **6**, 285-335 (1955). https://doi.org/10.1007/bf02024393

[53] L. Liyan, and D. Jiulin, "Ion acoustic waves in the plasma with the power-law q-distribution in nonextensive statistics," Physica A: Statistical Mechanics and its Applications, **387**(19-20), 4821-4827 (2008). https://doi.org/10.1016/j.physa.2008.04.016

[54] C. Tsallis, "Nonadditive entropy and nonextensive statistical mechanics-an overview after 20 years," Brazilian Journal of Physics, **39**, 337-356 (2009). https://doi.org/10.1590/s0103-97332009000400002

[55] A. Saha, "Bifurcation, periodic and chaotic motions of the modified equal width-Burgers (MEW-Burgers) equation with external periodic perturbation," Nonlinear Dynamics, **87**(4), 2193-2201 (2017). https://doi.org/10.1007/s11071-016-3183-5

[56] L. Mandi, K.K. Mondal, and P. Chatterjee, "Analytical solitary wave solution of the dust ion acoustic waves for the damped forced modified Korteweg-de Vries equation in q-nonextensive plasmas," The European Physical Journal Special Topics, **228**(12), 2753-2768 (2019). https://doi.org/10.1140/epjst/e2019-900047-4

[57] A. Saha, and P. Chatterjee, "Solitonic, periodic and quasiperiodic behaviors of dust ion acoustic waves in superthermal plasmas," Brazilian Journal of Physics, **45**, 419-426 (2015). https://doi.org/10.1007/s13538-015-0329-8

[58] N.S. Saini, "Dust ion acoustic rogue waves in superthermal warm ion plasma," Journal of Plasma Physics, **81**(3), 905810316 (2015). https://doi.org/10.1017/s0022377815000082

[59] K. Nozaki, and N. Bekki, "Chaos in a perturbed nonlinear Schrödinger equation," Physical review letters, **50**(17), 1226 (1983). https://doi.org/10.1103/physrevlett.50.1226

[60] W. Beiglbock, J.P. Eckmann, H. Grosse, M. Loss, S. Smirnov, *et al*. *Concepts and Results in Chaotic Dynamics*, (Springer, Berlin 2000).

[61] P. Chatterjee, R. Ali, and A. Saha, "Analytical solitary wave solution of the dust ion acoustic waves for the damped forced Korteweg–de Vries equation in superthermal plasmas," Zeitschrift für Naturforschung A, **73**(2), 151-159 (2018). https://doi.org/10.1515/zna-2017-0358

[62] A.S. Bains, M. Tribeche, and T.S. Gill, "Modulational instability of ion-acoustic waves in a plasma with a q-nonextensive electron velocity distribution," Physics of Plasmas, **18**(2), (2011). https://doi.org/10.1063/1.3554658

ВПЛИВ ТЕМПЕРАТУРИ ІОНІВ НА ДИНАМІКУ АНАЛІТИЧНОГО РІШЕННЯ ВІДОКРЕМЛЕНОЇ ХВИЛІ ДЛЯ ПИЛОВИХ ІОННИХ АКУСТИЧНИХ ХВИЛЬ ДЛЯ ЗАГАСАЮЧОГО РІВНЯННЯ КОРТЕВЕГА-ДЕ ВРІЗА В Q-НЕЕКСТЕНСИВНІЙ ПЛАЗМІ

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У цій статті досліджуються динамічні властивості аналітичного розв'язку одиночних хвиль пилово-іонної акустики (DIA), індукованих затухаючим вимушеним рівнянням Кортевега-де Фріза (DFKdV) у ненамагніченій зіткнювальній пиловій плазмі, що містить нейтральні частинки, q-неекстенсивні електрони, позитивно заряджені іони та негативно заряджені пилові частинки за наявності зовнішньої періодичної сили. Для отримання затухаючого вимушеного рівняння Кортевега-де Фріза (DFKdV) було розроблено підхід редуктивних збурень. Помічено, що для цієї моделі плазми можливі як стискаючі, так і розріджені одиночні хвильові розв'язки пилово-іонної акустики (DIA). Враховується вплив низки фізичних параметрів: ентропійного індексу (q), частоти зіткнень іонів пилу (ν_{id0}), швидкості біжучої хвилі (M), частоти періодичної сили (ω), співвідношення температур іонів до електронів (σ), параметра, що є співвідношенням між незбуреними густинами іонів пилу та електронів (μ), сили та частоти зовнішньої періодичної сили (f_0). Спостерігається, що ці параметри мають значний вплив на структури затухаючих вимушених акустичних одиночних хвиль іонів пилу. Результати цього дослідження можуть бути важливими для розуміння динаміки одиночних хвиль пил-іонів (DIA) у лабораторній плазмі, а також у космічному плазмовому середовищі.

Ключові слова: *іонно-акустичний солітон, одиночна хвиля, пилова плазма, метод редукційних збурень, неекстенсивний електрон, затухаюче вимушене рівняння KdV*