

## MODIFIED QCD GHOST SCALAR FIELD DARK ENERGY IN ANISOTROPIC AND INTERACTING UNIVERSE MODELS

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In this work, we study the Bianchi type-III interacting framework of modified QCD ghost dark energy with cold dark matter is being considered for illustrating the accelerated expansion of the Universe. The equation of state parameter shows evolution of the Universe completely varies in quintessence region only. The dynamics of scalar field and corresponding potential of various scalar field models shows consistence behavior with the accelerated expansion phenomenon. Also, the kinetic energy term of k-essence models lies within the range where equation of state parameter represents the accelerated expansion of the Universe.

**Keywords:** *Bianchi type-III; QCD ghost dark energy; Cold dark matter; Scalar field models*

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### 1. INTRODUCTION

Dark Energy (DE) is a captivating and enigmatic component of the Universe, widely believed to be responsible for its accelerated expansion. This phenomenon has been strongly supported by numerous observational datasets, including those from Ref [1, 2]. DE exhibits a repulsive gravitational effect, yet its fundamental nature remains largely unknown. The cosmological constant is the earliest and simplest candidate for DE. However, it suffers from two major theoretical challenges: the ‘cosmic coincidence’ problem and the ‘fine-tuning’ issue [3], which limit its acceptance in current discussions of DE. To overcome these limitations, a wide range of dynamical DE models have been proposed, including quintessence, k-essence, and various perfect fluid models [4]. Among these, perfect fluid models are particularly notable for their specific forms of the equation of state, encompassing frameworks such as the chaplygin gas family [5, 6], holographic DE [7, 8], new agegraphic DE [9], and the power-law entropy-corrected models like pilgrim dark energy (PDE) [10, 11, 12, 13]. Another intriguing approach is the quantum chromodynamics (QCD) ghost DE model, presented in various versions [14, 15, 16, 17]. Comprehensive reviews of these dynamical models can be found in Copeland et al. [18] and Bamba et al. [19]. In the framework of QCD, a dynamical DE model known as ‘Veneziano ghost dark energy’ has been proposed, inspired by the concept of the Veneziano ghost. This model originates from attempts to resolve the long-standing  $U(1)$  axial anomaly problem in QCD. It has been suggested that the Veneziano ghost can induce non-trivial physical effects in a Friedmann-Robertson-Walker (FRW) Universe [20, 21]. Specifically, the QCD ghost contributes to the vacuum energy density through a term proportional to  $\Lambda_{\text{QCD}}^3 H$ , where  $\Lambda_{\text{QCD}} \sim 100$  MeV represents the QCD energy scale and  $H$  is the Hubble parameter. Although this contribution is relatively small, it plays a significant role in the dynamics of the Universes evolution.

Importantly, this model has been proposed as a potential solution to two major theoretical challenges of the cosmological constant: the fine-tuning problem and the cosmic coincidence problem [22, 23, 24]. The theoretical investigations of this model via various cosmological parameters have been conducted in several works [25, 26, 27, 28, 29, 30]. Additionally, its compatibility with observational data has been explored [31], further supporting its relevance in modern cosmology. Moreover, the Veneziano ghost field in QCD, expressed in the form  $H + O(H^2)$ , can provide sufficient vacuum energy to account for the accelerated expansion of the Universe [32, 33]. However, the conventional ghost dark energy model incorporates only the leading-order term.

A modified version of this model was later proposed, in which the energy density of generalized ghost dark energy (QCD) ghost dark energy is associated with the radius of the trapping horizon [34]. The energy density in this formulation is defined as:

$$\rho_D = \alpha(1 - \epsilon) \frac{1}{\tilde{r}_T^2} = \alpha(1 - \epsilon) \left( H^2 + \frac{k}{a^2} \right), \quad (1)$$

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where the parameter  $\epsilon$  is given by

$$\epsilon \equiv \frac{\dot{r}_T}{2H\tilde{r}_T}, \quad (2)$$

and  $\tilde{r}_T$  denotes the trapping horizon radius,  $H$  is the Hubble parameter,  $k$  is the spatial curvature, and  $a$  is the scale factor.

Scalar field models have also been extensively explored as alternatives to DE, including quintessence, tachyon, k-essence, and dilaton fields. These models have played a significant role in explaining the late-time accelerated expansion of the Universe. The dynamics of these scalar field models, along with their corresponding potentials, have been thoroughly investigated in the context of various DE frameworks, such as the holographic dark energy (HDE) model with Hubble, future event horizon, and Granda-Oliveros infrared (IR) cutoffs, in both flat and non-flat Universe scenarios [35, 36, 37, 38, 39, 40, 41, 42, 43, 44]. These studies have yielded interesting and insightful results regarding the behavior of scalar fields and their potentials, offering a valuable theoretical understanding of the Universe's accelerated expansion. In our investigation, we have also reconstructed scalar field models in the context of interacting HDE with the Granda-Oliveros IR cutoff in a non-flat Universe.

There is significant observational support for the possibility of an interaction between DM and DE, with the interaction typically described through modified conservation equations for their respective energy densities [46]. Observations of the cosmic microwave Background radiation (CMB), particularly from the Wilkinson Microwave Anisotropy Probe (WMAP), indicate that the present Universe is largely homogeneous and isotropic on large scales. However, residual anisotropies, such as those seen in CMB temperature fluctuations, still persist. The study of anisotropic cosmological models is therefore important for gaining deeper insights into the formation and evolution of the Universe. Among these, Bianchi-type models are particularly useful, as they represent spatially homogeneous but anisotropic solutions to Einstein's field equations, allowing for directional dependencies in cosmic expansion.

Abdul et al. [47] examined the scalar field models such as quintessence, dilaton, tachyon and k-essence in the background of flat FRW Universe. In the presence of interaction, they studied the dynamics of scalar field models with their corresponding potentials. Sheykhi [48] focused on interacting HDE models and their correspondence with scalar field models in a flat Universe. In a non-flat Universe, Sharif and Jawad [49] examined the interaction between HDE and different scalar field models, including quintessence, tachyon, k-essence, and dilaton, considering various values of the power-law DE parameter. Sharif and Shamir [50] have studied exact solutions of Bianchi-type I and V space times in the context of modified theory of gravity. Additionally, Sharif and Zubair [51] investigated anisotropic Universe models involving a perfect fluid and scalar fields in the modified theories of gravity, concluding that  $f(R)$  gravity exhibits similar behavior under varying constraints. Thorsrud et al. [52] have studied cosmology of a scalar field coupled to matter and an isotropy-violating Maxwell field. García-Salcedo et al. [53] have analyzed interacting DE with a trapping horizon in Bianchi models.

Zubair and Abbas [54] have analyzed reconstructing QCD ghost models in the background of modified theory of gravitation. Das et al. [55] have investigated a magnetized anisotropic ghost dark energy cosmological model. Azimi and Barati [56] have analyzed instability of interacting GDE model in an anisotropic Universe. Reddy et al. [57] have investigated dynamics of Bianchi type-II anisotropic DE cosmological model in the presence of scalar-meson fields. Hossienkhani et al. [58] have investigated anisotropy effects on QCD ghost dark energy using the cosmological data. Javed et al. [59] have studied reconstruction of interacting generalized anisotropic scalar field models. Gómez et al. [60] have discussed anisotropic scalar field DE with a disformally coupled Yang-Mills field. Javed et al. [61] have analyzed interacting generalized anisotropic scalar field models. Talole et al. [62] have studied QCD-modified scalar field models of DE, in the presence of both interaction and viscosity, with varying gravitational constant. Bhardwaj and Yadav [63] have investigated observational constraints on scalar field cosmological model in anisotropic Universe. Sharif and Ajmal [64] have studied generalized GDE in the framework of modified theory Gravity. Very recently, Archana and Srivastava [65] have examined the kinematical and geometrical properties of the model as well as interacting ghost scalar field models of DE in Bianchi type-II Universe.

In this work, we investigate the cosmological evolution of an anisotropic Universe within the framework of general theory of relativity, using the redshift parameter  $z$  as a reference. The structure of the paper is as follows: in Section 2, we present the foundational concepts and define key cosmological parameters, including the energy density of dark energy and the equation of state parameter. Section 3 provides a graphical analysis of the Bianchi type III model in conjunction with various scalar field dark energy models, namely quintessence, tachyon, and k-essence. The final section summarizes and concludes the key findings of this study.

## 2. METRIC AND FIELD EQUATIONS

The gravitational field in our model is given by a Bianchi type-III metric as

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2nx}dy^2 - C^2(t)dz^2, \quad (3)$$

with  $A$ ,  $B$  and  $C$  being functions of the cosmic 't' only.

The Einstein's field equations (with gravitational units,  $8\pi G = 1 = C$ )

$$R_{ij} - \frac{1}{2}Rg_{ij} = (T_{ij} + \bar{T}_{ij}), \quad (4)$$

where  $g_{ij}$  is the metric tensor,  $R$  is the Ricci scalar and  $R_{ij}$  is the Ricci tensor,  $T_{ij}$  and  $\bar{T}_{ij}$  are the energy-momentum tensors of matter and dark energy respectively, and they are defined as

$$T_{ij} = \rho_m x_i x_j \quad (5)$$

and

$$\bar{T}_{ij} = (\rho_{DE} + p_{DE})x_i x_j - p_{DE}g_{ij}, \quad (6)$$

where  $\rho_m$  and  $\rho_{DE}$  are the energy densities of matter and dark energy respectively and  $p_{DE}$  is the pressure of the DE.

From equations (5) and (6), we have

$$T_1^1 = T_2^2 = T_3^3 = 0, \quad T_4^4 = \rho_m \quad (7)$$

$$\bar{T}_1^1 = \bar{T}_2^2 = \bar{T}_3^3 = -p_{DE}, \quad \bar{T}_4^4 = \rho_{DE}. \quad (8)$$

The field (4), for the metric (3) with the help of (5) and (6), can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -p_{DE} \quad (9)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p_{DE} \quad (10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{n^2}{A^2} = -p_{DE} \quad (11)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{n^2}{A^2} = \rho_m + \rho_{DE} \quad (12)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \quad (13)$$

where overhead dot denote differentiation with respect to cosmic time  $t$ .

Now solving (13), we get

$$A = c_1 B \quad (14)$$

where  $c_1$  is an integration constant and without loss of generality, we take  $c_1 = 1$ , we have

$$A = B \quad (15)$$

In order to determine the average anisotropy parameter  $A_h$  for a generalized anisotropic cosmological model, it is defined as follows:

$$A_h = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2, \quad (16)$$

where  $H$  is the mean Hubble parameter, and  $H_i$  are the directional Hubble parameters along each spatial direction.

For the specific case under consideration, we take:

$$H_x = H_y = \frac{\dot{A}}{A}, \quad H_z = \frac{\dot{B}}{B},$$

where  $A(t)$  and  $B(t)$  are the directional scale factors in the  $x, y$  and  $z$  directions, respectively.

Further, we discuss some solution regarding anisotropic Universe model with scalar field models.

### 3. SOLUTION OF THE FIELD EQUATIONS

The field equations (9) to (12) are a system of four highly non-linear differential equations in five unknowns  $A, C, p_{DE}, \rho_m, \rho_{DE}$ . The system is thus initially undetermined. We need one extra physical condition to solve the field equations completely.

We assume that the expansion scalar ( $\theta$ ) is proportional to the shear scalar ( $\sigma^2$ ) [66, 67]. The condition leads to

$$A = C^k, \quad (17)$$

where ( $k > 1$ ).

Now solving equations (9), (11), (15) and (17), we get

$$A = B = (bt + c), \quad C = (bt + c)^{\frac{1}{k}}, \quad (18)$$

where  $b = \frac{nk}{\sqrt{k^2-1}}$ , and  $c = kc_2$ .

From equations (18) in metric (3), we have

$$ds^2 = dt^2 - (bt + c)^2 dx^2 - (bt + c)^2 e^{-2nx} dy^2 - (bt + c)^{\frac{2}{k}} dz^2 \quad (19)$$

Equation (19) represents Bianchi type-III interacting modified QCD ghost scalar field model of DE.

The interacting between CDM and QCD dark energy [68, 69, 70, 71], the continuity equations turn out to be

$$\dot{\rho}_m + 3H\rho_m = Q, \quad \dot{\rho}_{DE} + 3H(1 + \omega_{DE})\rho_{DE} = -Q, \quad (20)$$

where  $\omega_{DE} = \frac{p_{DE}}{\rho_{DE}}$  and  $Q$  stands for the equation of state (EoS) parameter and the interaction term respectively, we choose the interaction as  $Q = 3d^2H\rho_m$  and  $d^2$  is a coupling constant. From (20), we get

$$\rho_m = \rho_{m0}(bt + c)^{\frac{(2k+1)(d^2-1)}{k}} \quad (21)$$

From equation (12), we get

$$\rho_{DE} = \frac{b^2(k+2)}{k(bt+c)^2} - \frac{n^2}{(bt+c)^2} - \rho_{m0}(bt+c)^{\frac{(2k+1)(d^2-1)}{k}} \quad (22)$$

From equation (20), we get

$$\omega_{DE} = -1 + \frac{k}{2k+1} \left[ \frac{2\left(\frac{b^2(k+2)}{k} - n^2\right) - \rho_{m0}(2k+1)(d^2-2)(bt+c)^{\frac{(2k+1)(d^2-2)}{k}-4}}{\frac{b^2(k+2)}{k} - n^2 - \rho_{m0}(bt+c)^{\frac{(2k+1)(d^2-1)}{k}-2}} \right] \quad (23)$$

**Redshift dependence:** In cosmological models, the redshift  $z$  is a measure of the expansion of the universe and is related to the average scale factor  $a(t)$  by

$$1 + z = \frac{1}{a(t)} \quad (24)$$

where the mean scale factor is given by

$$a(t) = (ABC)^{1/3}. \quad (25)$$

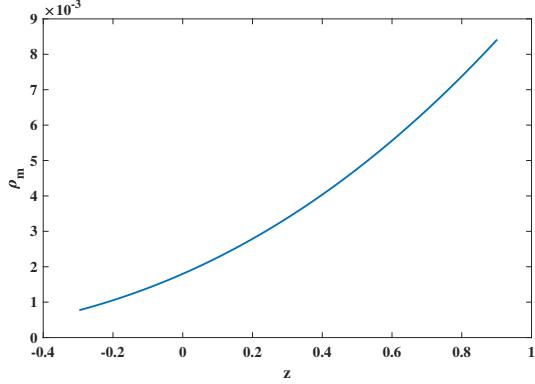
Using equations (18) and (25), we obtain

$$a(t) = (bt + c)^{\frac{2+1/k}{3}}. \quad (26)$$

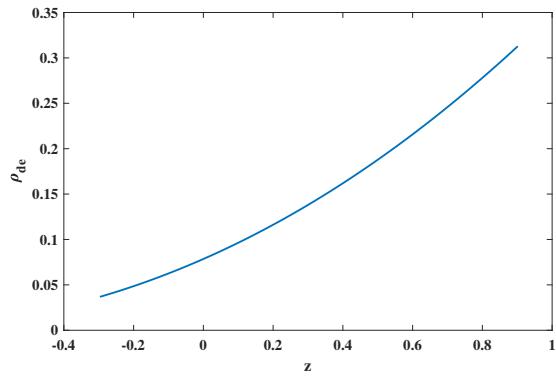
Hence, the relation between cosmic time and redshift is

$$bt + c = (1 + z)^{-\frac{3}{2+1/k}}. \quad (27)$$

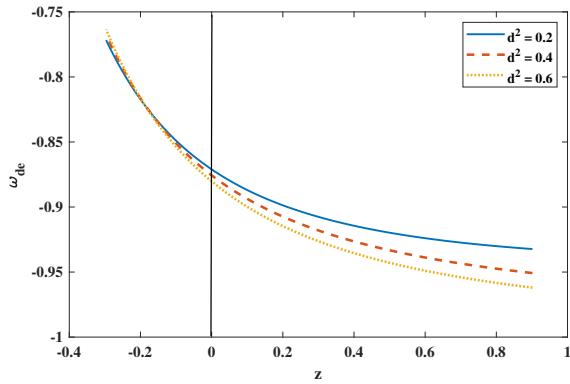
It is observed from Figures 1 and 2 that the energy densities of DM and QCD ghost dark energy remain positive throughout cosmic evolution. They are decreasing functions in redshift  $z$ . They start with a positive value and they tend to very close to zero when  $z$  approaches to negative value. The graphical behavior of the EoS parameter versus redshift ( $z$ ) for the different values of  $d^2$  is shown in Figure 3. It can be observed that the EoS parameter ( $\omega_{de}$ ) completely varies in the quintessence region for various values of  $d^2 = 0.2, 0.4, 0.6$ .



**Figure 1.** Plot of  $\rho_m$  versus  $z$  for  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$



**Figure 2.** Plot of  $\rho_{de}$  versus  $z$  for  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$



**Figure 3.** Plot of  $\omega_{de}$  versus  $z$  for the different values of coupling constants ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$

#### 4. CORRESPONDENCE WITH SCALAR FIELD MODELS IN ANISOTROPIC UNIVERSE

Our objective here is to examine whether a minimally coupled scalar field with a specific action or Lagrangian can replicate the dynamics of the GDE model. This approach aims to establish a possible connection between the GDE framework and a more fundamental theory, such as string theory or M-theory, which often involves scalar fields. To this end, it is meaningful to reconstruct the scalar field dynamics  $\phi$  and the corresponding potential  $V(\phi)$  such that they exhibit key features of the GDE model. Following the method proposed in [72], we establish a correspondence between the GDE model and various scalar field models by equating their respective energy densities and equations of state. Through this correspondence, we reconstruct both the field dynamics and the scalar potential, thereby gaining deeper insight into the theoretical underpinnings of the GDE framework.

##### 4.1. Reconstructing ghost quintessence model

We adopt the viewpoint that the quintessence scalar field model of DE are effective theories of an underlying theory of DE [73, 74, 75]. The energy density and pressure for the quintessence scalar field can be written as

$$\rho_q = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (28)$$

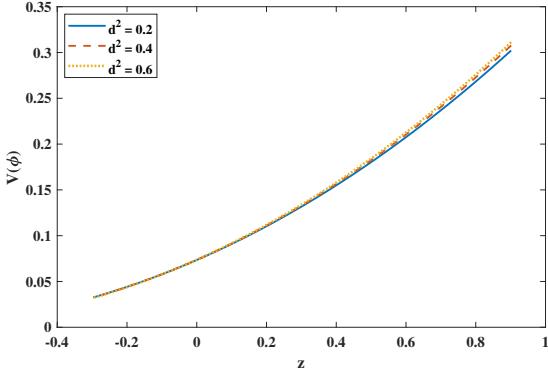
$$p_q = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (29)$$

Then, we can obtain the scalar potential and the kinetic energy term as

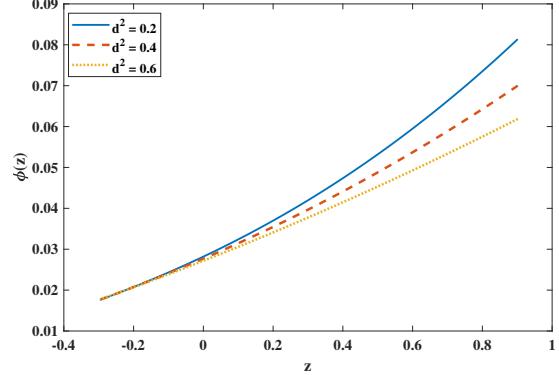
$$V(\phi) = \left[ 2(2k+1) \left( \frac{b^2(k+2)}{k} - n^2 - k\rho_{m0}(bt+c)^{\frac{(2k+1)(d^2-1)}{k}-2} \right) - k \left( 2\frac{b^2(k+2)}{k} - 2n^2 \right) - \rho_{m0}(2k+1)(d^2-1) \right] \left[ 2k(2k+1)(bt+c)^2 \right]^{-1} \quad (30)$$

$$\dot{\phi} = \sqrt{\frac{2\left(\frac{b^2(k+2)}{k} - n^2\right) - \rho_{m0}(2k+1)(d^2-2)(bt+c)^{\frac{(2k+1)(d^2-1)}{k}-4}}{(2k+1)(bt+c)^2}} \quad (31)$$

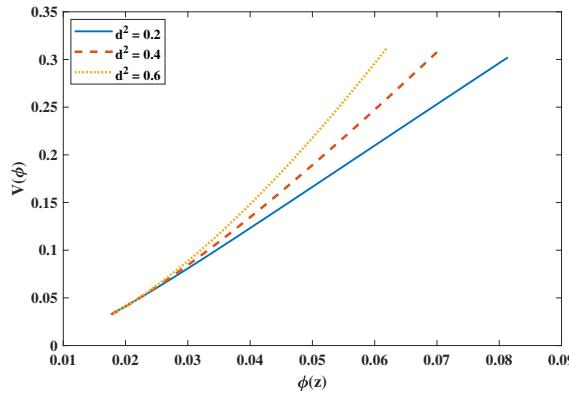
Where  $\omega q = \frac{p_q}{\rho_q}$ . For establishing the correspondence between present DE with quintessence scalar field, we identify  $\rho_{DE} = \rho_q$  and  $\omega_{DE} = \omega_q$ . The evolution trajectories of potential function and scalar field versus redshift ( $z$ ) in ghost



**Figure 4.** Plot of  $V(\phi)$  versus  $z$  in ghost quintessence model for the different values of coupling constants ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$



**Figure 5.** Plot of  $\phi(z)$  versus  $z$  in ghost quintessence model for the different values of coupling constant ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$



**Figure 6.** Plot of  $V(\phi)$  versus  $z$  in ghost quintessence model for the different values of coupling constant ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$

quintessence model for different values of  $d^2$  are shown in Figures 4 and 5. For  $d^2 = 0.2, 0.4, 0.6$ , the quintessence ghost dark energy both potential function and scalar field decreases during of the Universe. The plot of quintessence potential in terms of scalar field is shown in Figure 6 representing increasing behavior. The gradually decreasing kinetic energy while potential remains positive for quintessence model represents accelerate expansion of the Universe for different values of  $d^2$ .

#### 4.2. Reconstructing ghost tachyon model

The tachyon field is another approach for explaining DE [76, 77, 78, 79, 80, 81]. The tachyon energy density and pressure are

$$\rho_T = -T_1^1 = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad (32)$$

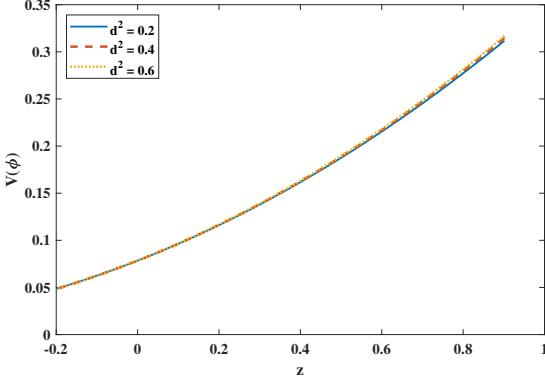
$$p_T = T_i^i = -V(\phi)\sqrt{1 - \dot{\phi}^2}. \quad (33)$$

$$p_T = -V(\phi)\sqrt{1 - \frac{2\left(\frac{b^2(k+2)}{k} - n^2\right) - \rho_{m0}(2k+1)(d^2-2)(bt+c)^{\frac{(2k+1)(d^2-1)}{k}-4}}{(2k+1)(bt+c)^2}}, \quad (34)$$

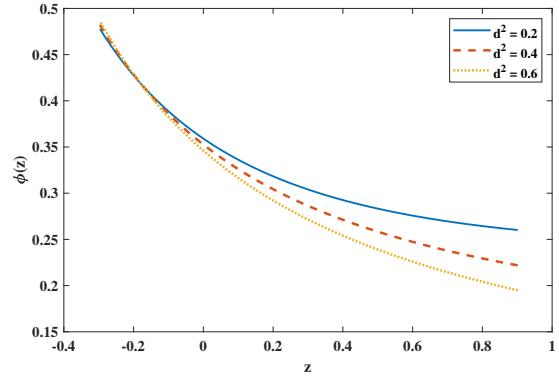
The EoS parameter of tachyon field takes the form

$$\omega_T = \frac{2 \left( \frac{b^2(k+2)}{k} - n^2 \right) - \rho_{m0}(2k+1)(d^2-2)(bt+c) \frac{(2k+1)(d^2-1)}{k} - 4}{(2k+1)(bt+c)^2} - 1. \quad (35)$$

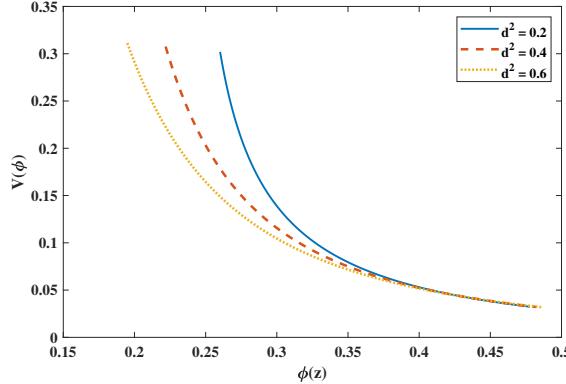
We plot potential function  $V(\phi)$  and scalar field  $\phi$  of ghost dark energy tachyon model as shown in Figures 9 and 7.



**Figure 7.** Plot of  $V(\phi)$  versus  $z$  in ghost tachyon model for the different values of coupling constant ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$



**Figure 8.** Plot of  $\phi(z)$  versus  $z$  in ghost tachyon model for the different values of coupling constant ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$



**Figure 9.** Plot of  $V(\phi)$  versus  $\phi(z)$  in ghost tachyon model for the different values of coupling constant ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$

The evolution of this model is much similar to quintessence model. The scalar field represents increasing behavior versus redshift and indicates more steeper behavior for  $d^2 = 0.2, 0.4$ . This leads to the decreasing kinetic energy.

We plot potential function  $V(\phi)$  and redshift  $\phi(z)$  of ghost tachyon model as shown in Figure 8. The corresponding potential function expresses decreasing but positive behavior with respect to redshift. Its decreasing behavior from maxima gives inverse proportionality to scalar field for the later times. This type of behavior corresponds to scaling solutions in the brane-world cosmology.

#### 4.3. Reconstructing ghost k-essence model

The k-essence scalar field model of DE is characterized by a scalar field with a non-canonical kinetic energy [82, 83, 84, 85, 86]. The density and corresponding pressure of k-essence model are of the form R

$$\rho_k = V(\chi)(-\chi + 3\chi^2), \quad p_k = V(\phi)(-\chi + \chi^2). \quad (36)$$

where  $\chi = \frac{\phi^2}{2}$ . The EoS parameter has the form

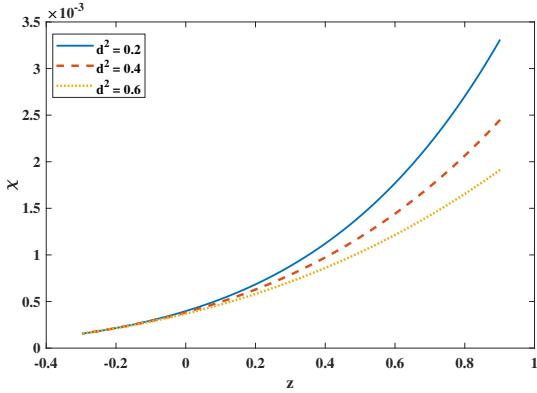
$$\omega_k = \frac{p_k}{\rho_k} = \frac{\chi - 1}{3\chi - 1} \quad (37)$$

in which  $\chi$  experienced the accelerated expansion of the Universe in the interval  $(\frac{1}{3}, \frac{2}{3})$ . Taking  $\omega_k = \omega_{DE}$ , we obtain

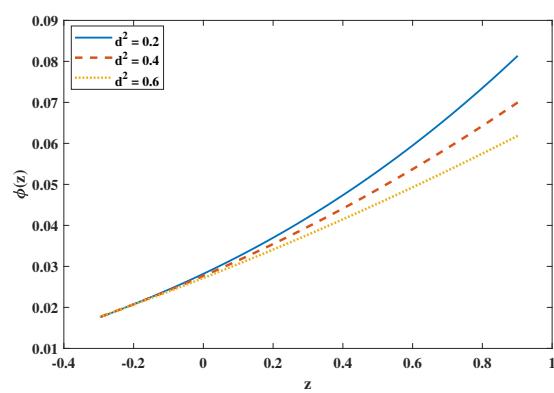
$$\chi = \frac{1}{2(2k+1)(bt+c)^2} \left[ 2 \left( \frac{b^2(k+2)}{k} - n^2 \right) - \rho_{m0}(2k+1)(d^2-2)(bt+c) \frac{(2k+1)(d^2-1)}{k} - 4 \right], \quad (38)$$

The EoS parameter has the form

$$\omega_k = \frac{\frac{1}{2(2k+1)(bt+c)^2} \left[ 2 \left( \frac{b^2(k+2)}{k} - n^2 \right) - \rho_{m0}(2k+1)(d^2-2)(bt+c) \frac{(2k+1)(d^2-1)}{k} - 4 \right] - 1}{3 \cdot \frac{1}{2(2k+1)(bt+c)^2} \left[ 2 \left( \frac{b^2(k+2)}{k} - n^2 \right) - \rho_{m0}(2k+1)(d^2-2)(bt+c) \frac{(2k+1)(d^2-1)}{k} - 4 \right] - 1}. \quad (39)$$

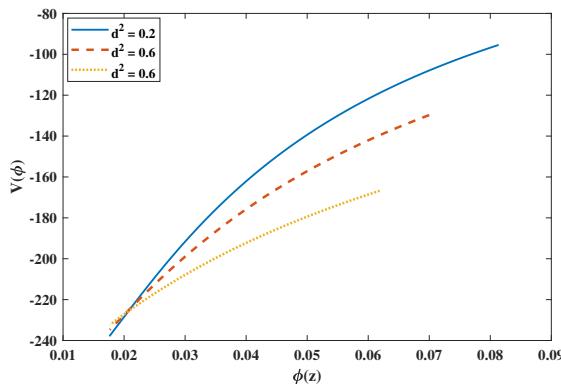


**Figure 10.** Plot of  $\chi$  versus  $z$  in ghost k-essence model for the different values of coupling constant ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$



**Figure 11.** Plot of  $\phi(z)$  versus  $z$  in ghost k-essence model for the different values of coupling constant ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$

The plot of  $\chi$  versus redshift is shown in Figure 10 for various values of  $d^2$ . It can be observed that the region within the range where the EoS parameter of k-essence ghost dark energy model shows consistency with the accelerated Universe. We plot scalar field  $\phi$  of ghost dark energy k-essence model as shown in Figure 11, representing decreasing behavior in the present epoch for the various values of  $d^2$ . The potential function versus scalar field is shown in Figure 12, indicates



**Figure 12.** Plot of  $V(\phi)$  versus  $\phi(z)$  in ghost k-essence model for the different values of coupling constant ( $d^2$ ),  $b=0.5$ ,  $k=9.7$ ,  $c=100$ ,  $n=2.521$

the increase in potential with increase in scalar field but k-essence scalar field decreases with expansion of the Universe.

## 5. CONCLUSIONS

In this work, we study the Bianchi type-III interacting framework of modified QCD ghost dark energy with cold dark matter is being considered for the accelerated expansion of the Universe. We have studied various cosmological parameters to analyze the viability of the models and our conclusions are the following:

- The energy densities of DM and QCD ghost dark energy remain positive throughout cosmic evolution. They are decreasing and start with a positive value and they tend to very close to zero when  $z$  approaches to negative value. The graphical behavior of the Eos parameter ( $\omega_{de}$ ) completely varies in the quintessence region for various values of  $d^2 = 0.2, 0.4, 0.6$ .
- For ghost quintessence model, the evolution trajectories of potential function and scalar field for different values of  $d^2$  are shown in Figures 4 and 5. For  $d^2 = 0.2, 0.4, 0.6$ , the quintessence ghost dark energy both potential function and scalar field decreases during of the Universe. The plot of quintessence potential representing increasing behavior. The gradually decreasing kinetic energy while potential remains positive for quintessence model represents accelerate expansion of the Universe for different values of  $d^2$ .
- For ghost tachyon model, the evoution of this model is much similar to quintessence model. The scalar field represents increasing behavoir versus redshift and indicates more steeper behavior for  $d^2 = 0.2, 0.4$ . This leads to the decreasing kinetic energy. The corresponding potential function expresses decreasing but positive behavior with respect to redshift. Its decreasing behavior from maxima gives inverse proportionality to scalar field for the later times. This type of behavior corresponds to scaling solutions in the brane-world cosmology.
- For ghost k-essence model, The plot of  $\chi$  can be observed that the region within the range where the EoS parameter of k-essence ghost dark energy model shows consistency with the accelerated Universe. We plot scalar field  $\phi$  of ghost dark energy k-essence model representing decreasing behavior in the present epoch for the various values of  $d^2$ . The potential function versus scalar field is indicates the increase in potential with increase in scalar field but k-essence scalar field decreases with expansion of the Universe.

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## МОДИФІКОВАНА ТЕМНА ЕНЕРГІЯ КХД-ПРИМАРНОГО СКАЛЯРНОГО ПОЛЯ В АНІЗОТРОПНИХ ТА ВЗАЄМОДІЮЧИХ МОДЕЛЯХ ВСЕСВІТУ

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У цій роботі ми вивчаємо взаємодіючу структуру Біанкі типу III модифікованої КХД-темної енергії-привіда з холодною темною матерією для ілюстрації прискореного розширення Всесвіту. Параметр рівняння стану показує, що еволюція Всесвіту повністю змінюється лише в області квінтесенції. Динаміка скалярного поля та відповідний потенціал різних моделей скалярного поля демонструють узгоджену поведінку з явищем прискореного розширення. Також член кінетичної енергії к-есенційних моделей знаходиться в діапазоні, де параметр рівняння стану відображає прискорене розширення Всесвіту.

**Ключові слова:** Біанкі типу III; темна енергія-привід КХД; холодна темна матерія; моделі скалярного поля