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EFFECT OF ANISOTROPIC DUST PRESSURE ON THE FORMATION AND PROPAGATION OF ARBITRARY AMPLITUDE DUST ACOUSTIC SOLITARY WAVES (DASW) IN A MAGNETIZED DUST-ION-ELECTRON PLASMA

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Arbitrary amplitude dust acoustic solitary waves (DASW) in a dusty magneto-plasma with anisotropic dust pressure and nonthermal distribution of ions and electrons has been investigated. Sagdeev pseudopotential technique is used to derive an energy balance equation and to analyze various properties of dust acoustic solitons. The effects of anisotropic dust pressure, dust number density ratio, non-thermality, etc., are investigated numerically for the propagation of DASWs. It is found that rarefactive solitons can exist for negatively charged dust, and compressive solitons can exist for positively charged dust. The present study could be useful for the understanding of DASWs in various astrophysical environments.
Keywords: *Dust acoustic wave; Dusty plasma; Anisotropic pressure; Sagdeev pseudopotential; Solitary waves; Magnetized plasma; Non-thermal electrons and ions*
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1. INTRODUCTION

Dusty plasmas are abundant in various astrophysical situations, such as interstellar media, cometary tails, ionospheres, planetary rings, etc. [1-6]. These are also relevant in industrial context [7-9]. The propagation of nonlinear waves in multispecies plasmas consisting of electrons, ions and massive dust particles is a fertile research area. Dust particles that spread in plasma media, acquire either positive or negative charges according to different mechanisms. In most cases, dust particles become negatively charged due to higher mobility of electrons compared to ions. On the other hand, dust particles can gain positive charge by emission of electrons from surface of dust particles by photo-electron emission, secondary electron emission and radiation [8]. From various studies [9-19] it has been observed that the presence of charged dust particles in plasma not only modifies the properties of existing plasma wave modes but also introduces new modes. Many authors showed that the role of dust charge fluctuation cannot be ignored. But it is important to mention here that the nonlinear study of dusty plasmas become too complicated when the charging process are taken into account. Therefore, for simplicity I limit my investigation in a dusty plasma without considering the charging process. The fundamental nature of waves does not change when dust charging and other collisional processes are ignored [20].

Often, magnetic fields [21-23] are involved with dusty plasma. The investigations of linear and non-linear propagation of dust acoustic waves with external magnetic fields have been performed by various researchers [24-28]. The existence of a magnetic field in plasma causes pressure anisotropy, i.e. the pressure in the perpendicular direction to the magnetic field is different from pressure in the parallel direction. The pressure anisotropy can be found in pulsar winds, planetary magnetosphere and near earth's magnetosheaths etc. The effect of anisotropic pressure of ions on solitary waves has been studied by many researchers using Chew-Golberger-Low (CGL) description [29].

The influence of anisotropic ion pressure on dust ion acoustic solitary waves and double layers in a magnetized dusty plasma has been studied by Choi et al. [30] and Chatterjee et al. [31] using the Sagdeev potential method. But this theory is not limited to ion dynamics only. We can also use it to describe the pressure anisotropy of dust particles in the plasma. In many places, like earth's magnetosphere, the dust pressure may not be negligible, and it can play an important role in the formation of various nonlinear structures. We incorporate the anisotropy of dust pressure so that the study may help in understanding the behaviors of nonlinear coherent structures in plasma. Bashir et al. [32] investigated the effect of anisotropic dust pressure and superthermal electrons on propagation and stability of dust acoustic waves. They employed the Reductive-perturbation technique and derived ZK equation for dust acoustic solitary waves in magnetized plasma. They found that the DASW is rarefactive for negative dust and compressive for positive dust.

Various studies have pointed out that highly energetic electrons and ions are present in several astrophysical plasma environments. Among non-thermal distributions, one of the frequently encountered distributions is Cairns' distribution [33-36]. Mamun et al. [33] studied dust acoustic waves in unmagnetized dusty plasma with non-thermal ions. Berbri and Tribeche [36] have considered non-thermal electrons to investigate weakly nonlinear dust ion acoustic shock waves. Choudhury [28] discussed the propagation of DASW in a magnetized plasma with nonthermal electrons and ions using the Sagdeev potential method.

To study nonlinear solitary waves in plasmas, reductive perturbation technique (RPT) and Sagdeev pseudopotential method are mainly used. With RPT only small amplitude nonlinear waves can be studied while Sagdeev pseudopotential

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method can handle arbitrary amplitude waves [37]. In Sagdeev pseudopotential method, fully nonlinearity of the system is considered. Various researchers have employed Sagdeev pseudopotential method to dusty plasma to study electrostatic nonlinear solitary modes.

To the best of our knowledge, the effect of anisotropic dust pressure on the formation and propagation of arbitrary amplitude dust acoustic solitary waves in a magnetized plasma has not been reported in the literature to date. The goal of the present work is, to study the effect of anisotropic dust pressure on the solitary dust acoustic waves in a three-component dusty plasma with nonthermal ions and electrons. Dust grains are assumed to be uniform in mass and charge and plasma is considered to be collisionless. The present study could be useful for the understanding of properties of DASWs in various astrophysical environments like Earth's magnetopause.

The manuscript is organized in the following way: in section I, the usual 'Introduction' is considered; 'Basic set of governing equations' is included in section II; 'Energy integral' is derived in section III; 'Condition of existence' of Dust Acoustic Waves are incorporated in section IV; 'Parametric Analysis' of the results are described in section V; and lastly 'Conclusions' are deducted in section VI which is followed by 'References'.

2. BASIC SET OF GOVERNING EQUATIONS

We consider the propagation of dust acoustic waves in a magnetized, collisionless, three component plasma consisting of non-thermal ions and electrons and dust fluids. The pressure tensor of dust fluid is assumed to be anisotropic and modelled by Chew-Goldberger-Low [27] description. The ambient magnetic field is assumed to be uniform and directed along z-axis, i.e. $\mathbf{B} = B_0\hat{z}$. The equations of continuity and motion governing the dust dynamics in this plasma system are described by

$$\frac{\partial n_d}{\partial t} + \nabla.(n_d \mathbf{v}_d) = 0 \tag{1}$$

$$\frac{\partial \mathbf{v}_d}{\partial t} + (\mathbf{v}_d.\nabla)\mathbf{v}_d = \frac{j\, z_d e}{m_d}\nabla\phi - \mathbf{v}_d \times \frac{j\, z_d e}{c m_d}\mathbf{B} - \frac{1}{m_d\, n_d}\nabla.\widetilde{\mathbf{P}} \tag{2}$$

respectively. Here n_d is dust number density and $\mathbf{v}_d$ is the dust fluid velocity. m_d is the dust mass, e is the magnitude of the electron charge, c is the speed of light in vacuum, z_d is dust charge and ϕ is the electrostatic potential. $j = 1$ for negatively charged dust and $j = -1$ for positively charged dust.

The dust pressure tensor $\widetilde{P}$ takes the form $\widetilde{\mathbf{P}} = p_\perp \hat{\mathbf{I}} + (p_\parallel - p_\perp)\widehat{\mathbf{B}}\widehat{\mathbf{B}}$ where $\hat{\mathbf{I}}$ is unit dyad and $\widehat{\mathbf{B}}\widehat{\mathbf{B}}$ is the dyad form of unit vector along the magnetic field. We can evaluate $\nabla.\widetilde{\mathbf{P}}$ as

$$\nabla.\widetilde{\mathbf{P}} = \boldsymbol{\nabla} p_\perp + (\mathbf{B}.\boldsymbol{\nabla})\left[(p_\parallel - p_\perp)\frac{\mathbf{B}}{B_0^2}\right] \tag{3}$$

Using Chew-Golberger-Low (CGL) description for constant magnetic field, the perpendicular and parallel pressure terms can be obtained as

$$p_\perp = p_{\perp 0}\left(\frac{n_d}{n_{d0}}\right) \text{ and } p_\parallel = p_{\parallel 0}\left(\frac{n_d}{n_{d0}}\right)^3. \tag{4}$$

where n_{d0} equilibrium number density of dust fluid.

The electrons and ions are assumed to be inertia less due to their lighter masses. The expression for nonthermal electron density and ion density are

$$n_e = n_{e0}\left[1 - \beta_e \frac{e\phi}{KT_e} + \beta_e\left(\frac{e\phi}{KT_e}\right)^2\right] Exp\left[\frac{e\phi}{KT_e}\right] \tag{5}$$

$$n_i = n_{i0}\left[1 + \beta_i \frac{e\phi}{KT_i} + \beta_i\left(\frac{e\phi}{KT_i}\right)^2\right] Exp\left[\frac{-e\phi}{KT_i}\right] \tag{6}$$

The symbols β_e and β_i are nonthermal parameters of electrons and ions respectively that characterize nonthermal effect and n_{e0} and n_{i0} are equilibrium number densities of electrons and ions.

The charge neutrality condition is

$$n_e - n_i + j z_d n_d = 0 \tag{7}$$

The wave is assumed to be evolve and propagate in $x - z$ plane. We have normalized densities by corresponding equilibrium densities, velocities by dust acoustic velocity $C_d = \left(\frac{KT_i Z_d}{m_d}\right)^{\frac{1}{2}}$, potential by $\frac{KT_i}{e}$, time by gyroperiod $\Omega_\mathrm{d}^{-1} = \frac{m_d c}{\mathrm{B_0} Z_d e}$ and distances by gyroradius $C_d\Omega_\mathrm{d}^{-1}$, K being the Boltzmann constant. Then the normalized set of governing equations are given by

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_{dx})}{\partial x} + \frac{\partial (n_d v_{dz})}{\partial z} = 0 \tag{8}$$

$$\frac{\partial v_{dx}}{\partial t} + v_{dx}\frac{\partial v_{dx}}{\partial x} + v_{dz}\frac{\partial v_{dx}}{\partial z} = j\frac{\partial \phi}{\partial x} - j\, v_{dy} - \frac{P_\perp}{n_d}\frac{\partial n_d}{\partial x} \tag{9}$$

$$\frac{\partial v_{dy}}{\partial t} + v_{dx}\frac{\partial v_{dy}}{\partial x} + v_{dz}\frac{\partial v_{dy}}{\partial z} = j\, v_{dx} \tag{10}$$

$$\frac{\partial v_{dz}}{\partial t} + v_{dx}\frac{\partial v_{dz}}{\partial x} + v_{dz}\frac{\partial v_{dz}}{\partial z} = j\,\frac{\partial \phi}{\partial z} - P_\parallel n_d \frac{\partial n_d}{\partial x} \tag{11}$$

where $P_\perp = \frac{p_{\perp 0}}{KT_i n_{d0}}$ and $P_\parallel = \frac{3\, p_{\parallel 0}}{KT_i n_{d0}}$
and the normalized number densities of nonthermal electron and ions are given by

$$n_e = (1-f)(1 - \beta_e \alpha\phi + \beta_e \alpha^2\phi^2)e^{\alpha\phi} \tag{12}$$

$$n_i = (1 + \beta_i\phi + \beta_i\phi^2)e^{-\phi} \tag{13}$$

where $f = \frac{z_d n_{d0}}{n_{i0}}$ and $\alpha = \frac{T_i}{T_e}$
The normalized quasi-neutrality condition is

$$fj\, n_d + (1-f)n_e = n_i \tag{14}$$

3. DERIVATION OF ENERGY INTEGRAL

To obtain the energy integral, we consider a wave propagating obliquely to the external magnetic field $B_o\hat{z}$ in moving frame defined by $\xi = xk_x + zk_z - Mt$ with $M = \frac{V}{C_d}$, $k_x^2 + k_z^2 = 1$ where M is the Mach Number (normalized soliton velocity) defined in terms of soliton velocity V and dust acoustic velocity C_d and k_x, k_z are direction cosines, so that

$$\frac{\partial}{\partial t} \to -M\frac{d}{d\xi}, \frac{\partial}{\partial x} \to k_x\frac{d}{d\xi} \text{ and } \frac{\partial}{\partial z} \to k_z\frac{d}{d\xi}. \tag{15}$$

Using these equations, equations (8-11) can be reduced to a set of ordinary differential equations

$$\frac{d}{d\xi}(-Mn_d + k_x n_d v_{dx} + k_z n_d v_{dz}) = 0 \tag{16}$$

$$-\frac{M}{n_d}\frac{dv_{dx}}{d\xi} = j\left(k_x\frac{d\phi}{d\xi} - v_{dy}\right) - \frac{P_\perp}{n_d}k_x\frac{dn_d}{d\xi} \tag{17}$$

$$-\frac{M}{n_d}\frac{dv_{dy}}{d\xi} = jv_{dx} \tag{18}$$

$$-\frac{M}{n_d}\frac{dv_{dz}}{d\xi} = jk_z\frac{d\phi}{d\xi} - P_\parallel n_d k_z\frac{dn_d}{d\xi} \tag{19}$$

After some lengthy but straightforward algebra, the set of equations (16)-(19), along with the equations (12)-(14), can be deduced to a single dimensionless nonlinear differential equation

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + S(\phi) = 0 \tag{20}$$

The boundary condition using in deriving the equation (20) are $\phi \to 0, \frac{d\phi}{d\xi} \to 0, v_{dx} \to 0, v_{dy} \to 0, v_{dz} \to 0$ as $\xi \to \infty$
Here,

$$S(\phi) = \frac{1}{H^2}\Bigg[jG(\phi) - j\,\phi + \frac{k_z^2}{M^2}\frac{(G(\phi))^2}{2} - \frac{P_\parallel}{3}\frac{k_z^2}{M^2}\big(K(\phi) - G(\phi)\big) - M^2\left(\frac{1}{n_d} - 1\right) - P_\perp k_x^2(n_d - 1) - \frac{P_\parallel}{3}k_z^2(n_d^3 - 1) + \frac{M^2}{2}\left(\frac{1}{n_d^2} - 1\right) + P_\perp k_x^2 \log n_d + \frac{P_\parallel}{2}k_z^2(n_d^2 - 1) - jk_z^2\left(\frac{G(\phi)}{n_d} - \phi\right) - j\frac{k_z^2}{M^2}P_\perp k_x^2\big(G(\phi)n_d - J(\phi)\big) - j\,\frac{k_z^2}{M^2}P_\parallel\, k_z^2\big(n_d^3 G(\phi) - K(\phi)\,\big) - \frac{P_\parallel}{3}\frac{k_z^2}{M^2}\left(\frac{M^2}{2}(n_d^2 - 1) + M^2\left(\frac{1}{n_d} - 1\right) - \frac{P_\perp k_x^2}{4}(n_d^4 - 1) + P_\perp k_x^2(n_d - 1) - \frac{P_\parallel k_z^2}{6}(n_d^6 - 1) + \frac{P_\parallel k_z^2}{3}(n_d^3 - 1)\right)\Bigg] \tag{21}$$

is the Sagdeev potential, where:

$$H = 1 + j\left(\frac{M^2}{n_d^3} - \frac{P_\perp k_x^2}{n_d} - P_\parallel\, k_z^2 n_d\right)\frac{\partial n_d}{\partial \phi},$$

$$n_d = \frac{1}{jf}\left[(1+\beta_i\phi+\beta_i\phi^2)e^{-\phi} - (1-j\,f)(1-\beta_e\alpha\phi+\beta_e\alpha^2\phi^2)e^{\alpha\phi}\right],$$

$$G(\phi) = \int_0^{\phi} n_d\, d\phi,$$

$$J(\phi) = \int_0^{\phi} n_d^2\, d\phi,$$

$$K(\phi) = \int_0^{\phi} n_d^4\, d\phi.$$

The values of H, $G(\phi)$, $J(\phi)$ and $K(\phi)$ are calculated using MATHEMATICA-11.

This result is in agreement with the expression forwarded by my earlier paper [28] when I consider only negatively charged ($j = 1$) and cold dust ($P_\parallel$, $P_\perp = 0$).

Choi et al. [30] and Chatterjee et al. [31] had considered negatively charged stationary dust particles forming only background plasma and used ion dynamics to derive Sagdeev potential. In our paper we have considered fully dynamic dust particles and used dust dynamics to derive the Sagdeev potential. So, our result is not directly comparable to these results.

The amplitude ϕ_m of solitary waves is determined by assigning different values of the parameters $\alpha, \beta_i, \beta_e, k_z, f, P_\parallel$, $P_\perp$ and M in the non-linear dispersion relation viz $S(\phi) = 0$.

4. CONDITION OF EXISTENCE OF SOLITARY DUST ACOUSTIC WAVES

The existence criterion of solitary wave formation is that the Sagdeev potential $S(\phi)$ must satisfy the following conditions,

$$S(\phi) = 0, \frac{dS(\phi)}{d\phi} = 0, \frac{d^2S(\phi)}{d\phi^2} < 0 \text{ at } \phi = 0$$

$S(\phi) = 0$ at $\phi = \phi_m$ and $S(\phi) < 0$ for ϕ lying between 0 and ϕ_m, where ϕ_m is the maximum amplitude of the solitons. If $\phi_m < \phi < 0$, rarefactive solitary waves exists, and if $0 < \phi < \phi_m$ compressive solitary waves exist.

From equation (21) it can be easily verified that the conditions $S(\phi) = 0$, $\frac{dS(\phi)}{d\phi} = 0$ at $\phi = 0$ are automatically satisfied. From the condition $\frac{d^2S(\phi)}{d\phi^2} < 0$ at $\phi = 0$ we find that the dust acoustic soliton solution may exist in the interval

$$k_z\sqrt{\frac{f}{(1-\beta_i)+(1-j\,f)(1-\beta_e)\alpha} + P_\parallel} < M < \sqrt{\frac{f}{(1-\beta_i)+(1-j\,f)(1-\beta_e)\alpha} + P_\perp(1-k_z^2) + P_\parallel k_z^2} \qquad (22)$$

5. PARAMETRIC ANALYSIS

To observe the existence of solitary wave, the range of Mach number for existence of DASWs and Sagdeev potential $S(\phi)$ have been plotted against ϕ for different values of plasma parameters. In this investigation, it is found that rarefactive waves travelling at both subsonic and supersonic speeds for negative dust and compressive waves travelling at subsonic speed for positive dust exist in this magnetized plasma. The effects of the parameters $P_\parallel$, $P_\perp$, f, β_i, M and k_z on the formation of dust acoustic solitons have also been studied.

The range of Mach number for nonlinear arbitrary amplitude dust acoustic solitons is investigated numerically from equation (22). It was proven that β had an upper limit 4/7 [38]. When the values of β are above 4/7, the Cairns distribution ceases to be monotonically decreasing. So, all present calculations were performed for values of β not exceeding 4/7. For fixed plasma parameters $\beta_i = 0.2$, $\beta_e = 0.2$, $\alpha = 0.1$, $k_z = 0.75$, and $P_\perp = 0.03$, Fig 1 (a) and (b) show the Mach number M values versus dust-ion density ratio f that supports the existence of arbitrary amplitude dust acoustic solitons in a magnetized plasma with anisotropic dust pressure for two different values of $P_\parallel = 0.1$ (solid) and 0.01(dashed). In figure 1(a) we considered negative dust and in Figure 1(b) we considered positive dust. For both positive and negative dust, we get similar range (not exactly same) of Mach number for existence of solitons.

Figure 2(a) and (b) show the variation of the Sagdeev potential $S(\phi)$ against the electrostatic potential ϕ for different values of parallel dust pressure $P_\parallel$ for negative and positive dust respectively. Other parameters are taken as fixed. From these figures it has been noted that the parallel pressure variation is quite effective for negative dust. As $P_\parallel$ increases the amplitude of the rarefactive solitons decrease very fast. On the other hand, for positive dusts, the amplitude of the compressive soliton increases. In Figure 2(c) and (d) show the variation of ϕ against ξ which has been computed numerically by integrating the Sagdeev potential for same parameters used in figure 2(a) and (b) respectively.

In figure 3(a) and (b), the variation of the Sagdeev potential $S(\phi)$ against the electrostatic potential ϕ for different values of perpendicular dust pressure $P_\perp$ is plotted for negative and positive dust respectively. From these two figures, it can be seen that the increase of perpendicular dust pressure $P_\perp$ increases(decreases) the amplitude of solitons very little for negative(positive) dust.

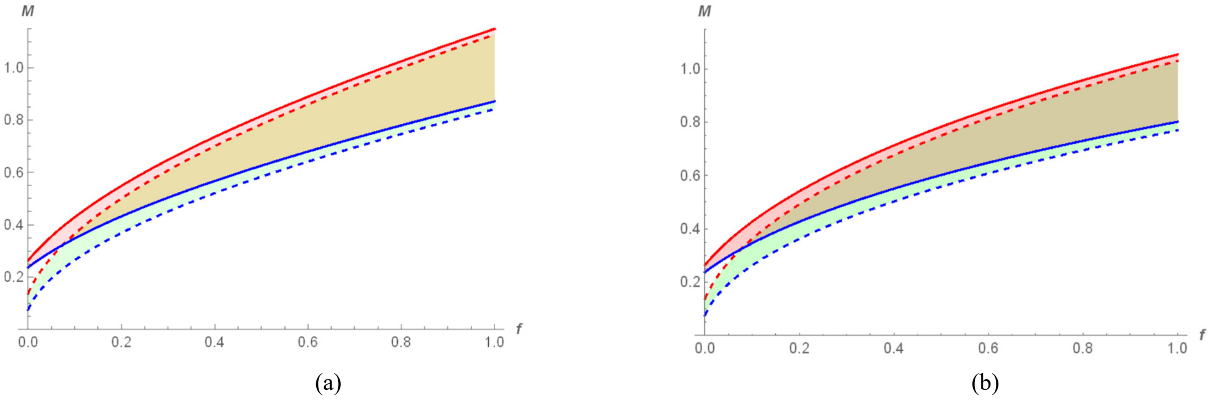


Figure 1. The range of Mach number for existence of dust acoustic waves against dust-ion density ratio f for $P_{\parallel} = 0.1$ (solid curves) and 0.01(dashed curves). The subfigure (a) for negative dust and (b) for positive dust

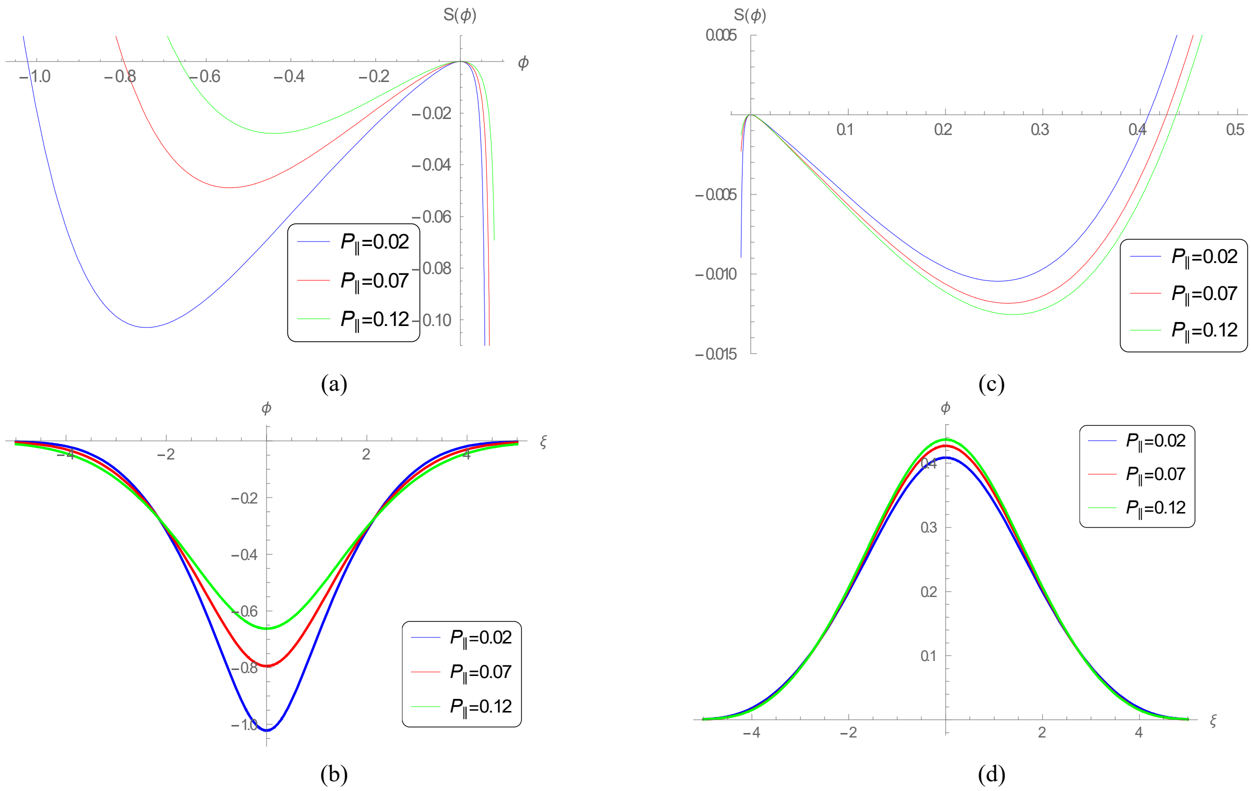


Figure 2. In panels: (a) Variation of Sagdeev potential $S(\phi)$ against ϕ and (b) the resulting soliton profile ϕ are depicted for different $P_{\parallel} = 0.02, 0.07, 0.12$ for negative dust, [Here $M = 0.98, f = 0.75, k_z = 0.6, \alpha = 0.1, \beta_i = 0.3$, $\beta_e = 0.3$ and $P_{\perp} = 0.02$]. Similarly, in panels (c) and (d) for positive dust [Here $M = 0.77, f = 0.5, k_z = 0.7, \alpha = 0.1, \beta_i = 0.3$, $\beta_e = 0.3$ and $P_{\perp} = 0.02$.]

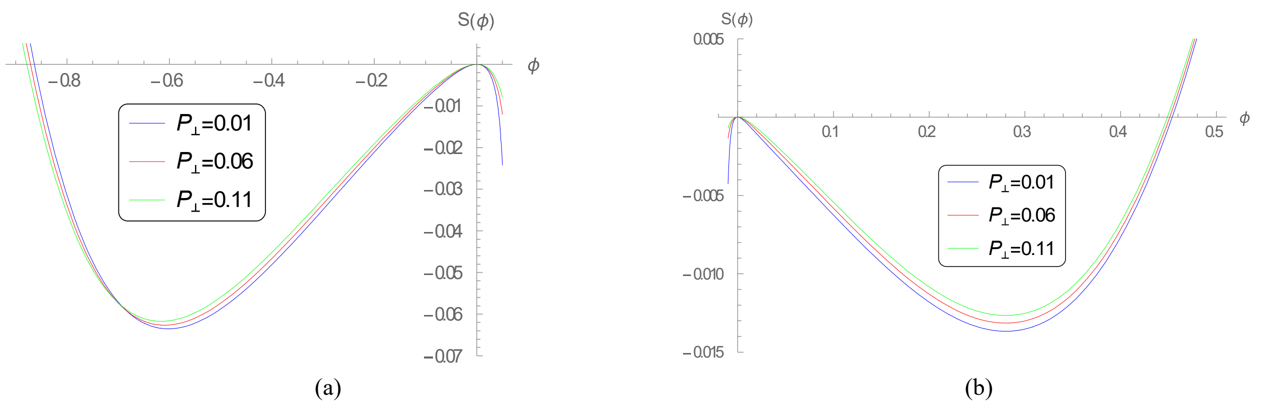
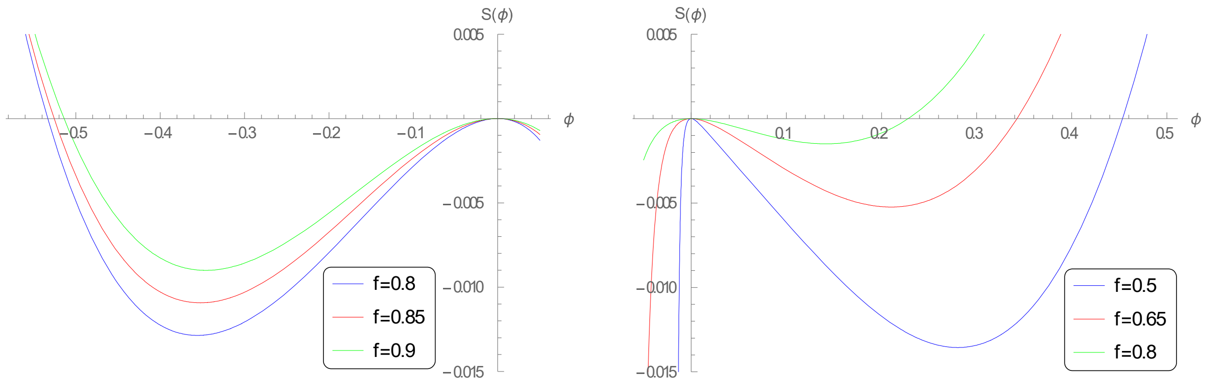


Figure 3. Variation of Sagdeev potential $S(\phi)$ against ϕ for different $P_{\perp} = 0.01, 0.06, 0.11$ for (a) $M = 0.98, f = 0.75, k_z = 0.6, \alpha = 0.1, \mathrm{j} = 1$ (negative dust), $\beta_i = 0.3$, $\beta_e = 0.3$ and $P_{\parallel} = 0.05$. (b) $M = 0.78, f = 0.5, k_z = 0.7, \alpha = 0.1, \mathrm{j} = -1$(positive dust), $\beta_i = 0.3$, $\beta_e = 0.3$ and $P_{\perp} = 0.1$

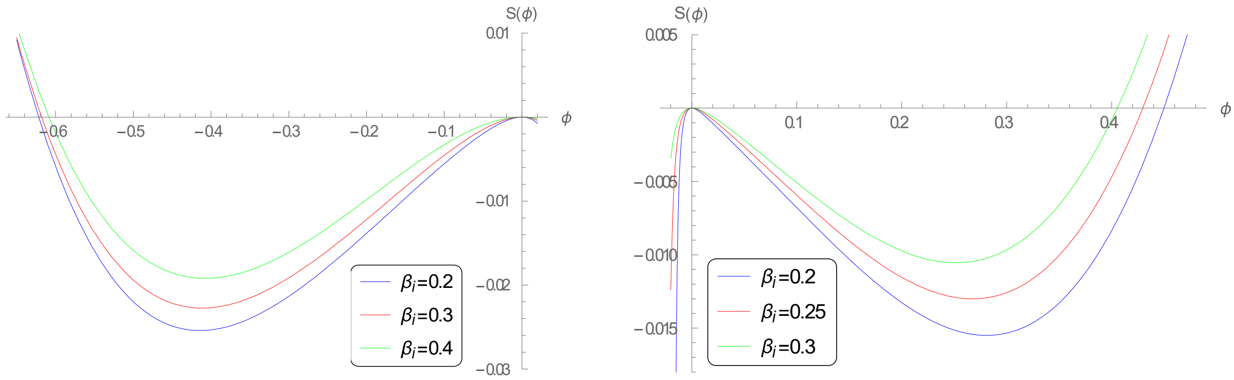
Figure 4(a) and (b) displays variation of Sagdeev potential $S(\phi)$ against ϕ for different values of density ratio of dust to ions f. It can be seen that the increase of f leads to decrease in amplitude of solitary waves.

The effect of nonthermal parameter β_i of ions on the Sagdeev potential is shown in the Figure 5(a) and (b) by plotting Sagdeev potential $S(\phi)$ against ϕ for different values of β_i. As β_i increases the amplitude of the dust acoustic solitary waves decreases for both positive and negative dust.



(a) (b)

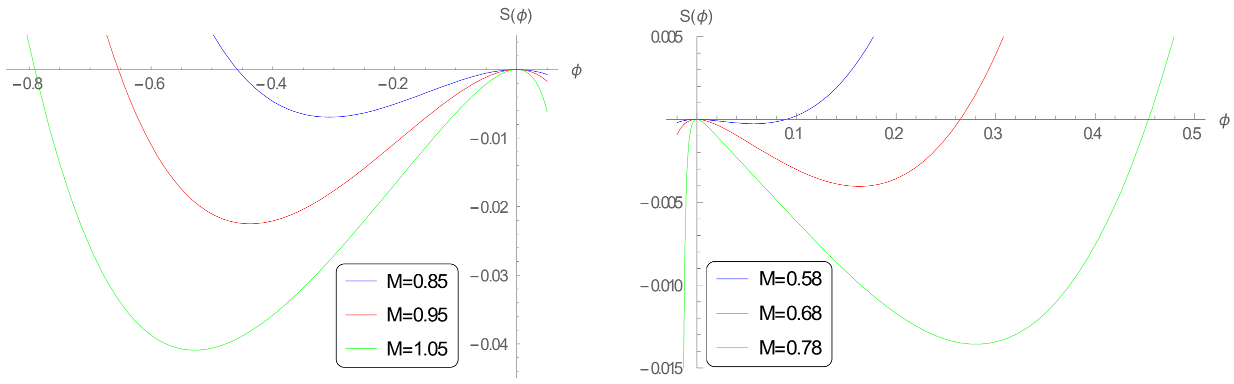
Figure 4. Variation of Sagdeev potential $S(\phi)$ against ϕ (a) for different $f = 0.8, 0.85, 0.9$ and $M = 0.85, k_z = 0.6, \alpha = 0.1, \mathrm{j} = 1$ (negative dust), $\beta_i = 0.3$, $\beta_e = 0.3$, $P_\perp = 0.01$ and $P_\parallel = 0.11$. (b) for different $f = 0.5, 0.65, 0.8$ and $M = 0.78, k_z = 0.7, \alpha = 0.1, \mathrm{j} = -1$(positive dust), $\beta_i = 0.3$, $\beta_e = 0.3$, $P_\perp = 0.02$ and $P_\parallel = 0.1$



(a) (b)

Figure 5. Variation of Sagdeev potential $S(\phi)$ against ϕ (a) for different $\beta_i = 0.2, 0.3, 0.4$ and $M = 0.93, k_z = 0.6, \alpha = 0.1, \mathrm{j} = 1$ (negative dust), $f = 0.75$, $\beta_e = 0.2$, $P_\perp = 0.02$ and $P_\parallel = 0.11$. (b) for different $\beta_i = 0.2, 0.25, 0.3$ and $M = 0.73, k_z = 0.68, \alpha = 0.1, \mathrm{j} = -1$(positive dust), $f = 0.5$, $\beta_e = 0.3$, $P_\perp = 0.02$ and $P_\parallel = 0.1$

Mach number is related to the speed of dust acoustic waves. Figure 6(a) and (b) show the variation of the Sagdeev potential $S(\phi)$ against ϕ for different values Mach number M for negative and positive dust respectively. From these figures it is shown that, for both positive and negative dust, increase of Mach number leads to increase in the amplitude of solitary waves.



(a) (b)

Figure 6. Variation of Sagdeev potential $S(\phi)$ against ϕ (a) for different $M = 0.85, 0.95, 1.05$ and $f = 0.85, k_z = 0.6, \alpha = 0.1, \mathrm{j} = 1$ (negative dust), $\beta_i = 0.3$, $\beta_e = 0.3$, $P_\perp = 0.01$ and $P_\parallel = 0.11$. (b) for different $M = 0.58, 0.68, 0.78$ and $f = 0.5, k_z = 0.7, \alpha = 0.1, \mathrm{j} = -1$(positive dust),$\beta_i = 0.3$, $\beta_e = 0.3$, $P_\perp = 0.02$ and $P_\parallel = 0.1$

In Figure 7(a) and (b) we have plotted the Sagdeev potential $S(\phi)$ against ϕ for different value of k_z for positive and negative dust respectively. We can see as k_z increases i.e. as direction of solitary waves approaches the direction of magnetic field, the amplitude of the solitary waves gradually decreases.

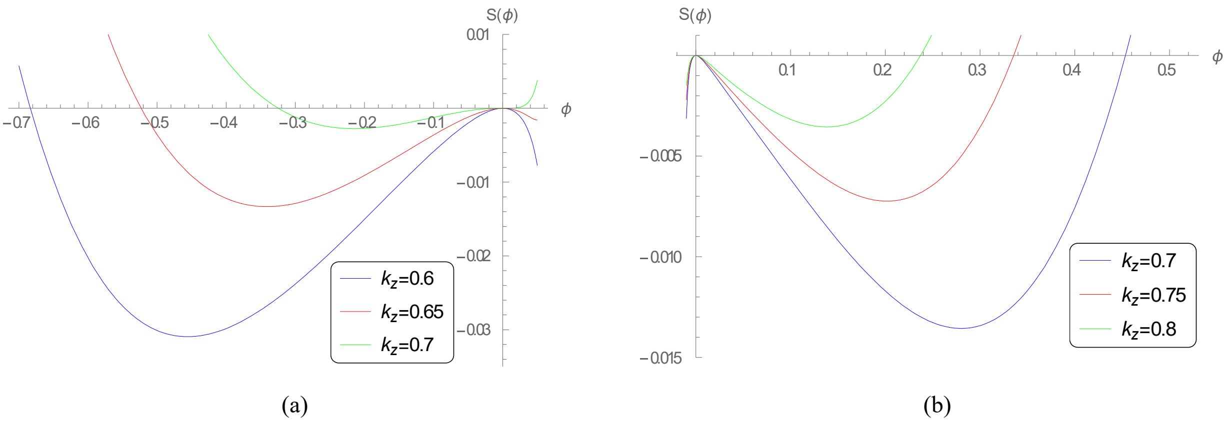


Figure 7. Variation of Sagdeev potential $S(\phi)$ against ϕ (a) for different , $k_z = 0.6, 0.65, 0.7$ and $f = 0.75, M = 0.98, \alpha = 0.1$, j = 1 (negative dust), $\beta_i = 0.3$, $\beta_e = 0.3$, $P_\perp = 0.01$ and $P_\parallel = 0.11$. (b) for different $k_z = 0.7, 0.75, 0.8$ and $f = 0.5, M = 0.78, \alpha = 0.1$, j $= -1$(positive dust),$\beta_i = 0.3$, $\beta_e = 0.3$, $P_\perp = 0.02$ and $P_\parallel = 0.1$

6. CONCLUSION

In this investigation, we have studied the effect of anisotropic dust pressure on the formation of dust acoustic solitary waves in a three-component magnetized plasma, consisting of dust fluid and nonthermal ions and electrons.

Although, Bashir et al. [32] studied the dusty plasma with anisotropic dust pressure as in this work but they used Reductive Perturbation Method. In our work we have used Sagdeev Potential Method which is more comprehensive method that accounts all nonlinearities and give more accurate results. Using Sagdeev pseudo-potential method, an energy integral has been derived for dust acoustic solitons. Bashir et al. [32] got very small amplitude solitons but we got solitons with larger amplitude. Also, we have calculated the range of Mach number for existence of solitary waves.

It is observed that, various parameters viz. parallel and perpendicular dust pressure, non-thermality of ions etc. significantly modify the arbitrary amplitude dust acoustic solitons. Some important findings are summarized as:

a) As parallel dust pressure $P_\parallel$ increases, for negative(positive) dust, the amplitude of the rarefactive(compressive) solitons decrease (increases).
b) The increase of perpendicular dust pressure $P_\perp$ increases(decreases) the amplitude of solitons very little for negative(positive) dust.
c) The increase of density ratio $f = \frac{z_d n_{d0}}{n_{i0}}$ leads to decrease in amplitude of solitary waves.
d) As nonthermal parameter β_i incrases the amplitude of the dust acoustic solitary waves decreases for both positive and negative dust.

Clusters of sub-micron and micron-sized dust particles within Earth's magnetosphere are detected by various satellites. Electrostatic solitary waves are also observed in various space and astrophysical plasma possessing the pressure anisotropy arising due to strong magnetic field [30, 39-40]. Theoretically, it is seen that in earth's magneto-tail region, electrostatic solitary waves exit [41-43], where the pressure anisotropy arises due to earth's magnetic field. Despite the theoretical plausibility and indirect evidence, the nature of dust particle swarms and associated DAWs in the magnetosheath remains an open question. The result of the present investigation may be effective to explore the various aspects of nonlinear structures of in those regions, where non-thermal electrons with pressure anisotropy can exist.

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ВПЛИВ АНІЗОТРОПНОГО ТИСКУ ПИЛУ НА ФОРМУВАННЯ ТА ПОШИРЕННЯ ПИЛОВИХ АКУСТИЧНИХ ОДИНОКИХ ХВИЛЬ (DASW) ДОВІЛЬНОЇ АМПЛІТУДИ В НАМАГНІЧЕНІЙ ПИЛОВО-ІОННО-ЕЛЕКТРОННІЙ ПЛАЗМІ

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Досліджено пилові акустичні одиночні хвилі (DASW) довільної амплітуди в запиленій магнітоплазмі з анізотропним тиском пилу та нетепловим розподілом іонів та електронів. Метод псевдопотенціалу Сагдеєва використовується для виведення рівняння балансу енергії та аналізу різних властивостей пилових акустичних солітонів. Чисельно досліджуються анізотропний тиск пилу, коефіцієнт щільності пилу, нетеплові ефекти тощо при поширенні DASW. Встановлено, що розріджені солітони можуть існувати для негативно зарядженого пилу, а стискаючі солітони можуть існувати для позитивно зарядженого пилу. Це дослідження може бути корисним для розуміння DASW у різних астрофізичних середовищах.

Ключові слова: *пилова акустична хвиля; пилова плазма; анізотропний тиск; псевдопотенціал Сагдеєва; поодинокі хвилі; намагнічена плазма; нетеплові електрони та іони*