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FLRW MODEL COUPLED WITH MASSLESS SCALAR FIELD IN $f(T)$ GRAVITY – TWO FLUID SCENARIO

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In this work, we investigate a cosmological model within the framework of modified teleparallel gravity, known as *f*(*T*) gravity, by considering a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) Universe filled with two fluids-barotropic matter and a dark fluid-alongside a massless scalar field. We study an interacting case of the fluids, deriving exact solutions of the field equations under a time-dependent deceleration parameter scenario. The model demonstrates a viable cosmological sequence: early decelerating expansion followed by late-time acceleration. The torsion scalar *T*, its function *f*(*T*), and the scalar field ϕ all evolve dynamically, transitioning from dominant roles in the early Universe to diminished effects at late times. The dark fluid energy density remains nearly constant, supporting accelerated expansion, while the matter density decreases with cosmic time. The effective equation of state (EoS) parameter evolves from a matter-like behavior to negative values, suggesting a natural transition from matter domination to a dark energy-dominated phase. These results affirm that *f*(*T*) gravity coupled with a scalar field can explain cosmic acceleration and provide an alternative to the standard ΛCDM model without invoking exotic energy components.
Keywords: *Two fluids; f(T) gravity; Linear deceleration parameter; Cosmology*
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1. INTRODUCTION

Observations have consistently demonstrated the Universe's accelerating expansion in its later stages [1, 2], with supernova type Ia (SN Ia) data providing strong confirmation [3, 4]. Analyses of the cosmic microwave background (CMB) and large-scale structure (LSS) point towards this accelerated expansion being driven by a component with negative pressure, termed dark energy (DE), within a spatially flat Universe. The behavior of DE is typically described using the equation of state (EoS) parameter, defined as the ratio of pressure (p) to energy density (ρ). Current cosmological data, particularly from SN Ia, suggest that the EoS parameter (w) is close to -1. Values precisely at -1, slightly above, or slightly below correspond to standard cosmology, quintessence, or phantom regions, respectively, while values significantly greater than -1 are disfavoured. Numerous researchers have explored DE models in diverse contexts [5-9]. The exploration of two-fluid cosmological models has gained significant traction in contemporary cosmology, offering a framework to understand the complex interplay of cosmic constituents. These models typically consider the Universe as a composition of distinct fluids, such as baryonic matter, dark matter, dark energy, or radiation, and examine their interactions and evolution. Recent research has delved into the dynamics of these models within modified gravity theories, to account for the observed accelerated expansion of the Universe. For example, studies have investigated scenarios where one fluid represents ordinary matter and the other models dark energy, or where radiation is considered alongside matter, allowing for a more nuanced understanding of cosmic evolution. Investigations into interacting and non-interacting scenarios between DE and barotropic fluids have also been conducted, with examples including Chen and Wang's study [10] on interacting viscous DE in Einstein cosmology using the Friedmann-Robertson-Walker (FRW) Universe, Avellino's work [11] on DE interactions with bulk viscosity in a flat FRW Universe, and Saha's exploration [12] of two-fluid DE models within FRW cosmology. Further research by Amirhashchi *et al*. [13, 14] has examined interacting two-fluid viscous DE in non-flat and anisotropic Universes. These investigations aim to reconcile theoretical predictions with observational data from sources like the Cosmic Microwave Background and supernovae, providing the fundamental nature of the Universe.

Researchers have investigated various $f(T)$ functions to better fit observational data, with particular attention to the late-time accelerated expansion of the Universe. Some studies have probed the interaction between dark energy and dark matter within the $f(T)$ framework, seeking to resolve tensions in the standard cosmological model. Additionally, there's been increased interest in analyzing the stability and viability of $f(T)$ models through phase-space analysis and perturbation studies. These investigations aim to constrain the free parameters of $f(T)$ theories and to distinguish them from other modified gravity approaches. For instance, studies have explored the viability of specific $f(T)$ models using data from the Pantheon+ Supernovae dataset and CMB observations, while others have focused on the theoretical implications of $f(T)$ gravity for early-Universe cosmology and the resolution of cosmological singularities.

However, the late-time accelerated expansion, supported by substantial observational evidence, presents a challenge to the framework of General Relativity (GR). An alternative approach to explain this phenomenon involves

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modifications to GR, tracing back to Einstein's 1928 attempt to unify gravity and electromagnetism via the introduction of a tetrad (vierbein) field and the concept of absolute parallelism, also known as Teleparallel Gravity (TG) or $f(T)$ gravity [15]. In $f(T)$ gravity, the gravitational field equations are formulated using torsion rather than curvature [16], offering the advantage of second-order field equations. Various aspects of $f(T)$ theory have been studied [17-20]. For instance, Jamil *et al*. [21, 22] investigated the dark matter (DM) problem within $f(T)$ gravity, successfully modelling flat rotation curves of galaxies and deriving DM density profiles. Additionally, they explored interacting DE models within this framework for a specific choice of $f(T)$. Setare and Darabi [23] examined power-law solutions in the phantom phase, identifying exit solutions for certain $f(T)$ scenarios. Setare and Houndjo [24] investigated particle creation in a flat FRW Universe within $f(T)$ gravity, and Chirde and Shekh [25] studied barotropic bulk viscous cosmological models in $f(T)$ gravity.

The investigation of interacting fields, particularly those involving zero-mass scalar fields, is pivotal for addressing the unification of gravitational and quantum theories. Recent decades have seen a resurgence of interest in gravitational theories incorporating zero-mass scalar fields. Maniharsingh [26], Singh and Bhamra [27], and Singh [28, 29] have explored various one-fluid models coupled with scalar fields. Singh and Deo [30] examined zero-mass scalar field interactions in the presence of a gravitational field for FRW spacetime within GR, demonstrating that the 'Big-Bang' singularity might be avoided through the introduction of such fields. Further research by authors [31, 32] has investigated cosmological models involving zero-mass scalar fields. Building upon the preceding analysis, this paper explores a two-fluid cosmological model within $f(T)$ gravity, incorporating a scalar field to uncover the underlying dynamics of this Universe, with a particular focus on the interacting scenario.

2. A BRIEF REVIEW OF $f(T)$ COSMOLOGY

The line element of the Riemannian manifold is given by

$$dS^2 = g_{\mu\nu}dx^\mu dx^\nu. \tag{1}$$

This line element can be converted to the Minkowskian description of the transformation called tetrad, as follows

$$dS^2 = g_{\mu\nu}dx^\mu dx^\nu = \eta_{ij}\theta^i\theta^j, \tag{2}$$

$$dx^\mu = e_i^\mu\theta^i, \;\; \theta^i = e_\mu^i dx^\mu, \tag{3}$$

where $\eta_{ij} = diag[\,-1,\ 1,\ 1,\ 1\,]$ and $e_i^\mu e_\mu^i = \delta_\nu^\mu$ or $e_i^\mu e_\mu^j = \delta_i^j$.

The root of metric determinant is given by $\sqrt{-g} = det[\,e_\mu^i] = e$. For a manifold in which the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the non-zero torsion terms exist, the Weitzenbocks connection components are defined as

$$\Gamma_{\mu\nu}^\alpha = e_i^\alpha\partial_\nu e_\mu^i = -e_\mu^i\partial_\nu e_i^\alpha. \tag{4}$$

Through the connection, we can define the components of the torsion and contorsion tensors as

$$T_{\mu\nu}^\alpha = \Gamma_{\mu\nu}^\alpha - \Gamma_{\mu\nu}^\alpha = e_i^\alpha\left(\partial_\mu e_\nu^i - \partial_\mu e_\mu^i\right), \tag{5}$$

$$K^{\mu\nu}{}_\alpha = \left(-\tfrac{1}{2}\right)\left(T^{\mu\nu}{}_\alpha - T^{\nu\mu}{}_\alpha - T_\alpha{}^{\mu\nu}\right). \tag{6}$$

For facilitating the description of the Lagrangian and the equations of motion, we can define another tensor $S_\alpha^{\mu\nu}$ from the components of the torsion and contorsion tensors, as

$$S_\alpha{}^{\mu\nu} = \left(\tfrac{1}{2}\right)\left(K^{\mu\nu}{}_\alpha + \delta_\alpha^\mu T^{\beta\nu}{}_\beta - \delta_\alpha^\nu T^{\beta\mu}{}_\beta\right). \tag{7}$$

The torsion scalar T is

$$T = T^\alpha{}_{\mu\nu}S_\alpha{}^{\mu\nu}. \tag{8}$$

Now, we define the action by generalizing the Teleparallel Theory i.e. f(T) theory as [18]

$$S = \int [T + f(T) + L_{matter}]\; e\, d^4x. \tag{9}$$

Here, f(T) denotes an algebraic function of the torsion scalar T**.** Making the functional variation of the action (9) with respect to the tetrads, we get the following equations of motion

$$S_\mu^{\nu\rho}\partial_\rho T f_{TT} + \left[e^{-1}e_\mu^i\partial_\rho\left(ee_i^\alpha S_\alpha^{\nu\rho}\right) + T^\alpha{}_{\lambda\mu}S_\alpha{}^{\nu\lambda}\right](1 + f_T) + \tfrac{1}{4}\delta_\mu^\nu(T + f) = 16\pi T_\mu^\nu, \tag{10}$$

where T_μ^ν is the energy momentum tensor, $f_T = df(T)/dT$ and, by setting $f(T) = a_0 =$ constant this is dynamically equivalent to the General Relativity.

3. METRIC AND FIELD EQUATIONS

We consider the spatially homogeneous and isotropic flat Friedman- Robertson-Walker (FRW) line element in the form

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2], \tag{11}$$

where a be the metric potential or average scale factor and $d\Omega^2 = d\theta^2 + sin^2\theta\, d\varphi^2$.

The energy momentum tensor T_μ^ν for the fluid distribution with zero-mass scalar fields is taken as

$$T_\mu^\nu = (p+\rho)\, u^\nu u_\mu + p g_\mu^\nu + \phi_{,\mu}\phi^{,\nu} - \frac{1}{2} g_\mu^\nu \phi_{,\lambda}\phi^{,\lambda}, \tag{12}$$

together with co-moving co-ordinates

$$u^\nu = (0,0,0,1) \text{ and } u^\nu u_\nu = -1 \tag{13}$$

where u^ν is the four-velocity vector of the cosmic fluid, p and ρ are the isotropic pressure and energy density of the matter respectively and ϕ be the zero-mass scalar field.

The Friedman equation for two fluid scenarios with zero mass scalar fields can be written as

$$\frac{\dot a}{a}\dot T f_{TT} + \left\{\frac{\ddot a}{a} + 2\frac{\dot a^2}{a^2}\right\}(1+f_T) + \frac{1}{4}(T+f) = (16\pi)p + 2\dot\phi^2, \tag{14}$$

$$3(1+f_T)\frac{\dot a^2}{a^2} + \frac{1}{4}(T+f) = (16\pi)(-\rho) - 2\dot\phi^2, \tag{15}$$

$$\ddot\phi + 3\frac{\dot a}{a}\dot\phi = 0. \tag{16}$$

The overhead dot represents the differentiation with respect to time t.

Here we consider $p = (p_m + p_D)$, $\rho = (\rho_m + \rho_D)$,

where p_m and ρ_m are pressure and energy density of barotropic fluid, p_D and ρ_D are pressure and energy density of dark fluid respectively also $p_m = w_m\rho_m$ and $p_D = w_D\rho_D$.

We assume that EoS parameter of the perfect fluid to be a constant, which is already considered by Akarsu [33] and Kumar [34] i.e.

$$w_m = \frac{p_m}{\rho_m} = \text{constant}, \tag{17}$$

while w_D has been admitted to be a function of time.

Next, we extend the discussion for interacting case to study the changing aspects of physical behavior of the Universe, we first assume that the perfect fluid and DE components interact minimally. Therefore, the energy momentum tensors of the two sources may be conserved separately. In this case, the densities of DE and matter no longer satisfy independent conservation laws, they obey instead

$$(\dot\rho_m) + 3\frac{\dot a}{a}(\rho_m + p_m) = Q, \tag{18}$$

$$(\dot\rho_D) + 3\frac{\dot a}{a}(\rho_D + p_D) = -Q. \tag{19}$$

The quantity Q ($Q > 0$) expressed the interaction term between the DE barotropic matter components. Since we are interested to investigate the interaction between DE and matter, it should be noted that an ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction Q. In our work we consider the interaction term in the form $Q\ \alpha\ H\rho_m$ which is already considered in [12, 13]

$$Q = 3Hk\rho_m \tag{20}$$

where k is a coupling coefficient which can be considered as a constant or variable parameter of redshift.

4. SOLUTION OF THE FIELD EQUATIONS

In constructing a physically meaningful cosmological model, the Hubble parameter and the deceleration parameter are crucial measurable quantities. The current value of the Hubble parameter reveals the present rate at which the Universe is expanding, while the current value of the deceleration parameter indicates that the observable Universe's expansion is accelerating, rather than slowing down. This suggests that a viable model should incorporate a phase of decelerating expansion during the early matter-dominated era, allowing for the formation of large-scale structures, followed by the observed late-time acceleration. Solutions to Einstein's field equations, derived using a law governing the Hubble parameter's variation that results in a constant deceleration parameter, have been extensively investigated. However, other researchers have proposed forms of the Hubble parameter as a function of the scale factor. Observations of Type Ia supernovae and anisotropies in the cosmic microwave background have indicated a transition from a decelerating expansion in the past to the current accelerated expansion. Consequently, for a Universe exhibiting this transition, the deceleration parameter must undergo a sign change, which is a key reason for considering a time-varying deceleration parameter. Hence, in the present study, we consider the deceleration parameter to be a suitable linear function of Hubble's parameter as

$$q = -1 + \beta H, \tag{21}$$

which yields a scale factor of the form $a = Exp\left[\left(\frac{\sqrt{2\beta t + k}}{\beta}\right)\right]$, where β and k are the constants. From the above equations the Hubble's parameter H is observed as $H = \frac{1}{\sqrt{1+2t\beta}}$.

5. PHYSICAL PARAMETERS

Here in this section, we find some other parameter of dark fluid/energy in interacting two fluid models such as: The Torsion scalar of the model is observed as

$$T = 6H^2. \tag{22}$$

The Figure 1, illustrates the behavior of the Torsion scalar in $f(T)$ gravity with $\beta = 0.5$, showing an decreasing trend over time towards a constant value.

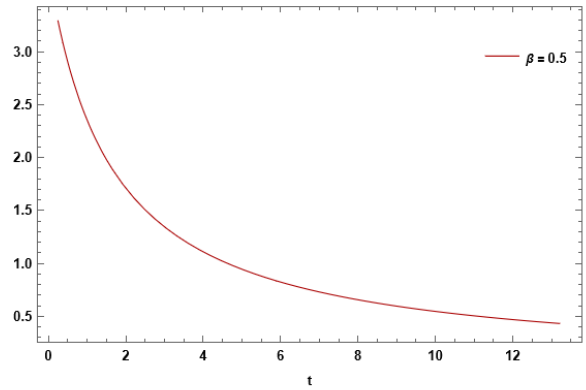

Figure 1. The behavior of Torsion scalar (T) versus time (t)

This suggests an evolution from a torsion-dominated early Universe to a later phase where torsion's influence diminishes, potentially leading to a de Sitter-like expansion. The asymptotic behavior indicates that the Torsion scalar may settle into a constant, possibly related to the late-time acceleration of the Universe, offering an alternative cosmological scenario to standard General Relativity.

The function of torsion scalar is observed as

$$f(T) = (6H^2)^n. \tag{23}$$

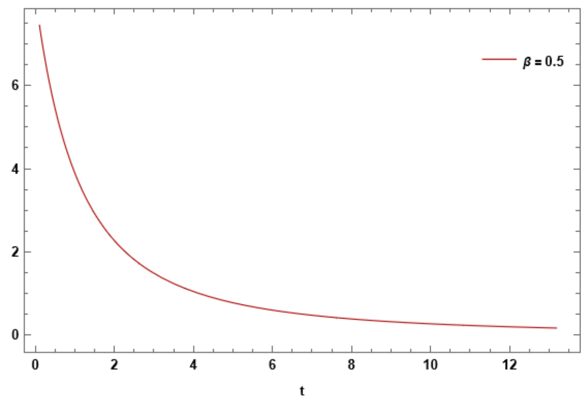

Figure 2. The behavior of function of torsion scalar (f(T)) versus time (t)

Figure 2 shows the evolution of the function $f(T)$, which modifies the teleparallel equivalent of general relativity. Unlike standard teleparallel gravity, which uses the linear form $f(T) = T^n$, modified gravity theories consider nonlinear functions to explain cosmic acceleration without a cosmological constant. From the plot, $f(T)$ shows a decreasing trend over time, echoing the behaviour of T. This indicates that the deviation from standard teleparallel gravity is most pronounced in the early Universe. As time progresses, $f(T)$ tends to a less dominant or nearly linear regime, suggesting that the Universe approaches a state similar to standard GR or TEGR (teleparallel equivalent of general relativity). This behaviour has significant implications. It supports the notion that modified gravity effects are necessary to explain early cosmic acceleration or inflation but may not be required or may even diminish in the late Universe. The evolution of $f(T)$ thus mirrors the transition from an early Universe dominated by torsion-induced acceleration to a dark energy-dominated cosmos at late times.

The scalar field ϕ is obtained as

$$\dot{\phi} = \phi_0 a^{-3}, \tag{24}$$

where ϕ_0 be the constant of integration.

The Figure 3, depicts the behavior of a zero-mass scalar field in $f(T)$ gravity with $\beta = 0.5$, showing a rapid decay from a significant initial value towards zero as time progresses. This indicates a transient influence of the scalar field, prominent in the early Universe but becoming negligible in the late-time evolution. The field's diminishing effect suggests it plays a crucial role in the initial cosmological dynamics, potentially representing additional degrees of freedom or dynamical fields that quickly lose relevance as the Universe expands, ultimately leaving the late-time evolution dominated by other factors within the $f(T)$ framework.

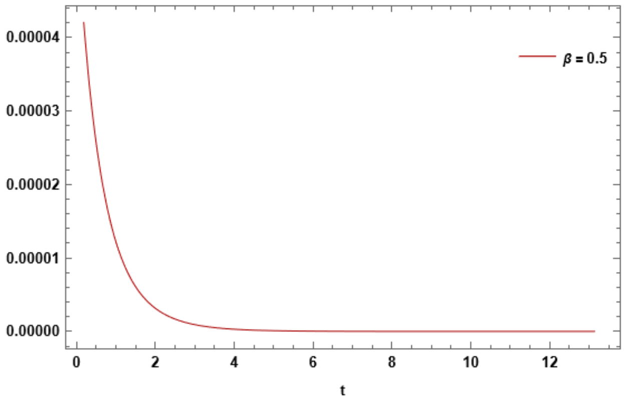

Figure 3. The behavior of scalar function (ϕ) versus time (t)

We obtain an energy density of barotropic fluid for interacting case as

$$\rho_m = \rho_0 (a)^{-3(1+w_m-k)}, \tag{25}$$

where ρ_0 be the constant of integration.

Matter density (ρ_m) represents the contribution of ordinary and dark matter to the total energy content of the Universe. As expected from the standard cosmological principle and shown in Figure 4, ρ_m decreases over time.

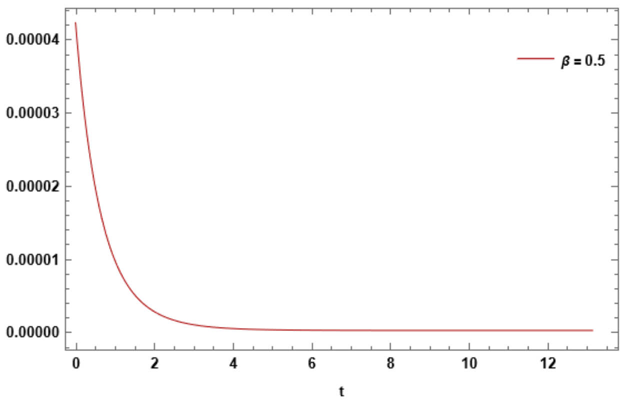

Figure 4. The behavior of matter density (ρ_m) versus time (t)

This decrease is due to the expansion of the Universe: as the volume increases, the matter density dilutes as (ρ_m varies with a^{-3}), where a is the scale factor. This trend is consistent with observations and reflects the diminishing dominance of matter as the Universe evolves from a matter-dominated era to a dark energy-dominated era. In a two-fluid scenario, this behavior is contrasted with that of the dark fluid. The declining ρ_m supports the transition required for cosmic acceleration, wherein the dark fluid's energy density must evolve differently either remaining constant (as with a cosmological constant) or decaying more slowly. This contrast underpins the need for modified gravity or dark energy components in cosmological models.

The Energy density is observed as,

$$\rho_D = \frac{18H^2+6^n\left(H^2\right)^n(1+2n)+4(4\pi\rho_m+\dot{\phi}^2)}{64\pi} \tag{26}$$

The dark fluid energy density ρ_D, shown in Figure 5, is crucial in explaining late-time acceleration. Unlike ρ_m, ρ_D exhibits a nearly constant or slowly varying behavior over time, which is a hallmark of dark energy.

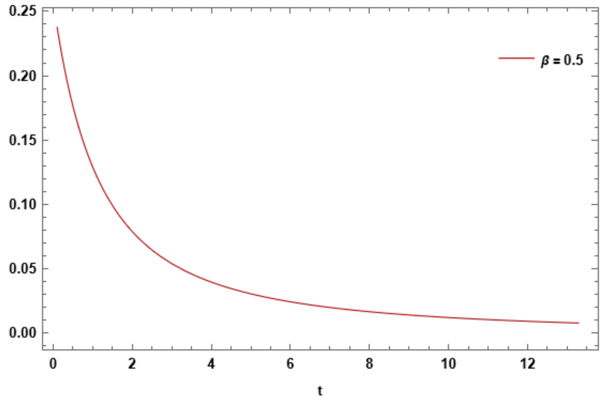

Figure 5. The behavior of energy density (ρ_D) versus time (t)

This behavior mimics that of a cosmological constant, which has a constant energy density and negative pressure, driving accelerated expansion. In modified gravity models like $f(T)$, the dark fluid can arise from geometric corrections rather than invoking a separate energy component. Thus, ρ_D relative constancy suggests that the model effectively produces an acceleration-driving component that matches observations, such as those from supernovae, the CMB, and large-scale structure surveys.

Such a dark fluid may correspond to an effective field emerging from the scalar sector or from the torsional modifications. Its energy density becomes increasingly dominant as ρ_m fades, leading to the observed late-time acceleration.

Isotropic pressure,

$$p_D = -\frac{30H^4+6^n\left(H^2\right)^n(2\dot{H}(-1+2n)n)+H^2(3+2n))+12H^2(\dot{H}-16\pi p_m-2\dot{\phi}^2)}{192H^2\pi}. \tag{27}$$

The isotropic pressure p_D of the dark fluid plays a vital role in dictating the dynamics of cosmic acceleration in modified gravity models like $f(T)$ gravity. As observed in Figure 6, p_D exhibits an interesting behavior: it starts off as negative and then gradually transitions into positive values as the Universe evolves. This transition holds significant physical implications.

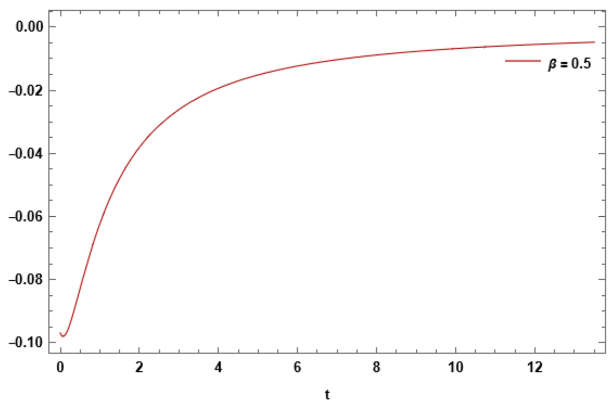

Figure 6. The behavior of isotropic pressure (p) versus time (t)

Initially, the negative and then positive pressure corresponds to initially dark energy and then a phase dominated by ordinary (baryonic) matter, where gravitational attraction governs the dynamics. During this epoch, the Universe experiences decelerated expansion, which aligns with our understanding of the matter-dominated era following radiation domination. As the Universe expands further, the torsional corrections and the influence of the scalar field become more prominent. This is reflected in the transition of p_D from negative to positive values. The emergence of negative pressure marks a critical shift; this is the regime where dark fluid effects take over, generating a repulsive gravitational component that drives accelerated expansion. Negative pressure is essential for this behavior, as it counters the gravitational pull of matter, a phenomenon supported by Type Ia supernovae observations and large-scale structure data initially.

In the framework of $f(T)$ gravity, the isotropic nature of this pressure is also consistent with the Cosmological Principle, ensuring uniformity in all directions of space, a foundational requirement for the FLRW metric. Overall, the behavior of p_D depicted in Figure 6 effectively demonstrates the Universe's transition from a dark energy-dominated phase to a matter-dominated phase, underlining the success of the $f(T)$ model in capturing both acceleration and deceleration within a unified framework.

EoS Parameter,

$$w_D = -\frac{(30H^4+6^nH^{2n}(2\dot{H}n(-1+2n)+H^2(3+2n))+12H^2(\dot{H}-16\pi\text{pm}-\dot{\phi}^2))}{3H^2(18H^2+6^nH^{2n}(1+2n)+64\pi\rho_m+4\dot{\phi}^2)} \tag{28}$$

The equation of state (EoS) parameter is a fundamental cosmological quantity defined by the ratio of pressure to energy density of a fluid component in the Universe. This parameter critically determines the dynamical behavior of the Universe, including whether it is accelerating, decelerating, or undergoing exotic evolution.

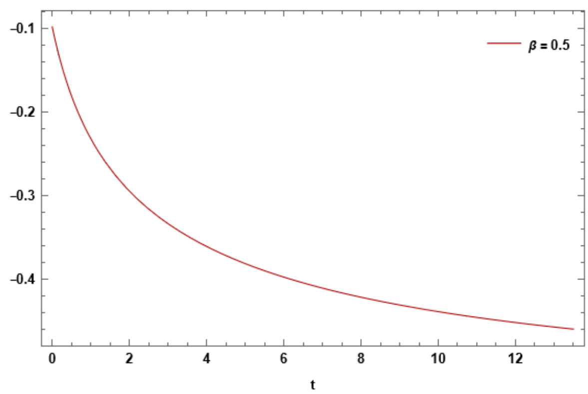

Figure 7. The behavior of equation of state parameter (w) versus time (t)

In Figure 7, w exhibits a clear evolution with time, indicating a transition in the nature of the dominant cosmic component. Initially, w starts in the negative regime, suggesting a dark energy like behaviour. Over time, it drops and present into the negative region, reflecting a shift towards dark energy dominance. Such behavior is highly relevant and consistent with modern cosmological observations. As, the range of w is:

$\boldsymbol{w = 0}$ – Dust-like matter (pressureless). This regime corresponds to non-relativistic matter such as baryons and cold dark matter, where pressure is negligible. It signifies the matter-dominated era of the Universe.

$\boldsymbol{w = 1/3}$ – Radiation. This applies to ultra-relativistic particles (photons, neutrinos). It dominates the early Universe before matter becomes significant.

$\boldsymbol{w > -1/3}$ – No accelerated expansion. The deceleration parameter q is positive in this regime, meaning gravitational attraction dominates and expansion slows down.

$\boldsymbol{-1 < w < -1/3}$ – **Quintessence**-like behavior. This is the sweet spot for dark energy driving **accelerated expansion**. It arises from scalar field models with canonical kinetic terms. The Universe transitions to accelerated expansion in this range.

$\boldsymbol{w = -1}$ – **Cosmological Constant (Λ)**. This case corresponds to a constant vacuum energy density with negative pressure, the simplest form of dark energy in the ΛCDM model. It leads to a de Sitter-like exponential expansion.

$\boldsymbol{w < -1}$ – **Phantom Energy**. This regime predicts **super-acceleration**, where the expansion rate increases without bound. Such behavior can lead to a future cosmic singularity known as the "Big Rip", where all bound structures are torn apart.

$\boldsymbol{w > 1}$ – **Stiff fluid**, less common but possible in early Universe models.

In your model, as seen in Figure 7, w starts near zero and gradually becomes negative. This evolution shows the model effectively **mimics the cosmological sequence** from matter domination to dark energy domination.

- The **early phase**, where w is close to zero or slightly positive, supports a **matter-dominated decelerating expansion**, essential for structure formation (galaxies, clusters, etc.).
- The **later evolution**, where w drops below $-1/3$, indicates the onset of **accelerated expansion**.
- If w continues to evolve past -1, entering the **phantom regime**, the model can describe **exotic cosmic fates**, such as the Big Rip scenario.

Such dynamical behavior is advantageous over constant –w models like ΛCDM, which assume a static $w = -1$. Dynamical dark energy allows for better fits to datasets and can explain tensions like the Hubble tension.

6. CONCLUSION

In this study, we explored a two-fluid cosmological model within the framework of $f(T)$ gravity, considering a massless scalar field and the flat FLRW metric. We analyzed an interacting between barotropic and dark fluids, aiming to understand the dynamic evolution of the Universe and its current acceleration. The model assumes a time-varying deceleration parameter, which leads to a scale factor consistent with observational data showing a transition from decelerated to accelerated expansion. Our solution confirms that the torsion scalar T starts with a large negative value and gradually approaches a constant value over time, implying a transition from a torsion-dominated early Universe to a more stable late-time phase resembling de Sitter expansion.

The scalar field ϕ demonstrates a rapid decay with time, indicating its significant role in early Universe dynamics but a negligible contribution at late times. This aligns well with the concept that scalar fields may drive early inflationary or transitional epochs before dark energy effects dominate.

The behavior of the function $f(T)$ indicates significant deviations from standard teleparallel gravity in the early Universe, with these deviations diminishing over time. This evolution is crucial in explaining early acceleration and smoothly transitioning into a standard-like regime consistent with late-time observations.

From the matter sector, the barotropic fluid's energy density consistently decreases with time, in line with cosmic expansion, and the energy density of the dark fluid remains nearly constant. This nearly constant behavior of energy density of the dark fluid effectively plays the role of a dark energy component driving the accelerated expansion of the Universe.

The isotropic pressure of the dark fluid exhibits a critical transition: it starts negative as dark energy dominates and turns positively in the matter-dominated phase, mirroring the dynamics required for cosmic acceleration. This pressure evolution provides clear evidence of a shift from gravitational attraction to repulsion as expansion accelerates.

A particularly notable result is the dynamic evolution of the equation of state parameter w. Initially near zero, indicating dark energy-like behaviour, w progressively drops into negative values. This transition supports the idea that the Universe has governed by dark energy-like behavior. Should w evolve further into the phantom region ($w < -1$), it could even allow the model to describe exotic scenarios such as the "Big Rip".

Overall, this model successfully captures the essential features of cosmic history from matter domination and structure formation to present-day accelerated expansion within the unified framework of $f(T)$ gravity and scalar field dynamics. It confirms the viability of modified torsional gravity as a compelling alternative to the ΛCDM model, capable of addressing cosmic acceleration without resorting to a cosmological constant or exotic matter fields.

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МОДЕЛЬ FLRW, ПОВ'ЯЗАНА З БЕЗМАСОВИМ СКАЛЯРНИМ ПОЛЕМ В ГРАВІТАЦІЇ *f*(*T*) – СЦЕНАРІЙ ДВОХ РІДИН

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У цій роботі ми досліджуємо космологічну модель в рамках модифікованої телепаралельної гравітації, відомої як гравітація *f*(*T*), розглядаючи просторово плоский Всесвіт Фрідмана-Леметра-Робертсона-Вокера (FLRW), заповнений двома рідинами - баротропною матерією та темною рідиною - поряд з безмасовим скалярним полем. Ми вивчаємо взаємодіючий випадок рідин, отримуючи точні розв'язки рівнянь поля за сценарієм параметра уповільнення, що залежить від часу. Модель демонструє життєздатну космологічну послідовність: раннє уповільнення розширення, за яким слідує прискорення наприкінці часу. Торсійний скаляр T, його функція f(T) та скалярне поле ϕ динамічно еволюціонують, переходячи від домінантних ролей у ранньому Всесвіті до зменшених ефектів наприкінці часу. Густина енергії темної рідини залишається майже постійною, підтримуючи прискорене розширення, тоді як густина матерії зменшується з космічним часом. Параметр ефективного рівняння стану (EoS) еволюціонує від поведінки, подібної до матерії, до від'ємних значень, що свідчить про природний перехід від домінування матерії до фази, де домінує темна енергія. Ці результати підтверджують, що гравітація *f*(*T*) у поєднанні зі скалярним полем може пояснити космічне прискорення та запропонувати альтернативу стандартній моделі ΛCDM без використання екзотичних компонентів енергії.

Ключові слова: *дві рідини; гравітація f(T); параметр лінійного уповільнення; космологія*