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EXPLORING PLANE SYMMETRIC SPACE-TIME IN *f*(R) MODIFIED GRAVITATIONAL THEORY

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This paper investigates a plane symmetric cosmological model (PSCM) in the context of modified *f*(R) gravity theory, incorporating both vacuum and non-vacuum scenarios. A perfect fluid is assumed as the matter source. To obtain the solutions, we consider the premise of both constant and nonconstant scalar curvature. By applying the conservation law for Einstein's field equation, $T^{ij}_{;j} = 0$, and the power-law assumption, we retrieve some well-known solutions. We solved the field equations by making a specific assumption that involved a transformation $A^2B = \mathcal{U}$. This study explores the physical and kinematic characteristics of specific cosmological models, along with an examination of the statefinder diagnostic—a key tool for analysing the Universe's evolutionary trajectory. The work provides important insights into the behaviour of anisotropic models within the context of modified *f*(R) gravity. It highlights the interplay between matter distribution and spacetime geometry, particularly emphasizing how assuming constant and nonconstant scalar curvature aids in simplifying and solving the corresponding field equations. The resulting solutions enhance our understanding of cosmic evolution governed by modified *f*(*R*) gravity.
Keywords: *Plane symmetric; f(R) gravitation theory; Perfect fluid; Statefinder; Anisotropic Universe*
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1. INTRODUCTION

The groundbreaking theory of modern physics called General Theory of Relativity (GR) brings numerous solutions to understand cosmic phenomena. The physical processes which GR explains are numerous yet there exist multiple unresolved astrophysical and cosmic problems outside its existing boundaries. One of the most significant challenges is the observed late-time acceleration of the Universe, which GR alone cannot adequately explain. To address such limitations, researchers have proposed modifications to the Einstein-Hilbert action—the foundational element of GR. Among these modifications, the *f*(R) theory of gravity stands out. This theory generalizes the Einstein-Hilbert action by introducing a function *f*(R), where R is the Ricci scalar representing spacetime curvature.

In the past decade, *f*(R) gravity has become a leading approach to modifying General Relativity (GR). This theory helps scientists find new solutions to describe how the Universe evolves. By adding a function of the Ricci scalar, *f*(R), to the gravitational action, the theory allows for more flexible models of gravity. These models can explain the observed cosmic acceleration without needing dark energy. *f*(R) gravity extends GR and offers new ideas for understanding the Universe. A key aim of the theory is to explain both the rapid expansion in the early Universe and the current acceleration in a single framework.

Hans Adolph Buchdahl introduced the *f*(R) gravity theory in 1970 [1]. Bertolami et al. [2] later extended it by coupling a function of the Ricci scalar R with the matter Lagrangian $L_{m,}$ while Carroll et al. [3] used it to explain the Universe's late-time acceleration. A fundamentally different image of our Universe has been revealed by astrophysical data from a variety of sources, including Cosmic Microwave Background (CMB) variations [4], type-Ia supernovae experiments [5], X-ray experiments [6], as well as large-scale structure observations [7]. All these data point to a faster expansion of the cosmos.

Some astrophysicists believe that modified gravity theories could explain the phenomena of dark matter (DM) and dark energy (DE), which appear to be causing the continuous expansion of the Universe. Numerous astrophysical models with varying aspects have been developed and analysed over the past decade. Perlmutter et al. [8] provided the first evidence for a Universe with a positive cosmological constant, indicating its expansion, based on observations of 40 high-redshift supernovae. Bamba et al. [9] explored various methods for testing DE and alternative extended gravity models using cosmography. One approach to addressing this problem involves replacing the conventional Einstein-Hilbert action in the standard part of the Einstein field equations with a general function of Ricci Scalar R [10]. Mishra et al. [11] demonstrated a transition from early deceleration to significant late-time acceleration by using different deceleration values with a hybrid scale factor.

The investigation of DE can be examined through its equation of state (EoS) parameter $\omega = \frac{p}{\rho}$. It's important to note that perfect fluid is a significant component of the Universe, undergoes variation and contraction in its action. Fakhereh MD [12], studied the dynamical behaviour of an anisotropic Universe in extended gravity using two cosmological models.

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Shamir [13], explored the Plane symmetric model in *f*(R) gravity, and obtained some well-known solutions. Raut et al. [14], obtained vacuum solutions of plane symmetric model with the help of special form of deceleration parameter (DP) in *f*(R) gravity theory. Agrawal et al. [15], studied gravitational baryogenesis models' comparison in *f*(R) gravity by considering perfect fluid as a matter. Singh et al. [16], studied Bianchi type-I five- dimensional cosmological models with massive string in GR and found that sum of energy density and string tension density was null. Agrawal et al. [17], investigated black holes and wormholes by considering spherical symmetric space-time beyond classical GR. Karim MR [18], studied Bianchi type-I model in the context of an anisotropic Universe within the Saez-Ballester theory of gravitation and found the formula for computing the Universe's entropy in terms of viscosity. Al-Haysah and Hasmani [19], assumed *f*(R) $= R + \alpha R^2$ and obtained the solution for higher dimensional Bianchi type-I cosmological model by considering string as a matter field by combining *f*(R) gravity theory and Kaluza-Klein (KK) theory. Ladke and Mishra [20,21], studied five-dimensional plane symmetric and static interior plane symmetric solutions in *f*(R) gravity theory. Thakare et al. [22], derived the precise solutions of non-vacuum, higher-dimensional, plane symmetric model in f (R, T) gravity theory by means of quadratic EoS. Sharma et al. [23], investigates the stability of the transition from early deceleration to recent acceleration in a perfect fluid LRS Bianchi-I cosmological model within f(R, T) theory. Dabre et al. [24], explores a bulk viscous fluid with a cloud of strings using the plane symmetric LRS Bianchi type I metric in the context of *f*(R) gravity. Pawar et al. [25], explored string cosmological plane symmetric model considering bulk viscosity and find solution using relation between metric potential $B = RA^n$. Turrion et al. [26], studied the physical non-viability of a wide class of *f*(R) models and their constant-curvature solutions, and show that most *f*(R)-exclusive constant- curvature solutions also exhibit a variety of unphysical properties. Larranaga A. [27], obtained a rotating charged solution in *f(R)* theory of gravity with constant curvature representing a black hole. Calza et al. [28], studied vacuum *f*(R) gravity solutions which are characterized by constant Ricci scalar and investigated black hole solutions in detail.

This paper explores vacuum and non-vacuum solutions for plane symmetric spacetime within the *f*(R) gravity framework. We obtain general solutions for the field equations of plane symmetric spacetime under the premise of constant and nonconstant scalar curvature. Additionally, we applied Einstein's field equation, $T^{ij}_{;j} = 0$ to solve non-vacuum field equations. The physical behaviour of these solutions is analysed using various physical quantities.

2. *f*(R) GRAVITATIONAL THEORY

The gravitational *f*(R) theory is regarded as the best straightforward example of an extended gravity theory, first proposed by Hans Adolph Buchdahl.

The action for *f*(R) gravity theory is given by [29,30,31],

$$s = \int \sqrt{-g}\ \left(\frac{1}{16\pi G}\, f(R) + L_m\right) d^4 x \tag{1}$$

Here, the Lagrangian matter is denoted by L_m, and a function that depends on the Ricci scalar R is represented as *f*(R). The above action is formulated by simply replacing R with *f*(R) in the traditional Einstein- Hilbert action expression.

Now, changing (1) with respect to the metric g_{ij} ,we obtain

$$F(R)R_{ij} - \frac{1}{2} f(R) g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \Box F(R) = kT_{ij} \tag{2}$$

Here, $F(R) = \frac{df(R)}{dR}$, $\Box \equiv g^{ij}\nabla_i\nabla_j$ is the d'Alembert operator, ∇_i represent the covariant derivative operator, T_{ij} denotes the standard matter energy- momentum tensor derived from the Lagrangian L_m and k is the coupling constant in gravitational units.

Solving the above equation (2), we get

$$F(R)R - 2f(R) + 3\Box F(R) = kT \tag{3}$$

In vacuum case (i.e., for $T = 0$) equation (3), reduce to

$$F(R)R - 2f(R) + 3\Box F(R) = 0 \tag{4}$$

This connection between *f*(R) and *F(R)* is crucial for simplifying the field equations.

With a constant Ricci scalar $R = R_0$, equation (4) becomes:

$$F(R_0)R_0 - 2f(R_0) = 0 \tag{5}$$

The term "constant curvature condition" describes this situation.

When we simplify equation (3), we obtain

$$f(R) = \frac{F(R)R + 3\Box F(R) - kT}{2} \tag{6}$$

Solving equations (2) and (6), we get,

$$\left(\frac{F(R)R-\Box F(R)-kT}{4}\right)=\frac{F(R)R_{ij}-\nabla_i\nabla_jF(R)-kT_{ij}}{g_{ij}} \tag{7}$$

As an LHS of equation (7) is not depend on index i, we have

$$K_i=\left(\frac{F(R)R_{ii}-\nabla_i\nabla_iF(R)-kT_{ii}}{g_{ii}}\right) \tag{8}$$

Where, $K_i-K_j=0$, for all i and j, K_i just a notation of traced quantity.

For perfect fluid the energy-momentum tensor is,

$$T_{ij}=(p+\rho)u_iu_j-\,pg_{ij} \tag{9}$$

Which follows the EoS,

$$p=\omega\rho \tag{10}$$

Where $-1\le\omega\le1$, ρ and p represents energy density and pressure of dark energy.

It is widely accepted that by the end of the inflationary period, the geometry of the Universe was homogeneous and isotropic [32], where the FLRW models plays an important role in representing both spatially homogeneous and isotropic Universe. Observational studies have confirmed these symmetries. Yet, theoretical arguments and unexpected anomalies in the cosmic microwave background (CMB) suggest that an anisotropic phase may have existed in the early Universe. After the announcement of the Plank probe results [33], it is believed that the early universe may not have been exactly uniform. Thus, the existence of inhomogeneous and anisotropic properties of the universe has increased popularity when it is come to constructing cosmological models under the supervision of anisotropic background.

3. METRIC AND THE FIELD EQUATIONS

The general non-static, anisotropic and spatially homogeneous plane symmetric spacetime metric is given by [14],

$$ds^2=dt^2-A^2(dx^2+dy^2)-B^2dz^2 \tag{11}$$

Where $A=A(t),B=B(t)$.
We denote the coordinates x,y,z,t as x^1,x^2,x^3,x^4 respectively.
The Ricci scalar of metric (11) is,

$$R=-2\left[\frac{2\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}+\frac{2\dot{A}\dot{B}}{AB}\right] \tag{12}$$

Where $\dot{A}=\frac{dA}{dt}$, $\ddot{A}=\frac{d^2A}{dt^2}$ etc.

From equations (8), for vacuum field,

$$K_i=\left(\frac{F(R)R_{ij}-\nabla_i\nabla_jF(R)}{g_{ij}}\right) \tag{13}$$

Where, $K_i-K_j=0, \forall\ i,j$

For $K_4-K_1=K_4-K_2=0$ and $K_4-K_3=0$ we get

$$\left[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{A}^2}{A^2}-\frac{\dot{A}\dot{B}}{AB}\right]F+\frac{\dot{A}}{A}\dot{F}-\ddot{F}=0 \tag{14}$$

$$\left[\frac{2\ddot{A}}{A}-\frac{2\dot{A}\dot{B}}{AB}\right]F+\frac{\dot{B}}{B}\dot{F}-\ddot{F}=0 \tag{15}$$

With three unknowns, A, B, and F, we are left with two non-linear DE (14) and (15). These are challenging, nonlinear equations to solve. However, we look into a few solutions utilizing the idea of constant curvature assumption.

4. SOLUTIONS WITH THE CONSTANT CURVATURE ASSUMPTION

Under the assumption of constant curvature, the field equations in *f*(R) gravity are analysed. This approach allows for the recovery of several well-known solutions that are already established in GR, but now within the context of significant *f*(R) gravity models.

Case-I:
Say for a constant curvature solution $R=R_0$, we have

$$\dot{F}(R_0)=0=\ddot{F}(R_0) \tag{16}$$

Equations (14) and (15) can be used to get the following form:

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{A}^2}{A^2}-\frac{\dot{A}\dot{B}}{AB}=0 \tag{17}$$

$$\frac{2\ddot{A}}{A}-\frac{2\dot{A}\dot{B}}{AB}=0 \tag{18}$$

The power law assumption can be used to solve these problems. i.e.$A \propto t^p$ and $B \propto t^q$, where p and q are real numbers.
Thus, $A = k_1 t^p$ and $B = k_2 t^q$, where k_1 and k_2 are proportionality constants.
From equations (17) and (18), we get,

$$p=\frac{2}{3}, q=-\frac{1}{3} \tag{19}$$

And hence from (11) and (19) the solution becomes

$$ds^2 = dt^2 - {k_1}^2 t^{\frac{4}{3}}(dx^2+dy^2) - {k_2}^2 t^{-\frac{2}{3}} dz^2 \tag{20}$$

This can be shown that, these values of p and q leads to $R = 0$. This represents the simplest possible solution and is somewhat trivial in the case of constant scalar curvature.
We reinterpret the specifications, i.e. ${k_1}^2 dx^2 = dX^2, {k_1}^2 dy^2 = dY^2, {k_2}^2 dz^2 = dZ^2$
The above metric can be written as,

$$ds^2 = dt^2 - t^{\frac{4}{3}}(dX^2+dY^2) - t^{-\frac{2}{3}} dZ^2 \tag{21}$$

This corresponds to Taub's Metric [13].

Case-II:
Now we assume that $B = A^n$. Then subtracting equation (17) from (18), we get

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}-\frac{\dot{A}\dot{B}}{AB}=0 \tag{22}$$

After solving above equation we get,

$$A = [(n+2)(k_3 t + k_4)]^{\frac{1}{(n+2)}} \tag{23}$$

Hence

$$B = [(n+2)(k_3 t + k_4)]^{\frac{n}{(n+2)}} \tag{24}$$

Where k_3 and k_4 are constants of integration.
With these values of A and B, metric takes the form

$$ds^2 = dt^2 - [(n+2)(k_3 t + k_4)]^{\frac{2}{(n+2)}}(dx^2+dy^2) - [(n+2)(k_3 t + k_4)]^{\frac{2n}{(n+2)}} dz^2 \tag{25}$$

This equation gives Taub's Metric for $n = -\frac{1}{2}$.

Case-III: For non-vacuum perfect fluid solutions:
In this instance, the metric in equation (11) has a non-vacuum solution.
The energy-momentum tensor for a single fluid source is

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij} \tag{26}$$

With

$$g^{ij}u_i u_j = 1, u_i = (0,0,0,1) \tag{27}$$

where ρ is the fluid's energy density and p is the isotropic pressure.
u_i is the four-velocity of the time-like vector satisfying (27).
Using (8), (26) and (27), we get
For $K_1 - K_4 = K_2 - K_4 = 0$ and for $K_3 - K_4 = 0$, we get,

$$\left[-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}\right]F-\frac{\dot{A}}{A}\dot{F}+\ddot{F}+k(p+\rho)=0 \tag{28}$$

$$\left[-\frac{2\ddot{A}}{A}+\frac{2\dot{A}\dot{B}}{AB}\right]F-\frac{\dot{B}}{B}\dot{F}+\ddot{F}+k(p+\rho)=0 \tag{29}$$

Using equations (16), equations (28) and (29) becomes,

$$\left[-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}\right]+\frac{k}{F}(p+\rho)=0 \tag{30}$$

$$\left[-\frac{2\ddot{A}}{A}+\frac{2\dot{A}\dot{B}}{AB}\right]+\frac{k}{F}(p+\rho)=0 \tag{31}$$

Solving (30) and (31), we get

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}-\frac{\dot{A}\dot{B}}{AB}=0 \tag{32}$$

Integrating (32), we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_5}{A^2B} \tag{33}$$

where k_5 is the integration constant. Making use of (33) and the conversion

$$A^2B=\mathcal{U} \tag{34}$$

Where $\mathcal{U}=\mathcal{U}(\mathrm{t})$ and $A=A(\mathcal{U}), B=B(\mathcal{U})$ are expressed as follows:

$$A=\beta\mathcal{U}^{(1/3)}e^{(k_5/3)\int(dt/\mathcal{U})}\ , B=\delta\mathcal{U}^{(1/3)}e^{(-2k_5/3)\int(dt/\mathcal{U})} \tag{35}$$

Where β and δ are integration constants.

$$ds^2=dt^2-\beta^2\mathcal{U}^{(2/3)}e^{(2k_5/3)\int(dt/\mathcal{U})}(dx^2+dy^2)-\delta^2\mathcal{U}^{(2/3)}e^{(-4k_5/3)\int(dt/\mathcal{U})}dz^2 \tag{36}$$

The standard conservation law for Einstein's field equation $T^{ij}_{;j}=0$ (Here semicolon (;) indicates covariant divergence) gives,

$$\dot{\rho}+(p+\rho)\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=0 \tag{37}$$

Solving (37), using equations (10) and (34) we obtain,

$$\rho=\xi\mathcal{U}^{-(1+\omega)} \tag{38}$$

Where ξ is integration constant and $-1\le\omega\le 1$.
The cosmological parameters are given by,
Spatial volume, $V=\mathcal{U}$
Hubble parameter, $H=\frac{1}{3}\frac{\dot{\mathcal{U}}}{\mathcal{U}}$
Expansion scalar, $\theta=\frac{\dot{\mathcal{U}}}{\mathcal{U}}$
Shear scalar, $\sigma^2=\frac{1}{3}\frac{{k_5}^2}{\mathcal{U}^2}$
Deceleration parameter, $q=\frac{[2\,\dot{\mathcal{U}}^2-3\,\ddot{\mathcal{U}}\,\mathcal{U}]}{\dot{\mathcal{U}}^2}$

Three different circumstances arise based on the parameter $\mathcal{U}$.

1. $\boldsymbol{\mathcal{U}=constant=c}$.
   In this case $A=\beta' e^{(k't/3)}$ and $B=\delta' e^{-(2k't/3)}$, $\rho=\xi c^{-(1+\omega)}$
   where $\beta'=\beta c^{(1/3)}$, $\delta'=\delta c^{(1/3)}$, $k'=k_5/c$
   Also $H=0,\ \theta=0,\ \sigma^2=\frac{k'^2}{3},\ q$ is not defined. Here ρ and σ are constant. For $k_5=0,\ A$ and B are constant and σ become significant. For $k_5>0, A$ increases and B decreases exponentially with time.

2. $\boldsymbol{\mathcal{U}=t}$
   In this case $A=\beta t^{((k_5+1)/3)}$ and $B=\delta t^{((1-2k_5)/3)}$, $\rho=\xi t^{-(1+\omega)}$, scale factor $a=t^{(1/3)}$, H =(1/3t), θ =(1/t), $\sigma^2=\left({k_5}^2/3t^2\right)$ and $q=2$.

3. $\boldsymbol{\mathcal{U}=t^n}$
   In this case $A=\beta t^{(n/3)}e^{\left(k_5t^{(1-n)}/3(1-n)\right)}$, $B=\delta t^{(n/3)}e^{-\left(2k_5t^{(1-n)}/3(1-n)\right)}$, $\rho=\xi t^{-n(1+\omega)}$,
   θ =(n/t), $\sigma^2=\left({k_5}^2/3t^{2n}\right)$ and $q=((3-n)/n)$, where $n\neq 1$.
   The dimensionless rate of change of the third derivative of the field parameter with respect to the cosmic time is known as the jerk parameter in cosmology. It is given by

$$j(t)=q+2q^2-\frac{\dot{q}}{H}\ \text{and hence}\ j(t)=(18/n^2)-(9/n)+1$$

In this case, the expansion scalar θ remains positive for $n>0$, confirming that the Universe is expanding. As cosmic time increases, θ decreases and asymptotically approaches zero, indicating a gradual slowdown in the expansion rate. The shear scalar σ^2 also diminishes over time and vanishes in the asymptotic limit, implying that the Universe evolves toward isotropy. The deceleration parameter is given by $q=((3-n)/n)$, which becomes negative for $n>3$, reflecting an accelerated expansion phase consistent with current cosmological observations. The energy density follows the relation

$\rho = \xi t^{-n(1+\omega)}$, and decreases with time for $\omega > -1$. Notably, for $\omega = -1$, the density remains constant, imitating the behaviour of a cosmological constant and aligning with the ΛCDM model. These results support a scenario in which the Universe transitions from an anisotropic, decelerating phase to an isotropic, accelerating one at late times.

5. SOLUTIONS WITHOUT THE CONSTANT CURVATURE ASSUMPTION

Field equations (28) and (29) are linearly independent with five unknowns A, B, p, ρ and F. The deceleration parameter describes the evolution of the universe. The cosmological models of the evolving universe transits from early deceleration phase. ($q > 0$) to the current accelerating phase ($q < 0$). Whereas the models can be classified based on the time dependence of DP. To solve the system of equations, we assume the deceleration parameter (q) varies linearly with the Hubble parameter [34,35].

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \gamma + \alpha H \tag{39}$$

Here γ and α are arbitrary constants.
For $\gamma = -1$ in equation (39)

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \alpha H$$

Which yields the following differential equation,

$$\frac{a\ddot{a}}{\dot{a}^2} + \alpha\frac{\dot{a}}{a} - 1 = 0 \tag{40}$$

Integrating equation (40) we get

$$a(t) = e^{\frac{1}{\alpha}\sqrt{2\alpha t + k_6}} \tag{41}$$

Where k_6 is a constant of integration.

Assuming the shear scalar σ is proportional to the expansion scalar θ we obtain the relationship between scale factor A and B as follows:

$$A = B^n \tag{42}$$

Where n is constant and $n \neq 1$
Using (34), (41) and (42) the metric components are

$$A(t) = e^{\frac{3n}{\alpha(2n+1)}\sqrt{2\alpha t + k_6}} \tag{43}$$

and

$$B(t) = e^{\frac{3}{\alpha(2n+1)}\sqrt{2\alpha t + k_6}} \tag{44}$$

The metric (11) is reduced to,

$$ds^2 = dt^2 - e^{\frac{6n}{\alpha(2n+1)}\sqrt{2\alpha t + k_6}}(dx^2 + dy^2) - e^{\frac{6}{\alpha(2n+1)}\sqrt{2\alpha t + k_6}}dz^2 \tag{45}$$

Equation (45) represents PSCM with variable deceleration parameter.
The directional Hubble parameter H_x, H_y and H_z are given by

$$H_x = H_y = \frac{3n}{(2n+1)\sqrt{2\alpha t + k_6}}$$

$$H_z = \frac{3}{(2n+1)\sqrt{2\alpha t + k_6}}$$

Hubble parameter (H) and expansion scalar (θ) is given by

$$H = \frac{1}{\sqrt{2\alpha t + k_6}} \tag{46}$$

$$\theta = \frac{3}{\sqrt{2\alpha t + k_6}} \tag{47}$$

The spatial volume (V) and anisotropic parameter (Δ) is given by

$$V = e^{\frac{3}{\alpha}\sqrt{2\alpha t + k_6}} \tag{48}$$

$$\Delta = \frac{2(n-1)^2}{(2n+1)^2} = constant \; (\Delta \neq 0 \text{ for } n \neq 1) \tag{49}$$

The shear scalar (σ^2) is given by

$$\sigma^2 = \frac{3(n-1)^2}{(2n+1)^2(2\alpha t+k_6)} \tag{50}$$

From equations (47) and (50), we get

$$\lim_{t\to\infty}\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(2n+1)^2} = constant(\neq 0\, for\, n \neq 1) \tag{51}$$

The energy density ρ and isotropic pressure p using equations (10) and (38) are given by

$$\rho = \xi\left[e^{\frac{-3(1+\omega)}{\alpha}\sqrt{2\alpha t+k_6}}\right] \text{ and } p = \omega\xi\left[e^{\frac{-3(1+\omega)}{\alpha}\sqrt{2\alpha t+k_6}}\right] \tag{52}$$

Where ξ is integration constant and $-1 \leq \omega \leq 1$.
The deceleration parameter (q) is given by

$$q = -1 + \frac{\alpha}{\sqrt{2\alpha t+k_6}} \tag{53}$$

From equation (12), the Ricci scalar R is found as

$$R = 6\left\{\frac{\alpha}{(2\alpha t+k_6)^{3/2}} - \frac{3(3n^2+2n+1)}{(2n+1)^2(2\alpha t+k_6)}\right\}, \tag{54}$$

In this model, the Universe's initial time is given by $t = -\frac{k_6}{2\alpha} = \mathcal{T}$ $\quad(k_6 \geq 0, \alpha > 0)$, which can be shifted to $t = 0$ by setting $k_6 = 0$. At this initial moment, the scale factor a remains constant, indicating that the Universe in this case begins from a non-singular state.

At the initial time $(t = \mathcal{T})$, the Hubble's parameter (H), expansion scalar (θ), shear scalar (σ) are all infinite. Additionally, the isotropic pressure (p) depends on EoS parameter (i.e. $p = \omega\xi$) while the energy density (ρ) remains constant (i.e. $\rho = \xi$).

Also, the energy density remains positive throughout cosmic evolution but gradually declines over time. In contrast, the isotropic pressure increases, it starts from a negative value in the early universe and approaches zero at the present epoch. Observationally, this negative pressure is attributed to dark energy, which, despite having positive energy density, drives accelerated cosmic expansion.

As time t progresses, the scale factor a increases, while the physical parameters H, θ, σ gradually decrease. In the limit of large t, the scale factor a becomes infinitely large, while the parameters H, θ, σ approach zero. This implies that the Universe in this model originates from a non-singular state and expands exponentially over cosmic time.

Furthermore, the deceleration parameter q is positive for $t < \frac{\alpha^2-k_6}{2\alpha}$, indicating an early decelerating phase of the Universe. Conversely, q is negative for $t > \frac{\alpha^2-k_6}{2\alpha}$, predicting the accelerating phase of the Universe's expansion in this model. However as $t \to \infty, q\ approches - 1$, which is characteristic of de Sitter-like exponential expansion. This indicates a transition from deceleration to acceleration, consistent with current observations of cosmic evolution.

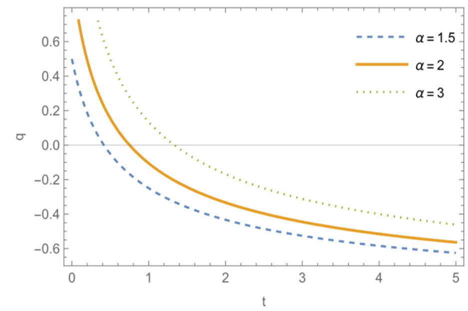

Figure 1. Plot of (q) vs. time t

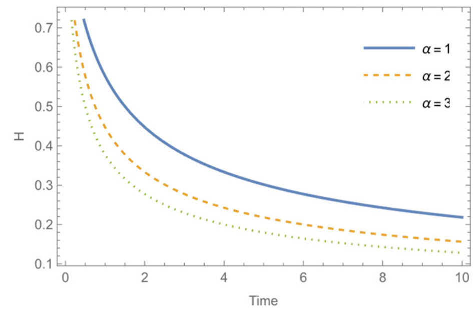

Figure 2. Plot of (H) vs. time t

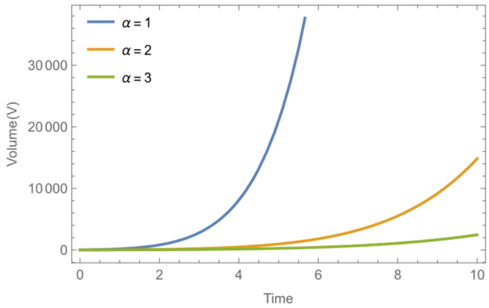

Figure 3. Plot of (V) vs. time t

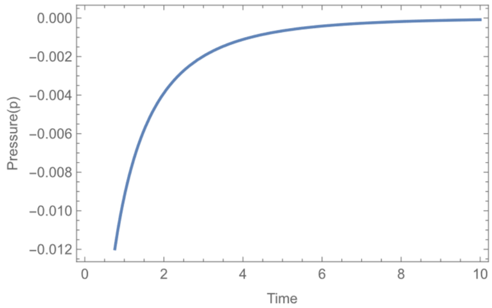

Figure 4. Plot of (p) vs. Time t

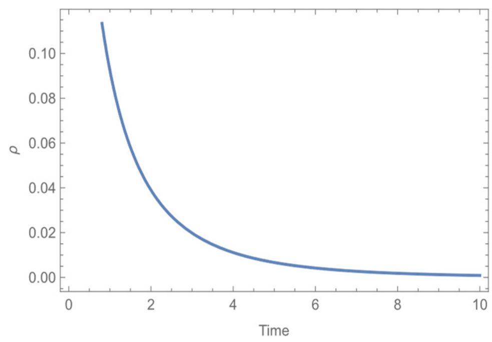

Figure 5. Plot of (ρ) vs. Time t

Statefinder diagnostic: A geometric statefinder pair {r, s}, whose notable characteristic is their dependence on the scale factor and its time derivatives, was introduced by Sahni et al. and Alam et al. [36,37]. The statefinder diagnostic method serves as effective tool for distinguishing different DE models in cosmology. This diagnostic pair also characterizes the ΛCDM model where the cosmological constant Λ act as DE. The ΛCDM model is considered the standard framework for studying the evolution of the accelerating Universe and is identified by the fixed point {r, s} = {1, 0}.
The state-finder diagnostic pair is defined as [38,39,40]

$$r = \frac{\dddot{H}}{H^3} - 3q - 2 \text{ and } s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \tag{55}$$

We employ the statefinder diagnostic method to evaluate our model's dynamics and contrast its behaviour with that of the ΛCDM model. The expressions for the statefinder parameters {r, s} in our framework are derived as follows:

$$r = 1 + \frac{3\alpha^2}{2\alpha t + k_6} - \frac{3\alpha}{\sqrt{2\alpha t + k_6}} \text{ and } s = \frac{\left[\frac{\alpha}{\sqrt{2\alpha t + k_6}} - 1\right]}{\left[1 - \frac{3}{2}\frac{\sqrt{2\alpha t + k_6}}{\alpha}\right]} \tag{56}$$

In our model, the state-finder pair {r, s} tends toward {1,0} at late time, thus the model is compatible with ΛCDM model [41]

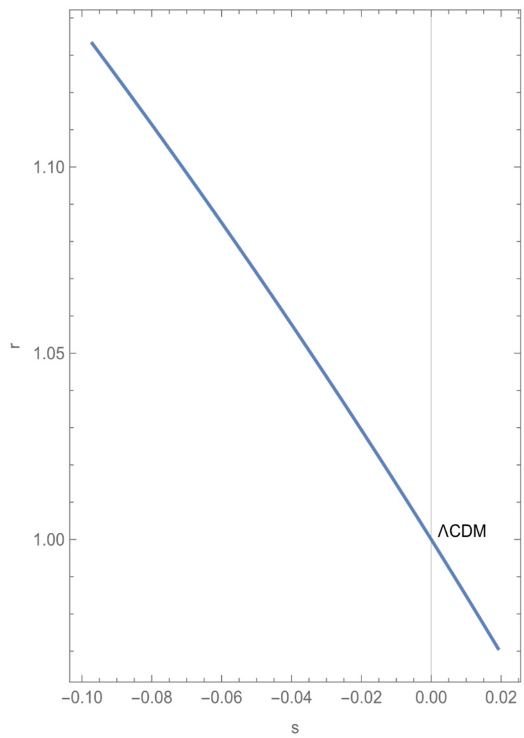

Figure 6. The figure shows the state-finder parameter

Energy conditions: Here, the time-varying behaviour of the energy conditions is explored. The energy conditions are used in many approaches to understand the evolution of the universe. The study of singularities in the spacetime was based on energy conditions. Some of the important energy conditions are given as follows:

Weak energy conditions (WEC) ⇒ $\rho \geq 0, \rho - P \geq 0$
Dominant energy conditions (DEC) ⇒ $\rho + P \geq 0$
Strong energy conditions (SEC) ⇒ $\rho + 3P \geq 0$

Using equation (52), we analyse the behaviour of above-mentioned energy conditions. Figures (7-9) show the behaviour of WEC, DEC and SEC with proper choice of constants respectively and it is observed that all the energy conditions are satisfied for this model.

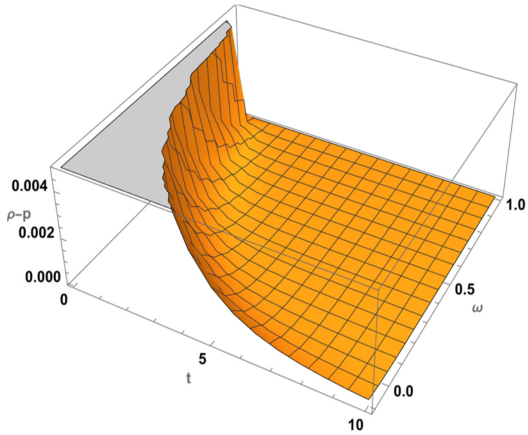

Figure 7. Behaviour of WEC vs. t and ω with $\xi = 1, \alpha = 2, k_6 = 1$

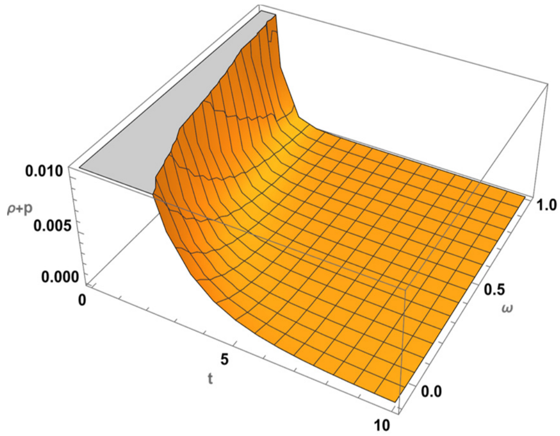

Figure 8. Behaviour of DEC vs. t and ω with $\xi = 1, \alpha = 2, k_6 = 1$

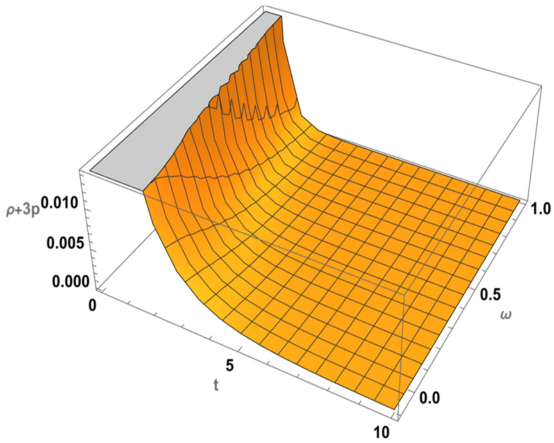

Figure 9. Behaviour of SEC vs. t and ω with $\xi = 1, \alpha = 2, k_6 = 1$

6. CONCLUSIONS

We have investigated non-static plane-symmetric model in the framework of *f*(R) gravity theory in this work, looking at both scenarios with and without matter (vacuum and non-vacuum conditions). To determine precise answer to the field equations, we have taken into consideration the metric form of the theory.

In the first two cases, we considered vacuum conditions and found solutions using constant curvature and power law assumptions. These solutions matched the well-known Taub's solution.

In the third case, we considered non-vacuum condition and found solutions using the energy momentum tensor equation. We solved the field equations by making a specific assumption that involved a transformation $A^2B = \mathcal{U}$ and using the EoS, $p = \omega\rho$. For the non-vacuum scenario, we considered three possibilities of $\mathcal{U}$, where $\mathcal{U}$ is constant, t, and t^n. Particularly, in the case where $\mathcal{U} = t^n$ with $0 < n < 1$, we observed that the ratio σ/θ does not approach to zero as $t \to \infty$. This implies that the shear decreases at a slower rate compared to the expansion scalar, implying that anisotropic remains significant over time. This persistent anisotropy suggests that our model remains anisotropic, representing the early evolutionary phase of the Universe.

These findings underscore the potential of *f*(R) gravity to effectively describe the Universe's dynamics, particularly during its early stages when anisotropic effects are expected to be prominent.

In this section, without assuming constant curvature, we assumed a scale factor $a(t) = e^{\frac{1}{\alpha}\sqrt{2\alpha t + k_6}}$ (where α and k_6 are positive constants), to derive an exact solution of the Einstein field equations. This model remains anisotropic throughout cosmic evolution for $n \neq 1$, as it is independent of the cosmic time t. It describes an exponentially expanding Universe that originates with a finite volume at the big bang (t=0) and later transitions into an accelerated expansion phase. The behaviour of the DP is shown in Fig. 1 for $\alpha = 1.5, 2, 3$. The deceleration parameter q in this model is found to be time dependent, exhibiting a transition from an early decelerating phase to the current accelerating phase of the Universe. Fig. 2 and 3 represents the time-varying behaviour of the Hubble parameter (H) and volume (V), respectively. Fig. 4 and Fig. 5 illustrate the variation of isotropic pressure p and energy density ρ w.r.t. the cosmic time t. The energy density and pressure gradually diminish and approach zero as t becomes large. Fig. 6 shows the evolution of r-s trajectory. The state-finder pair {r, s} tends towards {1,0} at late time, indicating that the model asymptotically behaves like the ΛCDM (cosmological constant Λ + Cold Dark Matter) model. Fig. 7,8, and 9 represents the time varying behaviour of WEC, DEC, and SEC, showing that all the energy conditions (WEC, DEC, SEC) are satisfied throughout the evolution.

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ДОСЛІДЖЕННЯ ПЛОСКОГО СИМЕТРИЧНОГО ПРОСТОРУ-ЧАСУ В МОДИФІКОВАНІЙ ГРАВІТАЦІЙНІЙ ТЕОРІЇ *f*(R)

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У цій статті досліджується плоскосиметрична космологічна модель (PSCM) у контексті модифікованої теорії гравітації *f*(R), включаючи як вакуумні, так і невакуумні сценарії. Джерелом матерії вважається ідеальна рідина. Щоб отримати розв'язки, ми розглядаємо передумову як постійної, так і непостійної скалярної кривини. Застосовуючи закон збереження для рівняння поля Ейнштейна, $T^{ij}_{;j} = 0$, та припущення степеневого закону, ми отримуємо деякі відомі розв'язки. Ми розв'язали рівняння поля, зробивши специфічне припущення, яке включало перетворення $A^2B = \mathcal{U}$. Це дослідження досліджує фізичні та кінематичні характеристики конкретних космологічних моделей, а також розглядає діагностику за допомогою методу пошуку станів – ключового інструменту для аналізу еволюційної траєкторії Всесвіту. Робота надає важливе розуміння поведінки анізотропних моделей у контексті модифікованої *f*(R) гравітації. Вона підкреслює взаємодію між розподілом матерії та геометрією простору-часу, особливо підкреслюючи, як припущення постійної та непостійної скалярної кривини допомагає спростити та розв'язати відповідні рівняння поля. Отримані рішення покращують наше розуміння космічної еволюції, що керується модифікованою *f*(R) гравітацією.

Ключові слова: *плоско-симетричний; теорія гравітації f(*R*); ідеальна рідина; взначник станів; анізотропний Всесвіт*