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SPECTRA OF SOME CHARMED HADRONS IN A NON-RELATIVISTIC MODEL

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In a non-relativistic framework the mass spectra of $c\bar{c}$, cc, ccc and ccu systems are investigated. The potential consists of the Cornell potential along with a logarithmic correction term as suggested from lattice QCD. We analyze the S, P, and D wave charmonium states and, S and P wave cc diquark states and have compared them with existing results from experiments and other potential models. Using the quark-diquark model, we have evaluated the S-wave spectra of doubly charmed baryon Ξ_{cc}^{++} and the triply charmed baryon Ω_{ccc}. These masses are compared with other theoretical studies.

Keywords: *Non-relativistic potential model; LQCD corrections; Charmed hadrons; Diquark-quark model; Mass spectrum*

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1. INTRODUCTION

Since the discovery of J/ψ, the first charmonium ($c\bar{c}$) state, at SLAC [1] and BNL [2] in 1974, charmed hadrons have been continuously investigated both theoretically and experimentally. At present a large number of charmed states have been experimentally discovered [3] and even more states have been theoretically predicted. Other than the mesonic and baryonic states, unanticipated states, classified as X, Y and Z, exotic states such as tetraquarks and pentaquarks, which cannot be classified under the conventional quark model have been detected [4, 5, 6, 7]. In 2017, the LHCb collaboration discovered a resonance in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum [8]. This was identified as the doubly charmed baryon Ξ_{cc}^{++} and its mass was measured to be $3621.40 \pm 0.72 \pm 0.27$ MeV. This has led to an extensive study of doubly and also triply heavy baryons theoretically. In literature, there are various studies [9, 10] that discuss the prospects for the disvovery of multi charmed baryons in future experiments at LHC. Hence, it becomes important to study charmed baryons theoretically. Charmed hadrons are important systems for they provide valuable information about the strong and weak interactions in the standard model [11].

In this work, we have investigated the spectra of charmed hadrons including charmonium ($c\bar{c}$), cc diquarks and, doubly charmed (ccu) and triply charmed (ccc) baryons: Ξ_{cc}^{++} and Ω_{ccc}. We make use of diquark-quark model to evaluate the spectra of baryons [12]. A diquark is a bound state of two quarks. Since they exist as colored states, they are not experimentally detected as isolated entities. Nonetheless, diquark models provide qualitative explanations for various puzzles of exotic hadron spectroscopy [13, 14]. In the case of baryons, it is found that the constituent quark model predicts more states than those observed experimentally, which is generally referred as the missing resonance problem [15, 16]. However, the diquark model addresses this issue as the effective degrees of freedom is less compared to constituent quark model [17, 18]. Also, the diquark-quark interaction in baryons can be visualized similar to the quark-antiquark interaction in mesons [18].

This article is structured as follows. In section 2, we have given the theoretical framework where we have explained our potential model formalism. In section 3, we discuss our results and in section 4, we give our summary.

2. THEORETICAL MODEL

In the present work, the mass spectra are estimated by solving the non relativistic Schrodinger equation,

$$\left[\frac{1}{2\mu}\left(-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right)+V(r)\right]\psi(r)=E\psi(r), \tag{1}$$

where μ is the reduced mass and l is the (relative) orbital angular momentum of the system. Here the potential $V(r)$ we use, is of the form

$$V(r)=\frac{\kappa\alpha_s(Q^2)}{r}+br+k\ln(ar) \tag{2}$$

The first term in the potential is a coulomb-like term that dominates at small distances, second term is the linear confinement term that dominates at large distances. The last term in Eq.(2) corresponds to the relativistic corrections to the static

Cite as: T. Harsha, C.A. Bokade, R. Kaushal, Bhaghyesh, East Eur. J. Phys. 3, 41 (2025), https://doi.org/10.26565/2312-4334-2025-3-04 quarkonium potential at order $1/m$ in the quark mass as suggested from various studies such as lattice [19, 20, 21, 22, 23] and effective string theory [24]. Models incorporating a quark-antiquark potential modified by a $\ln(r)$ term have successfully described quarkonium spectra [21, 22, 25]. In Eq.(2) $\alpha_s(Q^2)$ is the QCD coupling constant, κ is the color factor whose value is $-\frac{4}{3}$ for quark-antiquark (and also quark-diquark) interaction and $-\frac{2}{3}$ for quark-quark interactions [26], b is the QCD string tension, k and a are phenomenological constants. The $\alpha_s(Q^2)$ is evaluated using the formula [27],

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f)\ln(\frac{Q^2}{\Lambda^2})} \tag{3}$$

Here n_f denotes the number of active quark flavors, Q is the renormalization scale, and Λ is the QCD scale parameter, which is taken as $0.168\ GeV$ [28]. The model parameters are fitted by minimizing the chi square for some of the available experimental states for charmonium. The up quark mass is fitted by minimizing the chi square for Ξ_{cc}^{++}. The values of the parameters used in our model are listed in Table 1. The Schrodinger equation 1 is solved numerically using the Runge-Kutta method [29]. The eigenvlaues (E) of Eq.(1) corresponds to the spin-averaged masses. The spin-dependent contributions to the masses are incorporated perturbatively. The spin-dependent potentials: spin-spin ($V_{SS}(r)$), spin-orbit ($V_{LS}(r)$), and tensor ($V_T(r)$) potentials are given by [30, 31],

$$V_{SS}(r) = \frac{8\pi|\kappa|\alpha_s}{3m_1m_2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 exp(-\sigma^2r^2)\mathbf{S_1}\cdot\mathbf{S_2} \tag{4}$$

$$V_{LS}(r) = \frac{1}{2m_c^2r}\left(3\frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr}\right)\mathbf{L}\cdot\mathbf{S}. \tag{5}$$

$$V_T(r) = \frac{1}{12m_c^2}\left(\frac{1}{r}\frac{dV_V(r)}{dr} - \frac{d^2V_V(r)}{dr^2}\right)S_{12}. \tag{6}$$

where,

$$S_{12} = 12\left((\mathbf{S}_1\cdot\hat{r})(\mathbf{S}_2\cdot\hat{r}) - \frac{1}{3}\mathbf{S}_1\cdot\mathbf{S}_2\right). \tag{7}$$

Here, we take the Lorentz vector term $V_V(r) = \frac{\kappa\alpha_s(Q^2)}{r}$ and the Lorentz scalar term $V_S(r) = br$. The masses of charmonium states are given by

$$M_{c\bar{c}} = 2m_c + E_{c\bar{c}} + \langle V_{SS}(r) + V_{LS}(r) + V_T(r)\rangle_{c\bar{c}}, \tag{8}$$

where $E_{c\bar{c}}$ are the eigenvalues (spin-averaged charmonium masses) obtained by solving the Schrodinger equation (Eq.(1)) taking $\mu = m_c/2$ and using the potential given in Eq.(2) with $\kappa = -4/3$. Using the eigenfunctions corresponding to these eigenvalues, we evaluate the expectation values in Eq.(8). For charmonium, $m_1 = m_2 = m_c$ in Eq.(4), where m_c is the charm quark mass.

We use diquark-quark model to evaluate the mass spectra of doubly and triply charmed baryons: Ξ_{cc}^{++} and Ω_{ccc}. For this, we first estimate the diquark spectrum and then once again solve the Schrodinger equation for a two-body system composed of diquark and quark. In both Ξ_{cc}^{++} and Ω_{ccc}, we assume the two c quarks to form the cc diquark. For modelling the quark-quark interaction (V_{qq}) in a diquark, we use the general rule [32, 33, 29],

$$V_{qq}(r) = \frac{V_{q\bar{q}}(r)}{2}, \tag{9}$$

where ($V_{q\bar{q}}$) is the quark-antiquark potential. Hence, the central potential for diquarks becomes,

$$V_{qq}(r) = -\frac{2\alpha_s(Q^2)}{3r} + \frac{br}{2} + k\ln(ar). \tag{10}$$

Here we have not considered the factor of 1/2 for the $O(1/m)$ correction term present in $V_{q\bar{q}}$. In diquark-quark model, baryons are treated as bound states of diquark and quark. The diquark-quark interaction in baryons is assumed to be similar to the quark-antiquark interaction in mesons [18]. Therefore, the diquark-quark potential within baryons is taken to be,

$$V_{dq}(r) = -\frac{4\alpha_s(Q^2)}{3r} + br + k\ln(ar). \tag{11}$$

Due to the Pauli exclusion principle, for the diquarks in the $l = 0$ (S-wave) state, the total spin $S = 1$ (spin-triplet) and for the diquarks in the $l = 1$ (P-wave) state, the total spin $S = 0$ (spin-singlet). There will be no contribution to the masses

Table 1. Values of parameters

m_c $[GeV]$	m_u $[GeV]$	b $[GeV^2]$	a $[GeV]$	k $[GeV]$	σ $[GeV]$
1.298	0.175	0.136	1.105	0.195	2.600

from the spin-orbit and tensor interactions for $l = 0$ and $S = 0$ states. Hence, the masses of (S and P wave) diquark states are given by

$$M_{cc} = 2m_c + E_{cc} + \langle V_{SS}(r) \rangle_{cc}, \tag{12}$$

where E_{cc} are the eigenvalues (spin-averaged diquark masses) obtained by solving the Schrodinger equation (Eq.(1)) taking $\mu = m_c/2$ and using the potential given in Eq.(10). Using the eigenfunctions corresponding to these eigenvalues, we evaluate the expectation value in Eq.(12). For diquarks, $m_1 = m_2 = m_c$ in Eq.(4).

In this work, we only consider the radial excitations of the diquark-quark system with diquark in ground state. Hence, for baryons we will not be considering the spin-orbit and tensor interactions. The masses of baryons in our diquark-quark model are given by the relation

$$M_{dq} = M_{cc} + m_q + E_{dq} + \langle V_{SS}(r) \rangle_{dq}, \tag{13}$$

where E_{dq} are the eigenvalues obtained by sloving the Schrodinger equation (Eq.(1)) taking $\mu = (M_{cc}m_q/M_{cc} + m_q)$ and using the potential given in Eq.(11). $m_q = m_u$ in the case of Ξ^{++}_{cc} and $m_q = m_c$ in the case of Ω_{ccc}. Using the eigenfunctions corresponding to these eigenvalues, we evaluate the expectation value in Eq.(13). Here we take $m_1 = M_{cc}, m_2 = m_q$ in Eq.(4). For the $l = 0$ states, Ω_{ccc} has $J^P = \frac{3}{2}^+$ whereas Ξ^{++}_{cc} have $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$.

3. RESULTS AND DISCUSSIONS

3.1. Charmonium and cc diquark

The mass spectra of S, P, and D-wave charmonium states are presented in Tables 2,3, and 4 respectively, where we have compared our results with exisiting results from experiments and other potential models. In Figure 1, we have compared our $c\bar{c}$ spectra with results from experiments. From Figure 1, we can see that the masses of states below the $D\bar{D}$ threshold predicted from our model are in reasonable agreement with the experimental results. In literarture, the $3\,^3S_1$ and $4\,^3S_1$ states are suggested to be the experimentally detetcted $\psi(4040)$ and $\psi(4415)$ states respectively [31]. The masses of $3\,^3S_1$ and $4\,^3S_1$ states from our model are 4165 MeV and 4530 MeV respectively, which are approximately 125 MeV higher than the masses of $\psi(4040)$ and $\psi(4415)$ states. It is to be noted that since $\psi(4040)$ and $\psi(4415)$ lie above the $D\bar{D}$ threshold, there will be large $S-D$ mixing in these states [34]. Hence $\psi(4040)$ and $\psi(4415)$ cannot be considered as pure $^3S_1$ states [34, 35].

Table 2. S-wave charmonium spectrum (in MeV).

State	J^{PC}	PDG [3]	Our Work	[36]	[37]	[38]	[27]
$1\,^1S_0$	0^{-+}	2984.1 ± 0.4	2985	3004	2981	2982	2989
$1\,^3S_1$	1^{--}	3096.900 ± 0.006	3093	3086	3096	3090	3094
$2\,^1S_0$	0^{-+}	3637.7 ± 0.9	3657	3645	3635	3630	3602
$2\,^3S_1$	1^{--}	3686.097 ± 0.010	3722	3708	3685	3672	3681
$3\,^1S_0$	0^{-+}		4101	4124	3989	4043	4058
$3\,^3S_1$	1^{--}	$\psi(4040)$	4165	4147	4039	4072	4129
$4\,^1S_0$	0^{-+}		4478	4534	4401	4384	4448
$4\,^3S_1$	1^{--}	$\psi(4415)$	4530	4579	4427	4406	4514
$5\,^1S_0$	0^{-+}		4807	4901	4811	4685	4799
$5\,^3S_1$	1^{--}		4853	4942	4837	4703	4863

For $1P$ charmonium states ($h_c(1P)$, $\chi_{c0}(1P)$, $\chi_{c1}(1P)$, and $\chi_{c2}(1P)$) the masses predicted from our model are in agreement with the expreimental results and shows only small deviations (< 15 MeV). In some theoretical models [39, 40], $2\,^3P_0$, $3\,^3P_0$, $2\,^3P_2$ and $3\,^3P_1$ are suggested to be the experimentally detected $\chi_{c0}(3860)$, $X(4160)$, $\chi_{c2}(3930)$ and $X(4274)$ states respectively, with measured masses of 3862^{+26+40}_{-32-13} MeV, 4153^{+23}_{-21} MeV, 3922.5 ± 1 MeV and 4286^{+8}_{-9} MeV [3]. Our obtained masses for $2\,^3P_0$, $3\,^3P_0$, $2\,^3P_2$ and $3\,^3P_1$ states deviates from the measured masses of $\chi_{c0}(3860)$, $X(4160)$, $\chi_{c2}(3930)$ and $X(4274)$. This may be due to the fact that, for states close or above the open-charm thresholds, the coupled-channel effects are significant [35]. The mass of $1\,^3D_1$ state obtained from our analysis is close to the mass of $\psi(3770)$ state, which is assigned as $1\,^3D_1$ in some potential models [31]. But the inconsistency in the predicted leptonic decay width of this state in potential models, compels to interpret $\psi(3770)$ as an $S-D$ mixture [35]. In literature, the states

Table 3. P-wave charmonium spectrum (in MeV).

State	J^{PC}	PDG [3]	Our Work	[36]	[37]	[38]	[27]
$1\,^3P_0$	0^{++}	3414.71 ± 0.30	3420	3440	3413	3424	3428
$1\,^3P_1$	1^{++}	3510.67 ± 0.05	3496	3492	3511	3505	3468
$1\,^1P_1$	1^{+-}	3525.37 ± 0.14	3526	3496	3525	3516	3470
$1\,^3P_2$	2^{++}	3556.17 ± 0.07	3565	3511	3555	3549	3480
$2\,^3P_0$	0^{++}		3924	3932	3870	3852	3897
$2\,^3P_1$	1^{++}		3994	3984	3906	3925	3938
$2\,^1P_1$	1^{+-}		4061	3991	3926	3934	3943
$2\,^3P_2$	2^{++}		4023	4007	3949	3965	3955
$3\,^3P_0$	0^{++}		4284	4394	4301	4201	4296
$3\,^3P_1$	1^{++}		4351	4401	4319	4271	4338
$3\,^1P_1$	1^{+-}		4417	4410	4337	4279	4344
$3\,^3P_2$	2^{++}		4380	4427	4354	4309	4358
$4\,^3P_0$	0^{++}		4621	4722	4698	4509	4653
$4\,^3P_1$	1^{++}		4686	4771	4728	4576	4696
$4\,^1P_1$	1^{+-}		4752	4784	4744	4585	4704
$4\,^3P_2$	2^{++}		4715	4802	4763	4614	4718

Table 4. D-wave charmonium spectrum (in MeV).

State	J^{PC}	PDG [3]	Our Work	[36]	[37]	[38]	[27]
$1\,^3D_3$	3^{--}		3830	3798	3813	3805	3755
$1\,^3D_2$	2^{--}		3839	3814	3795	3800	3772
$1\,^3D_1$	1^{--}	3773.7 ± 0.7	3836	3815	3783	3785	3775
$1\,^1D_2$	2^{-+}		3834	3806	3807	3799	3765
$2\,^3D_3$	3^{--}		4236	4273	4220	4165	4176
$2\,^3D_2$	2^{--}		4237	4248	4190	4158	4188
$2\,^3D_1$	1^{--}		4230	4245	4150	4141	4188
$2\,^1D_2$	2^{-+}		4235	4242	4196	4158	4182
$3\,^3D_3$	3^{--}		4585	4626	4574	4481	4549
$3\,^3D_2$	2^{--}		4583	4632	4544	4472	4557
$3\,^3D_1$	1^{--}		4573	4627	4507	4455	4555
$3\,^1D_2$	2^{-+}		4582	4629	4549	4472	4553

$\psi_2(3823)$, $X(3842)$, $\psi(4160)$ and $\psi(4360)$ are assigned as $1\,^3D_2$, $1\,^3D_3$, $2\,^3D_1$ and $3\,^3D_2$ respectively [31, 41, 42, 43, 44]. But other interpretations for some of these states to be either mixed states, hybrids, tetraquarks etc., cannot be completely ruled out.

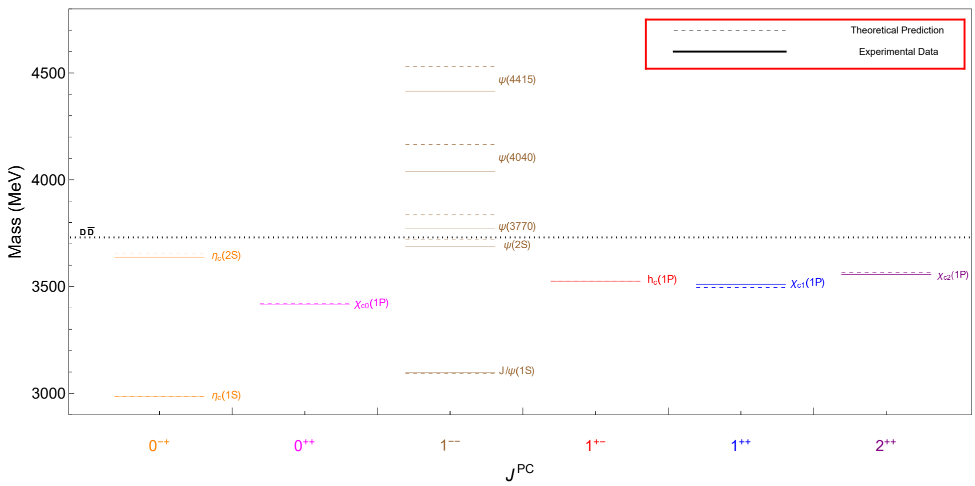

Figure 1. Comparison of predicted $c\bar{c}$ spectra with experiment.

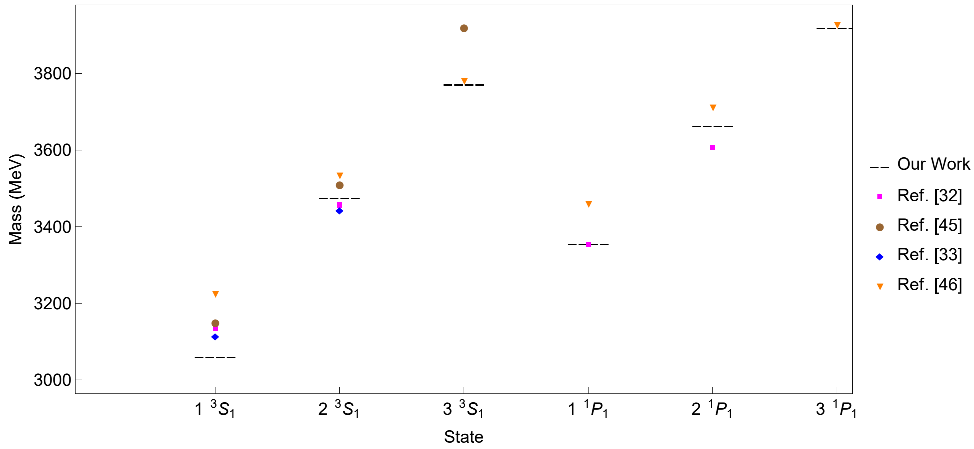

Figure 2. Comparison of cc-diquark spectrum with other models.

In Table 5 and Figure 2, we have compared the mass spectra of cc-diquarks obtained from our model with other theoretical models. We can observe from Figure 2 that the cc-diquark masses varries from model to model. The ground state mass of cc diquark obtained from our model is 3063 MeV. The ground state diquark mass obtained from our model is used further to determine the S-wave mass spectra of Ξ_{cc}^{++} and Ω_{ccc} baryons.

Table 5. cc diquark spectrum (in MeV).

State	Our Work	[32]	[45]	[33]	[46]
$1\,^3S_1$	3063	3133	3150	3114	3226
$1\,^1P_1$	3358	3353			3460
$2\,^3S_1$	3478	3456	3510	3443	3535
$2\,^1P_1$	3666	3606			3712
$3\,^3S_1$	3774		3920		3782
$3\,^1P_1$	3921				3928

3.2. Ξ_{cc}^{++} and Ω_{ccc} baryons

The S-wave mass spectra of Ξ_{cc}^{++} and Ω_{ccc} baryons are evaluated using diquark-quark model. The Ξ_{cc}^{++} baryon is considered as a bound state of cc-diquark and u quark, and Ω_{ccc} baryon is considered as a bound state of cc-diquark and c quark. The S-wave mass spectrum of Ξ_{cc}^{++} baryons are presented in Tables 6, 7 for $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ respectively. In Figures 3 and 4, we show the comparison of S-wave mass spectra of Ξ_{cc}^{++} baryons obtained from our model with other theoretical models. The ground state mass of Ξ_{cc}^{++} obtained from our model is 3621 MeV which is very close to the experimental value of 3621.6 ± 0.4 MeV. The authors in Ref [18] suggest that uc-diquark clustering is more favourable than that of cc-diquark clustering. In the present work we have considered the cc-diquark clustering which is reasonable within the heavy quark symmetry suggesting that quarks with equal masses form a compact configuration [48]. The ground state mass of Ω_{ccc} obtained from our model is 4779 MeV. We have compared the ground state mass of Ω_{ccc} obtained from our model with other theoretical models in Table 8. We see that our predicted Ω_{ccc} ground state mass is in agreement with the LQCD predictions. The S-wave mass spectrum of Ω_{ccc} are presented in Table 9.

Table 6. S-wave Ξ_{cc}^{++} spectrum corresponding to $J^P = \frac{1}{2}^+$ (in MeV).

State	Our Work	PDG [3]	[18] $\{cc\}u$	[18] $\{uc\}c$	[47]
$1S$	3621	3621.55 ± 0.23	3621	3687	3511
$2S$	4705		4478	4274	3920
$3S$	5412		5026	4666	4159
$4S$	5991		5482	4992	4501
$5S$	6501		5888	5281	4748

Table 7. S-wave Ξ_{cc}^{++} spectrum corresponding to $J^P = \frac{3}{2}^+$ (in MeV).

State	Our Work	[18] $\{cc\}u$	[18] $\{uc\}c$	[47]
1S	4057	4019	3773	3687
2S	4970	4670	4339	3983
3S	5631	5170	4725	4261
4S	6187	5602	5048	4519
5S	6681	5993	5334	4759

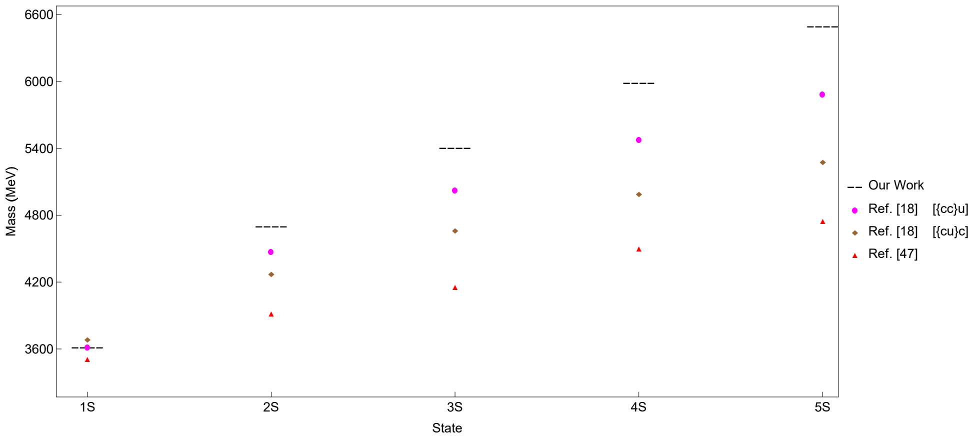

Figure 3. Comparison of S-wave mass spectra of ccu baryon corresponding to $J^P = \frac{1}{2}^+$ with other models.

Table 8. Comparison of ground state mass of Ω_{ccc} (in MeV).

Approach	Mass
Our Work	4779
LQCD [49]	4796
LQCD [50]	4769
LQCD [51]	4789
LQCD [52]	4761
LQCD [53]	4733
Hypercentral CQM [54]	4806
Constituent QM [55]	4965
Bag model [56]	4777
Variational Cornell [57]	4799
QCD sum rules[58]	4670
Relativistic Quark model [59]	4803
Quark diquark model [60]	4760

Table 9. S-wave Ω_{ccc} spectrum corresponding to $J^P = \frac{3}{2}^+$ (in MeV).

State	Our Work	[54]
1S	4779	4806
2S	5362	5300
3S	5764	5865
4S	6097	6488
5S	6390	7037

4. SUMMARY

We have evaluated the mass spectra of $c\bar{c}$, cc, Ξ_{cc}^{++}, and Ω_{ccc} systems using a phenomenological potential model incorporating a logarithmic correction term. The evaluated mass spectra of charmonium are consistent with the experimental results. The ground state mass of Ξ_{cc}^{++} obtained from our model is close to the experimental value, which validates the diquark-quark model and also supports our assumption that the Ξ_{cc}^{++} baryon can be treated as a bound state of cc-diquark and u quark. The S-wave mass spectrum of Ω_{ccc} baryon have been evaluated. The Ω_{ccc} ground state mass is comparable

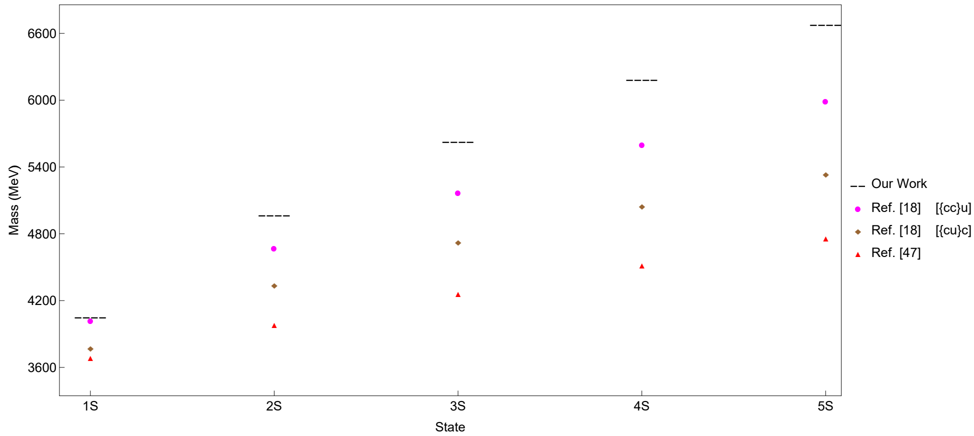

Figure 4. Comparison of S-wave mass spectra of ccu baryon corresponding to $J^P = \frac{3}{2}^+$ with other models.

with the LQCD predictions. Since we have no experimental data for Ω_{ccc} baryon, we need to rely on theoretical models to investigate its properties.

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СПЕКТРИ ДЕЯКИХ ЧАРІВНИХ АДРОНІВ У НЕРЕЛЯТИВІСТСЬКІЙ МОДЕЛІ

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У нерелятивістських рамках досліджуються мас-спектри систем $c\bar{c}$, cc, ccc та ccu. Потенціал складається з потенціалу Корнелла разом із логарифмічним коригувальним членом, як це запропоновано з ґраткової КХД. Ми аналізуємо хвильові стани чармонію S, P та D, а також хвильові стани cc дикварка S та P та порівнюємо їх з існуючими результатами експериментів та інших потенційних моделей. Використовуючи кварк-дикваркову модель, ми оцінили S-хвильові спектри подвійно зачарованого баріона Ξ_{cc}^{++} та тричі зачарованого баріона Ω_{ccc}. Ці маси порівнюються з іншими теоретичними дослідженнями.

Ключові слова: *нерелятивістська потенціальна модель; поправки LQCD; зачаровані адрони; модель дикварка-кварка; мас-спектр*