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# TWO-PHASE INCLINED MHD BLOOD FLOW IN POROUS TUMOR REGION WITH CONCENTRATION AND VOLUME FRACTION

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There are different approaches for treating invasive and non-invasive tumors. The flowing fluid (blood) provides the required nutrients to the tumors and absorbs the suspensions. Drug delivery systems are dependent on the medium (flowing fluid) that carries the drugs. The blood vessels usually carry drugs to the targeted regions that treat the affected region. This situation varies the concentration of the tumor surrounding the medium. The system is monitored under a magnetic field that is applied at an angle ($0 < \alpha < 90$). The system of the blood flow surrounding a tumor is governed by partial differential equations (PDEs). The Governing equations are solved using the mathematical function PDEPE in MATLAB. The effects of different parameters, concentration parameters, inclined magnetic field, porosity, on fluid (blood) velocity, and medication (drug) velocity in the presence of volume fraction. Flow patterns so obtained show significant effects that help to treat the deceased regions clinically. The numerical results are interpreted through the graphs drawn.
**Keywords:** *Two-Phase Inclined MHD Blood Flow; Porous Tumor Regions; Concentration; Volume Fraction*
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## INTRODUCTION

Cancer in the present is a big threat to the whole society, as this is the only cause of death compared to HIV, TB, and Malaria altogether. The cancer region (tumor region) is treated using different therapies like radiotherapy, immunotherapy, chemotherapy, and targeted therapy. There are ways to send the drugs, like external agents (medicines/drug particles) through the drug carriers, like blood flow. Life expectancy is very unpredictable due to huge deaths from cancer, heart attacks, and many other diseases, even after a massive development in medical science. This challenge is mounting day by day, so the required services and infrastructure are needed more [1]. Several deaths are predicted due to the late detection of tumors, which are fed by the flowing fluid (blood-carrying suspensions) surrounding the tumor spheroids. Surrounding the tumor spheroids multiphase fluid flow system occurs. Tumors are found at the apex of bifurcation in bifurcated arteries. The volume fraction is the space occupied by each phase, and the laws of conservation of mass and momentum are satisfied by each phase individually.

Plenty of researchers have tried to contribute to the understanding of the Two-phase flow behavior of fluid. Singh and Singh [2] studied a fluid suspended with dust particles. Heat and mass transfer effects were seen in the presence of volume fraction along with a magnetic field applied normally to the flow channel. Byrne and King [3] studied two-phase solid tumor growth. Bogdanov et al. [4] investigated two-phase flow through porous media. Prakash et al. [5] investigated the blood flow behavior in bifurcated arteries. The authors applied an external heat source to the flowing system. The authors provided the blood flow variations in the presence of different parameters. Chaudhary et al. [6], Abbas et al. [7], and Madhura and Shweta [8] provided their investigations on two-phase studies with volume fractions. Volume fractions play a vital role when fluid flows through porous media. There are pores in a medium that absorb the suspension-like drug, which is carried by blood in case of flow through the tumor spheroids. That enhances the size of the tumor. Chen et al. [9] used positron emission tomography to understand tumor permeability and blood flow transactions. Thulborn et al. [10] studied the view of chemoradiation, the number of drugs used during chemotherapy, tumor cell volume, and amount of cell killing volume. Thus, further studies of volume fraction along with heat stress create significant scope in tumor studies. Wirthl et al. [11] and Bera et al. [12] examined tumor development under the multiphase system. Flowing fluids carrying the nanoparticle suspension. The magnetic field is applied under thermal conditions. Kumar et al. [13] studied blood flowing through the apex of a bifurcated artery with a tumor. The amount of drug is applied that creates chemical reactions, and this supports tumor treatments. Heat transfer is applied where an inclined magnetic field supports the study.

Darvishi et al. [14] provided a cost-effective drug delivery system for tumor treatments using magnetized nanoparticles, which is nothing but monitoring of drugs as a model of volume fraction in the presence of heat and mass transfer. Most recently Wang et al. [15] investigated the impacts of porosity and volume fraction to understand the atomic behavior of cancer cells. The authors have used the molecular dynamics method in the formation of hematogenous metastasis. Yadav et al. [16] have investigated a two-phase blood flow study through the porous channel.

Maslov et al. [17] discussed drug deliveries in terms of their concentration. The authors focused on heavy concentrations of the drugs affecting drug distribution and the local region biologically. The findings say that high

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concentrations give good agreement with high concentrations. Shakya [18] identified the role of chemistry in cancer treatment. The harmful disease for the whole of humanity, cancer, is treated by several therapies using external drugs in the form of tablets. Chemical reactions are observed to serve the badly affected regions. Recent chemistry roles are developing the efficacy of drug delivery systems, and it has been proven to be cost-effective, too. Ezike et al. [19] investigated and compared the systematic drug delivery systems with the usual ones. The authors did a critical review of their observations that the targeted, biocompatible, biologically acceptable drugs are giving better results in the form of nanomaterials. They found that the clinically administered drugs that change and affect the biological environment can be administered well to treat the patients without harming them. In the critical review of Jain [20], fluid(blood) transport plays a key role in growth, metastasis, identification, and survival from tumor decreases. Radiotherapy requires a rich amount of oxygen in the tumor environment. In contrast, chemotherapy and immune therapy require a significant number of drugs that can be carried out through the blood vessels, along with blood. Thus, this study of blood transport with drug suspensions with different parameters becomes a necessity. In the present investigation, a single-phase study of blood flow in a bifurcated region is further studied with some drug-loaded (concentration) suspension with fluid. Thus, the two-phase flow behavior of flowing biofluid in the tumor-surrounded region is investigated as a drug delivery system.

## MATHEMATICAL FORMULATION

The flow channel is considered as a conducting parallel plate channel at 2b apart. The flowing fluid blood of density $\rho$ is considered Newtonian, homogeneous, and viscous. The flowing channel has a tumor region of rectangular shape, having pores surrounding it. The non-conducting drug mixture is loaded into the blood. The flowing region is under an external magnetic field $B_0$ applied at an inclination of angle $\alpha$. Mostly, these tumors are found at the apex of the bifurcation of the arterial flow (channel), where the angle of bifurcation is zero. The magnetic field produced due to induction is negligible, and the Reynolds number Re is kept low to maintain the flow laminar.

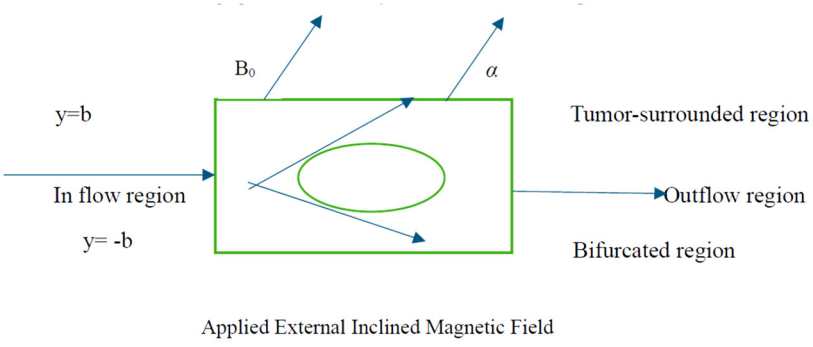


**Figure 1**. Schematic diagram of the flowing fluid surrounding the tumor spheroid

Blood flow through a porous medium under a magnetic field and mass transfer, followed by the assumptions considered in the mathematical formulation, is taken as two-dimensional boundary layers. Let $u^*$ and $v^*$ are the velocity of blood and suspension (drug), respectively, at time t in the flow field. $\eta$ is the viscosity of the blood while $p^*$ is pressure. $\beta'$ is the volumetric expansion parameter for concentration, $\alpha$ is the angle of inclination of the applied magnetic field, and $K^*$ is a porosity parameter.

$$(1-\phi^*)\frac{\partial u^*}{\partial t^*} = (1-\phi^*)\left(-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \frac{\eta}{\rho}\frac{\partial^2 u^*}{\partial y^{*2}}\right) + g\beta'(C - C_\infty)^* + K'N(u^* - v^*) - \frac{\sigma B_0^2 \sin^2\alpha}{\rho}u^* - \frac{1}{K^*}u^* \quad (1)$$

$$m_p\frac{\partial v^*}{\partial t} = K'(u^* - v^*) \quad (2)$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \quad (3)$$

D is for diffusion of drug particles
The non-dimensional variables are

$$x = \frac{x^*}{b}, y = \frac{y^*}{b},\ \ u = \frac{u^*}{m/2b\rho},\ v = \frac{v^*}{m/2b\rho},\ h = \frac{-(dp^*/dx^*)}{(\eta m/2b^3\rho)},\ \tau = \frac{\eta}{\rho},\ t = \frac{t^*}{\left(\frac{b^2\rho}{\eta}\right)},\ C = \frac{C^*(2b^3\rho^2)}{m\eta},\ K = \frac{K^*}{\left(\frac{b^2\rho}{\eta}\right)}$$

The non-dimensional governing equations of motion are:

$$\frac{\partial u}{\partial t} = h + \frac{\partial^2 u}{\partial y^2} + \frac{1}{(1-\phi)}\left(g\beta' C - M^2 \sin^2\alpha\, u - R(u-v) - \frac{u}{K}\right) \quad (4)$$

$$G\frac{\partial v}{\partial t} = u - v, \quad (5)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad (6)$$

Sc (Schmidt number) $= \frac{\eta}{D\rho}$, $M^2$ (Magnetic Field Parameter) $= \frac{kn}{\sigma b^2}$, (Porosity parameter) $= \frac{kn}{\sigma b^2}$, $G$ (Particle Mass Paprameter) $= \frac{m_p \mu}{\rho h^2 K'}$, $R$ (Particle Concentration Parameter) $= \frac{K' N h^2}{\mu}$

The boundary conditions are:

$$u = e^{-\lambda^2 t}, v = e^{-\lambda^2 t} \ , \ C = e^{-\lambda^2 t} \quad at\ y = -1$$
$$u \to 0, \quad v \to 0, \quad C = 0 \quad at\ y = 1 \tag{7}$$

The governing PDEs are solved by the function PDEPE in MATLAB. The PDEPE solver available in MATLAB is an easy and practical tool for simulating and visualizing models, serving as an alternative approach for solving initial boundary value problems (IBVP) related to parabolic and hyperbolic equations. The PDEPE is a built-in function that employs a second-order spatial discretization technique, which is developed by utilizing the $x$-Mesh and t-Span parameters. The accuracy, consistency, and expense of the computational approach rely on the mesh size employed. The function PDEPE operates in tandem with the stiff ODE solver ode15 for effective calculations, utilizing time step values that are dynamically modified to maintain the precision of the PDE solutions.

## RESULTS AND DISCUSSION

The transport of biofluid (blood) and the suspensions (drug material) is investigated in terms of velocity profiles for the different phases against different parameters through Figures 2 to 10. In this investigation of numerical results, some of the values are considered fixed values $t = 1, \tau = 0.25$, h=0.5, K = 2, R = 0.5(concentration), G = 0.8(particle mass), $\lambda = 1$, M=0.5, $\alpha$ =$30^0$, $\phi = 0.2$.

The amount of drug is loaded in the fluid for treating the affected tumor-surrounded region, which is mathematically formulated as a concentration amount. Thus, fluid concentration will change the viscosity of the fluid even if the flow is Newtonian. Figures 2-7 represent the velocity profiles for the drug-carrying fluid (blood) and suspension (drug) in the affected region for the magnetic field intensity M(0.5, 1, 1.5), concentration of drug amount R (0.5, 1, 1.5), and decay parameter $\lambda$ (0.8, 0.9, 1.0), angle of inclination of the applied magnetic field $\alpha(30^\circ, 45^\circ, 60^\circ)$, added particle mass G (0.8, 1, 1.2) and the Schmidt number Sc (2, 2.4, 2.8). The flow regions are considered in the entire channel region between $y = -1$ and $y = 1$. Velocity profiles are showing decreasing trends against increasing values of the parameters, respectively. Suspension particle (drug) velocity is less than that of the drug carrier. Velocity at the walls is almost zero due to the drag force at the slip region.

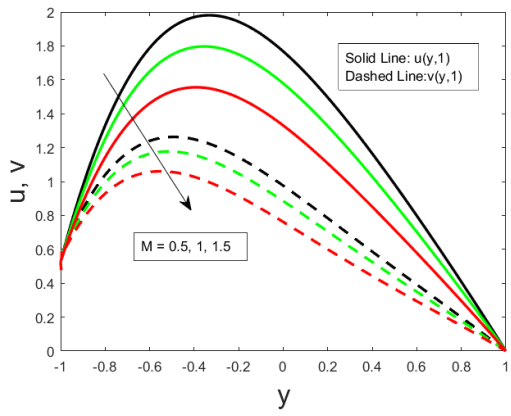


**Figure 2.** Fluid and suspension velocity for magnetic field M

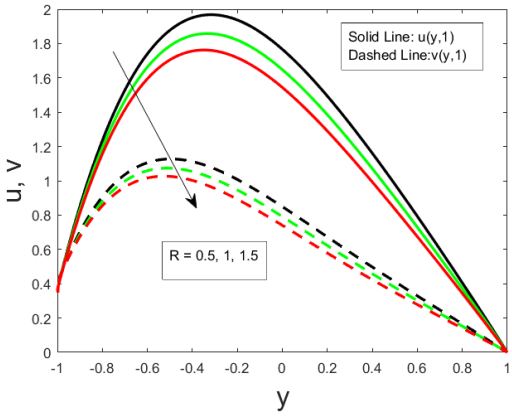


**Figure 3.** Fluid and suspension velocity for particle concentration R

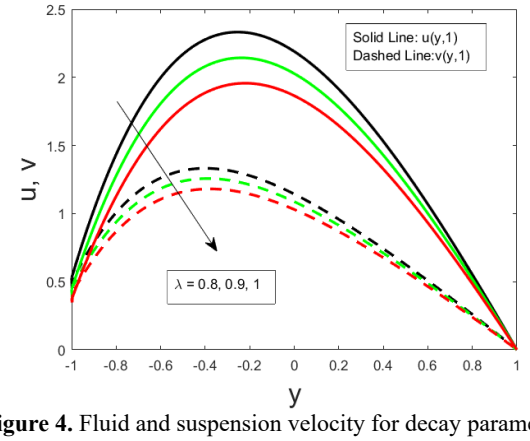


**Figure 4.** Fluid and suspension velocity for decay parameter λ.

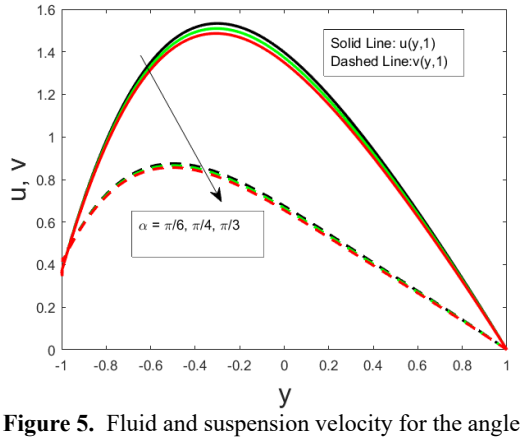


**Figure 5.** Fluid and suspension velocity for the angle of inclination α

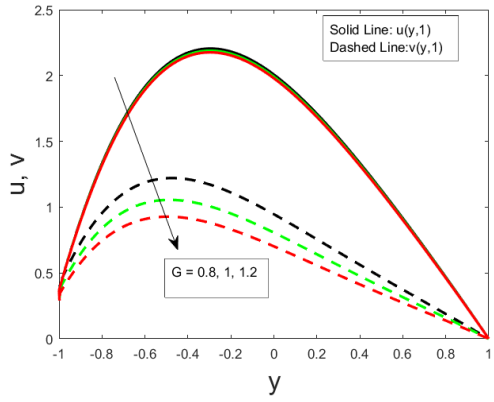


**Figure 6**. Fluid and suspension velocity for particle mass parameter G

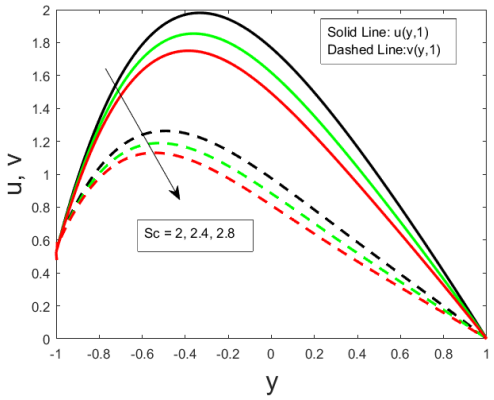


**Figure 7**. Fluid and suspension velocity to Schmidt number Sc

Figures 8-10 are structured to represent the velocity profiles for the drug-carrying fluid (blood) and medication (drug) in the affected region for the volumetric expansion coefficient $\beta'(1.5, 2, 2.5)$, volume fraction parameter $\phi(0.01, 0.1, 0.2)$ and porosity parameter K (1.5, 2, 2.5). The flow regions are considered the entire channel region between $y = -1$ and $y = 1$. Velocity profiles are showing decreasing trends against increasing values of the parameters, respectively. Suspension particle (drug) velocity is less than that of the drug carrier. Velocity at the walls is almost zero due to the drag force at the slip region.

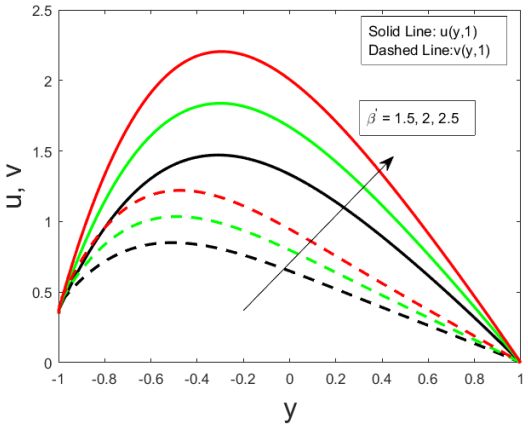


**Figure 8**. Fluid and suspension velocity for volumetric expansion due to concentration $\beta'$

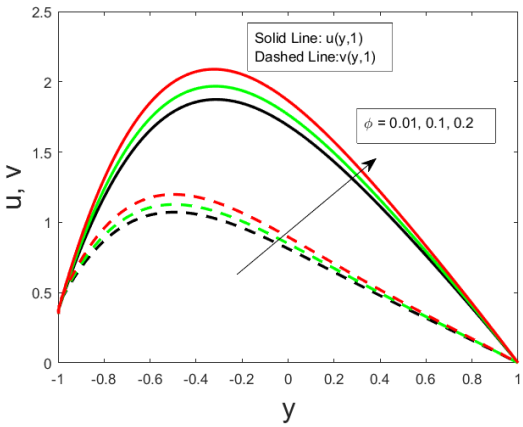


**Figure 9.** Fluid and suspension velocity for volume fraction ϕ

Figures 11 and 12 are depicted for the concentration profile against the Schmidt number Sc (2, 2.4, 2.8) and the decay parameter $\lambda$ (1, 1.1, 1.2). Both parameters decrease the concentration profile for increasing values. This indicates that once the diffusion of the drug is reduced, the concentration will also be reduced. Moreover, due to the flow force fraction, one of the boundaries of the tumor region is concentrated more, which increases the drug load. This will improve the efficacy of drug delivery.

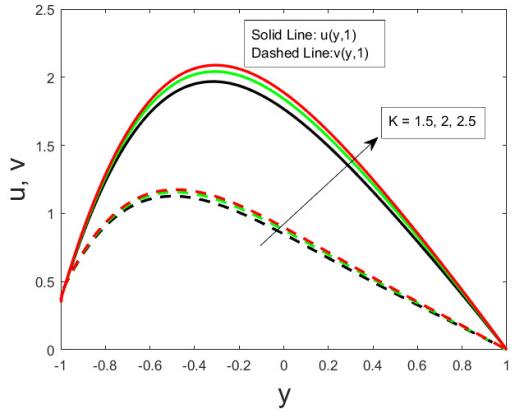


**Figure 10.** Fluid and suspension velocity for porosity parameter K

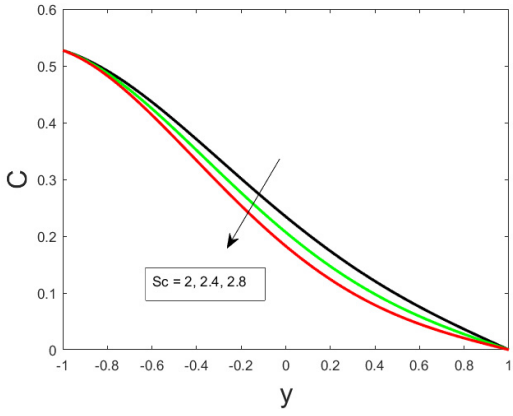


**Figure 11**. Concentration profile to Schmidt number Sc

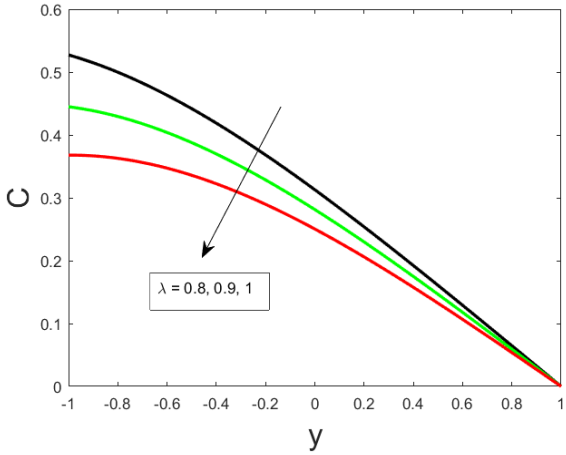


**Figure 12.** Concentration profile for decay parameter λ

## CONCLUSION

The present investigation is conducted to see the effect of volume fraction and high drug concentration on two-phase flow behavior in the tumor surrounding regions. Effects of various parameters on flow profiles of blood-carrying medications (drug) and concentration profiles are concluded as:

- The concentrated drug is flowing in the entire affected region along with a mixture of many flowing components.
- The flowing two-phase fluid (blood) along with the medications (drug particles) mixed with are, is transported with varying velocity profiles with a pulsating nature, with almost zero at the boundaries and maximum in the middle of the channel.
- The drug-carrying fluid supports the transport of the drug in the affected region as the medication velocity is less than that of the drug carrier.
- The increasing magnetized region (M) that is applied at different increasing angles ($\alpha$) gives a reduced flow as the high concentration is used within the flow channel to treat the affected region.
- Increasing values of R (particle concentration parameter), $\lambda$(decay parameter), G (suspension mass parameter), Sc (Schmidt Number) reduces the velocity of both phases. In the presence of dense medications (heavy drug load), low diffusions, and reduced flow rates give good indications to reduce inflammations.
- Rising values of K (porosity parameter), $\beta'$(volumetric expansion parameter for concentration), $\emptyset$ (volume fraction parameter) increases the velocity for both the fluid phases. These increasing velocities need to be monitored during treatment as flow rates will increase, which will lead to a jump in pressure on the walls of the fluid carrier.
- Increasing $\lambda$, and Sc (Schmidt number) reduces the concentration profile of the affected area as diffusion is reduced.

The above results are based on a mathematical formulation and computational approach. Further, these numerical approaches need to be validated clinically by the medical community.

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**Conflict of Interest**

There is no conflict of interest.

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## ДВОФАЗНИЙ ПОХИЛИЙ МГД-ПОТОК КРОВІ В ПОРИСТІЙ ДІЛЯНЦІ ПУХЛИНИ З КОНЦЕНТРАЦІЄЮ ТА ОБ'ЄМНОЮ ФРАКЦІЄЮ

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Існують різні підходи до лікування інвазивних та неінвазивних пухлин. Рідина, що тече (кров), забезпечує пухлини необхідними поживними речовинами та поглинає суспензії. Системи доставки ліків залежать від середовища (рідини, що тече), яке переносить ліки. Кровоносні судини зазвичай переносять ліки до цільових ділянок, які лікують уражену ділянку. Ця ситуація змінює концентрацію пухлини навколо середовища. Система контролюється під дією магнітного поля, яке прикладається під кутом (0<\α<90). Система кровотоку навколо пухлини визначається диференціальними рівняннями в частинних похідних (РЧП). Визначальні рівняння розв'язані за допомогою математичної функції PDEPE в MATLAB. Вплив різних параметрів, параметрів концентрації, похилого магнітного поля, пористості, на швидкість рідини (крові) та швидкість ліків (препаратів) за наявності об'ємної фракції. Отримані таким чином картини потоку демонструють значний вплив, що допомагає клінічно лікувати уражені ділянки. Числові результати інтерпретуються за допомогою побудованих графіків.

**Ключові слова:** *двофазний похилий МГД-кровотік; пористі ділянки пухлини; концентрація; об'ємна частка*