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EVENT FRACTIONS AND PROBABILITIES FOR ^{11}Be + ^{209}Bi REACTION IN THE MULTIBODY 3-STAGE CLASSICAL MOLECULAR DYNAMICS APPROACH

Vipul B. Katariya[a*], **Subodh S. Godre**[a], **Pinank H. Jariwala**[b]
[a] *Veer Narmad South Gujarat University, Surat - 395007, INDIA*
[b] *Department of Physics, Navyug Science College, Surat - 395009, INDIA*
**Corresponding Author e-mail: vipulkatariya5611@gmail.com*
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Unique structures that differ radically from ordinary nuclear matter have been demonstrated by Halo nuclei. Among other halo nuclei, the ^{11}Be nucleus is one of the most studied halo nuclei, and it has a well-established one-neutron halo structure with neutron separation energy S_n = 0.501 MeV. We have studied ^{11}Be + ^{209}Bi reaction in the multibody 3-Stage Classical Molecular Dynamics (3S-CMD) model, where ^{11}Be is constructed as a cluster of tightly bound ^{10}Be and one neutron. The separation between ^{10}Be and neutron is adjusted to set the ion-ion potential between them equal to the experimental neutron separation energy. For this reaction, we have calculated fractions of events *F(b)* for given impact parameter, *b* and collision energy E_{CM} and event probabilities *P(*E_{CM}*)* by integrating *F(b)* over all the values of $b \leq b_{max}$ (b_{max} is impact parameter above which all trajectories results in scattering) for the given E_{CM}. Here in present calculations, it is found that for near barrier energies, neutron transfer is significant for lower impact parameters, but at far above barrier energies, complete fusion dominates for lower impact parameters, while for slightly higher impact parameters, neutron transfer is accountable.

Keywords: *Fusion reactions; Halo nuclei; Heavy-ion collisions; Weakly-bound nuclei; Classical Molecular Dynamics*

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1. INTRODUCTION

There is a remarkable class of nuclei known as Halo Nuclei, which is similar to weakly bound nuclei in many ways, but their structure is strikingly different from normal nuclear matter. These light nuclei are situated near the very boundary of nuclear stability. Several remarkable features were revealed by the groundbreaking experiments [1–4] that were conducted to study characteristics of halo nuclei. Notably, these nuclei exhibit very high interaction cross sections, possess loosely bound outer nucleons with low separation energies (resulting in new surface densities and unique structures), and display exotic modes of collective vibration in the nucleus due to their large dissociation cross sections by high-Z targets. Similar to other weakly bound nuclei, halo nuclei exhibit high breakup probabilities as their outer orbiting protons or neutrons, forming a halo surrounding the core nucleus, have very low separation energy, which can lead to fusion with the target, resulting in incomplete fusion (ICF).

Much interest has been around halo nuclei is due to the development of radioactive beam facilities, which allow studying nuclides at the limits of stability. Among halo nuclei, ^{11}Be is one of the most studied ones and has well well-established one-neutron halo structure having very low neutron separation energy S_n = 0.501 MeV [5]. Due to the low separation energy of the valence neutron, this neutron has a relatively high probability of being knocked out. Such breakup effects can be better investigated with a heavier target (i.e., ^{209}Bi) due to the increasing predominance of long-range Coulomb interaction compared to the nuclear potential [6]. Here, the ^{11}Be + ^{209}Bi reaction is studied using the multibody 3-stage Classical Molecular Dynamics (3S-CMD) model, for which ^{11}Be is constructed as a cluster of tightly bound ^{10}Be and one neutron. In the multibody 3S-CMD model, by systematically removing rigidity constraints on the target and projectile, calculations are carried out. As a result, this model can account for a direct reaction like a neutron transfer from the projectile in the present case. Therefore, Event fractions, *F(b)* and event probabilities, *P(*E_{CM}*)* as a function of impact parameter, *b* and center of mass frame energy, E_{CM} are extracted from this calculation.

In this paper, the details of the model are presented in Section – 2 , while the details and the results for event fractions for ^{11}Be + ^{209}Bi are discussed in Section – 3. Finally, conclusions are summarized in Section – 4.

2. MODEL DETAILS

The multibody 3-Stage Classical Molecular Dynamics (3S-CMD) model is used to simulate the ^{11}Be + ^{209}Bi reaction. The projectile ^{11}Be is constructed as a cluster of tightly bound ^{10}Be and one neutron. Tightly bound ^{10}Be and ^{209}Bi are constructed using variational potential energy minimization STATIC [7] code and "cooled" using DYNAMIC [7] code,

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for which a purely phenomenological soft-core Gaussian potential is used, which is given by,

$$V_{ij}(r_{ij}) = -V_0\left(1 - \frac{C}{r_{ij}}\right)\exp\left(-\frac{r_{ij}^2}{r_0^2}\right) \quad (1)$$

where the typical form of the Coulomb potential between protons is,

$$V_C\left(r_{ij}\right) = \frac{1.44}{r_{ij}}(\text{MeV}) \quad (2)$$

and the potential parameter set V_0 = 710.0 MeV, C = 1.88 fm and r_0 = 1.15 fm is used to produce ground state properties of nuclei. Using a "dynamic cooling" method by carrying out rigid body dynamics [8] like procedure and setting the cluster velocities and their angular moment zero after every time-step and thus obtaining the equilibrium orientation and position of the centre of mass of these constituents (^{10}Be + *n*). Now, the distance between the center of mass of ^{10}Be and the neutron is adjusted in such a way that the typical ion-ion potential between them is equal to the experimental neutron separation energy of ^{11}Be. Ground-state properties of generated nuclei are mentioned in Table 1 below.

Table 1. Ground-state properties.

Nucleus	Calculated		Experimental	
	B.E. (MeV)	R (fm)	B.E. (MeV) [9]	R (fm)
^{10}**Be**	59.68	2.06	65.97	2.28 [10]
^{11}**Be** (^{10}Be + *n*)	60.18	2.09	65.55	2.90 [10]
^{209}**Bi**	1606.6	5.55	1640.26	5.52 [11]

In this multibody 3S-CMD model, there are three stages involved in the simulation:

1. Rutherford Trajectories: The target and projectile are initially brought along classical Rutherford trajectories, considering their Coulomb interaction.

2. Classical Rigid Body Dynamics (CRBD): The system evolves dynamically to approach the fusion barrier, incorporating collective motion and interactions [8].

3. Classical Molecular Dynamics (CMD): The entire multibody system undergoes CMD evolution, allowing for interactions and dynamic evolution of the system [7].

In this model, for the first and second stages, the projectile and target are treated as complete rigid bodies, while during the third stage, this rigidity constraint on the target and projectile can be relaxed. In the present calculation, during the third stage of simulation in the multibody 3S-CMD model, for ^{11}Be (^{10}Be + *n*), the bond between ^{10}Be and the neutron is relaxed and considered non-rigid, while ^{10}Be is treated as a rigid body. Also, the rigidity constraint on ^{209}Bi is relaxed during the third stage.

Previous applications of the multibody 3S-CMD model to the ^{11}Be + ^{209}Bi reaction have yielded fusion cross-sections in good quantitative agreement with experimental data (Figure 1) [12]. Furthermore, these studies emphasized the significant influence of rigidity constraints on the calculated fusion outcomes. In Figure 1, SBPM represents the results of the Static Barrier Penetration Model [13] calculation, where all degrees of freedom are suppressed for the target and projectile. (a) ^{11}Be[^{10}Be(R)-R-n] + ^{209}Bi(NR) represents a case where the ^{11}Be is treated as completely rigid and the target ^{209}Bi is treated as non-rigid. (b) ^{11}Be[^{10}Be(R)-NR-n] + ^{209}Bi(NR) represents a case where the ^{10}Be core of ^{11}Be is treated as rigid, but the bond between the ^{10}Be core and the neutron is treated as non-rigid, and the target ^{209}Bi is treated as non-rigid. So, we used the multibody 3S-CMD approach here to investigate event fractions and probabilities across different event channels. The calculation details of event fractions and probabilities, along with the results for the ^{11}Be + ^{209}Bi reaction, are discussed in Section – 3.

3. EVENT FRACTIONS AND PROBABILITIES

During the collision between weakly bound or halo nuclei and a heavy target, different events like complete fusion, incomplete fusion, and direct reactions like nucleon transfer can take place. Here, for such possible events, the fraction of

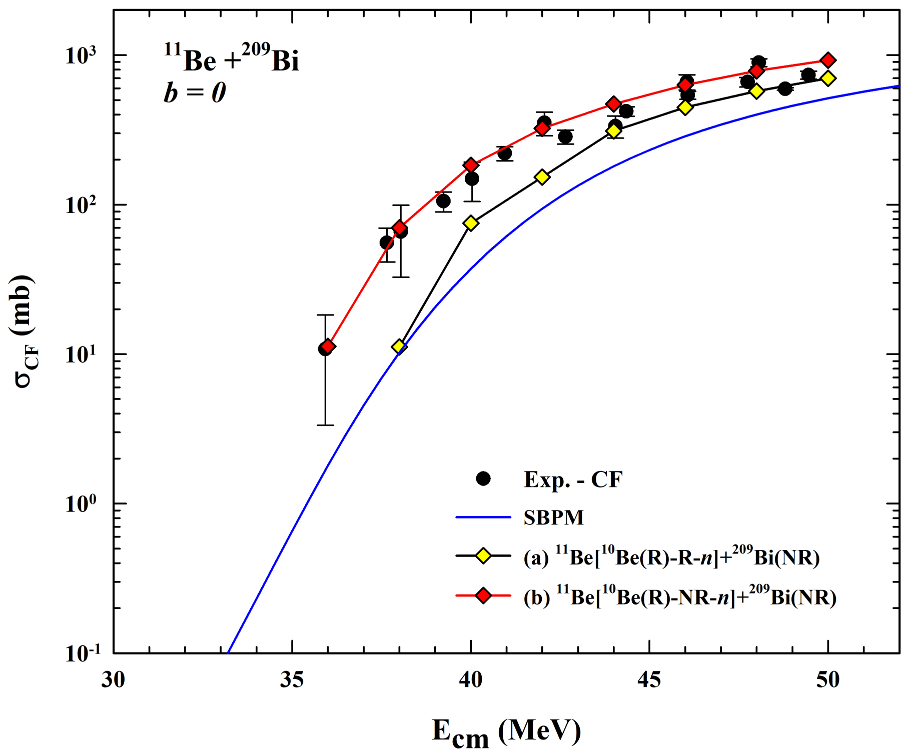

Figure 1. Complete fusion cross sections for the ^{11}Be + ^{209}Bi reaction. Exp. – CF [5].

events, *F(b)*, for the given E_{CM} and b can be defined as,

$$F\left(b, E_{CM}\right) = \frac{N_{events}}{N_{total}} \tag{3}$$

Where, N_{total} = Total number of trajectories for initially random orientations for given E_{CM} and b. N_{events} = total number of events leading to complete fusion, a direct reaction like a neutron transfer, scattering, etc. The event fraction is a function of both b and E_{CM}, but as we have calculated event fractions as a function of b at a given E_{CM}, therefore, for the sake of convenience, we have used the notation *F(b)* instead of $F(b, E_{CM})$ to denote event fractions.

The event probabilities, $P(E_{CM})$, for all possible events can be found by integrating *F(b)* over all the values of $b \leq b_{max}$ for given E_{CM},

$$P\left(E_{CM}\right) = \left[\frac{1}{\left(b_{max}/\Delta b\right) + 1}\right] \int_{b}^{b_{max}} F(b) db \tag{4}$$

We have calculated fraction of event, *F(b)* for different three different collision energies, (i) E_{CM} = 38 MeV (Figure 2a) (ii) E_{CM} = 40 MeV (Figure 2b) and (iii) E_{CM} = 50 MeV (Figure 2c) in which 38 MeV and 40 MeV are near barrier energy and 50 MeV is far above barrier energy.

For E_{CM} = 38 MeV (Figure 2a), slightly below the barrier energy, it can be seen that for lower impact parameter neutron transfer from a projectile and direct complete fusion is quite significant while for higher impact parameter most trajectories result in the non-breakup scattering. For E_{CM} = 40 MeV (Figure 2b), slightly above the barrier energy, for lower impact parameters neutron transfer and direct complete reaction are more probable than non-breakup scattering while for higher impact parameters most trajectories result in the non breakup scattering. For E_{CM} = 50 MeV (Figure 2c) for lower impact parameters almost all trajectories result in direct complete fusion but for grazing-like collisions neutron transfer is accountable. For higher impact parameters trajectories results in non-breakup scattering.

For these three energies, Event probabilities, $P(E_{CM})$, are calculated, which are shown in Figure 3. From this, it can be understood that the probability of direct complete fusion increases as the collision energy increases, whereas the probability of no breakup scattering decreases as the collision energy increases. For the neutron transfer, the impact parameter plays a crucial role.

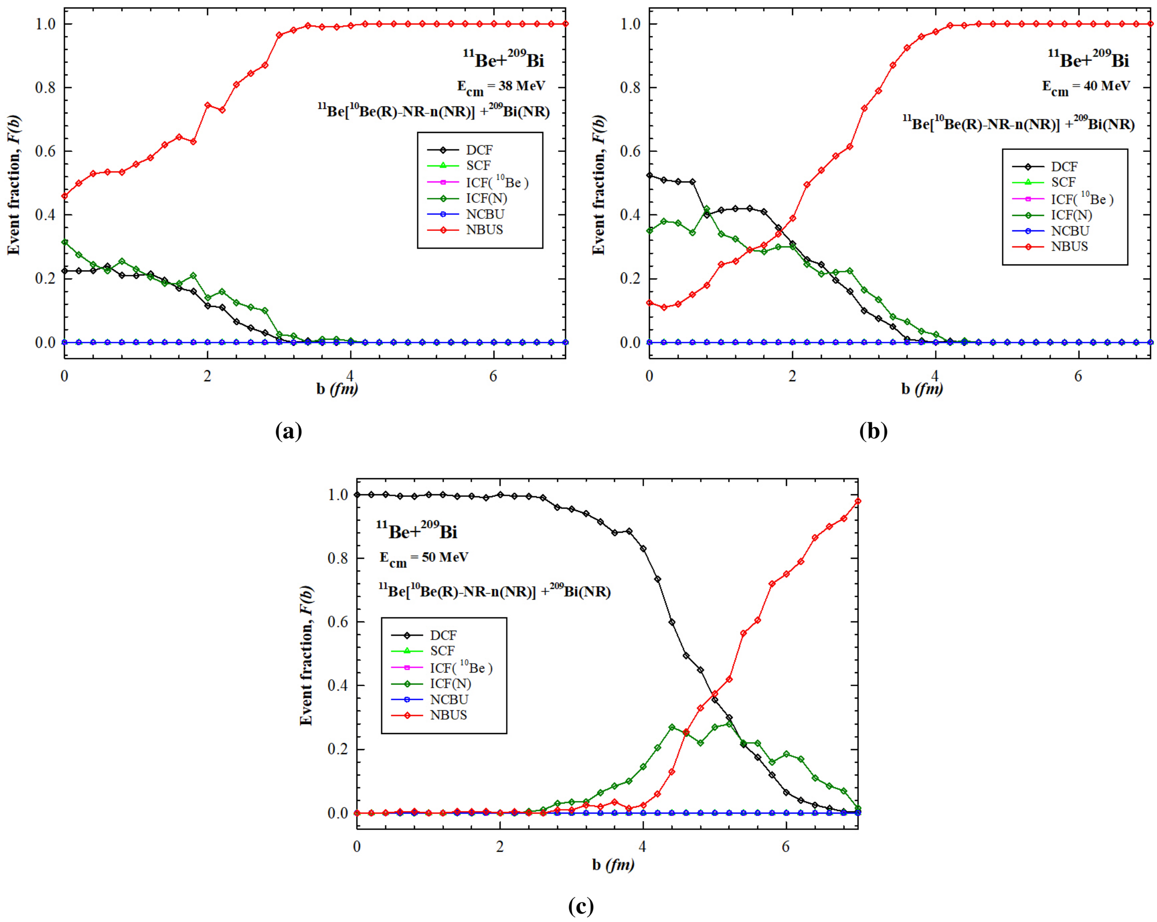

Figure 2. Fraction of event, *F(b)* for different three different collision energies, (a) E_{CM} = 38 MeV (b) E_{CM} = 40 MeV and (c) E_{CM} = 50 MeV.

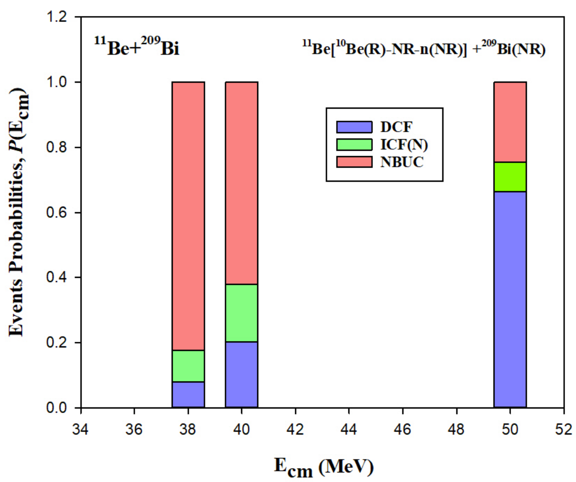

Figure 3. Event probabilities, $P(E_{CM})$ for the system ^{11}Be (^{10}Be + n) + ^{209}Bi for three different collision energy

4. CONCLUSION

The multibody 3S-CMD model calculations are carried out to investigate the ^{11}Be + ^{209}Bi reaction dynamics. The ^{11}Be halo nucleus is explicitly treated as a ^{10}Be core plus a weakly bound neutron, with their interaction potential matched to the experimental separation energy. In this model, the reactions can be simulated with various rigidity constraints on the projectile fragments, bond between them and the target, which enables the dynamical simulation of various outcomes like complete fusion (CF), neutron transfer, scattering, etc. We calculated event fractions, $F(b)$, versus impact parameter (b) and integrated event probabilities, $P(E_{CM})$, at near-barrier (E_{CM} = 38, 40 MeV) and above-barrier (E_{CM} = 50 MeV) energies. The results highlight a strong dependence of reaction mechanisms on both energy and impact parameter. Near the barrier, neutron transfer significantly competes with complete fusion, particularly in central collisions. At the higher energy (50 MeV), complete fusion dominates at low impact parameters, while neutron transfer becomes the most probable channel for grazing trajectories. The overall probability for CF increases with collision energy, counterbalanced by a decrease in scattering probability. The multibody 3S-CMD model effectively captured the interplay and energy-dependent competition between complete fusion, neutron transfer, and scattering channels in reactions induced by the ^{11}Be halo nucleus. Moreover, understanding the reaction dynamics of Halo nuclei projectiles like ^{11}Be with heavy targets such as ^{209}Bi provides insights into the behaviour of halo nuclei, which can help in improving the accuracy of reaction models used in astrophysical nucleosynthesis simulations, which are used for predicting element formation in stellar environments and understanding energy production in stars. The analysis of neutron transfer, complete fusion processes, and the energy dependence of reaction mechanisms can also facilitate in refining conditions for neutron-rich isotope production.

ORCID

Vipul B. Katariya, https://orcid.org/0009-0001-7215-8128; **Subodh S. Godre**, https://orcid.org/0000-0001-5677-648X; **Pinank H. Jariwala**, https://orcid.org/0000-0003-1141-0761

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ЧАСТКИ ПОДІЙ ТА ЙМОВІРНОСТІ ДЛЯ РЕАКЦІЇ ^{11}Be + ^{209}Bi В БАГАТОЧАСТИНКОВОМУ 3-СТАДІЙНОМУ КЛАСИЧНОМУ МОЛЕКУЛЯРНОМУ ДИНАМІЧНОМУ ПІДХОДІ

Віпул Б. Катарія[a], **Субодх С. Годре**[a], **Пінанк Х. Джарівала**[b]
[a] *Університет Віра Нармада Південного Гуджарата, Сурат - 395007, Індія*
[b] *Кафедра фізики, Науковий коледж Нав'юг, Сурат - 395009, Індія*

Ядра гало продемонстрували унікальні структури, які радикально відрізняються від звичайної ядерної матерії. Серед інших ядер гало, ядро ^{11}Be є одним з найбільш вивчених ядер гало, і воно має добре встановлену структуру гало з одним нейтроном та енергією розділення нейтронів S_n = 0,501 МеВ. Ми досліджували реакцію ^{11}Be + ^{209}Bi в багаточастинковій 3-стадійній моделі класичної молекулярної динаміки (3S-CMD), де ^{11}Be побудовано як кластер щільно зв'язаного ^{10}Be та одного нейтрона. Відстань між ^{10}Be та нейтроном регулюється таким чином, щоб встановити іон-іонний потенціал між ними, рівний експериментальній енергії розділення нейтронів. Для цієї реакції ми розрахували частки подій $F(b)$ для заданого параметра удару, b та енергії зіткнення E_{CM}, а також ймовірності подій $P(E_{CM})$ шляхом інтегрування $F(b)$ за всіма значеннями $b \leq b_{max}$ (b_{max} - параметр удару, вище якого всі траєкторії призводять до розсіювання) для заданого E_{CM}. У цих розрахунках виявлено, що для енергій поблизу бар'єру перенесення нейтронів є значним для нижчих параметрів удару, але при енергіях, що значно перевищують бар'єр, повне злиття домінує для нижчих параметрів удару, тоді як для дещо вищих параметрів удару перенесення нейтронів є відповідним.

Ключові слова: *реакції злиття; ядра гало; зіткнення важких іонів; слабкозв'язані ядра; класична молекулярна динаміка*