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FUSION CROSS SECTIONS FOR ^{24}Mg + ^{208}Pb REACTION IN THREE–STAGE CLASSICAL MOLECULAR DYNAMICS MODEL

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Fusion cross sections for heavy-ion reactions have been calculated in various classical and semi-classical models. In the classical approach fusion cross sections have been calculated using different model such as Classical Molecular Dynamics (CMD), Classical Rigid-Body Dynamics (CRBD), a 3-Stage Classical Molecular Dynamics (3S-CMD) model and a microscopic Static Barrier Penetration Model (SBPM). In the present work 3S-CMD model is used to calculate fusion cross section. This model combines the advantages of both CMD and CRBD models. This model uses ion-ion potential obtained from dynamically evolving classical microscopic configurations of nuclei with a suitable NN-potential. The 3S-CMD model calculation proceeds in the following three stages: (1) Rutherford trajectory calculation at very large separation, followed by (2) CRBD calculation with rigid-body constraint on both nuclei up to distances close to the barrier, followed by (3) finding the trajectories of all the nucleons in a full CMD calculation for further evolution by numerically solving Coupled Newton's equations of motion for all the point nucleons. In the present work we have calculated fusion cross sections for ^{24}Mg + ^{208}Pb system in 3S-CMD model. Fusion cross sections have been calculated using a soft-core Gaussian form of NN-potential with the parameter set New Potential (NP). We also investigated the effect of this potential for ^{16}O + ^{92}Zr reaction which agree very well with the experimental fusion cross sections. The use of this NP parameter set might give better agreement in the case of 3S-CMD calculation of classical fusion cross sections for ^{24}Mg + ^{208}Pb reaction.

Keywords: *Fusion cross sections; Classical microscopic approaches; Heavy-ion reactions; Deformed nuclei*

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1. INTRODUCTION

Fusion cross sections for heavy-ion reactions have been calculated in various classical and semi-classical models. In the classical approach fusion cross sections have been calculated using different model such as Classical Molecular Dynamics (CMD) [1], Classical Rigid-Body Dynamics (CRBD) [2], a 3-Stage Classical Molecular Dynamics (3S-CMD) [3] model and a microscopic Static Barrier Penetration Model (SBPM) [4]. In the present work 3S-CMD model is used to calculate the fusion cross section. This model combines the advantages of both CMD and CRBD models. This model uses ion-ion potential obtained from dynamically evolving classical microscopic configurations of nuclei with a suitable NN-potential.

The 3S-CMD model calculation [3] proceeds in three stages: (1) Rutherford trajectory calculation at very large separation, followed by (2) CRBD calculation with rigid-body constraint on both nuclei up to distances close to the barrier, followed by (3) finding the trajectories of all the nucleons in a full CMD calculation for further evolution by numerically solving Coupled Newton's equations of motion for all the point nucleons.

In the present work, we calculate fusion cross sections for the ^{24}Mg + ^{208}Pb system in the 3S-CMD model. Fusion cross sections are calculated using classical approximations and compared with the experiments. Fusion cross sections for many reactions have been calculated using a soft-core Gaussian form of NN-potential (eq.(1)), with the parameter set P4 [2]. However, the calculated fusion cross sections do not match with the experimental data for many reactions at different energies [5].

In the present work we have used a parameter set P4 and a new potential parameter set (called as NP) from ref. [6]. Using this NP potential, the fusion cross sections calculated for the ^{16}O + ^{92}Zr reaction in ref. [6] shows good agreement with the experimental data. The use of this NP parameter set might give better agreement in the case of 3S-CMD calculation of classical fusion cross sections for the ^{24}Mg + ^{208}Pb reaction.

2. CALCULATION DETAILS

2.1. Nucleon-Nucleon potential

The soft-core Gaussian form of NN-potential is given by

$$V_N(r_{ij}) = -V_0\left(1 - \frac{C}{r_{ij}}\right)\exp\left(-\frac{r_{ij}^2}{r_0^2}\right), \quad (1)$$

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where V_0, C and r_0 are respectively, the depth parameter, repulsive-core radius and range parameter. The parameters V_0, C and r_0 are chosen so that the NN-potential reproduces gross properties of the nuclei in their ground state such as, the ground state binding energy, the rms radius etc.

The Coulomb potential between protons has the form

$$V_C\left(r_{ij}\right) = \frac{1.44}{r_{ij}}(\text{MeV}) \tag{2}$$

is also added to the two-body potential.

^{24}Mg + ^{208}Pb reaction has been studied with the potential parameter set P4 [1] and New Potential (NP) [6], which is shown in Table 1. In the present 3S-CMD calculation, the individual nuclei are first generated using the variational potential energy minimization code STATIC [7] and are further "cooled" using DYNAMIC [7] method. The nuclear configurations are generated using the parameter set P4 and NP which reproduces the ground state properties close to the experimental values (see Table 2).

Fusion cross sections are calculated using the classical formula [1]

$$\sigma_{\text{fusion}} = \pi b_{cr}^2, \tag{3}$$

where b_{cr} is the maximum (critical) impact parameter for which the two nuclei fuse.

Table 1. P4 and New Potential Parameter Set for the NN-potential of eq. (1).

POTENTIAL	**V_0 (MeV)**	**C (fm)**	**r_0 (fm)**
P4	1155	2.07	1.2
NP	900	1.95	1.2

2.2. Construction of nuclei in their ground state

Utilizing the "STATIC" minimization technique around 2000 ground state configurations for ^{24}Mg and ^{208}Pb are produced using the P4 and NP potential parameter sets. The calculated ground state properties of the ^{24}Mg and ^{208}Pb nuclei are presented in Fig. 1 and Fig. 2. Out of these 2000 ground state configurations in each case, the ones having quadrupole deformation β_2 value close to the experimental value is selected, which is indicated in Fig. 1 and Fig. 2 by a red circle. The generated ground state properties using both potentials are shown in Table 2.

Table 2. Ground state properties of ^{24}Mg and ^{208}Pb nuclei used in present calculation.

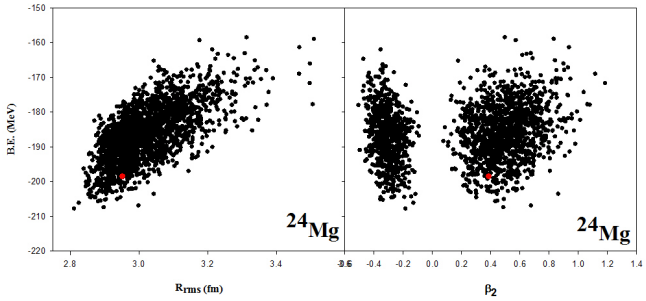
			BE (MeV)	**R (fm)**	β_2 **(fm)**
^{24}**Mg**	Cal.	P4	-198.72	2.94	0.37
		NP	-250.71	2.76	0.43
	Exp.		**-198.25[8]**	**3.07[9]**	**0.39[10]**
^{208}**Pb**	Cal.	P4	-1870.41	6.08	0.24
		NP	-2389.46	5.73	0.16
	Exp.		**-1636.46[8]**	**5.50[9]**	**0.00[10]**

From Table 2 it can be seen that the ground state binding energy, rms radius and β_2 of the chosen ^{24}Mg nucleus calculated using P4 potential are very close to the experimental values. None of the 2000 configuration generated for ^{208}Pb with potential P4 are very close to the experimental values of binding energy and rms radius. However, a configuration with quadrupole deformation $\beta_2 \approx 0$ is chosen which over estimates binding energy by 12% and rms radius by 6%.

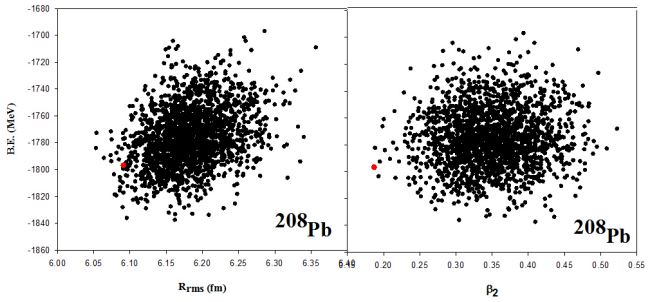
In a similar way a configuration with quadrupole deformation $\beta_2 \approx 0$ is chosen for ^{24}Mg and ^{208}Pb for NP potential. For ^{24}Mg the binding energy is over estimates by 21% and rms radius is under estimates by 11%. For ^{208}Pb the binding energy is over estimates by 31% and rms radius is over estimates by 4%.

3. RESULTS AND DISCUSSION

^{24}Mg + ^{208}Pb Reaction : Fusion cross sections for ^{24}Mg + ^{208}Pb reaction calculated using P4 and NP potential parameter set and using eq.(2.1) are shown in Fig. 3 and 4 are compared with the experimental data of ref. [11]. The dynamical simulation is carried out in the 3S-CMD model.

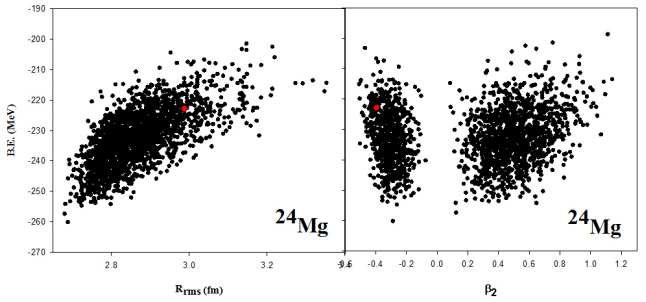


(a) Ground state configuration of ^{24}Mg

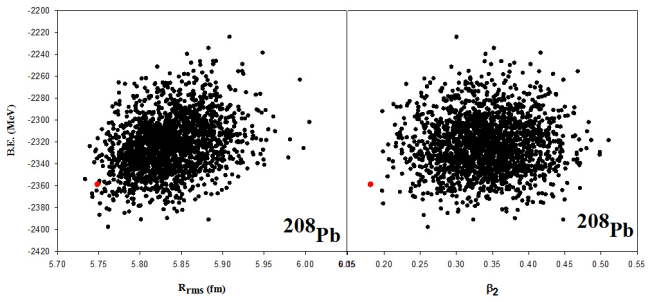


(b) Ground state configuration of ^{208}Pb

Figure 1. Ground state properties of the ^{24}Mg and ^{208}Pb nuclei generated by the "STATIC" method using P4 potential. Red circle indicate the ground state properties of the nuclei used in the present study.



(a) Ground state configuration of ^{24}Mg



(b) Ground state configuration of ^{208}Pb

Figure 2. Ground state properties of the ^{24}Mg and ^{208}Pb nuclei generated by the "STATIC" method using NP potential. Red circle indicate the ground state properties of the nuclei used in the present study.

For this system fusion cross-sections averaged over about 500 initial orientations for E_{CM} = 160-190 MeV and 1000 for E_{CM} < 160 MeV are determined using both potentials (P4 & NP) and are shown in Fig. 3.

Since the effect of reorientation is expected to be noticeable at collision energies near the Coulomb barrier, it is desirable to find fusion cross-sections at energies well below the lowest E_{CM} used in ref. [11]. Therefore, to clearly bring out the effect of reorientation, fusion cross-sections are determined in 3S-CMD calculation up to energies as low as 124.5 MeV, which is well below the lowest energy in the experiment by Back. For $E_{CM} \leq 124.0$ MeV, no pocket in the ion-ion potential is found in any of the 2000 initial random orientation that are considered. Therefore, fusion cross-section is assumed to be zero for $E_{CM} \leq 124.0$ MeV MeV in the 3S-CMD calculation for this system.

Fusion cross section calculated using potential parameter set P4 and eq.(2.1) in 3S-CMD model are highly over-estimated compared to the experimental data at all energy ranges. The reason for this overestimation is the larger rms radius of the ^{208}Pb produced in this calculation as compared to the experimental value (see Table 2). Fusion cross sections calculated using potential parameter set NP and eq.(2.1) in 3S-CMD model shows close agreement with the experimental data than the fusion cross sections calculated using P4 potential.

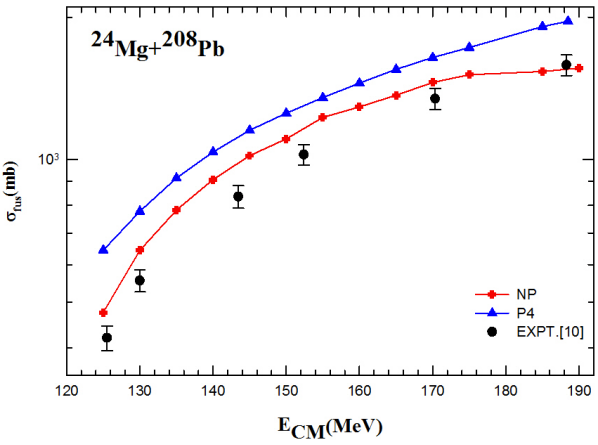


Figure 3. Fusion cross section for ^{24}Mg + ^{208}Pb reaction (log scale)

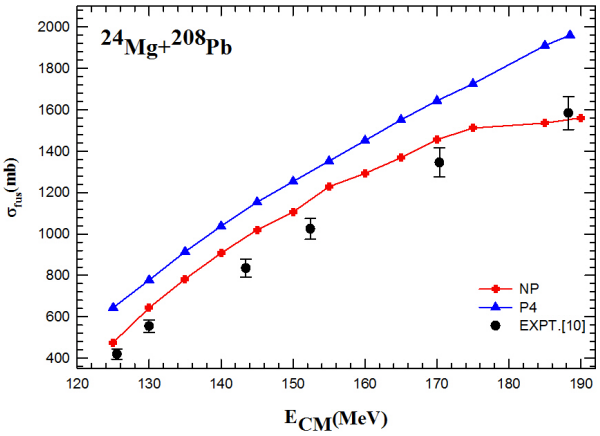


Figure 4. Fusion cross section for ^{24}Mg + ^{208}Pb reaction (linear scale)

4. CONCLUSIONS

Although fusion cross sections calculated using potential parameter set NP which gives good agreement for ^{16}O + ^{92}Zr reaction in ref.[6], however, it is slightly overestimated around the barrier energies for the medium heavy mass and heavy mass nucleus. i.e., ^{24}Mg + ^{208}Pb reactions.

However, it may be noted that the ground state properties of the nuclei generated using this potential (NP) are not close to the experiment values (see Table 2).

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ПЕРЕРІЗИ СИНТЕЗУ ДЛЯ РЕАКЦІЇ ^{24}Mg + ^{208}Pb У ТРИСТАДІЙНІЙ КЛАСИЧНІЙ МОДЕЛІ МОЛЕКУЛЯРНОЇ ДИНАМІКИ

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Перерізи синтезу для реакцій важких іонів були розраховані за допомогою різних класичних та напівкласичних моделей. У класичному підході перерізи синтезу були розраховані з використанням різних моделей, таких як класична молекулярна динаміка (CMD), класична динаміка твердого тіла (CRBD), 3-етапна модель класичної молекулярної динаміки (3S-CMD) та мікроскопічна модель проникнення статичного бар'єру (SBPM). У цій роботі для розрахунку перерізу синтезу використовується модель 3S-CMD. Ця модель поєднує переваги моделей CMD та CRBD. Ця модель використовує іон-іонний потенціал, отриманий з динамічно еволюціонуючих класичних мікроскопічних конфігурацій ядер з відповідним NN-потенціалом. Розрахунок моделі 3S-CMD відбувається у такі три етапи: (1) розрахунок траєкторії Резерфорда на дуже великій відстані, потім (2) розрахунок CRBD з обмеженням твердого тіла на обох ядрах до відстаней, близьких до бар'єру, а потім (3) знаходження траєкторій усіх нуклонів у повному розрахунку CMD для подальшої еволюції шляхом чисельного розв'язання зв'язаних рівнянь руху Ньютона для всіх точкових нуклонів. У цій роботі ми розрахували перерізи синтезу для системи ^{24}Mg + ^{208}Pb у моделі 3S-CMD. Перерізи синтезу були розраховані з використанням м'якої гауссової форми NN-потенціалу з набором параметрів Новий потенціал (NP). Ми також дослідили вплив цього потенціалу на реакцію ^{16}O + ^{92}Zr, які дуже добре узгоджуються з експериментальними перерізами синтезу. Використання цього набору параметрів NP може забезпечити кращу узгодженість у випадку розрахунку 3SCMD класичних перерізів синтезу для реакції ^{24}Mg + ^{208}Pb.

Ключові слова: *перерізи синтезу; класичні мікроскопічні підходи; реакції важких іонів; деформовані ядра*