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UNSTEADY FLUID MOTION BETWEEN INFINITELY STRETCHED PARALLEL HORIZONTAL PLATES WITH THE ABSENCE OF VISCOUS DISSIPATION: AN ANALYTICAL APPROXIMATION

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Unsteady fluid motion between two infinitely stretched parallel horizontal plates with the absence of viscous dissipation is explored in this present study. Plates are maintained at different constant temperatures and separated with the distance of $2h$ units. The flow is produced by a constant oscillating pressure gradient between the plates and parallel to the boundaries of the plates. The solution of the concerned flow equations with suitable boundary limitations have been obtained using most elegant analytical approximation method: A Perturbation technique. The impact of various flow field material parameters has been studied for velocity and temperature fields and deliberated with the help of Graphical interpretations. The obtained results are validated and compared with the results existing in the literature. The obtained results are identified to be exceptional agreement with literature results. The present study will be hopefully helpful to the various industrial applications especially in the nuclear industry for the emergency cooling of nuclear reactors. The researchers and scientists can utilize the methodology of the present work in their interest of study.
Keywords: *Thermo-viscous fluid; Thermo strain conductivity coefficient; Thermal-mechanical stress viscosity; Prandtl number*
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Nomenclature

$\alpha_1 = -p$	Fluid pressure
$\alpha_3 = 2\mu$	Newtonian coefficient of viscosity
$\alpha_5 = 4\mu_c$	Cross viscosity coefficient
α_6	Thermal-mechanical stress constant viscosity
α_8	Thermal stress viscosity coefficient
$-\beta_1 = k$	Thermo conductivity coefficient
β_3	Thermo strain conductivity coefficient
C_1	Non dimensional pressure gradient
C_2	Non dimensional temperature gradient
A_1	Non dimensional classical viscosity
A_6	Non dimensional thermal mechanical stress viscosity
B_3	Non dimensional thermo strain conductivity cofficient
p_r	Prandtl number
Greek Symbols	
η	Ordinate
σ	Period of oscillations
ρ	Density
γ	Energy

1. INTRODUCTION

Over a century ago, there was a great deal of research done on the non-Newtonian properties of fluids. However, the substantial research efforts to extend these analyses into the non-linearity principality have only been conducted in the recent six to seven decades, especially during the Second World War. Koh and Eringen [1] have done preliminary research on the construction of non-linear concept embodying the interplay and association between viscous and thermal impacts. Kelly [2] investigated some incompressible visco-metric flows of thermo viscous fluids. Jithender Reddy *et al.* [3] analyzed nano fluid flow through a stretched sheet with the impact of heat generation. Pothanna and P. Aparna *et al.* [4] investigated unsteady viscous flow with thermal effects in a permeable region over a stretched fluctuating plate. Pothanna *et al.* [5] used numerical and analytical approach on around an oscillating sphere for unsteady fluid flow. Pothanna *et al.* [6] study thermo viscous fluid flow in porous region confined between impervious horizontal plates using four step recursive method of algorithm. Pavan Kumar Reddy et *al.* [7] examined steady flow under sloping magnetic field with suction in a couple stress nature of a fluid via. Rectangular region. Pothanna *et al.* [8] explored steady thermo viscous flow in between infinitely stretched porous parallel horizontal plates. The flow of somewhat thermo viscous fluid in a permeable surface limited in two permeability parallel horizontal stretched plates was explored by

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Pothanna *et al.* [9]. The impact of thermo strain coefficient of conductivity on a moderately thermos viscous fluid in a permeable surface bounded with permeability stretched parallel plates was investigated by Pothanna *et al.* [10]. P. N. Rao and Pattabhi [11] evaluated the problem of stable flow of an order two thermos viscous fluid over extended plate. The uniform flow of a fluid that is viscous via a permeable sphere soaked with micropolar fluid was explored by Aparna *et al.* [12]. Examined by Aparna *et al.* [13] flow produced in a micro-polar fluid by the gradual, steady rotation of a holey sphere. The fluid that is viscous passing a permeable cylinder was studied by Aparna *et al.* [14]. In Aparna *et al.* [15] study, the fluctuations of a perishing sphere was examined for an incompressible coupled stress fluid were. Padmaja *et al.* [16] examined the numerical computations of multi parameter problem with singular perturbations with exponential spline technique. The numerical exploration of associated nonlinear equations for the problem of thermal viscous fluid movement in a cylindrical configuration was investigated by Pothanna *et al.* [17]. The exact and numerical studies of thermo viscous fluid flow between stretched horizontal plates that are parallel were examined by Pothanna *et al.* [18]. Srinivas Joshi *et al.* [19] investigated the linear behavior of a fluid with thermos viscous nature in a permeable surface that was confined between two immovable passable horizontal stretched parallel plates. Pothanna *et al.* [20] in the paper entitled unsteady forced oscillations of a fluid bounded by rigid bottom studied analytical solutions of governing flows. Srinivas *et al.* [21] examined slow steady motion of a thermo-viscous fluid between two parallel plates with constant pressure and temperature gradients. Srinivas *et al.* [22] studied flow of a thermo-viscous fluid in a radially non-symmetric constricted tube. Srinivas *et al.* [23] investigated peristaltic transport of a thermo-viscous fluid. Meghdadi *et al.* [24] observed mixing enhancement in micro channels using thermo-viscous expansion by oscillating temperature wave.

The incompressible flow of thermos viscous fluids in porous region satisfies the standard conservation equations.

Continuity Equation: $v_{i,i} = 0$

Momentum Equation: $\rho\left[\frac{\partial v_i}{\partial t} + v_k v_{i,k}\right] = \rho f_i + t_{ji,j} - \frac{\mu}{k^*} v_i$,

and the equation of energy: $\rho c \dot{\theta} = t_{ij} d_{ij} - q_{i,i} + \rho\gamma - \frac{\mu}{k^*} v_i v_i$,

where

$f_i = i^{th}$ Component external force/unit mass, $c =$ Heat constant, $\gamma =$ Energy source/unit mass, $q_i = i^{th}$ Component-Heat flux bivector $= \in_{ijk} h_{jk} / 2$.

The fundamental standard equations of thermo viscous second order fluids as given by Koh and Eringen for stress tensor and heat flux bivector are

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db - bd)$$

and

$$h = \beta_1 b + \beta_3 (bd + db)$$

where the coefficients $\alpha_i'^s$ and $\beta_i'^s$ are polynomials in terms of $tr\, d, tr\, d^2, tr\, b^2$. The explicit expressions for the constitutive coefficients $\alpha_i'^s$ and $\beta_i'^s$ for the theory of second order can be obtained as

$\alpha_1 = \alpha_{1000} + \alpha_{1010} tr\, d + \alpha_{1020} tr\, d^2 + \alpha_{1002}\, tr\, b^2$,

$\alpha_3 = \alpha_{3010} + \alpha_{3020} tr\, d$

$\alpha_5 = \alpha_{5020}$, $\quad \alpha_6 = \alpha_{6002}$, $\quad \alpha_8 = \alpha_{8011}$,

$\beta_1 = \beta_{1001} + \beta_{1011} tr\, d$, and $\quad \beta_3 = \beta_{1011}$

the secondary coefficients α_{isrt} and β_{isrt} are the functions involving ρ and θ.

In this work, the effects of different material characteristics on the thermos-viscous unsteady flow fields of a fluid through horizontal plates are attempted to be studied. The current study was very much helpful to the scientist and researchers to solve their engineering and research study problems. In industrial and technical systems, impermeable plates that are parallel are quite useful. Some of the application mentioned here are the human circulatory system, as well as a number of engineering instruments, including chromatography columns, chemical reactors, heat and mass exchangers, and other processing machinery. The last several decades have seen a huge increase in interest in the investigation of the flow features of these formations due to the vast range of applications.

Considering the expanding significance and use of non-Newtonian type flows in a chemical technology, industry, and geophysical fluid dynamics, this present work attempts to explore analytical approximation approach to thermo viscous flow of a fluid in between impermeable horizontal stretched plates bound in a porous medium. This current work has not yet been discussed in the literature.

2. PROBLEM STATEMENT AND ANALYSIS

An unsteady flow of thermo viscous fluid in order two can be characterized by constitute basic equations between horizontal stretched plates parallel to each other. The flow is produced by the pressure gradient oscillating with the direction parallel to the plates. The Cartesian coordinate system $O(X, Y, Z)$ with the origin centered between two plates, the $X - axis$ in the direction of flow and the $Y - axis$ perpendicular to the plates is considered. The two plates are separated by a distance $2h\ units$ and are kept at two different temperatures. The plates are denoted by $y = h$ and $y = -h$. The lower and upper plates are maintained with the constant temperatures θ_0 and θ_1 respectively.

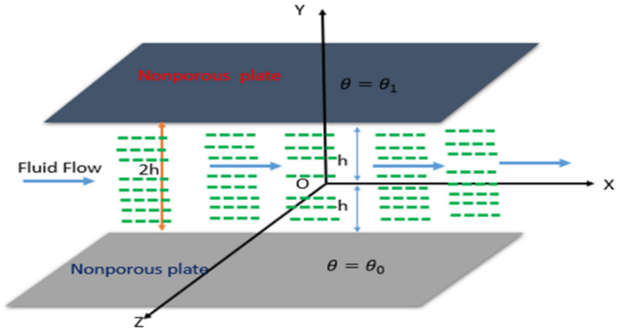


Figure 1. Schematic Representation of Flow

Let us assume the unsteady flow characterized by the velocity $[u(y,t), 0{,}0]$ and temperature $\theta[y,t]$. The continuity equation is satisfied by this assumed velocity selection. Without internal energy sources and external forces, the equation of motion reduces to

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \alpha_6 \frac{\partial \theta}{\partial x} \frac{\partial^2 \theta}{\partial y^2}, \tag{1}$$

and the energy equation reduces to

$$\rho c \left(\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x}\right) = \mu \left(\frac{\partial u}{\partial y}\right)^2 - \alpha_6 \frac{\partial \theta}{\partial x} \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial y} + k \frac{\partial^2 \theta}{\partial y^2} + \beta_3 \frac{\partial \theta}{\partial x} \frac{\partial^2 u}{\partial y^2}, \tag{2}$$

composed with the boundary limitations

$$u[-h,t] = 0,\ \theta[-h,t] = \theta_0, \tag{3}$$

$$u[h,t] = 0,\ \theta[h,t] = \theta_1. \tag{4}$$

Employing the following dimensionless quantities,

$$y = h\eta\ ,\ u = (\mu/\rho h)\ U,\ T = \frac{\theta - \theta_0}{\theta_1 - \theta_0}\ ,\ \frac{\partial \theta}{\partial x} = \frac{\theta_1 - \theta_0}{h} C_2\ ,\ -\frac{\partial p}{\partial x} = \frac{\mu^2}{\rho h^3} C_1. \tag{5}$$

The equations (1) and (2) can be reduced to

$$\frac{\partial U}{\partial t} = C_1 + \frac{\partial^2 U}{\partial \eta^2} - A_6 C_2 \frac{\partial^2 T}{\partial \eta^2}, \tag{6}$$

and

$$\frac{\partial T}{\partial t} + UC_2 = A_1 \left[\left(\frac{\partial U}{\partial Y}\right)^2 - A_6 C_2 \frac{\partial U}{\partial \eta} \frac{\partial T}{\partial \eta}\right] + \frac{1}{p_r} \frac{\partial^2 T}{\partial \eta^2} + B_3 C_2 \frac{\partial^2 U}{\partial \eta^2}, \tag{7}$$

with $p_r = \frac{c\mu}{k}$ (Prandtle number), $B_3 = \frac{\beta_3}{\rho h^2 c}$, $A_1 = \frac{\mu^2}{\rho h^2 c(\theta_1 - \theta_0)}$,

$$A_6 = \frac{\alpha_{6(\theta_1 - \theta_0)^2}}{\mu^2},\ C_2 = \text{constant temperature gradient} \tag{8}$$

and C_1 is the dimensionless pressure gradient oscillatory of period $\frac{2\pi}{\sigma}$ (say) i.e.

$$C_1 = p_0 \cos(\sigma t) = Re(p_0 e^{i\sigma t}) \text{ say} \tag{9}$$

The dissipation absence in the fluid flow, the momentum and energy equations reduce to

$$\frac{\partial U}{\partial t} = C_1 + \frac{\partial^2 U}{\partial \eta^2} - A_6 C_2 \frac{\partial^2 T}{\partial \eta^2} \tag{10}$$

and

$$\frac{\partial T}{\partial t} + UC_2 = \frac{1}{p_r} \frac{\partial^2 T}{\partial \eta^2} + B_3 C_2 \frac{\partial^2 U}{\partial \eta^2}. \tag{11}$$

3. METHODOLOGY AND SOLUTION

Let us consider the velocity and temperature of the flow in the form

$$U(\eta,t) = Re\,[g(\eta)e^{i\sigma t}],\ T(\eta,t) = Re[f(\eta)e^{i\sigma t}], \tag{12}$$

with the boundary limitations:

$$g(1) = 0,\ g(-1) = 0$$
$$f(1) = T_1,\ f(-1) = T_0 \tag{13}$$

Only the real parts of the functions in the above-mentioned equations need to be taken into account.
Using eqn. (12) in eqns. (10) and (11) we get

$$i\sigma g(\eta) = p_0 + g''(\eta) - A_6 C_2 f''(\eta), \tag{14}$$

$$i\sigma f(\eta) + C_2 g(\eta) = \frac{1}{p_r} f''(\eta) + B_3 C_2 g''(\eta). \tag{15}$$

Using A_6 as the perturbation parameter, the perturbation method solves the aforementioned eqns. (14) and (15).
Let $g(\eta)$ and $f(\eta)$ can be taken as

$$g(\eta)=g^{(0)}(\eta)+A_6 g^{(1)}(\eta)+A_6^2 g^{(2)}(\eta)+\dots \tag{16}$$

$$f(\eta)=f^{(0)}(\eta)+A_6 f^{(1)}(\eta)+A_6^2 f^{(2)}(\eta)+\dots \tag{17}$$

The following are the consecutive approximations obtained by gathering terms of same powers of A_6 and using eqns. (16) and (17) in eqns. (14) and (15).

The Zeroth order calculation

The equations of this estimation are becomes

$$i\sigma g^{(0)}(\eta) = p_0 + g^{(0)''}(\eta), \tag{18}$$

$$i\sigma f^{(0)}(\eta) + C_2 g^{(0)}(\eta) = \frac{1}{p_r} f^{(0)''}(\eta) + B_3 C_2 g^{(0)''}(\eta), \tag{19}$$

with boundary limitations

$$g^{(0)}(-1) = 0,\ g^{(0)}(1) = 0,$$
$$f^{(0)}(-1) = T_0,\ f^{(0)}(1) = T_1. \tag{20}$$

This gives

$$g^{(0)}(\eta) = \frac{p_0}{i\sigma}\left(1 - \frac{\cosh\sqrt{i\sigma}\,\eta}{\cosh\sqrt{i\sigma}}\right), \tag{21}$$

$$f^{(0)}(\eta) = \frac{1}{\sinh 2\sqrt{i\sigma p_r}}\left[T_0 \sinh\sqrt{i\sigma p_r}\,(1-\eta) + T_1 \sinh\sqrt{i\sigma p_r}\,(1+\eta)\right]$$
$$+\frac{p_0 p_r(1-i\sigma B_3)}{\sigma^2(1-p_r)\cosh\sqrt{i\sigma}}\left[\cosh\sqrt{i\sigma}\,\eta - \frac{\cosh\sqrt{i\sigma}}{\cosh\sqrt{i\sigma p_r}}\cosh\sqrt{i\sigma p_r}\,\eta\right] + \frac{p_0 p_r c_2}{\sigma^2}\left[1 - \frac{\cosh\sqrt{i\sigma p_r}\,\eta}{\cosh\sqrt{i\sigma p_r}}\right] \tag{22}$$

First order calculation (i.e. terms containing A_6)

The equations are

$$i\sigma g^{(1)}(\eta) = g^{(1)''}(\eta) - C_2 f^{(0)''}(\eta), \tag{23}$$

$$i\sigma f^{(1)}(\eta) + C_2 g^{(1)}(\eta) = \frac{1}{p_r} f^{(1)''}(\eta) + B_3 C_2 g^{(1)''}(\eta), \tag{24}$$

with boundary limitations:

$$g^{(1)}(-1) = 0,\ g^{(1)}(1) = 0$$
$$f^{(1)}(-1) = 0,\ f^{(1)}(1) = 0, \tag{25}$$

the above equations solutions yield

$$g^{(1)}(\eta) = \frac{p_0 C_2 i}{\sigma^2}\left[p_r B_3 \sigma - C_2(i + \sigma B_3) + i C_2 p_r^2\right]\left[1 - \frac{\cosh\sqrt{i\sigma}\,\eta}{\cosh\sqrt{i\sigma}}\right]$$
$$+\frac{p_0 p_r C_2(1-i\sigma B_3)}{2i\sigma\sqrt{i\sigma}\cosh\sqrt{i\sigma}}\left[C_2 + \frac{1}{i\sigma(1-p_r)}\right]\left[\frac{\sinh\sqrt{i\sigma}}{\cosh\sqrt{i\sigma}}\cosh\sqrt{i\sigma}\,\eta - \eta\sinh\sqrt{i\sigma}\,\eta\right]$$
$$+\frac{4p_0 C_2^2 p_r^2 \sinh\sqrt{i\sigma p_r}}{\sigma^2(p_r-1)}\left[1 + \frac{1-i\sigma B_3}{1-p_r}\right]\left[\frac{\cosh\sqrt{i\sigma p_r}}{\cosh\sqrt{i\sigma}}\cosh\sqrt{i\sigma}\,\eta - \cosh\sqrt{i\sigma p_r}\,\eta\right]$$
$$+\frac{T_0 p_r C_2}{(p_r-1)\sinh\sqrt{i\sigma p_r}}\left[\sinh\sqrt{i\sigma p_r}\,(1-\eta) - \frac{\sinh 2\sqrt{i\sigma p_r}}{\sinh 2\sqrt{i\sigma}}\sinh\sqrt{i\sigma}\,(1-\eta)\right]$$

$$+\frac{T_1 p_r C_2}{(p_r-1)\sinh\sqrt{i\sigma p_r}}\left[\sinh\sqrt{i\sigma p_r}\,(1+\eta)-\frac{\sinh 2\sqrt{i\sigma p_r}}{\sinh 2\sqrt{i\sigma}}\sinh\sqrt{i\sigma}\,(1+\eta)\right] \tag{26}$$

$$f^{(1)}(\eta)=\frac{p_0 p_r c_2^2}{i\sigma^3}\left[C_2(1-i\sigma b_3)-C_2 p_r+i\sigma b_3\right]\left[\frac{\cosh\sqrt{i\sigma p_r}\,\eta}{\cosh\sqrt{i\sigma p_r}}-1\right]$$

$$+\frac{p_0 p_r^2 C_2^2(1-i\sigma b_3)}{\sigma^3(1-p_r)\cosh\sqrt{i\sigma}}\left[\begin{array}{c} i\langle c_2-p_r C_2+i\sigma b_3(1-C_2)\rangle \\ -\dfrac{i}{2\sigma(1-p_r)}\dfrac{\sinh\sqrt{i\sigma}}{\cosh\sqrt{i\sigma}}(1-i\sigma b_3)\big(i-\sigma C_2(1-p_r)\big) \\ -\dfrac{2ip_r C_2\sinh 2\sqrt{i\sigma p_r}}{(1-p_r)^2}-\dfrac{iB_3(1+i\sigma)}{2\sigma(1-p_r)} \end{array}\right]$$

$$*\left[\cosh\sqrt{i\sigma}\,\eta-\frac{\cosh\sqrt{i\sigma}}{\cosh\sqrt{i\sigma p_r}}\cosh\sqrt{i\sigma p_r}\,\eta\right]$$

$$+\frac{2p_0 p_r^3 C_2^3(1-i\sigma b_3)(2-p_r-i\sigma b_3)}{\sigma^2\sqrt{i\sigma p_r}(1-p_r)^2}\sinh\sqrt{i\sigma p_r}\left[\begin{array}{c}\eta\sinh\sqrt{i\sigma p_r}\,\eta \\ -\dfrac{\sinh\sqrt{i\sigma p_r}}{\cosh\sqrt{i\sigma p_r}}\cosh\sqrt{i\sigma p_r}\,\eta\end{array}\right]$$

$$-\frac{p_0 p_r^2 C_2^2\big(i-\sigma C_2(1-p_r)\big)(1-i\sigma b_3)^2}{2\sigma^3\sqrt{i\sigma}(1-p_r)^2\cosh\sqrt{i\sigma}}\left[\eta\sinh\sqrt{i\sigma}\,\eta-\frac{\sinh\sqrt{i\sigma}}{\cosh\sqrt{i\sigma p_r}}\cosh\sqrt{i\sigma p_r}\,\eta\right]$$

$$+\frac{p_0 p_r^2 C_2^2\big(i-\sigma c_2(1-p_r)\big)(1-i\sigma b_3)^2}{i\sigma^4(1-p_r)^3\cosh\sqrt{i\sigma}}\left[\cosh\sqrt{i\sigma}\,\eta-\frac{\cosh\sqrt{i\sigma}}{\cosh\sqrt{i\sigma p_r}}\cosh\sqrt{i\sigma p_r}\,\eta\right]. \tag{27}$$

Up to the first order estimations, the expressions $g(\eta)$ and $f(\eta)$ in A_6 are as follows:

$$g(\eta)=g^{(0)}(\eta)+A_6\,g^{(1)}(\eta), \tag{28}$$

$$f(\eta)=f^{(0)}(\eta)+A_6\,f^{(1)}(\eta). \tag{29}$$

The solution of velocity and temperature as assumed above is restricted to the first order approximation only. It is common and useful to limit an analysis to first-order perturbation theory in many areas of applied mathematics, fluid dynamics, and physics, particularly when working with complicated or nonlinear systems for which an accurate solution is either impossible or very difficult. In fluid flow, first-order may provide adequate information for design or prediction about how velocity reacts to changes in temperature or pressure. Higher-order corrections may only slightly increase accuracy and greatly increase mathematical effort if the first-order answer matches the experimental data.

The velocity and temperature distribution are thus obtained as

$$U(\eta,t)=Re\,[g(\eta)\,e^{i\sigma t}]\ \text{and}\ T(\eta,t)=Re\,[f(\eta)\,e^{i\sigma t}]. \tag{30}$$

4. VALIDATION AND COMPARISON OF RESULTS

The Perturbation method of results for the associated equations of governing motion have been determined in terms of the temperature and velocity fields in this paper. The method have been assumed and calculated up to the first order approximations. The governed modelling equations of momentum and energy are highly non-linear, complex and coupled. The higher order approximation computations becomes difficult and tedious with manual calculations. The numerical computations have been obtained for velocity and temperature with the impact of various material parameters by generating the code of algorithm using *MATLAB* software. The obtained results have been compared with the results of Pothanna *et al.* [4]**.** The excellent agreement was achieved when compared with the present study results of perturbation technique. The results comparison shows the validity and the reliability of current study in the numerical computations.

Table 1. Validation and Comparison of results of velocity and temperature.

Y	Velocity Results of Pothanna et al. [4] for $S=0, a_6=0.001$, c=1, $\sigma=1$, ρ=1, μ=1		Present Results for $t=\frac{\pi}{2}, a_6=0.001$, c=1, $\sigma=1$, ρ=1, μ=1		Temperature Results of Pothanna *et al.* [4] for $S=0, a_6=0.001$ c=1, $\sigma=1$, ρ=1, μ=1		Present Results for $t=\frac{\pi}{2}, a_6=0.001$ c=1, $\sigma=1$, ρ=1, μ=1	
	$p_r=0.7$, $B_3=1$	$p_r=1.5$, $B_3=3$	$p_r=0.7$, $B_3=1$	$p_r=1.5$, $B_3=3$	$p_r=0.7$, $B_3=1$	$p_r=1.5$, $B_3=3$	$p_r=0.7$, $B_3=1$	$p_r=1.5$, $B_3=3$
0.0	0.1599	0.2174	0.1599	0.2174	0.0895	-1.6680	0.0895	-1.6680
0.1	0.1580	0.2149	0.1580	0.2149	0.0894	-1.6481	0.0894	-1.6481
0.2	0.1521	0.2076	0.1521	0.2076	0.0890	-1.5886	0.0890	-1.5886
0.3	0.1424	0.1954	0.1424	0.1954	0.0879	-1.4907	0.0879	-1.4907
0.4	0.1292	0.1786	0.1292	0.1786	0.0858	-1.3563	0.0858	-1.3563
0.5	0.1127	0.1574	0.1127	0.1574	0.0819	-1.1881	0.0819	-1.1881
0.6	0.0932	0.1322	0.0932	0.1322	0.0755	-0.9898	0.0755	-0.9898
0.7	0.0713	0.1034	0.0713	0.1034	0.0653	-0.7658	0.0653	-0.7658
0.8	0.0475	0.0716	0.0475	0.0716	0.0504	-0.5216	0.0504	-0.5216

0.9	0.0224	0.0372	0.0224	0.0372	0.0291	-0.2638	0.0291	-0.2638
1.0	0.0033	0.0011	0.0033	0.0011	0.0000	-0.0000	0.0000	-0.0000

5. DISCUSSION ON RESULTS

The impact of different material coefficients such as thermal conductivity coefficient (B_3), prandtl number (p_r), thermal stress coefficient (A_6) and time parameter (t) on velocity and temperatures of the flow field have been discussed and illustrated graphically for fixed parameters taking c=1, $\sigma=1$, ρ=1 and μ=1.

The effect of time parameter (t) on the velocity field is shown graphically in Fig. (2). It is observed that the velocity variations drifted towards right side of the region from the middle of the channel by increasing the value of 't' from t=0 to $\frac{\pi}{2}$. The fluid velocity increases along the flow direction up to the centre then decreases to attain the velocity of the upper plate. The velocity variations moving towards left of the flow region by increasing the value of 't' from $t=\pi$ to $\frac{3\pi}{2}$ from the centre of the flow region. In this case, the fluid velocity decreases along the flow direction up to the centre then decreases and attain its upper plate velocity. It is depicted that, the flow is oscillating with the period 'π' and also the symmetry of these velocity profile variations is observed with the difference of π period. Transient effects are introduced by the time parameter, which means that the flow and heat fields change over time instead of staying constant. In the flow development of channels or pipes, the velocity profile changes from flat (uniform) to parabolic (developed) over time. For oscillating or pulsatile flows, the velocity may vary periodically with time.

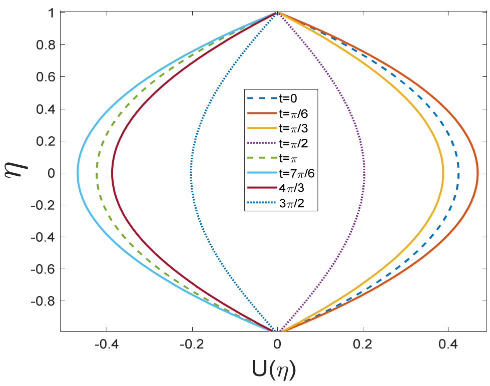
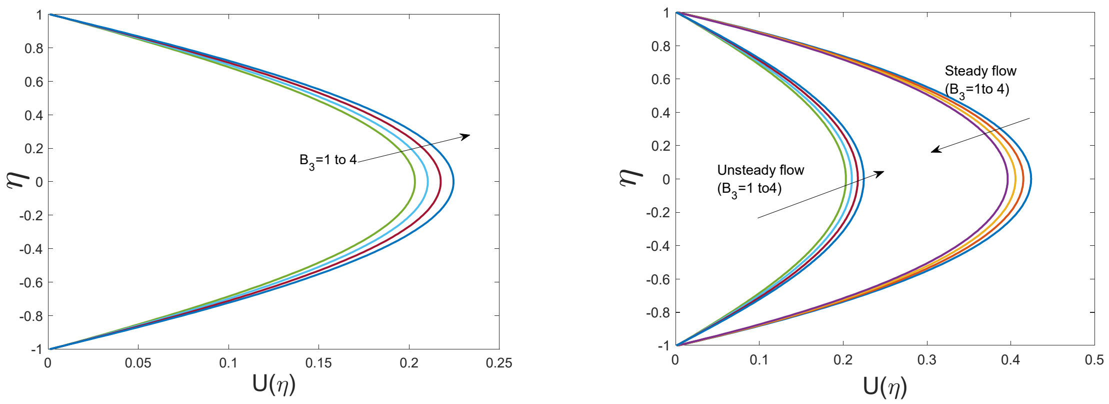


Figure 2. Velocity variations for various values of $'t'$ with fixed a_6 B_3 and p_r

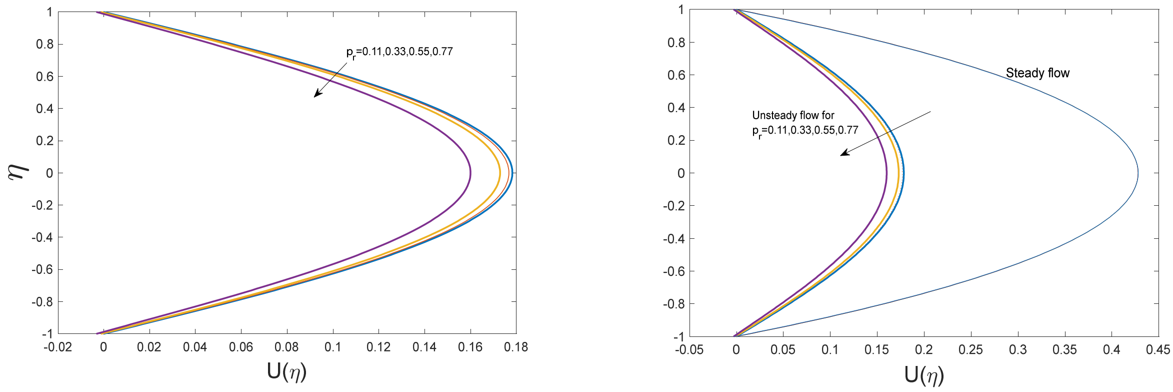
The influence of thermal conductivity coefficient (B_3) for both steady and unsteady flow on the velocity is depicted in Fig. 3(a) and 3(b). Fluid flows are significantly impacted by thermal conductivity, especially when heat transfer is involved. The ability of a material to conduct heat is measured by its thermal conductivity (k). It plays a crucial role in fluids in determining how well heat is transferred through the medium. It is observed that, the fluid velocity increases and moves towards right as the value of B_3 incresing from 1 to 4. The reverse effect has been observed for the steady flow and is depicted in Fig. 3(b). Steep temperature gradients can arise and heat transport is slower in materials with low thermal conductivity, such as oils and gases. Heat spreads faster in materials with strong thermal conductivity (like liquids), which results in more consistent temperature distributions. This may be treated as the natural occurrence of any fluid that will be observed in a flow region. Along the flow direction, the fluid velocity increases up to the middle channel then decreases and attains its upper plate velocity.



(a) (b)

Figure 3. Velocity variations for B_3 (a) Unsteady flow (b) Steady and Unsteady flow

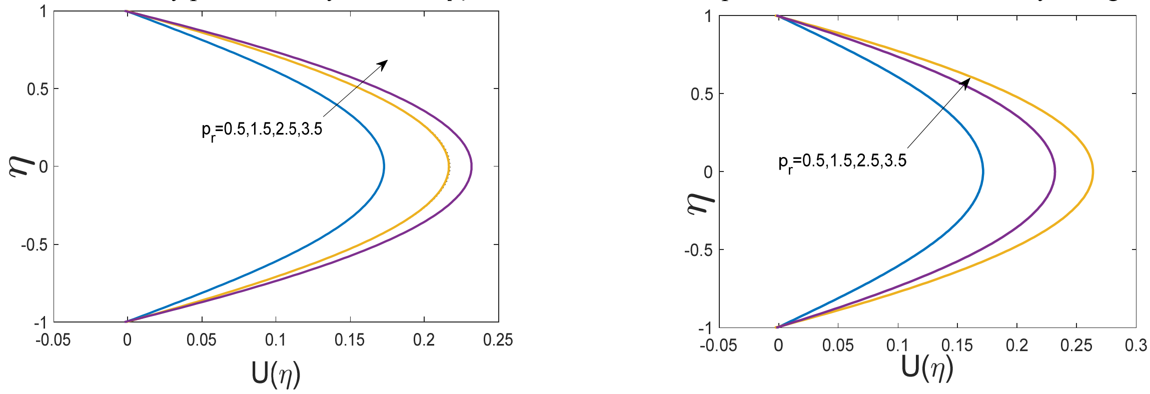
The impact of prandtle (p_r) number for small values on velocity field have illustrated graphically in Fig. 4(a) and 4(b) for steady and unsteady flow respectively.



a b

Figure 4. Velocity variations for p_r with $B_3 = 1$ (a) Unsteady flow (b) Steady and Unsteady flow

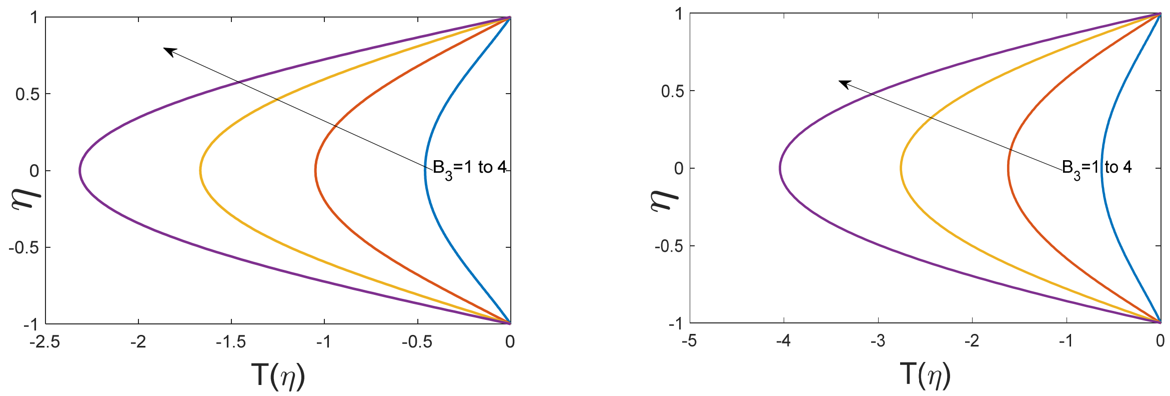
The effect of p_r for some large values have been shown in Fig. 5(a) and 5(b) for various values of thermal conductivity coefficient (B_3). The velocity variations decreases by increasing the p_r and for the steady flow, the increasing rate is very slow and all the profile variations are coincides which is observed in Fig. 4(a) and 4(b). The velocity variations increases with the increase of p_r and shown in Fig. 5(a) and 5(b). It is also observed from these figures that, the fluid velocity increases with the values of B_3 increases. For large values of p_r , the increasing rate of fluid velocity is more predominant when compare to small p_r values. The p_r doesn't directly affect the velocity profile in purely viscous flows, but it does in thermo-viscous coupling cases where temperature gradients change viscosity or drive buoyancy (natural convection). For the high values of p_r , the flow near the wall is strongly influenced by temperature changes and this affects the velocity profile locally. For low p_r , the more uniform temperature and smoother velocity changes occurs.



a b

Figure 5. Velocity variations for p_r with (a) $B_3 = 3$, (b) $B_3 = 5$

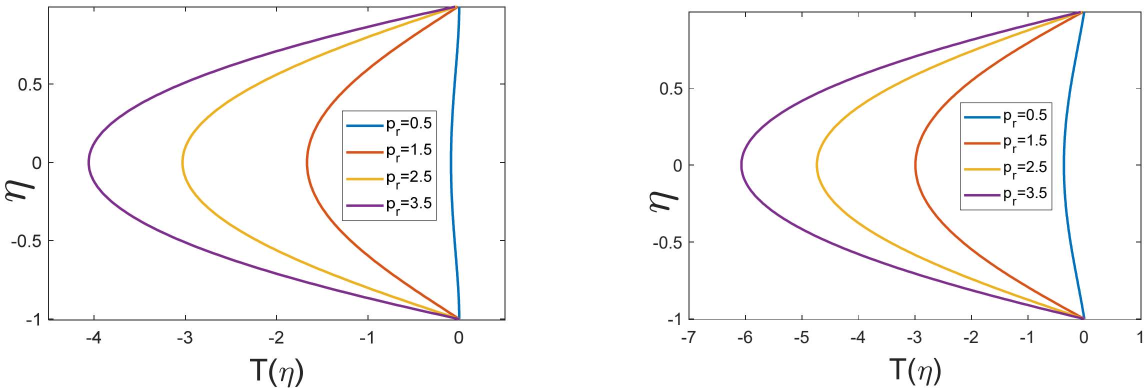
The impact of thermal conductivity coefficient (B_3) and thermal stress coefficient (A_6) on the flow temperature is depicted in Fig. 6(a) and 6(b).



a b

Figure 6. Temperature variations for B_3 with (a) $\alpha_6 = 0.001$ (b) $\alpha_6 = 0.005$

It is observed that, the fluid temperature decreases and moves towards left with increasing the value of B_3 from 1 to 4. The value of A_6 increases, then the temperature of the fluid decreases with the faster rate as it is observed from the Fig.6 (a) and 6(b). Along the flow direction, the fluid temperature decreases up to the middle channel then increases and attains its hotter plate temperature. This is due to the non-homogeneous and anisotropic nature of the fluid considered. The influence of thermal conductivity will differ depending on the location due to its anisotropic nature. The corresponding figures clearly show the same thing. This is particularly noticeable when the thermal conductivity coefficient takes very large values. Therefore, it may be concluded that the anisotropy will rise in situations for large values of thermal conductivity.

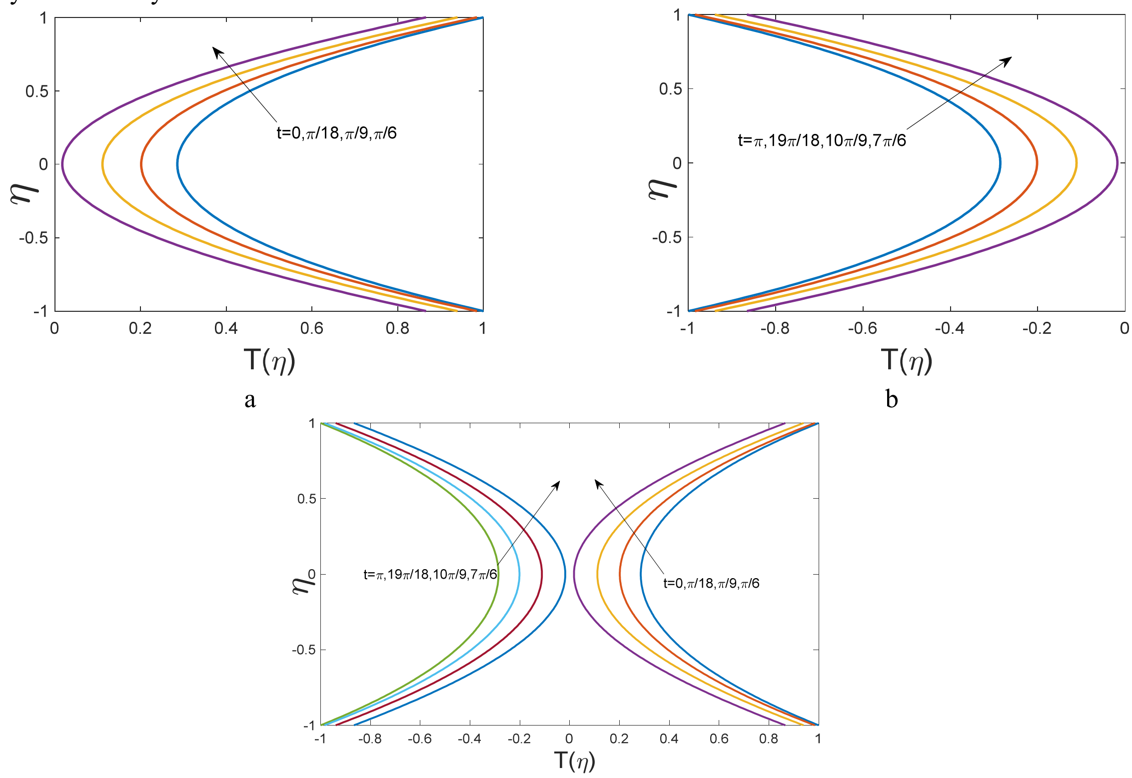


a b

Figure 7. Temperature variations for p_r with (a) $B_3 = 3$ (b) $B_3 = 5$

The influence of Prandtl (p_r) number on temperature field for large values have been illustrated in Fig. 7(a) and 7(b) for various thermal conductivity (B_3) values. The temperature variations decreases with the increase of p_r and shown in Fig. 7(a) and 7(b). For low value p_r fluids, the temperature gradients are spread over a large area which smoother temperature field. For high p_r fluids, the steep thermal gradients are near the surfaces and the thin thermal boundary layer becomes more localized heating.

It is also observed from these figures that the fluid temperature decreases at the faster rate with the values of B_3 increases. This is because the viscosity and specific heat have a direct correlation with the Prandtl number. The fluid domain under consideration in this work has a constant specific heat. Therefore, the skewness in the mirroring will be caused by the viscosity.



c

Figure 8. Temperature variations (a), (b) and (c) for different values of $'t'$ with fixed a_6 B_3 and p_r

The impact of time parameter (t) on the temperature field is shown graphically in Fig. (8a), (8b) and (8c). It is observed that the temperature variations increase and drifted towards center from right of the flow surface with the increasing values of time parameter (t) from 0 to $\frac{\pi}{6}$. The fluid velocity increases along the flow direction up to the centre then decreases and attains its upper plate velocity. The velocity profiles drifted towards middle of the with the increasing values of time parameter (t) from π to $\frac{7\pi}{6}$ from left side of the flow region. In this case, the fluid velocity decreases along the flow direction up to the centre then decreases and attain its upper plate velocity. The hyperbolic type profiles are realized for the temperature variations of the fluid. Heat sources, conduction, or convection cause temperature variations over time in transient heating. The temperature can fluctuate if exposed to time-varying heat input, the velocity increases, and the heated region spreads over time in the thermal boundary layer growth.

6. CONCLUSIONS

In this present work an unsteady fluid motion between two infinitely stretched parallel horizontal plates with the absence of viscous dissipation is examined. The following are the conclusions observed from the above graphical illustrations.

- The velocity profile variations are symmetrically oscillating with the difference of period π. Fluid particles in symmetric oscillatory motion move forward during the first half of the cycle and backward for the second half of the same distance. Long-term fluid movement is eliminated as a result, but momentum and energy can still be transferred.
- The fluid velocity increases and shifted towards right from the origin as the value of B_3 increases.
- The velocity variations decrease by increasing small values of p_r. The rate of increase is very slow and the velocity profiles coincides when fluid is steady. For low p_r, the more uniform temperature and smoother velocity changes occurs.
- For large values of p_r, the velocity variations increase with the faster rate. For high values of p_r, the flow near the wall is strongly influenced by temperature changes and this affects the velocity profile locally.
- The fluid temperature decreases as the value B_3 and A_6 increases. The real-world uses of industry, Low-thermal-conductivity fluids, such as oils or gases, need bigger surfaces or longer residence times in heat exchangers, while high-thermal-conductivity fluids, such as water or liquids, increase heat transfer efficiency. The influence of thermal conductivity will differ depending on the location due to its anisotropic nature.
- The hyperbolic type profiles are realized for the temperature variations as the value of $'t'$ increases.

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НЕСТАЦІОНАРНИЙ РУХ РІДИНИ МІЖ НЕСКІНЧЕННО РОЗТЯГНУТИМИ ПАРАЛЕЛЬНИМИ ГОРИЗОНТАЛЬНИМИ ПЛАСТИНКАМИ ЗА ВІДСУТНОСТІ В'ЯЗКОЇ ДИСИПАЦІЇ: АНАЛІТИЧНЕ НАБЛИЖЕННЯ

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У цьому дослідженні вивчається нестаціонарний рух рідини між двома нескінченно розтягнутими паралельними горизонтальними пластинами за відсутності в'язкої дисипації. Пластини підтримуються при різних постійних температурах і знаходяться на відстані 2 год одна від одної. Потік створюється постійним коливальним градієнтом тиску між пластинами та паралельно до меж пластин. Розв'язок відповідних рівнянь потоку з відповідними граничними обмеженнями був отриманий з використанням найелегантнішого методу аналітичної апроксимації: методу збурень. Вплив різних параметрів матеріалу поля потоку був досліджений для полів швидкості та температури та обговорений за допомогою графічних інтерпретацій. Отримані результати перевірені та порівняні з результатами, що існують у літературі. Отримані результати визначені як такі, що винятково узгоджуються з результатами літератури. Сподіваємося, що це дослідження буде корисним для різних промислових застосувань, особливо в ядерній промисловості для аварійного охолодження ядерних реакторів. Дослідники та науковці можуть використовувати методологію цієї роботи в інтересах своїх досліджень.

Ключові слова: *термов'язка рідина; коефіцієнт термодеформаційної провідності; термомеханічна в'язкість під напруженням; число Прандтля*