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SLOW SURFACE ELECTROMAGNETIC WAVES ON A MU-NEGATIVE CYLINDER

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This paper presents the theoretical study of metamaterials with negative magnetic permeability. The electrodynamics phenomenological description has been chosen. The dispersion properties of slow surface electromagnetic waves propagating along a circular cylinder made of mu-negative metamaterial are studied. We neglect energy losses in the metamaterial. Negativity of permeability occurs in relatively bounded frequency intervals, and for all modes, a normal dispersion occurs, regardless of parameter values. The values of phase velocities of these waves lie between c and 0.3c. The phase velocity dependencies of the studied modes versus their frequency have a diverse appearance. The directions of group and phase velocities coincide. The values of group velocity are less than 0.002c. The wave fields are the superposition of transverse-electric and transverse-magnetic parts and decay exponentially in a radial direction away from the separating boundary. The wave fields penetrate mu- negative metamaterial much weaker than into a vacuum. Wave propagation in the structure does not require an external magnetic field. The variety of these wave features on the cylinder parameters can be used for different applications.

Keywords: *Metamaterial; Negative permeability; Cylinder; Surface electromagnetic wave*

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1. INTRODUCTION

Metamaterials are artificially engineered materials consisting of special identical cells. These ones play role as artificial atoms for electromagnetic waves of wavelength much larger then cell size. The phenomenological description gives values of effective electric permittivity and magnetic permeability. Metamaterials may posses unique combinations of properties, that no exist in natural materials [1]-[3].

Most of the articles are devoted to the study of metamaterials in which both these characteristics are negative. Meanwhile, it is easier to create a metamaterial with single negative magnetic permeability [4]. Such metamaterials have been studied much less (see, for example [5],[6])

Consider the slow surface electromagnetic eigenwaves on a mu-negative cylinder. Here, the term ”slow” means that we consider waves with phase velocities less than the speed of light in vacuum (in the diagram on the right-hand of a light's line). Structure considered consists of a infinitely long circular cylinder of the ideal (lossless) homogeneous, isotropic and frequency dependent mu-negative medium ($r < R_c$) embedded in a vacuum ($r > R_c$) with permittivity $\epsilon_1 = 1$ and permeability $\mu_1 = 1$. Assume that the coordinate axis $\boldsymbol{OZ}$ and the cylinder axis coincide.

This mu-negative medium possess such parameters: the constant permittivity $\epsilon_2 = 1$ and the frequency dependent permeability $\mu_2(\omega) = 1 - a\omega^2/(\omega^2 - \omega_0^2)$, $\omega_0/2\pi = 4GHz$ - resonant frequency, $a = 0.56$ - geometric factor (for example, [4]). Parameters a, ω_0 are uniquely determined by the specific design of a particular metamaterial. Throughout this communication our study was limited by such values of frequency $\omega_0 < \omega < \omega_0/\sqrt{1-a}$, that inequality $\mu_2(\omega) < 0$ is fulfilled.

2. RESULTS

We want to search the possibility of propagation (along $\boldsymbol{OZ}$) the surface electromagnetic waves of angular frequency ω, wavenumber β and azimuthal number n. In what follows we assume the anzats

$$A_j^n(r,\phi,z) = A_{0j}^n(r)\Psi^n, j = 1,2 \tag{1}$$

where $\Psi^n = \exp i(\beta z - \omega t + n\phi)$.

As is a well-known, the cylindrical eigenwave fields are the superpositions of transverse-electric and transverse-magnetic modes [7]

$$E_{r,j}^n = \left[\frac{i\beta}{\kappa_j}F_n'(\kappa_j r)a_j^n - \frac{n\mu_j(\omega)\omega}{\kappa_j^2 r}F_n(\kappa_j r)b_j^n\right]\Psi^n \tag{2}$$

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$$E^n_{\phi,j} = -\left[\frac{n\beta}{\kappa_j^2 r}F_n(\kappa_j r)a^n_j + i\frac{\mu_j(\omega)\omega}{\kappa_j}F'_n(\kappa_j r)b^n_j\right]\Psi^n \tag{3}$$

$$E^n_{z,j} = F_n(\kappa_j r)a^n_j\Psi^n \tag{4}$$

$$H^n_{r,j} = \left[\frac{nk_j^2}{\mu_j(\omega)\omega\kappa_j^2 r}F_n(\kappa_j r)a^n_j + i\frac{\beta}{\kappa_j}F'_n(\kappa_j r)b^n_j\right]\Psi^n \tag{5}$$

$$H^n_{\phi,j} = \left[\frac{ik_j^2}{\mu_j(\omega)\omega\kappa_j}F'_n(\kappa_j r)a^n_j - \frac{n\beta}{\kappa_j^2 r}F_n(\kappa_j r)b^n_j\right]\Psi^n \tag{6}$$

$$H^n_{z,j} = F_n(\kappa_j r)b^n_j\Psi^n \tag{7}$$

here $\kappa_j = \sqrt{k_j^2 - \beta^2}$, $k_j^2 = k^2\epsilon_j\mu_j$, $k = \omega/c$, c - the speed of light in vacuum. In equations (2)-(7) the wave fields are presented in a compact form both for the vacuum ($j = 1, F_n = H^{(1)}_n$) and for mu-negative cylinder ($j = 2, F_n = J_n$). Sign $'$ denotes the derivative of cylindrical functions by the argument.

Satisfying of the boundary conditions for tangential wave field components at $r = R_c$

$$H^n_{z,1} = H^n_{z,2}, H^n_{\phi,1} = H^n_{\phi,2}, E^n_{z,1} = E^n_{z,2}, E^n_{\phi,1} = E^n_{\phi,2} \tag{8}$$

and the condition for solvability of the homogeneous system of equations for four coefficients $a^n_1, a^n_2, b^n_1, b^n_2$ give rise the dispersion equation for such wave disturbances

$$\left[\frac{\mu_2(\omega)}{u}\frac{J'_n(u)}{J_n(u)} - \frac{\mu_1}{v}\frac{H'_n(v)}{H_n(v)}\right]\left[\frac{\epsilon_2}{u}\frac{J'_n(u)}{J_n(u)} - \frac{\epsilon_1}{v}\frac{H'_n(v)}{H_n(v)}\right]\left[\frac{\omega}{c}\right]^2 - n^2\beta^2\left[\frac{1}{v^2} - \frac{1}{u^2}\right]^2 = 0 \tag{9}$$

$v = \kappa_1 R_c$, $u = \kappa_2 R_c$.

The solution of (9) at a given n gives rise to a pair (ω, β), so we get the dispersion curve $\omega(\beta)$ for nth mode. We introduce the dimensionless wave frequency $\Omega = \omega/\omega_0$, wavenumber $K = \beta c/\omega_0$ and the dimensionless radius $R = R_c\omega_0/c$.

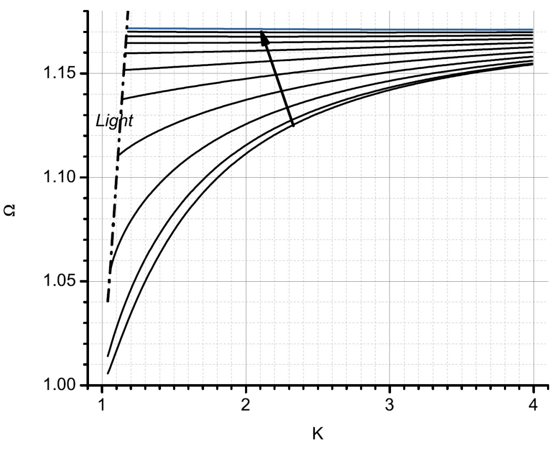

Figure 1. Dispersion curves for modes with different azimuthal number values n=0,1...10 (along in the direction of arrows) with R=2

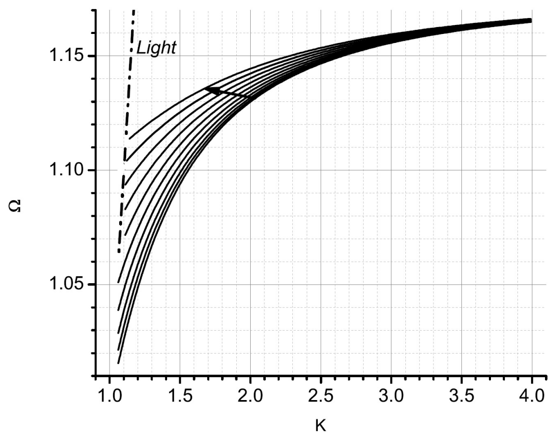

Figure 2. Dispersion curves for modes with different azimuthal number values n=0,1...10 (along in the direction of arrows) with R=8

In Fig.1,2 it has been shown the dispersion curves for modes with azimuthal number $n = 0, 1...10$ for cylinder with the dimensionless radius $R = 2$ and $R = 8$. The dash-dotted line correspond with the light dispersion. For all modes take

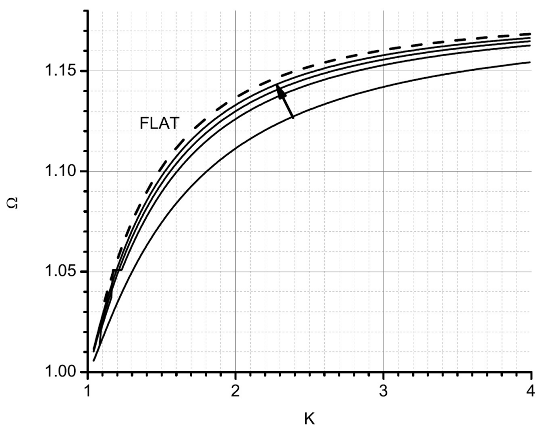

Figure 3. Dispersion curves for mode with azimuthal number n=0 for various values R=2,5,8,15 (along in the direction of arrows) and for planar surface mode [5] (dashed line)

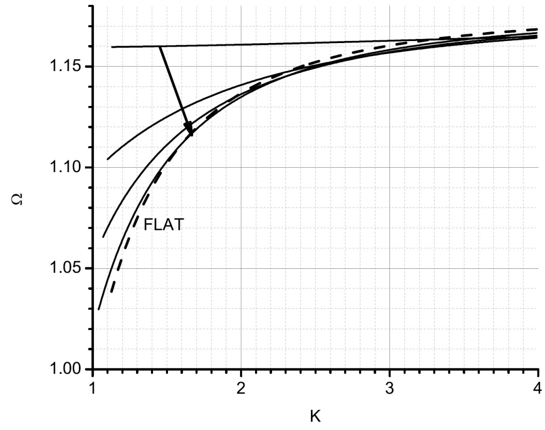

Figure 4. Dispersion curves for mode with azimuthal number n=6 for various values R=2,5,8,15 (along in the direction of arrows) and for planar surface mode [5] (dashed line)

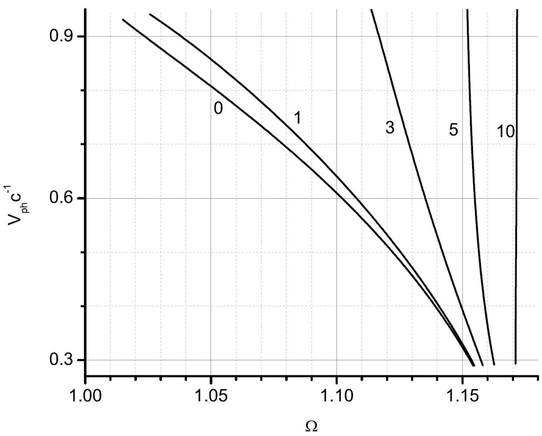

Figure 5. Phase velocity versus frequency for modes with azimuthal numbers n=0, 1, 3, 5, 10 for cylinder radius value R=2

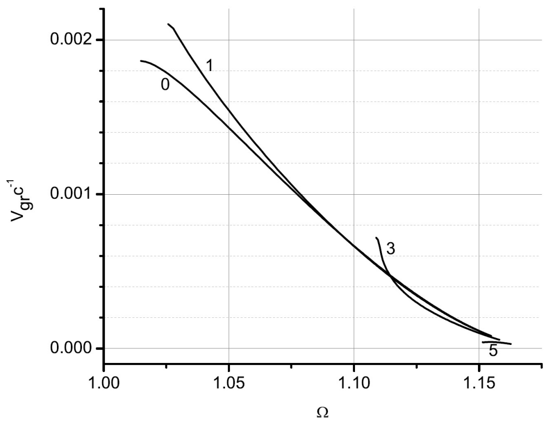

Figure 6. Group velocity versus frequency for modes with azimuthal numbers n = 0, 1, 3, 5 for cylinder radius value R=2

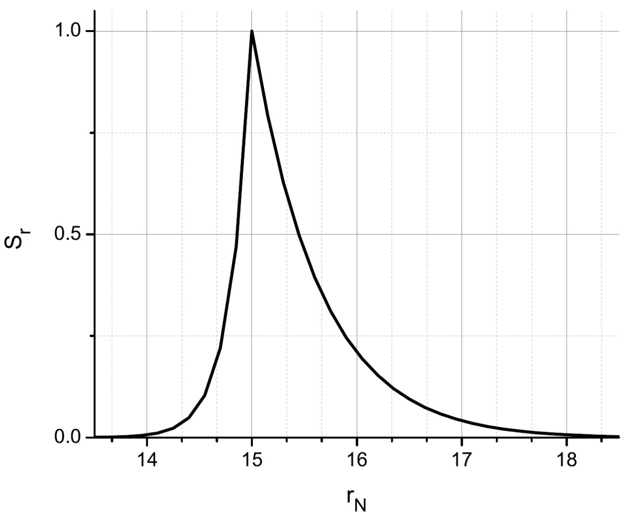

Figure 7. Radial Poynting vector S_r versus r_N for modes with azimuthal number n=6 at cylinder radius R=15 and wave frequency ω=1.054729

place a normal dispersion. The modes with higher value n have higher frequencies and exist in a narrower bandwidths. The transverse wave numbers $\kappa_{1,2}$ are purely imaginary values. Therefore the wave fields decays exponentially with distance from the cylinder surface in the radial direction. Similar dispersion curves were obtained for an epsilon-negative circular cylinder [8]).

In Figs.3,4 we present the dispersion curves evolution with increasing cylinder radius R for modes with $n = 0$ and $n = 6$. The dispersion curves for larger cylinder radii asymptotically approach the dispersion curve of a surface wave in the planar geometry obtained in [5] (dashed line). The dispersion curves of the symmetric mode ($n = 0$) change little with increasing radius R.

The phase velocities of these modes vary by an order of magnitude within the frequency intervals considered (Fig.5). Variety of types the dependency phase velocity of the mode versus its frequency may be used in different devices. As a example, an almost linear dependence will certainly come in handy for broadband pulse amplification [9], etc.

The directions of group and phase velocities coincide, so these modes are forward waves (Fig.6). The values of group velocities no more $0.002c$. The group velocities for modes with angular wavenumber $n > 5$ are less by several orders.

The electromagnetic fields of surface modes on a mu-negative cylinder are attenuated in the radial direction away from the interface 'metamaterial/vacuum'. In Fig.7 it presents the radial component of the Poynting vector $S_r \sim Re(E_z H^*_\phi)$ versus the dimensionless radial coordinate $r_N = r\omega_0/c$. It can be seen that the wave fields penetrate not as deeply into the mu-negative cylinder as into the vacuum. This difference becomes more noticeable with a larger cylinder radius. This property makes these modes with promising for interacting with charged particle beams [**?**] that do not must to move very close to the cylinder.

The distribution of energy flux over the angle ϕ is symmetric for $n = 0$, but consists of $2n$ identical narrow radiant sectors for $n > 0$. For higher n, the total number of these sectors increases, they become narrower and have a higher energy flux density.

3. CONCLUSIONS

We found that surface electromagnetic modes can propagate along a mu-negative cylinder. These waves are slow and forward, consisting of a superposition of transverse-electric and transverse-magnetic modes. Modes with higher azimuthal numbers occur at higher frequencies. A significant practical advantage of these waves—especially for controlling the movement of charged particles—is their ability to function without an external magnetic field. We hope that these surface electromagnetic waves will enhance the existing range of modes used for generating electromagnetic waves and managing the motion of charged particles, as well as for receiving and transmitting signals and other applications.

Declarations

Conflict of interest. None declared.
Ethical approval. Not required.

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ПОВІЛЬНІ ПОВЕРХНЕВІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ НА МЮ-НЕГАТИВНОМУ ЦИЛІНДРІ

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У цій статті ми представляємо теоретичні дослідження метаматеріалів з негативною магнітною проникністю. Вибрано феноменологічний опис електродинаміки. Досліджено дисперсійні властивості повільних поверхневих електромагнітних хвиль, які можуть поширюватися вздовж круглого циліндра з мю-негативного метаматеріалу. Ми нехтуємо втратами енергії в метаматеріалі. Негативність проникності має місце в досить обмеженому інтервалі частот. Для всіх режимів має місце нормальна дисперсія, незалежно від значень параметрів. Значення фазових швидкостей цих хвиль лежать між c і 0,3c. Залежності фазових швидкостей досліджуваних мод від їх частоти мають різноманітний вигляд. Напрями групових і фазових швидкостей збігаються. Значення групової швидкості менше 0,002c. Хвильові поля являють собою суперпозицію поперечно-електричної та поперечно-магнітної частин і експоненціально спадають у радіальному напрямку від межі розділення. Хвильові поля проникають в мю-негативний метаматеріал набагато слабше, ніж у вакуум. Для поширення хвиль в структурі не потрібне зовнішнє магнітне поле. Різноманітність цих хвильових характеристик параметрів циліндра можна використовувати для різних застосувань.

Ключові слова: *метаматеріал; негативна магнітна проникність; циліндр; поверхнева електромагнітна хвиля*