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A NEW IBM-1 HAMILTONIAN PARAMETERIZATION BASED ON DYNAMICAL SYMMETRY FOR EVEN-EVEN Se^{74-78}, $Ru^{104-106}$ AND $Mo^{106-108}$

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The present study investigates the dynamical symmetries of even-even Se^{74-78}, $Ru^{104-106}$ and $Mo^{106-108}$ isotopes within the framework of the Interacting Boson Model-1 (IBM-1). The analysis explores the three fundamental symmetry groups: SU(5) (vibrational), O(6) (gamma-soft), and SU(3) (rotational). Energy levels were calculated using newly optimized Hamiltonian parameterizations based on the U(6) unitary group structure in six dimensions, and the results were compared with experimental data, demonstrating excellent agreement. A key finding of this study is that for each nucleus, the predominant Hamiltonian parameter can be selectively adjusted to achieve an optimal theoretical-experimental match, particularly in relation to the $\frac{E4_1^+}{E2_1^+}$ energy ratio. This approach provides a systematic and efficient method for refining Hamiltonian parameterization, offering a standardized technique to enhance the accuracy of nuclear structure studies. The findings contribute to a deeper understanding of nuclear collective motion and may serve as a foundation for further advancements in theoretical nuclear physics.
Keywords: *IBM-1; Dynamical symmetry, Hamiltonian parameterization; Energy levels; Se-isotopes; Ru-isotopes; Mo-isotopes*
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INTRODUCTION

The Interacting Boson Model (IBM-1), developed by Arima and Iachello, revolutionizes the study of nuclear structure by offering a simplified yet powerful framework rooted in group theory [1]. In the six-dimensional space governed by the unitary group U(6), the IBM-1 model explains dynamical symmetry through its fundamental subgroups: SU(5) for vibrational nuclei, SU(3) for rotational nuclei, and O(6) for gamma-soft nuclei [2].

Several studies have utilized the IBM-1 model to investigate the nuclear structure of various isotopes. S.A. Abdulsahib et al. employed IBM-1 for even-even Se^{76-82} isotopes, considering them as O(6) shapes. Their study focused on the impact of the O(6) dynamical symmetry group chain on the Hamiltonian parameterization, particularly on (a_0, a_1, and a_3) [3]. Similarly, Yaseen, Mustafa T., et al. analyzed the Se^{72-80} isotopes, primarily highlighting the dominance of U(5) characteristics. They also noted a minor influence of the a_2 parameter, beginning from Se^{74} to Se^{80}. Their energy ratios indicated that Se^{72} is the closest to a typical vibrational limit, while the Se^{74-80} isotopes exhibit a gradual transition toward the rotational region along the U(5)-SU(3) limit [4].

Beyond selenium isotopes, Sharrad, Fadhil I. et al. investigated $Ru^{96,98}$ isotopes, demonstrating their U(5) dynamical symmetry. They highlighted the significant influence of the ϵ parameter in the IBM-1 Hamiltonian. Furthermore, the even-even $Ru^{108-112}$ isotopes were classified as transitional nuclei, positioned between the U(5) spherical vibrator and SO(6) γ-unstable rotor symmetries [6]. More broadly, several studies focus on refining the IBM-1 Hamiltonian to enhance its applicability to these isotopes.

In addition, Berun et al. have carried out significant research on the nuclear structure of Mo isotopes, including $^{94}_{42}Mo$ [7], $^{96-98}_{42}Mo$ [8], and $^{100}_{42}Mo$ [9], using the IBM-1 model. Their studies emphasized the vibrational nature of these nuclei under SU(5) symmetry, highlighting the crucial role of four Hamiltonian parameters (ε, a_1, a_3, and a_4) in determining their energy spectra. More recently, Ghafoor and Shwan (2024) explored the nuclear structure of $^{102}_{42}Mo$ within the IBM-1 framework, analyzing the impact of the Hamiltonian parameters a_0, a_1, and a_3. Their findings established a strong correlation between these parameters and the O(6) dynamical symmetry group, which characterizes gamma-soft nuclei [10].

The objective of the current study is to systematically investigate the Hamiltonian parameters for constructing the energy levels of even-even Se^{74-78}, $Ru^{104-106}$, and $Mo^{106-108}$ isotopes. A particular focus is placed on the $\frac{E4_1^+}{E2_1^+}$ energy ratio, which serves as a key indicator of nuclear dynamical symmetry. Furthermore, this study examines the specific dynamical symmetries exhibited by these nuclei within the IBM-1 framework, refining the parameterization approach for future nuclear structure studies.

THEORETICAL BACKGROUND

The Hamiltonian, which establishes connections between the basis states, is formulated using the principles of second quantization. As a result, it exclusively consists of combinations of the operators s, $s^\dagger$, d, and $d^\dagger$. Such a Hamiltonian operator ($\hat{H}$) comprises both one-body and two-body operators [1].

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$$\hat{H} = \varepsilon_s\, s^{\dagger}\tilde{s} + \varepsilon_d \sum_m d^{\dagger}\tilde{d} + V. \quad (1)$$

The parameters ε_s and ε_d represent the single-boson energies for the s- and d −bosons, respectively, and m represents the magnetic quantum number of the dd-boson states, which range from $m = -2, -1, 0, 1, 2$ corresponding to the angular momentum quantum number L = 2. while V denotes the boson-boson interaction potential. The operators $s^{\dagger}(\tilde{s})$ function as creation and annihilation operators for the s-boson state, whereas $d^{\dagger}(\tilde{d})$ serve as creation and annihilation operators for the d-boson state. The most widely utilized formulation of the IBM-1 Hamiltonian [11], which also provides the clearest insight into how each term influences the resulting nuclear structure, is known as the multipole expansion. In this approach, different boson-boson interactions are categorized systematically, allowing the Hamiltonian to be expressed in a structured form.

$$\hat{H} = \varepsilon\,(n_d) + a_0\,(\hat{P}.\,\hat{P}) + a_1(\hat{L}.\,\hat{L}) + a_2(\hat{Q}.\,\hat{Q}) + a_3\,(\hat{T}_3.\,\hat{T}_3) + a_4\,(\hat{T}_4.\,\hat{T}_4). \quad (2)$$

Here ε, a_0, a_1, a_2, a_3, and $a_4$represent the model parameters, while $\hat{P}.\,\hat{P}$, $\hat{L}.\,\hat{L}$, $\hat{Q}.\,\hat{Q}$, $\hat{T}_3.\,\hat{T}_3$, and $\hat{T}_4.\,\hat{T}_4$ correspond to the pairing, angular momentum, quadrupole, octupole, and hexadecapole operators, respectively. Additionally, n_d denotes the d-boson number operator.

RESULTS AND DISCUSSION

The first step in our calculations is to determine the total number of bosons. In the original IBM-1 model, there is no distinction between protons and neutrons [12], and the valence number is always counted relative to the nearest closed shells. To calculate the total boson count, we first determine the difference in proton or neutron numbers relative to the nearest closed shell, divide this value by two, and then sum the resulting proton and neutron bosons. Following this method, the total number of bosons for even-even isotopes is as follows: Se^{74-78} has 8, 7, and 6 bosons, respectively; $Mo^{106-108}$ has 11 and 12 bosons, respectively; and $Ru^{104-106}$ has 8 and 9 bosons, respectively. In the present study the $\frac{E4_1^+}{E2_1^+}$ ratio was used in order to determine the dynamical symmetry region for isotopes. Table 1 provides the IBM-1 Hamiltonian parameters essential for calculating the low-lying positive parity energy levels of presented nuclei. In the Interacting Boson Model-1 (IBM-1), the Hamiltonian plays a crucial role in describing nuclear structure by incorporating different interaction terms that govern the behavior of bosons. To ensure accurate results, the IBM-1 Hamiltonian is constructed and solved within the framework of dynamical symmetry regions. This approach allows for a systematic understanding of nuclear shape transitions, whether vibrational (U(5)), rotational (SU(3)), or gamma-soft (O(6)). The calculations are performed using the PHINT computer program [13], which efficiently handles the matrix diagonalization and provides the energy eigenvalues necessary for analyzing nuclear excitations.

Table 1. IBM-1 Hamiltonian parameterization for description of presented nuclei

Nucleus	Hamiltonian Parameters (MeV)						χ	$SO(6)$
	EPS	*PAIR*	*ELL*	*QQ*	*OCT*	*HEX*		
Se^{74}	0.2700	0.0070	0.0188	0.0000	0.0606	0.0850	0.0000	1.0000
Se^{76}	0.2670	0.0060	0.0188	0.0000	0.0506	0.0850	0.0000	1.0000
Se^{78}	0.1600	0.1160	0.0108	0.0000	0.3406	0.0550	0.0000	1.0000
Ru^{104}	0.5000	0.0500	0.0105	-0.0080	0.0000	0.0000	-1.3200	1.0000
Ru^{106}	0.4700	0.0500	0.0105	-0.0070	0.0000	0.0000	-1.3200	1.0000
Mo^{106}	0.4800	0.0500	0.0105	-0.0080	0.0000	0.0000	-1.3200	1.0000
Mo^{108}	0.4800	0.0400	0.0120	-0.0080	0.0000	0.0000	-1.3200	1.0000

Where the Parameters are related to its coefficients such $\varepsilon = EPS$, $a_0 = 2 * PAIR$, $a_1 = \frac{ELL}{2}$, $a_2 = \frac{QQ}{2}$, $a_3 = 5 * OCT$, $a_4 = 5 * HEX$ [14] The low-lying positive parity energy levels of Se isotopes have been investigated, and the theoretical results show strong agreement with experimental data [15-17], as illustrated in Figure 1.

The Hamiltonian parameters ε, a_1 and a_4 play a crucial role in accurately reproducing the energy levels for Se^{74} due to its SU(5) dynamical symmetry. However, in the case of Se^{76}, the sensitivity shifts towards a_0, where even a slight variation in a_0 yields the best agreement between theoretical and experimental values. This indicates that Se^{76} exhibits characteristics intermediate between SU(5) and O(6) dynamical symmetries. On the other hand, Se^{78} predominantly exhibits O(6)-like gamma-unstable behavior, with a_0 , a_1 and a_3 emerging as the most influential parameters in determining its energy structure. Additional energy levels that are not explicitly shown in Figure 1 have also been investigated, demonstrating strong agreement with experimental data. For Se^{74}, the calculated energy values for 2_2^+, 3_1^+, 6_1^+, 6_2^+, and 4_4^+ are 1.268 MeV, 1.0904 MeV, 2.294 MeV, 3.096 MeV, and 3.050 MeV, respectively. These results closely match the corresponding experimental values [15] of 1.269 MeV, 1.884 MeV, 2.232 MeV, 2.986 MeV, and 3.078 MeV. Similarly, for Se^{76}, the calculated energy levels for 2_2^+ and 4_3^+ are 1.216 MeV and 2.805 MeV, which align well with the experimental values [16] of 1.230 MeV and 2.840 MeV. For Se^{78}, the calculated energy values for 2_2^+, 4_2^+, 6_1^+, and 6_2^+, are 1.308 MeV, 2.191 MeV, 2.546 MeV, and 3.140 MeV, respectively. These values show excellent agreement with the experimental data [17], which are 1.378 MeV, 2.207 MeV, 2.464 MeV, and 3.123 MeV, respectively.

The energy levels of Ru^{104} and Ru^{106} are depicted in Figure 2 and compared with experimental data [18, 19], demonstrating excellent agreement.

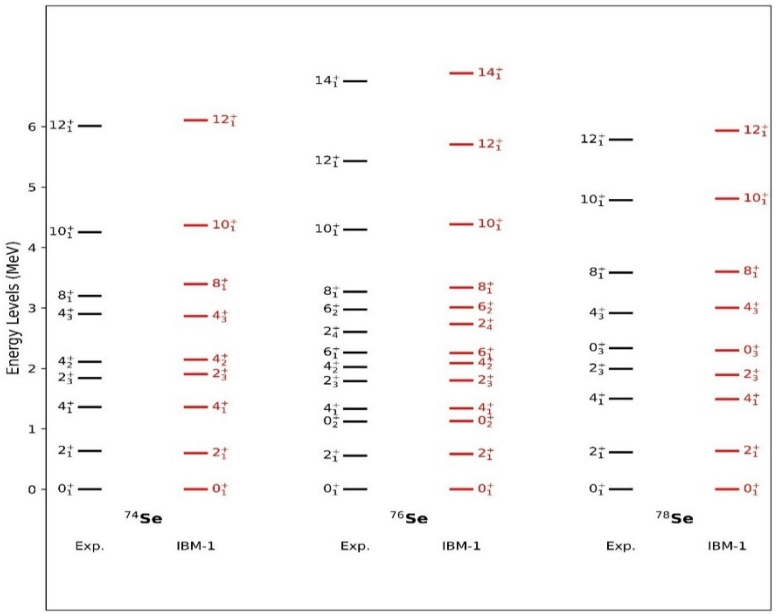

Figure 1. IBM-1 calculated energy level schemes for Se isotopes compared with experimental data [15-17].

The O(6) dynamical symmetry is predominant in Ru^{104}, making a_0 a highly sensitive parameter alongside a_3,. However, variations in the ε parameter can systematically alter the energy levels due to the vibrational nature of Ru^{104}, which aligns closely with SU(5) symmetry. In contrast, Ru^{104} also exhibits gamma-soft behavior, with O(6) symmetry remaining dominant. However, it slightly deviates from the O(6) group chain, displaying characteristics that gradually shift toward rotational motion. While maintaining the calibration of a_0 , a_1 and a_3, minor adjustments to the a_2, parameter is necessary. Additionally, in Figure 2, several energy levels have been analyzed using the optimized Hamiltonian parameterization. For Ru^{104}, the calculated values for 2_2^+, 0_2^+, 0_3^+, 4_2^+, 4_3^+, and 3_2^+ are 0.880 MeV, 0.899 MeV, 1.288 MeV, 1.451 MeV, 1.995 MeV, and 2.401 MeV, respectively, showing strong agreement with the experimental values [18] of 0.893 MeV, 0.988 MeV, 1.335 MeV, 1.502 MeV, 2.080 MeV, and 2.330 MeV, respectively. Similarly, for Ru^{106}, the calculated energy levels for 2_2^+, $0_2^+$and 4_2^+ are 0.702 MeV, 0.832 MeV, and 1.276 MeV, which align well with the experimental data [19] of 0.792 MeV, 0.991 MeV, and 1.306 MeV, respectively.

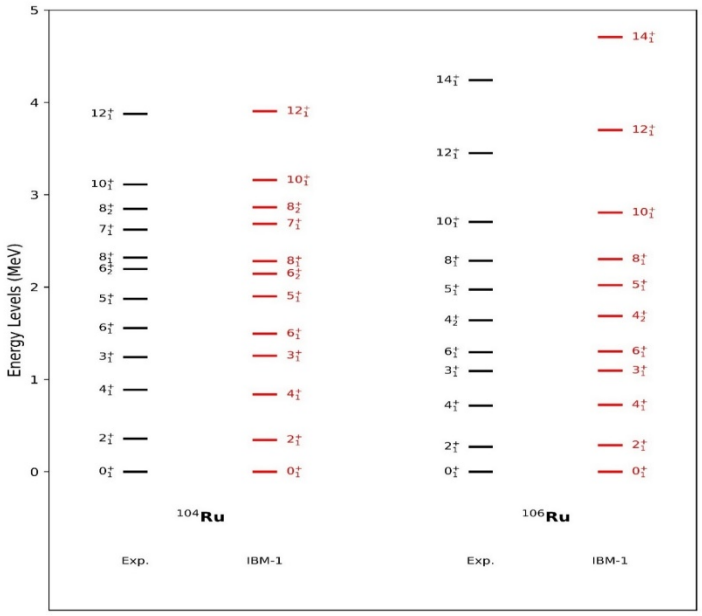

Figure 2. IBM-1 calculated energy level schemes for Ru isotopes compared with experimental data [18, 19].

The energy values of Mo^{106} and Mo^{108} are presented in Figure 3, demonstrating excellent agreement with experimental data [19, 20]. For Mo^{106}, the Hamiltonian parameters a_1 and a_2 exhibit the highest sensitivity, as the

nucleus is close to the SU(3) dynamical symmetry. Additionally, the parameters a_0 and a_3 show minor sensitivity, indicating the influence of O(6) dynamical symmetry. Consequently, $Mo^{106}$is positioned within the O(6)-SU(3) dynamical symmetry limit. On the other hand, Mo^{108} is highly sensitive to a_1 and a_2, confirming its strong proximity to SU(3) dynamical symmetry.

Furthermore, additional energy levels not listed in Figure 3 have been investigated. For Mo^{106}, the 4_2^+, 6_2^+, and the double gamma band state 7_1^+ are calculated as 1.105 MeV, 1.606 MeV, and 2.127 MeV, respectively, showing excellent agreement with the experimental values [19] of 1.068 MeV, 1.563 MeV, and 2.127 MeV. Similarly, for Mo^{108}, the 2_2^+, 4_2^+, and the two-phonon vibrational band state 9_1^+ are calculated as 0.644 MeV, 1.056 MeV, and 2.908 MeV, respectively, aligning well with the experimental data [20] of 0.586 MeV, 0.978 MeV, and 2.883 MeV.

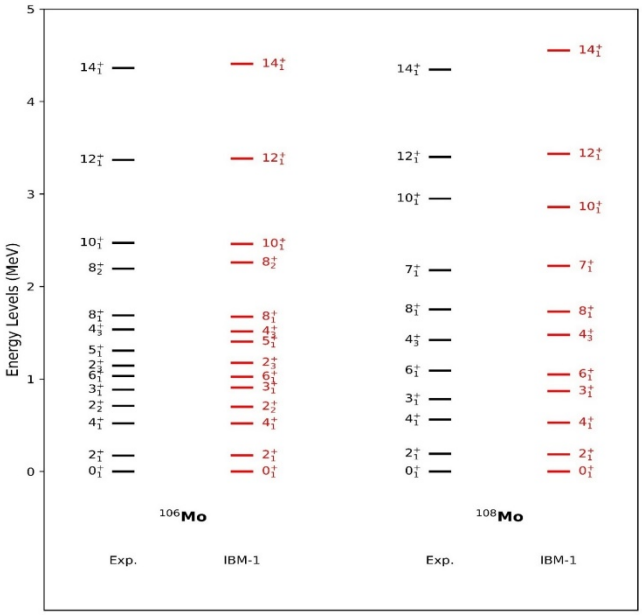

Figure 3. IBM-1 calculated energy level schemes for Mo isotopes compared with experimental data [19, 20].

The energy ratio $\frac{E4_1^+}{E2_1^+}$ plays a crucial role in diagnosing dynamical symmetry behavior. As previously mentioned, this ratio serves as a key indicator of nuclear structure: it is 2.0 for SU(5)-like vibrational nuclei, 2.5 for O(6) gamma-soft nuclei, and 3.33 for SU(3) rotational nuclei [21-24]. The calculated $\frac{E4_1^+}{E2_1^+}$ values for the investigated nuclei are presented in Figure 4. The figure reveals that these nuclei predominantly align with the SU(5) symmetry, deviating from the O(6) limit and gradually approaching the SU(3) region. Furthermore, the energy ratio $\frac{E4_1^+}{E2_1^+}$ provides insights into the evolution of Hamiltonian parameters, highlighting which interactions dominate within each symmetry group.

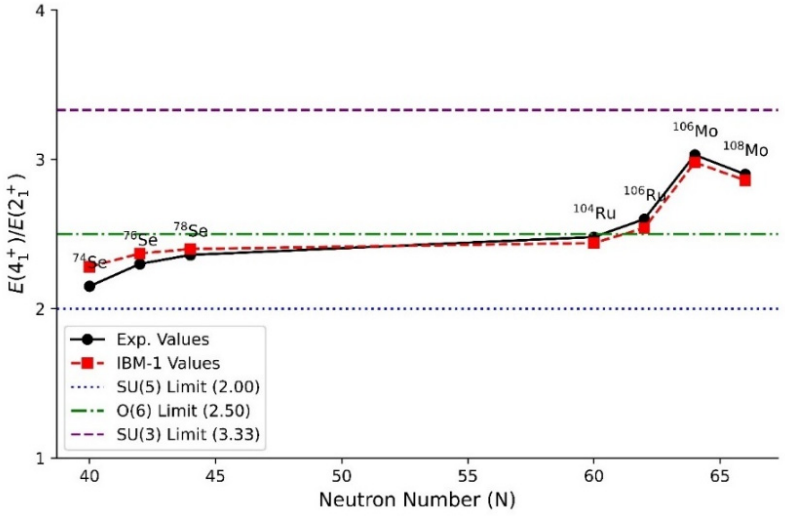

Figure 4. IBM-1 calculated value of $\frac{E4_1^+}{E2_1^+}$ compared with experimental data [20-25].

The energy ratio $\frac{E4_1^+}{E2_1^+}$ is crucial because it directly measures the collective motion inside the nucleus, differentiating between vibrational, gamma-soft, and rotational motions. Furthermore, it acts as a standard for verifying theoretical nuclear models, assisting in parameter refinement and improving the ability to predict of nuclear structure predictions.

CONCLUSION

This study systematically investigated the energy levels of even-even Se^{74-78}, $Mo^{106-108}$, and $Ru^{104-106}$ isotopes across the ground state, beta, and gamma bands using a specifically optimized IBM-1 Hamiltonian parameterization. Based on group theoretical analysis, the examined nuclei were categorized into three primary dynamical symmetry groups: SU(5) (vibrational), O(6) (gamma-soft), and SU(3) (rotational). The findings reaffirm that for SU(5) vibrational nuclei, the Hamiltonian parameters ε, a_1, a_3 and a_4 which correspond to the d-boson number, angular momentum, octupole, and hexadecapolar operators, respectively—are the most influential in accurately reproducing the nuclear energy spectra. In contrast, for O(6) gamma-soft nuclei, the dominant parameters are a_0 , a_1 and a_3, which are associated with pairing, angular momentum, and octupole interactions, respectively. Meanwhile, in the SU(3) rotational regime, the a_1 and a_2 parameters, linked to angular momentum and quadrupole operators, play the most significant role in Hamiltonian parameterization. This study highlights that ε (d-boson number operator) is the most sensitive parameter for vibrational nuclei, a_0 (pairing operator) is the key sensitivity factor for gamma-soft nuclei, and a_1 (angular momentum operator) is the dominant parameter for rotational nuclei. These insights contribute to a deeper understanding of nuclear structure and provide a refined approach for optimizing IBM-1 Hamiltonian parameterization in future nuclear physics research.

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НОВА ГАМІЛЬТОНОВА ПАРАМЕТРИЗАЦІЯ IBM-1 НА ОСНОВІ ДИНАМІЧНОЇ СИМЕТРІЇ ДЛЯ ПАРНИХ-ПАРНИХ Se^{74-78}, $Ru^{104-106}$ ТА $Mo^{106-108}$

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У цьому дослідженні досліджуються динамічні симетрії парно-парних ізотопів Se^{74-78}, $Ru^{104-106}$ і $Mo^{106-108}$ у рамках моделі взаємодіючого бозона-1 (IBM-1). Аналіз досліджує три основні групи симетрії: SU(5) (вібраційна), O(6) (гамма-м'яка) і SU(3) (обертальна). Рівні енергії були розраховані з використанням нещодавно оптимізованих гамільтонівських параметризацій, заснованих на структурі унітарної групи U(6) у шести вимірах, і результати порівнювалися з експериментальними даними, демонструючи чудову узгодженість. Ключовим висновком цього дослідження є те, що для кожного ядра переважаючий параметр Гамільтона можна вибірково регулювати для досягнення оптимального теоретико-експериментального збігу, зокрема щодо $\frac{E4_1^+}{E2_1^+}$ співвідношення енергії. Цей підхід забезпечує систематичний та ефективний метод уточнення гамільтонівської параметризації, пропонуючи стандартизовану техніку для підвищення точності досліджень ядерної структури. Отримані результати сприяють глибшому розумінню колективного ядерного руху і можуть стати основою для подальшого прогресу в теоретичній ядерній фізиці.

Ключові слова: *IBM-1; динамічна симетрія, гамільтонова параметризація; рівні енергії; Se-ізотопи; Ru-ізотопи; Mo-ізотопи*