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FLRW COSMOLOGICAL MODEL WITH QUADRATIC FUNCTIONAL FORM IN $f(R,T)$ THEORY OF GRAVITY

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This work investigates a spatially homogeneous and isotropic flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe within the context of $f(R,T)$ gravity as introduced by Harko, *et al.*, [Phys. Rev. D, 84, 024020 (2011)]. The present work deals with the functional form $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = R + \lambda_1 R^2$ and $f_2(T) = 2\lambda_2 T$ where λ_1 and λ_2 are arbitrary constants, R and T being the Ricci scalar and the trace of the stress-energy tensor T_{ij} respectively. We present a novel cosmological model in the framework of $f(R,T)$ gravity, exploring the dynamics of the FLRW universe through an exact solution to the gravitational field equations. By employing an innovative ansatz for the Hubble parameter, $H = \alpha\left(1 + \frac{1}{t}\right)$ where α is a positive constant, we capture a evolutionary history of the universe. This approach provides a natural pathway to investigate key cosmological parameters, such as the scale factor, deceleration parameter, jerk, snap, lerk parameters and energy conditions, revealing intriguing insights into the universe's expansion dynamics. We also discuss the statefinder diagnostic. Our results offer a deeper understanding of cosmic evolution within the $f(R,T)$ gravity framework.

Keywords: $f(R,T)$ *gravity; FLRW metric; Hubble parameter; Statefinder diagnostic*

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1. INTRODUCTION

Late time cosmic acceleration is one of the most significant and challenging discoveries in cosmology which might revolutionize the theories of gravitation and cosmology in near future. Over the last few decades, several crucial cosmological and astrophysical observations from high redshift supernovae type Ia (SNIa) [1–3], Wilkinson Microwave Anisotropy Probe (WMAP) [4–8], Large Scale Structure (LSS) [9], Cosmic Microwave Background (CMB) [10, 11] etc. have been providing strong evidence that the universe is currently undergoing a phase of accelerated expansion. As there is no theoretical explanation for this observed acceleration in the rate of expansion of the universe, many theories have been proposed in the literature to understand the cause behind it. The General Theory of Relativity has provided the foundation for numerous attempts with a prominent hypothesis that the acceleration is driven by Dark energy, an enigmatic form of energy with a high negative pressure, which itself significantly challenges conventional cosmology as cosmological origin and the true nature of dark energy have not been determined yet. In addition, according to the observational data, more than 95% of the total matter-energy budget of the universe is comprised of two dark components - dark energy (DE) and dark matter (DM) - the contributions from DE and DM being about 68.3% and 26.8% respectively. The characteristics of these two dark components are not distinctly recognized. However, the nature of dark matter appears to be partially known [12] as it can be inferred to exist from its gravitational effects on ordinary baryonic matter which contributes only about 4.9% of the total content of the universe. Dark energy, therefore, becomes one of the biggest mysteries and a challenging topic of research in modern cosmology.

The cosmological constant Λ is the most widely discussed candidate for dark energy. However, it encounters two significant theoretical challenges: Cosmic Coincidence Problem and Fine-Tuning Problem.

To address these issues, several candidates of dark energy such as quintessence, k-essence, tachyon, phantom, Chaplygin gas models, Holographic dark energy models etc. are proposed in the literature. Another approach to understand the true cause behind the presence of dark sector in the universe and the mechanism behind the cosmic acceleration in the recent past is the modification of Einstein's theory of gravity. Several modifications of General Theory of Relativity are available in the literature, popularly known as modified theories of gravity, which are extremely attractive. Some important modified theories of gravity are: $f(R)$ theory of gravity [13], where R is the Ricci scalar, the action of which is constructed by replacing R by an arbitrary function $f(R)$ in the Einstein-Hilbert Lagrangian. $f(R,T)$ theory of gravity [14], a generalization of $f(R)$ theory by introducing an arbitrary function $f(R,T)$ of R and T, where T is the trace of the stress-energy of tensor, Brans Dicke theory of gravity [15] in which gravity couples with a time-varying scalar field through a coupling parameter. $f(T)$ gravity theory [16], which uses the torsion scalar in place of the Ricci scalar. $f(G)$ theory of gravity [17] where G is the Gauss-Bonnet invariant. $f(Q)$ gravity [18] where Q is the non-metricity scalar.

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Some other modified theories of gravity are $f(R,G)$ gravity [19], $f(Q,T)$ gravity [20] etc. By exploring these diverse approaches, researchers aim to gain a deeper understanding of the fundamental nature of the universe and its evolution with an accelerated rate of expansion.

Harko *et al.* introduced the $f(R,T)$ theory of gravity and obtained the gravitational field equations for three explicit forms of the functional $f(R,T)$ viz. $f(R,T) = R + 2f(T)$, $f(R,T) = f_1(R) + f_2(T)$ and $f(R,T) = f_1(R) + f_2(R)f_3(T)$. Houndjo [21] developed the cosmological reconstruction of $f(R,T)$ theory of gravity for the functional $f(R,T) = f_1(R) + f_2(T)$ and discussed the transition of matter dominated era with decelerated expansion to the current era with an accelerated expansion. Since then many authors have explored various aspects of this theory in different contexts as this theory can be best applied to study several issues of current interest and also takes care of the early time inflation as well as the late time accelerated expansion. Bhattacharjee and Sahoo [22] studied redshift drift in $f(R,T)$ theory of gravity where they have used the functional $f(R,T) = R + \lambda T$. Pradhan *et al.* [23] studied FLRW model in $f(R,T)$ gravity using $f(R,T) = R + 2\lambda T$.

In the present work, we consider the functional form $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = R + \lambda_1 R^2$ and $f_2(T) = \lambda_2 T$ where λ_1 and λ_2 are arbitrary constants. Starobinsky's work [24] motivates us to consider the functional form $f_1(R) = R + \lambda_1 R^2$. The Starobinsky model follow the cosmological observational test and successfully predicts a spectrum of nearly scale-invariant curvature perturbations. The R^2 term in the functional form $f(R) = R + \alpha R^2$, α is a constant, in Starobinsky's original work demonstrates that the R^2 term could naturally drive inflation due to a slow-roll regime, leading to a nearly de Sitter expansion. This mechanism does not require an explicit scalar field as the additional degrees of freedom from the R^2 term behave like a scalar field.

A number of researchers also considered the functional form $f(R,T)$ of the type $f(R,T) = R + \lambda_1 R^2 + \lambda_2 T$ in various contexts. Zubair and Noureen [25] studied evolution of axially symmetric anisotropic sources, Noureen *et al.* [26] investigated shear-free condition and dynamical instability, Sahoo *et al.* [27,28] proposed a model of wormholes and also $f(R,T)$ gravity model as alternatives to cosmic acceleration by constructing three cosmological models that arise from the three different choices for $f_1(R)$, viz, $f_1(R) = R + \alpha R^2 - \frac{\mu^4}{R}$, $f_1(R) = R + k\ln(\gamma R)$ and $f_1(R) = R + me^{[-nR]}$ with α, μ, k, γ, m and n all free parameters. Vinuthaa and Kavya [29] studied Bianchi type cosmological models in $f(R,T)$ theory with quadratic functional form. Bishi *et al.* [30] studied domain walls and quark matter cosmological models in $f(R,T) = R + aR^2 + kT$ gravity. These studies affirm $f(R,T)$ gravity as a versatile and promising framework, capable of providing insights into a wide array of phenomena, from exotic matter distributions to cosmic expansion, while accommodating both isotropic and anisotropic configurations.

In this study, we consider a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe filled with a perfect fluid within the framework of $f(R,T)$ gravity with the functional form $f(R,T) = R + \lambda_1 R^2 + \lambda_2 T$, where λ_1, λ_2 are arbitrary constants. The outline of the present work is as follows: In section 2, we provide a concise overview of the metric formalism of $f(R,T)$ theory of gravity and present the basic equations. In section 3, we derive the gravitational field equations in terms of the Hubble parameter H. In section 4, we solve the field equations by choosing an ansatz for the Hubble parameter. In Section 5, we discuss the physical and kinematical properties of the model by plotting the cosmological parameters against cosmic time t and redshift parameter z. In Section 6, we analyze and discuss the jerk, snap and lerk parameters, the statefinder diagnostic and the energy conditions of our model. Finally, in Section 7, we conclude the paper with a summary of our findings and key insights.

2. $f(R,T)$ GRAVITY THEORY: BASIC EQUATIONS

The action of $f(R,T)$ gravity theory proposed by Harko *et al.* [14] is given by

$$S = \int \left[\frac{1}{16\pi} f(R,T) + L_m \right] \sqrt{-g}\, d^4x \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci Scalar R and the trace T of the stress-energy tensor and L_m is the matter Lagrangian density.

The stress-energy tensor of matter is defined as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{ij}} \tag{2}$$

and the trace T is given by $T = g^{ij} T_{ij}$. Considering the metric tensor components g_{ij} to be the sole ones influencing the amount L_m of matter and not its derivatives, the stress-energy tensor T_{ij} is obtained as

$$T_{ij} = g_{ij} L_m - 2 \frac{\partial L_m}{\partial g^{ij}} \tag{3}$$

By varying the action (1) with respect to the metric tensor components g^{ij}, the field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \tag{4}$$

where, $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$ and the covariant derivative with regard to the symmetric connection Γ associated to the metric g is represented by the operator ∇_i.

Here, Θ_{ij} is obtained by specifying the variation of T with respect to the metric tensor

$$\frac{\delta(g^{\alpha\beta}T_{\alpha\beta})}{\delta g^{ij}} = T_{ij} + \Theta_{ij} \tag{5}$$

For a known matter Lagrangian L_m, Θ_{ij} can be calculated as

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lk}} \tag{6}$$

For a perfect fluid, the stress-energy tensor of matter is provided by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{7}$$

where ρ is the energy density, p is the pressure and the four velocity u_i satisfies the conditions $u_i u^i = 1$ and $u^i \nabla_j u_i = 0$. The matter Lagrangian can be written as $L_m = -p$. Using eq (6), we get the expression for Θ_{ij} for the modification of stress-energy tensor of perfect fluid as

$$\Theta_{ij} = -2T_{ij} - p g_{ij} \tag{8}$$

Assuming

$$f(R,T) = f_1(R) + f_2(T) \tag{9}$$

where $f_1(R)$ and $f_2(T)$ are arbitrary functions of R and T respectively, from equation (4), if the matter source is a perfect fluid, then the field equations of $f(R,T)$ gravity become

$$f_1^{'}(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_1^{'}(R) = 8\pi T_{ij} + f_2^{'}(T)T_{ij} + \left[f_2^{'}(T)p + \frac{1}{2}f_2(T)\right]g_{ij} \tag{10}$$

where the prime denotes differentiation with respect to the argument.

For the choice of $f_1(R) = R + \lambda_1 R^2$ and $f_2(T) = 2\lambda_2 T$, where λ is a constant, the gravitational field equations of $f(R,T)$ gravity from eq (10) are obtained as

$$R_{ij} - \frac{1}{2}Rg_{ij} + 2\lambda_1 R\left(R_{ij} - \frac{1}{4}Rg_{ij}\right) + (g_{ij}\Box - \nabla_i\nabla_j)(1 + 2\lambda_1 R) = 8\pi T_{ij} + \lambda_2\left[2T_{ij} + (\rho - p)g_{ij}\right] \tag{11}$$

3. THE METRIC AND GRAVITATIONAL FIELD EQUATIONS

We consider the flat Friedmann-Lemaître-Robertson-Walker metric given by

$$ds^2 = dt^2 - a^2(dx^2 + dy^2 + dz^2) \tag{12}$$

where a is a function of cosmic time t only. Using comoving coordinates, the field equations (11) for the metric (12) are obtained as

$$\frac{\ddot{a}}{a} + \frac{1}{2}\frac{\dot{a}^2}{a^2} + 6\lambda_1 G_1(a, \dot{a}, \ddot{a}, \dddot{a}, \ddddot{a}) = -4\pi p + \frac{\lambda_2}{2}(\rho - 3p) \tag{13}$$

$$\frac{\dot{a}^2}{a^2} + 6\lambda_1 G_2(a, \dot{a}, \ddot{a}, \dddot{a}) = \frac{8\pi}{3}\rho + \lambda_2\left(\rho - \frac{p}{3}\right) \tag{14}$$

where

$$G_1(a, \dot{a}, \ddot{a}, \dddot{a}, \ddddot{a}) = -\frac{3}{2}\left[\left(\frac{\dot{a}}{a}\right)^4 + \frac{\ddot{a}^2}{a^2}\right] - 2\left[\frac{\dot{a}\dddot{a}}{a^2} - 3\frac{\dot{a}^2\ddot{a}}{a^3}\right] - \frac{\ddddot{a}}{a}$$

$$G_2(a,\dot{a},\ddot{a},\dddot{a}) = -2\left(\frac{\dot{a}}{a}\right)^2\left[\frac{\ddot{a}}{\dot{a}}+\frac{\ddot{a}}{a}-\frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2\right]+\left(\frac{\ddot{a}}{a}\right)^2$$

In terms of the Hubble parameter H, defined by $H=\frac{\dot{a}}{a}$, equations (13) and (14) can be expressed as

$$\dot{H}+\frac{3}{2}H^2+6\lambda_1 G_1(a,\dot{a},\ddot{a},\dddot{a},\ddddot{a}) = -4\pi p+\frac{\lambda_2}{2}(\rho-3p) \tag{15}$$

$$H^2+6\lambda_1 G_2(a,\dot{a},\ddot{a},\dddot{a}) = \frac{8\pi}{3}\rho+\lambda_2\left(\rho-\frac{p}{3}\right) \tag{16}$$

where

$$G_1(H,\dot{H},\ddot{H},\dddot{H}) = -\left(\dddot{H}+6H\ddot{H}+9H^2\dot{H}+\frac{9}{2}\dot{H}^2\right)$$

$$G_2(H,\dot{H},\ddot{H}) = \dot{H}^2-6H^2\dot{H}-2H\ddot{H}$$

The Ricci scalar curvature is

$$R=-\left(6\dot{H}+12H^2\right) \tag{17}$$

where an overhead dot denotes differentiation with respect to t.

From equations (15) and (16), we get

$$\rho=\frac{1}{(8\pi+4\lambda_2)(4\pi+\lambda_2)}\Big[12\pi H^2+\lambda_2(3H^2-\dot{H})+72\pi\lambda_1(\dot{H}^2-6H^2\dot{H}-2H\ddot{H})+ 3\lambda_1\lambda_2(2\dddot{H}+18\dot{H}^2-6H\ddot{H}-36H^2\dot{H})\Big] \tag{18}$$

$$p=\frac{1}{(8\pi+4\lambda_2)(4\pi+\lambda_2)}\Big[-(8\pi+3\lambda_2)\dot{H}-3(4\pi+\lambda_2)H^2+24\pi\lambda_1\Big(2\dddot{H}+12H\ddot{H}+ 18H^2\dot{H}+9\dot{H}^2\Big)+18\lambda_1\lambda_2(\dddot{H}+5\dot{H}^2+5H\ddot{H}+6H^2\dot{H})\Big] \tag{19}$$

4. EXACT SOLUTION OF THE FIELD EQUATIONS

To find an exact solution of the field equations, we consider the following ansatz for the Hubble parameter H:

$$H=\alpha\left(1+\frac{1}{t}\right) \tag{20}$$

where $\alpha>0$ is an arbitrary constant.

The Hubble parameter H is an observable parameter. It measures the rate of cosmic expansion. Using the definition $H=\frac{\dot{a}}{a}$, from equation (20), we obtain the scale factor a as

$$a(t)=a_0(te^t)^\alpha \tag{21}$$

where $a_0>0$ is a constants.

Then from equations (18) and (19), the energy density ρ and the pressure p are obtained as

$$\rho(t)=\frac{3\alpha^2}{8\pi+4\lambda_2}+\frac{\alpha}{(8\pi+4\lambda_2)(4\pi+\lambda_2)}\Bigg[\frac{(24\pi+6\lambda_2)\alpha}{t}+\frac{\lambda_2+3\alpha(4\pi+\lambda_2)(1+36\lambda_1\alpha)}{t^2}+ \frac{108\lambda_1\lambda_2\alpha^2+36\lambda_1\alpha(3\alpha-1)(8\pi+\lambda_2)}{t^3}+\frac{18\lambda_1(2\alpha-1)(12\pi\alpha+\lambda_2(3\alpha+2))}{t^4}\Bigg] \tag{22}$$

$$p(t)=\frac{-3\alpha^2}{8\pi+4\lambda_2}+\frac{\alpha}{(8\pi+4\lambda_2)(4\pi+\lambda_2)}\Bigg[\frac{-(24\pi+6\lambda_2)\alpha}{t}+\frac{(8\pi+3\lambda_2)-(4\pi+\lambda_2)(3\alpha+108\lambda_1\alpha^2)}{t^2}+ \frac{36\lambda_1\alpha((16\pi+5\lambda_2)-\alpha(24\pi+6\lambda_2))}{t^3}+\frac{18\lambda_1(\alpha(44\pi+15\lambda_2)-\alpha^2(24\pi+6\lambda_2)-(16\pi+6\lambda_2))}{t^4}\Bigg] \tag{23}$$

The equation of state (EoS) parameter is defined as the ratio of pressure to energy density: $\omega = \frac{p}{\rho}$. This parameter is crucial in understanding the nature of the universe's energy content.

Therefore, the equation of state parameter ω is obtained as

$$\omega(t) = \frac{\begin{array}{c}-3\alpha t^4 - (24\pi + 6\lambda_2)\alpha t^3 + \left\{(8\pi + 3\lambda_2) - (4\pi + \lambda_2)(3\alpha + 108\lambda_1\alpha^2)\right\} t^2 + \\ 36\lambda_1\alpha\left\{(16\pi + 5\lambda_2) - \alpha(24\pi + 6\lambda_2)\right\} t + 18\lambda_1\left\{\alpha(44\pi + 15\lambda_2) - \alpha^2(24\pi + 6\lambda_2) - (16\pi + 6\lambda_2)\right\}\end{array}}{\begin{array}{c}3\alpha t^4 + (24\pi + 6\lambda_2)\alpha t^3 + \left\{\lambda_2 + 3\alpha(4\pi + \lambda_2)(1 + 36\lambda_1\alpha)\right\} t^2 + \\ \left\{108\lambda_1\lambda_2\alpha^2 + 36\lambda_1\alpha(3\alpha - 1)(8\pi + \lambda_2)\right\} t + 18\lambda_1(2\alpha - 1)\left\{12\pi\alpha + \lambda_2(3\alpha + 2)\right\}\end{array}} \tag{24}$$

The deceleration parameter q, a dimensionless measure of the cosmic expansion, is defined by the relation $q = -\frac{a\ddot{a}}{\dot{a}^2}$. Thus, q is related to the Hubble parameter H by the relation $q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right)$. For our model, we obtain

$$q = -1 + \frac{1}{\alpha(1+t)^2} \tag{25}$$

The deceleration parameter exhibits the universe's expansion. For $q < 0$, it undergoes accelerated expansion and for $q > 0$, it undergoes decelerated expansion. Recent observations reveal that the universe transitioned from the decelerated expansion phase to an accelerated expansion phase in the recent past and currently passing through a phase of accelerated expansion. According to current observational data, $-1 \leq q < 0$.

5. PHYSICAL AND KINEMATICAL PROPERTIES OF THE MODEL

We aim to investigate the physical and kinematical properties of the model by studying the bahaviour of some cosmological parameters as the universe evolves. Cosmological parameters describe the kinematic properties of the universe's expansion and are essential for understanding its dynamic evolution. To develop a cosmological model that transitions from a decelerating phase to an accelerating phase, we focus on the range $0 < \alpha < 1$, as recommended by the equation (25). This range aligns with models that match observed cosmic acceleration patterns.

For our analysis, we consider four specific values of α viz $\alpha = 0.3, 0.4, 0.5$ and 0.6. These values allow us to explore how slight variations in α influence the behavior of the universe's expansion. Additionally, we set the arbitrary constants as $\lambda_1 = 0.2$ and $\lambda_2 = 0.1$.

Using these values, we compute and plot the behaviour of various cosmological parameters. These plots help us to visualize and analyze how the universe's expansion evolves over time and assess whether the model aligns with the expected transition from a decelerated expansion to an accelerated expansion phase. This transition is a critical feature of modern cosmological theories that explain the role of dark energy in driving the accelerated expansion of the universe.

The scale factor a measures the relative size of the universe at a given time. As the universe expands, the scale factor increases. In the context of cosmology, the volume V of the universe is directly related to the scale factor, as $V \propto a^3$, meaning that as the scale factor increases, the volume increases significantly.

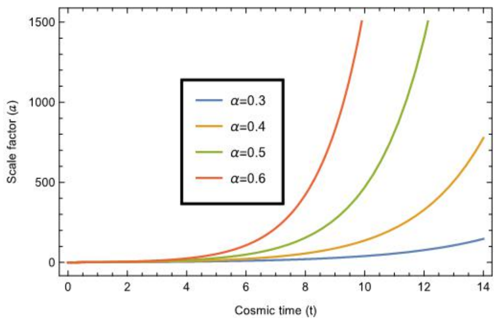

Figure 1. Variation of the scale factor a versus cosmic time t with $a_0 = 1$ and different values of α.

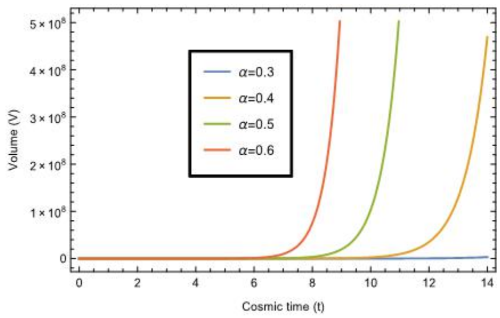

Figure 2. Variation of the volume V versus cosmic time t with different α.

From Figures 1 and 2, we see that the scale factor and volume grow significantly over time, especially in the late universe. This rapid increase in volume suggests that the universe's expansion has accelerated, particularly after a certain point. This acceleration can be attributed to the influence of dark energy, which became dominant in recent cosmic history, driving the universe's expansion at an increasingly rapid pace.

In order to have a better understanding of the properties of the universe corresponding to our model, it will be of great help if we also study the evolutionary behaviour of some cosmological parameters versus redshift z.

REDSHIFT AND COSMIC TIME RELATION

The redshift z is related to the scale factor $a(t)$ by:

$$z = \frac{a_0}{a(t)} - 1$$

which implies

$$a(t) = \frac{a_0}{1+z}$$

Substituting this into equation (21), we get

$$a_0 (te^t)^\alpha = \frac{a_0}{1+z}$$

To isolate te^t, we take the α-th root of both sides and get

$$te^t = (1+z)^{-1/\alpha}$$

The equation $te^t = C$, where $C = (1+z)^{-1/\alpha}$, is solved using the Lambert W function. The Lambert W function satisfies: $W(x)e^{W(x)} = x$

Thus, we can obtain the time-redshift relation as:

$$t = W\left[(1+z)^{-\frac{1}{\alpha}}\right] \tag{26}$$

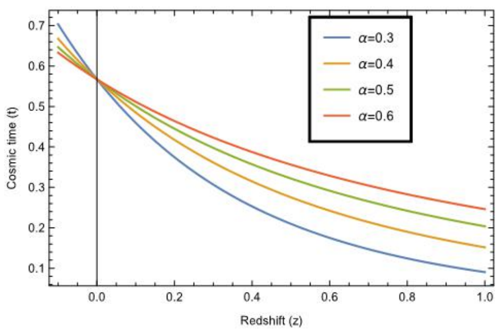

Figure 3. Variation of cosmic time t versus redshift z for different values of α.

The Hubble parameter and deceleration parameter in terms of redshift lead to

$$H(z) = \alpha\left[1 + \frac{1}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]}\right] \tag{27}$$

$$q(z) = -1 + \frac{1}{\alpha\left(1 + W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^2} \tag{28}$$

Using eqn (26), the expressions for the energy density (ρ), the pressure (p) and Equation of State (EoS) parameter ω in terms of redshift (z) are obtained as:

$$\rho(z) = \frac{3\alpha^2}{8\pi + 4\lambda_2} + \frac{\alpha}{(8\pi + 4\lambda_2)(4\pi + \lambda_2)}\left[\frac{(24\pi + 6\lambda_2)\alpha}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]} + \frac{\lambda_2 + 3\alpha(4\pi + \lambda_2)(1 + 36\lambda_1\alpha)}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]^2} + \right.$$
$$\left. \frac{108\lambda_1\lambda_2\alpha^2 + 36\lambda_1\alpha(3\alpha - 1)(8\pi + \lambda_2)}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]^3} + \frac{18\lambda_1(2\alpha - 1)(12\pi\alpha + \lambda_2(3\alpha + 2))}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]^4}\right] \tag{29}$$

$$p(z) = \frac{-3\alpha^2}{8\pi + 4\lambda_2} + \frac{\alpha}{(8\pi + 4\lambda_2)(4\pi + \lambda_2)}\left[\frac{-(24\pi + 6\lambda_2)\alpha}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]} + \frac{(8\pi + 3\lambda_2) - (4\pi + \lambda_2)(3\alpha + 108\lambda_1\alpha^2)}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]^2} + \right.$$
$$\left. \frac{36\lambda_1\alpha((16\pi + 5\lambda_2) - \alpha(24\pi + 6\lambda_2))}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]^3} + \frac{18\lambda_1(\alpha(44\pi + 15\lambda_2) - \alpha^2(24\pi + 6\lambda_2) - (16\pi + 6\lambda_2))}{W\left[(1+z)^{-\frac{1}{\alpha}}\right]^4}\right] \tag{30}$$

$$\omega(z)=\frac{\begin{array}{r}-3\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]^4-(24\pi+6\lambda_2)\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]^3+\left\{(8\pi+3\lambda_2)-(4\pi+\lambda_2)(3\alpha+108\lambda_1\alpha^2)\right\}W\left[(1+z)^{-\frac{1}{\alpha}}\right]^2+\\ 36\lambda_1\alpha\left\{(16\pi+5\lambda_2)-\alpha(24\pi+6\lambda_2)\right\}W\left[(1+z)^{-\frac{1}{\alpha}}\right]+18\lambda_1\left\{\alpha(44\pi+15\lambda_2)-\alpha^2(24\pi+6\lambda_2)-(16\pi+6\lambda_2)\right\}\end{array}}{\begin{array}{r}3\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]^4+(24\pi+6\lambda_2)\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]^3+\left\{\lambda_2+3\alpha(4\pi+\lambda_2)(1+36\lambda_1\alpha)\right\}W\left[(1+z)^{-\frac{1}{\alpha}}\right]^2+\\ \left\{108\lambda_1\lambda_2\alpha^2+36\lambda_1\alpha(3\alpha-1)(8\pi+\lambda_2)\right\}W\left[(1+z)^{-\frac{1}{\alpha}}\right]+18\lambda_1(2\alpha-1)\left\{12\pi\alpha+\lambda_2(3\alpha+2)\right\}\end{array}} \tag{31}$$

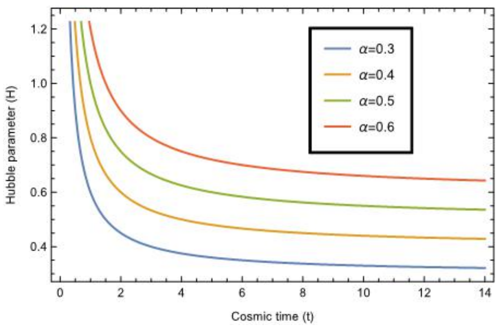

Figure 4. Variation of Hubble parameter H versus cosmic time t with different values of α.

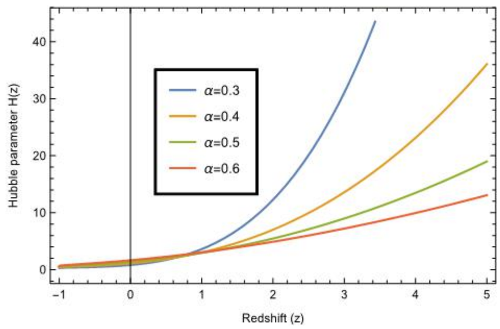

Figure 5. Variation of Hubble parameter H versus redshift z with different α.

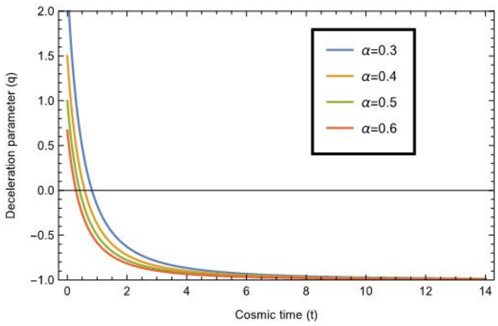

Figure 6. Variation of deceleration parameter q versus cosmic time t with different α.

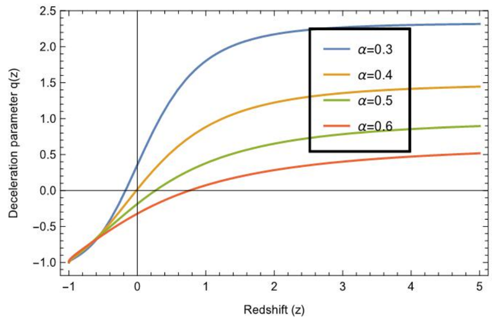

Figure 7. Variation of deceleration parameter q versus redshift z with different α.

From Figures 4 and 5, we observe that in the early universe, the Hubble parameter (H) is very large, and keeps decreasing as the universe evolves. This is consistent with the Big Bang theory, according to which the universe started from an extremely hot and dense state, and then began to expand. As time progresses, the value of H decreases, reflecting the fact that the rate of expansion slows down due to the gravitational pull of matter in the universe. In cosmological terms, redshift (z) corresponds to looking at the universe in the past. The Hubble parameter decreases with redshift (z). The decrease of H with respect to z refers to how the expansion rate of the universe slows down as the universe ages.

Figures 6 and 7 show that at the start of the universe, $q > 0$, meaning that the expansion was slowing down. As time progressed, due to the influence of matter and radiation, the deceleration continued for a long period. Initially, the universe is decelerating, but as the value of q moves toward negative values, the expansion switches to an accelerating phase. This matches observations of the current expansion rate, where the universe is observed to be accelerating due to the influence of dark energy.

From the above graphs, we see that the choice of $\alpha = 0.5$ reflects a best model where the deceleration parameter transitions smoothly into the accelerating phase. We choose this value for the plots of other cosmological parameters.

The energy density (ρ) of the universe consists of contributions from matter-energy content of the universe. Figure 8 shows that at the beginning of the universe (near time $t = 0$), the energy density is extremely high, exhibiting thereby that the universe was incredibly dense and compact. This is consistent with the idea of a hot, dense Big Bang origin. As the universe expands, the energy density decreases. This behaviour is expected in the standard cosmological model, where both matter and radiation contribute to the energy density but becomes less dense as the universe expands. The energy density asymptotically approaches zero as time progresses towards infinity, reflecting the fact that, although the

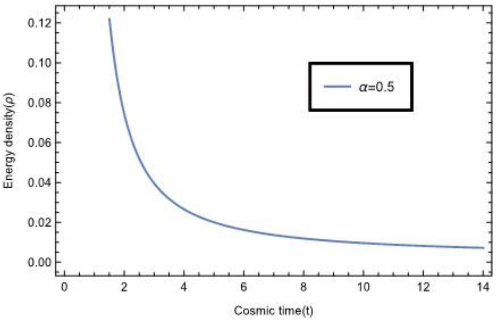

Figure 8. Variation of energy density ρ versus cosmic time t with $\alpha = 0.5$.

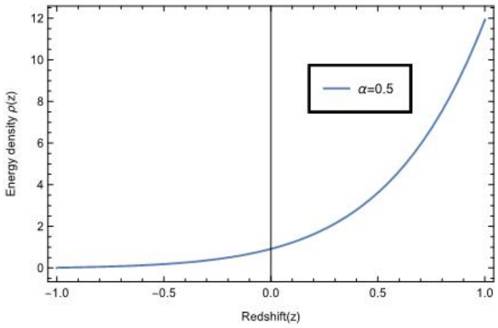

Figure 9. Variation of energy density ρ versus redshift (z) with $\alpha = 0.5$.

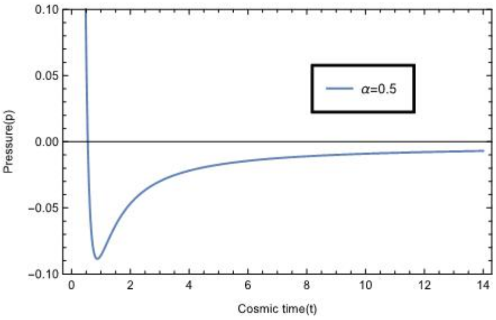

Figure 10. Variation of pressure p versus cosmic time t with $\alpha = 0.5$.

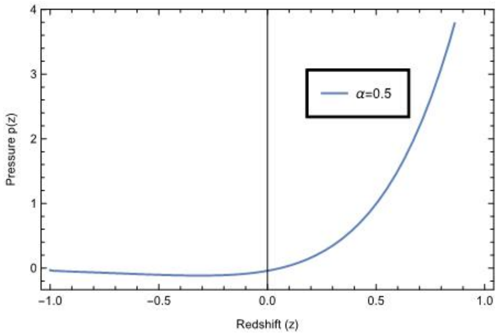

Figure 11. Variation of pressure p versus redshift (z) with $\alpha = 0.5$.

universe continues to expand, the contributions of matter and radiation become negligible over time, and dark energy (which remains constant or evolves slowly) dominates. From Figure 9, we see that the energy density ρ decreases against redshift z. This decrease in ρ describes how different components of the universe's energy content (like matter, radiation and dark energy) evolve as the universe expands.

The pressure (p) in the universe varies with time and plays an essential role in understanding the dynamics of the universe. In Figure 10, for $\alpha = 0.5$, the pressure is initially positive, which is typical for a matter dominated universe. However, as the universe expands, the pressure becomes negative. Negative pressure is associated with dark energy, which causes the accelerated expansion of the universe. This transition from positive to negative pressure is one of the signatures of the onset of the dark energy dominated phase. From Figure 11, we see that the pressure p decreases against redshift z.

In Figures 12 and 13, we observe that the value of ω lies between -1 and 1. This range is significant because $\omega \approx 0$ corresponds to matter domination, $\omega = \frac{1}{3}$ corresponds to radiation domination and $\omega \approx -1$ corresponds to a universe dominated by dark energy, as dark energy is modeled to have $\omega = -1$ in the simplest cosmological models.

Since ω lies within this range, it suggests that the universe is currently undergoing accelerated expansion, dominated by dark energy, which fits well with current cosmological observations of an accelerated expanding universe.

6. PHYSICAL ACCEPTABILITY OF THE SOLUTIONS

For the validity of the solution, we should check that our model is physically acceptable.

6.1. Jerk, snap and lerk parameters

In cosmological models, understanding the evolution of the universe's expansion is crucial for predicting its future behaviour and unraveling the underlying forces shaping its dynamics. While the deceleration parameter provides a foundational insight into how the expansion rate of the universe is changing, higher-order time derivatives of the scale factor such as the jerk, snap, and lerk offer a more detailed and refined understanding of the universe's expansion.

These parameters are derived from the Taylor series expansion of the scale factor $a(t)$, which describes the size of the universe as a function of time (t). By examining the higher derivatives of the scale factor, more complex aspects of the universe's expansion, such as changes in acceleration and the rate at which these changes are occurring can be captured.

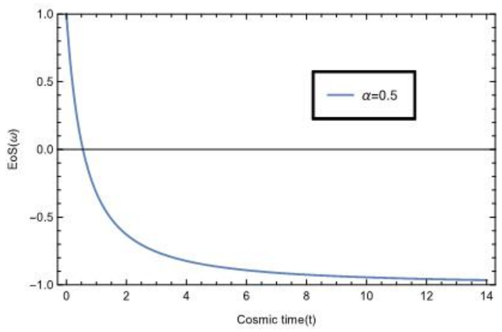

Figure 12. Variation of EoS parameter (ω) versus cosmic time t with $\alpha = 0.5$.

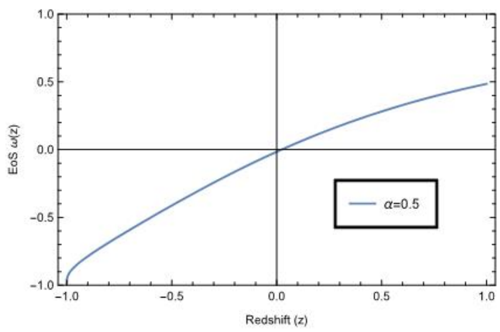

Figure 13. Variation of EoS parameter (ω) versus redshift (z) with $\alpha = 0.5$.

These higher-order derivatives - jerk, snap, and lerk are essential for predicting the future evolution of the universe and for understanding its past behavior with greater precision.

Jerk parameter

The jerk parameter is the third time derivative of the scale factor $a(t)$, denoted by $j(t)$. It measures the rate of change of the acceleration of the universe's expansion. In other words, it provides insight into whether the rate of acceleration is itself increasing or decreasing over time. The jerk parameter is particularly important because it helps predict the future behaviour of the expansion. If the jerk is positive, it indicates that the expansion is accelerating at an increasing rate. Conversely, a negative jerk suggests that the acceleration is decreasing.

Mathematically, the jerk parameter is expressed as:

$$j(t) = \frac{a^2}{\dot{a}^3}\frac{d^3a}{dt^3}$$

For our model, it is obtained as

$$j(t) = 1 - \frac{3}{\alpha(1+t)^2} + \frac{2}{\alpha^2(1+t)^3} \tag{32}$$

This equation reflects how the second derivative of the scale factor (acceleration) changes over time.

Snap parameter

The snap parameter is the fourth time derivative of the scale factor $a(t)$, denoted by $s(t)$. It measures how the jerk parameter *i.e* the rate of change of acceleration evolves over time. It is a higher-order derivative that provides even finer details about the acceleration of the universe's expansion. The snap is essential for identifying subtle transitions in the universe's expansion, such as shifts between accelerating and decelerating phases of expansion.

The snap parameter is mathematically defined as:

$$s(t) = \frac{a^3}{\dot{a}^4}\frac{d^4a}{dt^4}$$

For our model, it is obtained as

$$s(t) = 1 - \frac{6}{\alpha(1+t)^2} + \frac{8}{\alpha^2(1+t)^3} + \frac{3\alpha - 6}{\alpha^3(1+t)^4} \tag{33}$$

This equation reflects the evolving nature of the jerk parameter, which helps us understand the changing nature of the universe's acceleration in even greater detail.

Lerk parameter

The lerk parameter is the fifth time derivative of the scale factor $a(t)$, denoted by $l(t)$. As the fifth derivative, the lerk parameter measures how the snap parameter, the rate of change of the jerk parameter is changing over time. The lerk parameter provides the most detailed information about the expansion of the universe, capturing extremely subtle shifts in the acceleration and deceleration rates.

Mathematically, the lerk parameter is defined as:

$$l(t) = \frac{a^4}{\dot{a}^5}\frac{d^5a}{dt^5}$$

For our model, it is obtained as

$$l(t) = 1 - \frac{10}{\alpha(1+t)^2} + \frac{20}{\alpha^2(1+t)^3} + \frac{15\alpha - 30}{\alpha^3(1+t)^4} + \frac{24-20\alpha}{\alpha^4(1+t)^5} \tag{34}$$

This equation provides the highest level of detail regarding the changing behaviour of the universe's expansion, offering important insights into the acceleration dynamics that may not be immediately apparent from the jerk or snap parameter alone.

For our model, the expressions for jerk, snap and lerk parameter in terms of redshift (z) are obtained as:

$$j(z) = 1 - \frac{3}{\alpha\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^2} + \frac{2}{\alpha^2\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^3} \tag{35}$$

$$s(z) = 1 - \frac{6}{\alpha\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^2} + \frac{8}{\alpha^2\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^3} + \frac{3\alpha-6}{\alpha^3\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^4} \tag{36}$$

$$l(z) = 1 - \frac{10}{\alpha\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^2} + \frac{20}{\alpha^2\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^3} + \frac{15\alpha-30}{\alpha^3\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^4} + \frac{24-20\alpha}{\alpha^4\left(1+W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)^5} \tag{37}$$

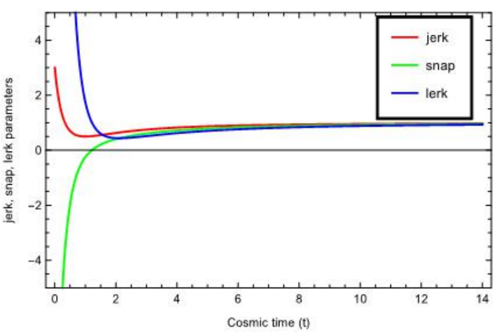

Figure 14. Variation of jerk, snap and lerk parameter versus cosmic time t with $\alpha = 0.5$

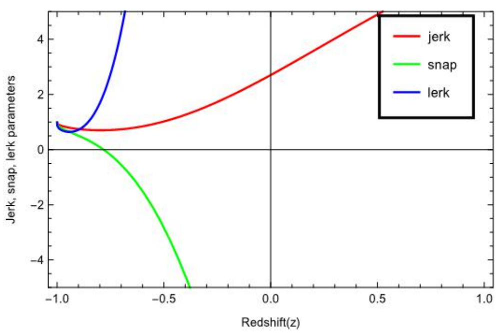

Figure 15. Variation of jerk, snap and lerk parameter versus redshift z with $\alpha = 0.5$

From Figures 14 and 15, we observe that the jerk and lerk parameters remain positive throughout the evolution of the universe. They exhibit diminishing tendencies as seen in the figures. The snap parameter has negative value in the beginning and occurs positive value at late cosmic time. This denotes an accelerated expansion of the universe.

6.2. Statefinder diagnostic

Statefinder parameter is a crucial geometrical diagnostic tool used to differentiate between different dark energy models. The two parameters of statefinder $\{r, s\}$ are dimensionless and geometrical since they are generated from the cosmic scale factor $a(t)$ alone, however they may be reconstructed in terms of dark energy and dark matter.

In table 1, various forms of statefinder pairs are displayed. Here, r measures the third derivative of the scale factor normalized by the Hubble parameter, quantifying jerk or snap and s provides a diagnostic to distinguish between dark energy models by normalizing r using q. ΛCDM model serves as the baseline: standard cosmological constant with cold dark matter model. Quintessence dark energy model represents a scalar field with varying energy density that drives the accelerated expansion of the universe. Phantom dark energy model is the model where the dark energy equation of state is $\omega < -1$, leading to super-accelerated expansion. Chaplygin gas model is a unified dark matter-energy model. Interacting Models are models involving interactions between dark energy and other components of the universe like cold dark matter.

Table 1. Statefinder diagnostic

Parameter	Definition
Deceleration Parameter (q)	$q = -\frac{\ddot{a}a}{\dot{a}^2}$
Statefinder Pair $\{r, s\}$	$r = \frac{\dddot{a}}{aH^3}$, $s = \frac{r-1}{3(q-\frac{1}{2})}$
ΛCDM Model	$\{r, s\}$= $\{1, 0\}$
Quintessence	$r < 1, s > 0$
Phantom Dark Energy	r, s vary based on parameters
Chaplygin Gas	$r > 1, s < 0$
Interacting Models	$r \neq 1, s \neq 0$

The statefinder pair $\{r, s\}$ [32] for our model are obtained as

$$r = \frac{a^2}{\dot{a}^3}\frac{d^3a}{dt^3} = 1 - \frac{3}{\alpha(1+t)^2} + \frac{2}{\alpha^2(1+t)^3} \tag{38}$$

$$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = \frac{4-6\alpha(1+t)}{6\alpha(1+t)-9\alpha^2(1+t)^3} \tag{39}$$

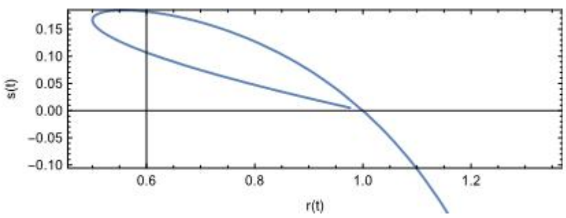

Figure 16. Variation of statefinder pair $\{r, s\}$

For our model, the statefinder pair $\{r, s\}$ has a present value of $\{r, s\}$= $\{0.976, 0.0054\}$ for $\alpha = 0.5$. Since, from Figure 16, we find $r < 1$ and $s > 0$, therefore, our model aligns with a quintessence dark energy model. Quintessence differs from the cosmological constant (Λ) as it evolves over time, leading to unique dynamics captured by the statefinder parameters. At late times, our model will behave like the ΛCDM model.

6.3. Energy Conditions

Energy conditions are sets of mathematical inequalities imposed on the energy-momentum tensor T_{ij}, which describes the matter and energy content of the universe in the framework of general relativity. These conditions provide a way to ensure the physical viability of a cosmological model and impose constraints on the behavior of matter and energy under gravitational interactions.

1. Null Energy Condition (NEC): The Null Energy Condition requires: $\rho + p \geq 0$. The NEC ensures that the energy density observed by a light-like observer (null vector) is non-negative. It is the most fundamental of all energy conditions, as the violation of the NEC often leads to unphysical scenarios such as exotic matter or superluminal signals. In an expanding universe, the NEC is closely linked to the second law of thermodynamics and the avoidance of unphysical singularities.

2. Weak Energy Condition (WEC): The Weak Energy Condition requires: $\rho \geq 0, \quad \rho + p \geq 0$. The WEC ensures that the energy density observed by any time like observer is non-negative. This condition is fundamental for a physically reasonable distribution of matter and energy. Satisfying the WEC indicates that matter behaves normally (e.g., no negative energy densities). It guarantees the normal gravitational attraction of matter and aligns with the observed dynamics of galaxies and cosmic structures.

3. Dominant Energy Condition (DEC): The Dominant Energy Condition requires: $\rho \geq |p|$. The DEC ensures that the flow of energy and momentum is causal, meaning that energy cannot propagate faster than the speed of light. Additionally, it implies that the energy density dominates over pressure contributions. Models satisfying the DEC respect causality and prevent the occurrence of unphysical faster-than-light phenomena. It is critical in describing the large-scale structure of the universe and the evolution of density perturbations.

4. Strong Energy Condition (SEC): The Strong Energy Condition requires: $\rho + 3p \geq 0$, $\rho + p \geq 0$. The SEC ensures that gravity is always attractive, implying that the combined effects of energy density and pressure act as a source of gravitational pull. The SEC is rooted in classical general relativity, where gravity is inherently attractive. In standard cosmology, the SEC is satisfied during the matter-dominated and radiation-dominated phases. However, during the accelerated expansion of the universe (e.g., the inflationary epoch or the current dark energy-dominated era), the SEC is violated. This violation is necessary to explain repulsive gravitational effects, such as the ones driving the universe's accelerated expansion.

For our model,

$$\rho(t)+p(t)=\frac{\alpha}{(4\pi+\lambda_2)t^4}\left[t^2+36\lambda_1\left(-1+2\alpha+\alpha t\right)\right] \tag{40}$$

$$\rho(t)-p(t)=\frac{6\alpha^2}{(8\pi+4\lambda_2)(4\pi+\lambda_2)}+\frac{\alpha}{(8\pi+4\lambda_2)t^4}\left[12\alpha t^3+(6\alpha+216\lambda_1\alpha^2-2)t^2+216\lambda_1\alpha(2\alpha-1)t+36\lambda_1(6\alpha^2-7\alpha+2)\right] \tag{41}$$

$$\rho(t)+3p(t)=\frac{\alpha}{(4\pi+2\lambda_2)(4\pi+\lambda_2)t^4}\left[-3\alpha t^4-(24\pi+6\lambda_2)\alpha t^3+\{(12\pi+5\lambda_2)-3\alpha(4\pi+\lambda_2)(1+36\lambda_1\alpha)\}t^2 + 36\lambda_1\alpha t\{(20\pi+7\lambda_2)-6\alpha(4\pi+\lambda_2)\}+18\lambda_1\left\{\alpha(60\pi+23\lambda_2)-(24\pi+10\lambda_2)-\alpha^2(24\pi+6\lambda_2)\right\}\right] \tag{42}$$

In terms of redshift, we obtain

$$\rho(z)+p(z)=\frac{\alpha}{(4\pi+\lambda_2)W\left[(1+z)^{-\frac{1}{\alpha}}\right]^4}\left[W\left[(1+z)^{-\frac{1}{\alpha}}\right]^2+36\lambda_1\left(-1+2\alpha+\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]\right)\right] \tag{43}$$

$$\rho(z)-p(z)=\frac{6\alpha^2}{(8\pi+4\lambda_2)(4\pi+\lambda_2)}+\frac{\alpha}{(8\pi+4\lambda_2)W\left[(1+z)^{-\frac{1}{\alpha}}\right]^4}\left[12\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]^3 + (6\alpha+216\lambda_1\alpha^2-2)W\left[(1+z)^{-\frac{1}{\alpha}}\right]^2+216\lambda_1\alpha(2\alpha-1)W\left[(1+z)^{-\frac{1}{\alpha}}\right]+36\lambda_1(6\alpha^2-7\alpha+2)\right] \tag{44}$$

$$\rho(z)+3p(z)=\frac{\alpha}{(4\pi+2\lambda_2)(4\pi+\lambda_2)W\left[(1+z)^{-\frac{1}{\alpha}}\right]^4}\left[-3\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]^4-(24\pi+6\lambda_2)\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]^3+ \{(12\pi+5\lambda_2)-3\alpha(4\pi+\lambda_2)(1+36\lambda_1\alpha)\}W\left[(1+z)^{-\frac{1}{\alpha}}\right]^2+36\lambda_1\alpha W\left[(1+z)^{-\frac{1}{\alpha}}\right]\{(20\pi+7\lambda_2)-6\alpha(4\pi+\lambda_2)\}+ 18\lambda_1\left\{\alpha(60\pi+23\lambda_2)-(24\pi+10\lambda_2)-\alpha^2(24\pi+6\lambda_2)\right\}\right] \tag{45}$$

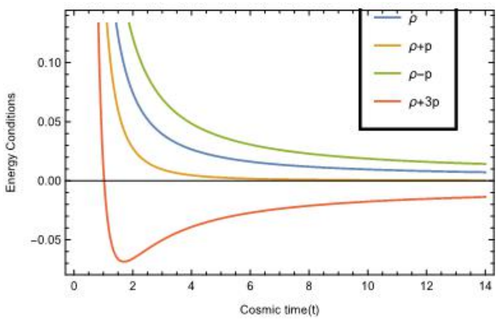

Figure 17. Variation of Energy Conditions versus cosmic time t with $\alpha = 0.5$.

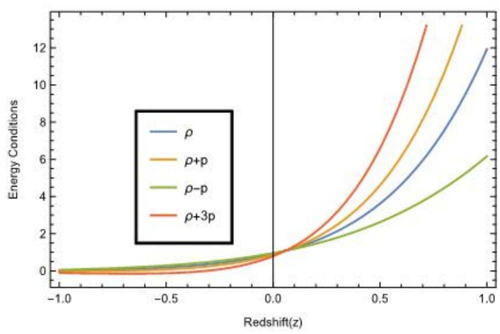

Figure 18. Variation of Energy Conditions versus redshift z with $\alpha = 0.5$.

From figures 17 and 18, we observe that the WEC, NEC, DEC are well satisfied whereas the SEC gets violated at approximately $t \approx 1$ and $z \approx 0.08$. This violation results in the accelerated expansion of the universe for our model.

7. CONCLUSIONS

In this study, we explore flat Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model within the context of an extended gravitational theory known as $f(R,T)$ gravity. By solving the non-linear field equaitons derived from this theory using the Hubble parameter as $H = \alpha\left(1 + \frac{1}{t}\right)$; $\alpha > 0$, we aim to understand the evolution of the universe and its expansion dynamics. Our findings provide several critical insights into the Hubble parameter, deceleration parameter, energy density, and other higher-order cosmological parameters. Furthermore, we evaluate the model's compatibility with standard cosmology, particularly the ΛCDM model, and test its consistency with various energy conditions.

The key features of the cosmological model corresponding to the solution obtained in Section 4 are as follows:

The Hubble parameter H, which quantifies the universe's expansion rate, exhibits significant changes over cosmic time: At the early stages of the universe ($t = 0$), H is exceptionally high, reflecting a rapid expansion. As time progresses, H decreases monotonically, consistent with the gradual slowing of expansion during the matter-dominated era, followed by a phase of accelerated expansion at late times driven by dark energy. When expressed in terms of redshift (z), H increases as z increases. This is a natural consequence of the relation between redshift and scale factor, where higher redshifts correspond to earlier cosmic epochs with faster expansion rates.

The deceleration parameter q is a critical quantity that describes the acceleration or deceleration of the universe's expansion: Initially, the universe is in a decelerating phase, dominated by the gravitational pull of matter and radiation. The model shows a transition from deceleration to acceleration at a specific point in cosmic time, corresponding to the dominance of dark energy or a similar repulsive component. For lower values of the model parameter α, this transition to acceleration occurs earlier (at lower redshifts), highlighting the sensitivity of the model to its parameters. At very late times ($t \to \infty$), the deceleration parameter asymptotically approaches -1, indicating a de Sitter like state with a constant rate of accelerated expansion, typical of dark energy-dominated cosmologies.

The behavior of the energy density ρ and pressure p in the model reveals key characteristics of the universe's evolution: At $t = 0$, the energy density ρ is extremely high, consistent with a big bang-type singularity, where the universe begins in a state of infinite density. Over time, ρ decreases monotonically but never reaches zero, even p as $t \to \infty$. This ensures that $\rho > 0$ throughout the evolution, satisfying fundamental physical requirements. The pressure is negative at all times, a crucial feature for explaining the observed accelerated expansion. Negative pressure is a hallmark of dark energy or similar components driving the late-time acceleration of the universe.

The study also examines the higher-order parameters derived from the scale factor: the jerk, snap, and lerk parameters (representing the third, fourth, and fifth derivatives of $a(t)$, respectively) which provide additional insights into the universe's expansion dynamics. These parameters show trends consistent with a universe transitioning from deceleration to acceleration, further validating the model's description of cosmic evolution.

Our analysis shows that the model behaves like a quintessence dark energy scenario at present times. Quintessence is a dynamic form of dark energy driven by a scalar field with a time-dependent equation of state, as opposed to the constant equation of state in the standard cosmological constant (Λ) model. At late times, the model aligns with the ΛCDM framework, suggesting that it can reproduce the well-observed behavior of the universe while providing additional flexibility in earlier epochs.

To evaluate the physical viability of the model, we tested it against the four standard energy conditions. In our model, initially the Strong Energy Condition (SEC) is satisfied but later it is violated. This is a typical feature of models describing an accelerating universe.

The presence of a point-type singularity at $t = 0$ is consistent with the Big Bang scenario, marking the universe's origin in a state of infinite density and temperature. As time progresses, the volume of the universe increases monotonically, reflecting the ongoing cosmic expansion. The model captures the key features of late-time acceleration, aligning with observations of the universe's current phase of accelerated expansion driven by dark energy. Its compatibility with quintessence-like behavior and eventual convergence to ΛCDM at late times ensures that it is consistent with the observational data for both current and early-universe.

By successfully explaining the transition from deceleration to acceleration, the evolution of energy density and pressure, and the higher-order cosmological parameters, this model demonstrates the potential of $f(R,T)$ gravity to serve as a viable extension of general relativity. The framework offers flexibility to accommodate a range of observational phenomena while maintaining consistency with fundamental physical laws.

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КОСМОЛОГІЧНА МОДЕЛЬ FLRW ІЗ КВАДРАТИЧНОЮ ФУНКЦІОНАЛЬНОЮ ФОРМОЮ У $f(R,T)$ ТЕОРІЇ ГРАВІТАЦІЇ

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У цій роботі досліджується просторово однорідний та ізотропний плоский всесвіт Фрідмана-Лемаȋтре-Робертсона-Уокера (FLRW) у контексті гравітації $f(R,T)$, представленої Harko, *et al.*, [Phys. Rev. D, 84, 024020 (2011)]. У цій роботі розглядається функціональна форма $f(R,T) = f_1(R) + f_2(T)$ з $f_1(R) = R + \lambda_1 R^2$ і $f_2(T) = 2\lambda_2 T$ де λ_1 і λ_2 довільні константи, R і T є скаляром Річчі та слідом тензора енергії напруги T_{ij} відповідно. Ми представляємо нову космологічну модель у рамках гравітації $f(R,T)$, досліджуючи динаміку Всесвіту FLRW через точне рішення рівнянь гравітаційного поля. Використовуючи інноваційний анзац для параметра Хаббла, $H = \alpha\left(1 + \frac{1}{t}\right)$, де α — додатна константа, ми фіксуємо еволюційну історію Всесвіту. Цей підхід забезпечує природний шлях для дослідження ключових космологічних параметрів, таких як масштабний фактор, параметр уповільнення, ривок, стрибок, параметри lerk та енергетичні умови, відкриваючи інтригуючу інформацію про динаміку розширення Всесвіту. Ми також обговорюємо діагностику вимірювача стану. Наші результати пропонують глибше розуміння космічної еволюції в рамках $f(R,T)$ гравітації.

Ключові слова: $f(R,T)$ *гравітація; метрика FLRW; параметр Хаббла; діагностика вимірювача стану*