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RELATIVISTIC IMPACT ON DUST-ELECTRON-ACOUSTIC SOLITARY WAVES IN AN UNMAGNETIZED PLASMA WITH NONEXTENSIVE IONS

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The nonlinear properties of the dust-electron-acoustic (DEA) solitary waves and their propagating behaviours are theoretically studied in an unmagnetized relativistic plasma model. Such plasma is composed by the weakly relativistic electrons, nonextensive distributed ions and negatively charged immobile dust particles. Staring from a set of unidirectional fluid equations for electrons and nonextensive distribution for ions with Poisson equation, the Korteweg-de Vries (KdV) and modified KdV (mKdV) equations are determined by using the reductive perturbation method technique and their soliton solutions, thus obtained, to analyse the existence regime and basic features of small amplitude DEA solitons. The effects of physical parameters namely ion-to-electron number density ratio (δ), relativistic streaming factor (v_0/c) and ion nonextensive parameter (q) on the dynamics of solitary formations are examined in detail. The result shows the existence of both compressive and rarefactive DEA KdV solitons and only compressive DEA mKdV solitons in the range $-1 < q < 3$, with various δ and v_0/c in the plasma. Additionally, the influences of all the physical parameters on the propagation of DEA solitary waves corresponding to the KdV and mKdV equations are numerically analysed within the paper. The results of this study might help clarify the basic characteristics of nonlinear travelling waves propagating in both laboratory and space plasma as well as astrophysical plasma environments.

Keywords: *Dust-electron-acoustic solitary wave; KdV and mKdV equations; Reductive perturbation technique; q-nonextensive ions; Relativistic plasma*

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1. INTRODUCTION

One of the most interesting and fascinating laboratories of nonlinear structures in space is the study of solitary waves in plasmas. Large dust grains in space plasma have a negative charge, while small grains have a positive charge. However, the polarity of the dust grains might differ. An exceptionally low-frequency dust wave mode known as dust acoustic waves (DAW) is produced by the new time scales connected to the slower dust component (larger).

It is found that the properties of plasma waves are significantly altered by the presence of dust charged particles. One intriguing area of current research is the study of electrostatic solitary waves in dusty plasma. An intriguing field of recent work has been the study of electrostatic solitary waves in dusty plasma.

The characteristics of plasma waves are discovered to be significantly altered by the presence of dust-charged particles.

Due to field emission, it is widely known that the presence of negatively charged dust particles with micrometer or sub micrometer sizes can alter the properties of plasma waves in space. Dusty plasmas are common in a wide range of astrophysical environments, including the interstellar medium, asteroid zones, cometary tails, planetary rings, the Earth's magnetosphere, and the vicinity of stars.

It's an intriguing application to evaluate low-frequency noise enhancement seen by the Vego and Goitto space missions in the dusty area of Haley's Comet. Dust density, temperature, particle size, and charge are just a few of the characteristics of dust grains that are reflected by the Cassini plasma spectrometer instrument over Saturn's major A and B rings. A wealth of knowledge regarding dusty plasmas can also be found in interstellar clouds in space. Many researchers [1–16] have provided a wonderful explanation of the role and influence of dusty plasma in astrophysical plasma and space environments. In dissipative plasma with superthermal electrons, Hanbaly *et al.* [17] studied nonlinear electron acoustic waves. Singh and Lakhina [18] have investigated the generation of electron-acoustic waves (EAWs) in the magnetosphere. Bansal *et al.* [19] have discussed the subject of oblique modulation of electron acoustic waves in nonextensive plasma. The research paper titled Dust-electron-acoustic shock waves originating from changes in dust charge was written by Mamun [20]. The relationship between electron-acoustic solitons in the auroral zone for an electron beam plasma system has been investigated by Jahangir *et al.* [21].

The relativistic effects are currently being incorporated into the research that is being done on solitary waves in plasmas that contain several components [22–33]. There have been a multitude of papers written over the course of the past few decades, each of which has a variety of compositions that cover a wide range of topics. The generation of weakly relativistic ion acoustic solitons in magnetized plasma is facilitated by unidirectional relativistic electrons with inertia, as

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researched by Kalita and Choudhury [34]. Kalita & Kalita [35] have investigated the mild relativistic effects of electrons and the implicit role of Cairns dispersed ions in the generation of dust acoustic waves in plasma. Moreover, Kalita and Das [36] have studied in a comparative analysis of modified Korteweg-de Vries solitons and dust ion acoustic Korteweg-de Vries solitons in dusty plasmas with different temperatures.

It is commonly known that the macroscopic ergodic equilibrium state is uniformly represented by the Maxwell distribution for ions. Over the last few decades, Maxwellian particle distributions have been utilized by many researchers. However, there has been a noticeable growth in interest in nonextensive statics, also known as Tsillis statistics, which are based on the Boltzmann-Gibbs-Shannon (BGS) method for studying particle dispersion in plasma in recent years. In the field of statistics, Renyi [37] and Tsallis [38] first introduced an appropriate nonextensive generalization of the BGS entropy. In this system, the entropic index q represents the degree of non extensivity. Currently, numerous researchers are examining the number density of particle plasma in the $q-$nonextensive distribution by referencing the following sources: [39–47]. Numerous important aspects of conventional statistical mechanics are still present in nonextensive statistics. Superextensivity is defined as having a composite system's generalized entropy greater than the sum of the entropies of its constituents; subextensivity is defined as having a composite system's generalized entropy smaller than the sum of its subsystems. A generalized particle distribution known as the q distribution function is the outcome of Tsallis nonextensive statistical mechanics. When $q < -1$ and the q-nonextensive distribution cannot be normalized, it might be helpful. Once more, the distribution function reduces to the typical Maxwellian-Boltzmann velocity distribution if $q = 1$. Dai *et al.* [48] investigated in nonextensive statistics the dust ion-acoustic instability with q-distribution. Awady & Moslem [49] carried out numerical studies on a plasma that contained nonextensive electrons and positrons. Amour & Tribeche [50] conducted research on Collisionless damping of dust-acoustic waves in a dusty plasma with nonextensive ions that varies in charge. Thus, our aim is to study the formation and properties of DEA solitary waves in an ummagnetized plasma in which electrons are relativistic and ions are nonextensive, in the presence of static dusts. Small amplitude DEA solitons are studied using the reductive perturbation approach. Our results should help to understand the basic features of nonlinear travelling waves propagating in dusty plasmas with relativistic and nonextensive particles. The paper is organized as follows: Section-1 contains the formal introduction; Section-2 contains basic equations governing the plasma model. The KdV and mKdV equations for DEA solitary waves has been derived respectively in Section-3 and Section-4. Results and parametric discussions are made in Section-5, while Section-6 presents a summary of our results.

2. BASIC GOVERNING EQUATIONS

We consider an unmagnetized homogeneous warm dusty plasma system comprising of electrons with weak relativistic effect, $q-$nonextensive ions and static dusts with negative charge. Also, the dust charge number z_d is taken to be constant [51], and their impacts of the dynamics of DEA waves is neglected. Therefore, in such a plasma system the dynamics of nonlinear one-dimensional DEA waves motion is governed by the following unnormalized fluid equations:

$$\frac{\partial N_e}{\partial T} + \frac{\partial}{\partial x}(N_e V_e) = 0 \tag{1}$$

$$\left(\frac{\partial}{\partial T} + V_e \frac{\partial}{\partial X}\right)(\gamma V_e) + \frac{k_b T_e}{N_e m_e}\frac{\partial N_e}{\partial X} = \frac{e}{m_e}\frac{\partial \Phi}{\partial X} \tag{2}$$

$$\frac{\partial^2 \Phi}{\partial X^2} = 4\pi e\left[N_e + z_d n_{d0} - N_i\right] \tag{3}$$

where N_e, V_e, and Φ are respectively the electron number density , electron fluid velocity and electrostatic potential. To normalize the set of equations (1)-(3), we consider the scaling variables as: $n_e = \frac{N_e}{n_{e0}}$; $n_i = \frac{N_i}{n_{i0}}$; $v_e = \frac{V_e}{C_e}$; $\phi = \frac{e\Phi}{k_b T_e}$; $x = \frac{X}{\lambda_D}$; $t = \frac{T}{\omega_{pe^{-1}}}$, and the normalized form of equations (1)-(3) can be written as

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0 \tag{4}$$

$$\left(\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}\right)(\gamma v_e) + \frac{1}{n_e}\frac{\partial n_e}{\partial x} = \frac{\partial \phi}{\partial x} \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + \mu - \delta n_i \tag{6}$$

Here $C_e = \sqrt{K_B T_e / m_e}$ is the electron acoustic speed; $\omega_{pe} = \sqrt{4\pi n_{e0} e^2 / m_e}$ the electron plasma frequency and $\lambda_D = \sqrt{K_B T_e / 4\pi n_{e0} e^2}$ the electron Debye length, and m_e, T_e, e, K_B are the electron mass, characteristic electron temperature, electronic charge and the Boltzmann constant. Also, the relativistic factor for electrons $\gamma = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{-1} \approx 1 + \frac{v^2}{2c^2}$, for weakly relativistic regime, c (normalized with C_e) speed of light. Moreover, we have defined $\mu = z_d n_{d0}/n_{e0}$ and $\delta = n_{i0}/n_{e0} > 1$. The overall charge neutrality condition that is $n_{i0} = n_{e0} + z_d n_{d0}$, gives $\mu = \delta - 1$.

The presence of ion density is assumed through the $q-$nonextensive distribution function [52–54], which is one dimensional equilibrium in nature and given as

$$f(v_i) = C_q \left[1 + (q-1)\left\{\frac{m_i v_i^2}{2T_i} - \frac{e\phi}{T_i}\right\}\right]^{\frac{1}{q-1}} \tag{7}$$

where the normalization constant C_q indicated by

$$C_q = n_{i0}\frac{\Gamma\left(\frac{1}{q-1}\right)}{\Gamma\left(\frac{1}{q-1}-\frac{1}{2}\right)}\sqrt{\frac{m_i(1-q)}{2\pi K_B T_i}}, \text{ for } -1 < q < 1$$

$$C_q = n_{i0}\left(\frac{1+q}{2}\right)\frac{\Gamma\left(\frac{1}{q-1}-\frac{1}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)}\sqrt{\frac{m_i(1-q)}{2\pi K_B T_i}}, \text{ for } q > 1$$

Here Γ is the well-known gamma function, the parameter q measures the ion nonextensivity of the system and the remaining parameters or variables maintain their usual meaning. As the gamma function is undefined for negative numbers, so for $q < -1$, the function (7) cannot be normalized, and it is noted that the function (7) is the specific distribution which optimizes the Tsallis nonextensive entropy and, as a result it follows to the laws of thermodynamics. Moreover, the distribution (7) reduces to the standard Maxwell–Boltzmann velocity distribution due to the limiting case $q \to 1$.

Integrating the distribution function (7), the expression for ion number density is obtained as

$$n_i = n_{i0}\left[1 - (q-1)\frac{e\phi}{K_B T_i}\right]^{\frac{1}{q-1}+\frac{1}{2}}$$

Hence, the normalized form of ion number density is

$$n_i = [1 - \sigma(q-1)\phi]^{\frac{q+1}{2(q-1)}} \tag{8}$$

Here, q is a real parameter that is higher than -1 and $\sigma = T_e/T_i$. For $\phi \ll 1$, expanding (8) upto the third order and substitution of (6), gives

$$\frac{\partial^2\phi}{\partial x^2} = n_e - 1 + k_1\phi - k_2\phi^2 + k_3\phi^3 + \ldots \tag{9}$$

where

$$\left.\begin{aligned} k_1 &= \frac{\delta\sigma(1+q)}{2} \\ k_2 &= \frac{\delta\sigma^2(1+q)(3-q)}{8} \\ k_3 &= \frac{\delta\sigma^3(1+q)(3-q)(5-3q)}{48} \end{aligned}\right\} \tag{10}$$

3. THE KDV EQUATION AND ITS SOLUTION

To derive KdV equation for small amplitude DEA solitary wave, the reductive perturbation approach is used. In resent times, reductive perturbation technique becomes very popular to study the nonlinear electrostatic waves of small amplitude limit in the field of plasma. To investigate the DEA solitary waves propagating through KdV equation, stretching and writing the independent variables as

$$\xi = \epsilon^{1/2}(x - Vt), \quad \tau = \epsilon^{3/2}t \tag{11}$$

where ϵ is a small dimensionless parameter and V the phase velocity of DEA waves. And the dependent variables are expanded about the equilibrium states in power series of ϵ as

$$\left.\begin{aligned} n_e &= 1 + \epsilon n_1 + \epsilon^2 n_2 + \epsilon^3 n_3 + \ldots \\ v_e &= v_0 + \epsilon v_1 + \epsilon^2 v_2 + \epsilon^3 v_3 + \ldots \\ \phi &= \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \ldots \end{aligned}\right\} \tag{12}$$

Substituting, the transformation (11) and expression (12), into equations (4),(5) and (9), and then collecting the coefficients of lowest order terms in ϵ, and after integration with the boundary conditions: $n_1 = 0,\ v_1 = 0,\ \phi_1 = 0$ at $|\xi| \to \infty$, we obtain the first order terms as

$$\left.\begin{aligned} n_1 &= -k_1\phi_1 \\ v_1 &= -k_1(V - v_0)\phi_1 \end{aligned}\right\} \tag{13}$$

along with the linear phase velocity expression for DEA waves,

$$V = v_0 \pm \sqrt{\frac{1+k_1}{Ak_1}} \tag{14}$$

where $A = 1 + \frac{3v_0^2}{2c^2}$. The positive and negative sign respectively refers to the fast and slow DEA mode. Although, we assumed the case of fast DEA mode in our numerical simulations, whereas the slow DEA mode is entirely ignored.

Again, collecting the coefficients of second higher order terms in ϵ, we obtain the subsequent equations

$$-S\frac{\partial n_2}{\partial \xi} + \frac{\partial n_1}{\partial \tau} + \frac{\partial v_2}{\partial \xi} + \frac{\partial (n_1 v_1)}{\partial \xi} = 0 \tag{15}$$

$$-AS\frac{\partial v_2}{\partial \xi} + A\frac{\partial v_1}{\partial \tau} + (A-2BS)v_1\frac{\partial v_1}{\partial \xi} - ASn_1\frac{\partial v_1}{\partial \xi} + \frac{\partial n_2}{\partial \xi} - \frac{\partial \phi_2}{\partial \xi} - n_1\frac{\partial \phi_1}{\partial \xi} = 0 \tag{16}$$

$$n_2 + k_1\phi_2 - k_2\phi_1^2 - \frac{\partial^2 \phi_1}{\partial \xi^2} = 0 \tag{17}$$

where $S = V - v_0$ and $B = 3v_0/2c^2$. Eliminating the terms n_2 and v_2 from equations (15)-(17) and after utilizing (13) and (14), the KdV equation for first order electrostatic potential $\phi_1(=\varphi)$ is found as

$$\frac{\partial \varphi}{\partial \tau} + M\varphi\frac{\partial \varphi}{\partial \xi} + N\frac{\partial^3 \varphi}{\partial \xi^3} = 0 \tag{18}$$

with the constants M and N given by

$$M = \frac{1}{2ASk_1}\left[2\left(\frac{k_2}{k_1}\right) - k_1 - AS^2k_1^2 + 2BS^3k_1^2\right] \quad \text{and} \quad N = \frac{1}{2ASk_1^2} \tag{19}$$

and called respectively nonlinear coefficient and dispersion coefficient of the KdV equation.

Equation (18) is a nonlinear partial differential equation, and is analytically solvable. To determine the stationary solitary wave solutions (18), we take a new transformation $\chi = \xi - C\tau$, where C is wave velocity in the linear χ-space. Using this variable, the KdV equation (18) can be integrated under the boundary condition: $\varphi = 0$, $d\varphi/d\chi = 0$ and $d^2\varphi/d\chi^2 = 0$ as $\chi \to \pm\infty$, to give the solitary wave solution as

$$\varphi = \varphi_0 \operatorname{sech}^2\left(\frac{\chi}{\Delta}\right) \tag{20}$$

where $\varphi_0 = 3C/M$ is the wave amplitude of the DEA soliton and it is proportional to the soliton speed C, and $\Delta = 2\sqrt{N/C}$ is the width of DEA soliton and it is inversely proportional to $\sqrt{C}$.

It reveals from equation (19) that the nonlinear coefficients M depends on the plasma parameters q, σ, δ and the electron relativistic streaming factor v_0/c. We find a compressive DEA solitary waves for $M > 0$, and while rarefactive for $M < 0$. Based on the Fig.-[1], we find a critical composition value of q_c (say) at a given value of parameters q, σ, δ and v_0/c, for which $M \approx 0$. As the nonliearity becomes zero at that critical point/region, as a result the amplitude $\varphi \to \infty$, thus the DEA solitary wave solution has infinite divergence. We can find the expression of q_c, by solving the equation $M = 0$, given in (19) for q and obtained as

$$q_c = \frac{-(a+\delta+1) \pm \sqrt{8a + (\delta+1)^2}}{a} \tag{21}$$

where $a = \sigma\delta^2S^2(A - 2BS)$. In this particular regime, the model is not adequately described by the KdV equation (18). In order to study the propagation properties of DEA solitary waves in the critical point/region, we take the higher order nonlinearity and proceed with modified KdV equation in the following section.

4. THE MODIFIED KDV EQUATION AND ITS SOLUTION

For the purpose of describing the system at or close to the critical nonextensive q_c given in (21), to derive modified KdV (mKdV) equation, for which we consider the same set of expression (9) but with different stretched coordinates as follows:

$$\xi = \epsilon(x - Vt)\ , \quad \tau = \epsilon^3 t \tag{22}$$

Therefore, Substituting (12) and (22) into equations (4),(5) and (9), and proceeding as in section-3, we obtain the same first order terms in ϵ as given in (13) and (14). For second order terms, collecting the coefficients of next higher order of

ϵ and then integrating with the boundary conditions: $n_2 = 0,\ v_2 = 0,\ \phi_2 = 0$ at $|\xi| \to \infty$, we obtain after the use of first order terms as

$$\left.\begin{array}{l} n_2 = k_2\phi_1^2 - k_1\phi_2 \\ v_2 = S\left[(k_2 - k_1^2)\phi_1^2 - k_1\phi_2\right] \end{array}\right\} \tag{23}$$

along with the condition as follows

$$2\left(\frac{k_2}{k_1}\right) - k_1 - AS^2k_1^2 + 2BS^3k_1^2 = 0 \tag{24}$$

Finally, collecting the coefficients of third highest order in ϵ, we find the following equations

$$-S\frac{\partial n_3}{\partial \xi} + \frac{\partial n_1}{\partial \tau} + \frac{\partial v_3}{\partial \xi} + \frac{\partial (n_1 v_2)}{\partial \xi} + \frac{\partial (n_2 v_1)}{\partial \xi} = 0 \tag{25}$$

$$-AS\frac{\partial v_3}{\partial \xi} + A\frac{\partial v_1}{\partial \tau} - ASn_1\frac{\partial v_2}{\partial \xi} + A\frac{\partial (v_1 v_2)}{\partial \xi} + An_1v_1\frac{\partial v_1}{\partial \xi} - ASn_2\frac{\partial v_1}{\partial \xi} - 2BS\frac{\partial (v_1 v_2)}{\partial \xi} + 2Bv_1^2\frac{\partial v_1}{\partial \xi} - 2BSn_1v_1\frac{\partial v_1}{\partial \xi} - \frac{3S}{2c^2}v_1^2\frac{\partial v_1}{\partial \xi} - \frac{\partial \phi_3}{\partial \xi} - n_1\frac{\partial \phi_2}{\partial \xi} - n_2\frac{\partial \phi_1}{\partial \xi} + \frac{\partial n_3}{\partial \xi} = 0 \tag{26}$$

$$n_3 + k_1\phi_3 - 2k_2\phi_1\phi_2 + k_3\phi_1^3 - \frac{\partial^2 \phi_1}{\partial \xi^2} = 0 \tag{27}$$

Eliminating the terms n_3 and v_3 from equations (25)-(27) and making use of the terms from (13), second-order terms from (23) and the relations (14) and (24) as well, we get the following mKdV equation

$$\frac{\partial \psi}{\partial \tau} + M_1\psi^2\frac{\partial \psi}{\partial \xi} + N_1\frac{\partial^3 \psi}{\partial \xi^3} = 0 \tag{28}$$

where, $\phi_1 = \psi,\ N_1 = N$ and the higher order nonlinear coefficient M_1 is given by

$$M_1 = \frac{1}{2ASk_1}\left[6AS^2k_1k_2 - 3AS^2k_1^3 + 6BS^3k_1^3 - 6BS^3k_1k_2 - \frac{3S^4k_1^3}{2c^2} + 3\left(\frac{k_3}{k_1}\right) + k_2\right] \tag{29}$$

Now, using the same transformation and proceeding with the same procedure as given in section[3], we can determine the solitary wave solution of the mKdV equation(28) as

$$\psi = \psi_0\,\mathrm{sech}\left(\frac{\chi}{W}\right) \tag{30}$$

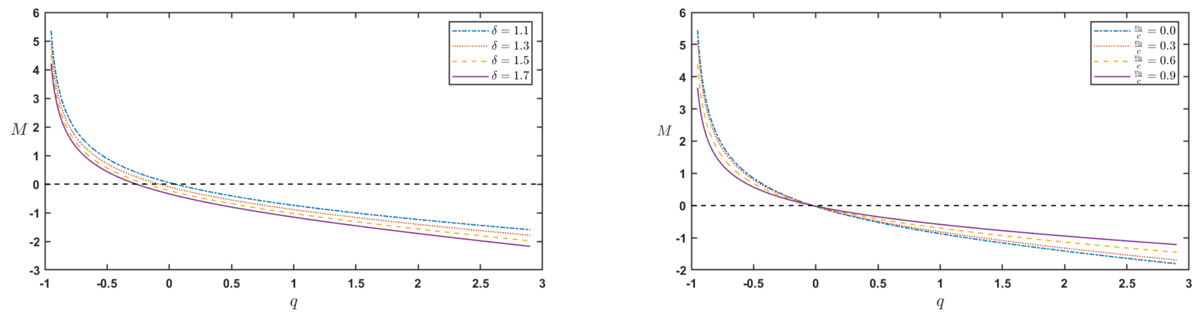
Where $\psi_0 = \sqrt{6C/M_1}$ is amplitude and $W = \sqrt{N/C}$, the width of the of DEA solitary waves represented by the mKdV equation(28) and C is the velocity of soliton.

5. RESULTS AND DISCUSSIONS

In this modal of dusty plasma, the dynamical properties of small amplitude DEA solitary wave have been studied in the context of relativistic electrons and nonextensive distributed ions. The influences of various plasma parameters namely, the ion-to-electron number density ratio δ, relativistic electron streaming factor v_0/c and the degree of ion nonextensive parameter q on the DEA solitary waves are numerically analyzed. Throughout the graphical analysis, we have looked at the case for $\sigma(= T_e/T_i) = 1$.

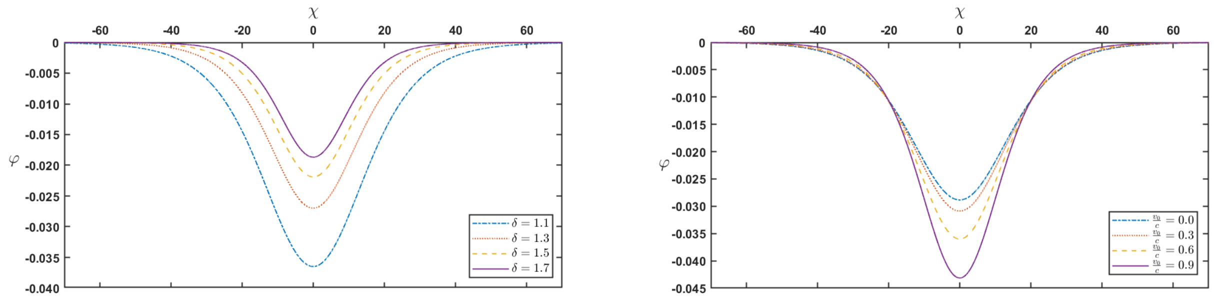
The ion nonextensive parameter q have a notable effect over the nonlinear coefficient M and dispersion coefficient N of KdV equation. Fig-[1] shows the sketch of M for two scenarios, that is for distinct values of $\delta(= 1.1,\ 1.3,\ 1.5,\ 1.7)$ and $v_0/c(= 0.0,\ 0.3,\ 0.6,\ 0.9)$, as q increases in the range $-1 < q < 3$. It is clear from both Fig-[1a] and Fig-[1b] that the nonlinear coefficient M can have values that are both positive and negative, while the dispersion coefficient N is observed to take always positive values (figures not included). This change of signs of the nonlinearity indicates the existence of two kind of DEA solitary waves having a positive and a negative potentials. It is traced out that $M \approx 0$ at $q = q_c$, whereas $M > 0$ in the range $-1 < q < q_c$, which is the parametric domain where compressive DEA solitons occur. Again, $M < 0$ in the range $q_c < q < 3$, which is the parametric domain where rarefactive DEA solitons occur. Therefore, in the current KdV model of plasma, both compressive and rarefactive DEA solitary structures can exist.

Now, we illustrate how the various physical parameters influences over the behavior of propagating DEA solitary waves. It is seen from Fig.-[2a] that, when the ratio of ion-to-electron number density δ increases with fixed $q = 0.5,\ v_0/c = 0.3$ and $C = 0.005$, the rarefactive DEA solitons, and both its amplitude and width of the negative potential DEA solitary waves decreases. Here, it can predicted that the structure of rarefactive DEA solitary waves is broader when



(a) (b)

Figure 1. **(a)** The variation of M with q for different δ and $\frac{v_0}{c} = 0.3$ and **(b)** The variation of M with q for different $\frac{v_0}{c}$ and $\delta = 1.2$. With $\sigma = 1$.



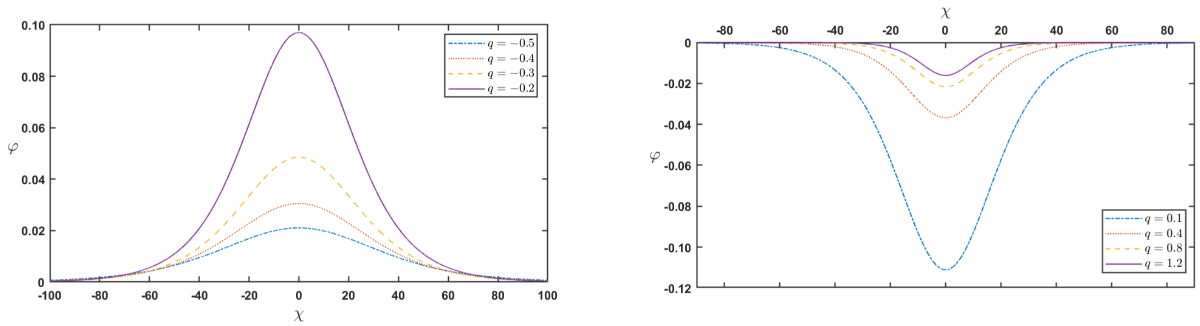
(a) (b)

Figure 2. **(a)** The variation of KdV solitons for different δ with $\frac{v_0}{c} = 0.3$ and **(b)** The variation of KdV solitons for different $\frac{v_0}{c}$ with $\delta = 1.2$. In both the panels, $q = 0.5(> q_c)$, $\sigma = 1$ and $C = 0.005$.

the population of ions and electrons are almost same in the present plasma. In Fig.-[2b], it is observed that when relativistic electron streaming factor v_0/c increases for fixed $q = 0.5$, $\delta = 1.2$ and $C = 0.005$, the DEA soliton is rarefactive and the amplitude (width) of the negative potential DEA solitary waves increases (decreases). That means the energy of the propagating DEA solitons enhances with increasing values of relativistic electron streaming factor in the present plasma.

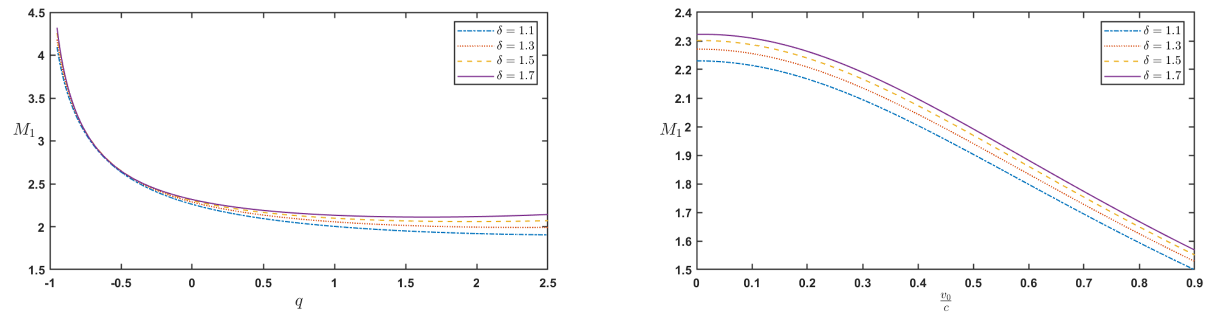
For fixed parameters as $\delta = 1.2$, $v_0/c = 0.3$ and $C = 0.005$, how the ion nonextensive parameter q effects on the propagating DEA solitary waves are shown in Fig.-[3]. The soliton type is observed to have changed from compressive to rarefactive at q depending on δ and v_0/c. With rising values of $q < q_c$, the soliton is compressive (Fig.-[3a]) and amplitude of positive potential DEA solitary waves increases notably while width decreases slightly. Conversely, for increasing values of $q > q_c$, the soliton is rarefactive (Fig.-[3b]) and decreases the pulse of negative potential DEA solitary waves in both amplitude and width. Here, for $\delta = 1.2$, $v_0/c = 0.3$ and $q = 0.5$, we find from (18) that $q_c \approx 0.07$. Additionally, it is clear that $M \approx 0$ at $q \approx q_c$, as a result, the KdV equation fails and the system can not be described.

The change of second order nonlinear coefficient M_1 of mKdV equation (25) against q and v_0/c for distinct values of δ have been shown in Fig.-[4] at q_c. In both the cases the value of M_1 is found to be positive, which indicates only compressive mKdV solitons can propagates at or near the critical point q_c in the present plasma model. Fig.-[5] shows the graph of solitary wave potential ψ of mKdV equation against the linear parameters χ, for various q, δ and v_0/c. It



(a) (b)

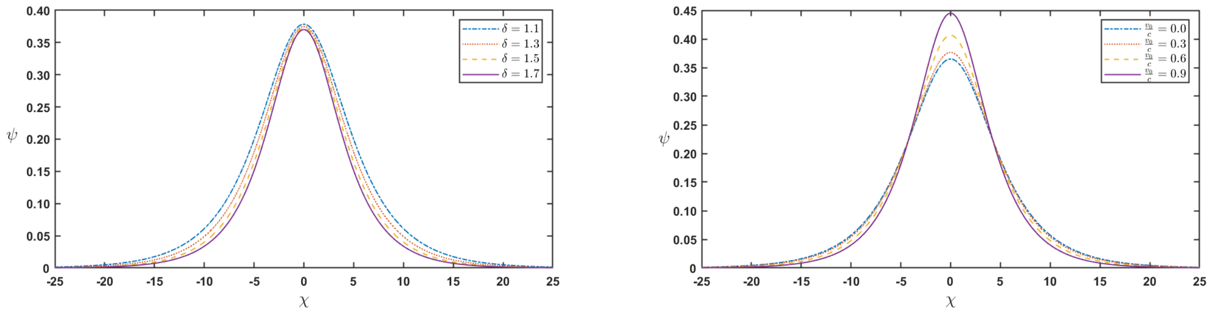
Figure 3. The variation of KdV solitons **(a)** for different $q < q_c$ and **(b)** for different $q > q_c$. where $\frac{v_0}{c} = 0.3$, $\sigma = 1$, $\delta = 1.2$ and $C = 0.005$.



(a) **(b)**

Figure 4. The variation of higher order nonlinear term M_1 **(a)** with q, for $\frac{v_0}{c} = 0.3$ and different δ and **(b)** with $\frac{v_0}{c}$ for $q = 0.5$ and different δ at q_c and $\sigma = 1$.

is evident from Fig.-[5a] that, when the ratio of ion-to-electron number density δ increases with fixed other parameters, the amplitude as well as width of the compressive modified DEA solitary waves to decrease. Moreover, rising value of relativistic electron streaming factor v_0/c and fixed other parameters, enhance the amplitude while decreases the width of the compressive modified DEA solitary waves as shown in Fig.-[5b]. The change of modified DEA solitary wave pulses are shown in Fig.-[6] for distinct values of ion nonextensive parameter q, with fixed other parameters. It is seen that the amplitude of compressive DEA solitary waves increases while the width is not notably changes by the increasing values of q.



(a) **(b)**

Figure 5. The variation of mKdV solitons **(a)** for different δ with $\frac{v_0}{c} = 0.3$; and **(b)** for different $\frac{v_0}{c}$ with $\delta = 1.2$; at q_c. where $q = 0.5,\ \sigma = 1,\ C = 0.05$.

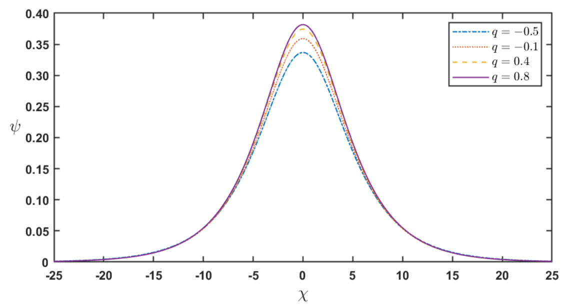


Figure 6. The variation of mKdV solitons for different q with $\delta = 1.2,\ \frac{v_0}{c} = 0.3$ at q_c. where $\sigma = 1,\ C = 0.05$.

6. CONCLUSION

In this manuscript, we have numerically analyzed the dynamical properties of the propagation of small amplitude DEA solitary waves in an unmagnetized plasma model, consisting of relativistic electrons, nonextensive ions and negatively charged static dust grains. Both the KdV and mKdV equations are determined using the reductive perturbation technique, and their solitary wave solutions are obtained. The effects of physical parameters such as ion-to-electron number density ratio δ, relativistic electron streaming factor v_0/c, and ion nonextensive parameter q over DEA solitary wave potentials represented by the KdV and mKdV equations are discussed numerically in the case when electron-to-ion temperature ratio $\sigma = 1$. The outcomes that have been noticed in this study can be contracted as follows:

1. In the present plasma model, two distinct sorts of wave modes are found to exist, namely fast DEA acoustic mode and slow acoustic modes. But only fast DEA modes are considered in the extraction of KdV and mKdV equations.
2. First order nonlinear coefficient M in the KdV equation can be a positive and a negative quantity, while the second order nonlinear coefficient M_1 of mKdV equation is a positive quantity, depending on the plasma parametric values. Therefore, there exist both compressive and rarefactive KdV solitons in the present plasma system.
3. The change in the soliton types from compressive to rarefactive or vice-versa is predicting mainly through the variation of ion nonextensive parameter q, depending on δ and v_0/c. It is seen that compressive and rarefactive solitons show the range of ion nonextensivity $-1 < q < q_c$ and $q_c < q < 3$ respectively.
4. At the critical q_c, we consider a second order nonlinearity and determine mKdV equation. Only compressive DEA solitary wave structures are feasible in present plasma system.
5. The increasing of ion-to-electron number density ratio δ in the plasma, lead to decrease both the amplitude and width of the pulses of the propagating DEA solitons.
6. Moreover, with the rising values of relativistic electron streaming factor v_0/c, enhances the energies of the propagating DEA solitons.

Finally, we draw the conclusion that our present theoretical findings should be useful for better understanding the dynamical nature of small but finite amplitude DEA solitons in both astrophysical and space contexts as well as in future laboratory investigations in which the present plasma model are occurred.

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РЕЛЯТИВІСТСЬКИЙ ВПЛИВ НА ПИЛОВО-ЕЛЕКТРОННО-АКУСТИЧНІ САМІТНІ ХВИЛІ В НЕНАМАГНІЧЕНІЙ ПЛАЗМІ З НЕЕКСТЕНСИВНИМИ ІОНАМИ

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Теоретично досліджено нелінійні властивості пилово-електронно-акустичних (DEA) одиночних хвиль та поведінку їх поширення в моделі ненамагніченої релятивістської плазми. Така плазма складається із слаборелятивістських електронів, невеликих розподілених іонів і негативно заряджених нерухомих частинок пилу. Виходячи з набору односпрямованих рівнянь рідини для електронів і неекстенсивного розподілу для іонів з рівнянням Пуассона, рівняння Кортевега-де Фріза (KdV) і модифіковані рівняння KdV (mKdV) визначаються за допомогою методу редуктивного збурення та їх солітонних рішень, отриманих таким чином, для аналізу режиму існування та основних характеристик солітонів DEA малої амплітуди. Детально розглянуто вплив фізичних параметрів, а саме співвідношення густини іонів до електронної кількості (δ), релятивістського коефіцієнта потоку (v_0/c) та параметра неекстенсивності іонів (q) на динаміку солітарних утворень. Результат показує існування як стислих, так і розріджених солітонів DEA KdV і лише стиснутих солітонів DEA mKdV в діапазоні $-1 < q < 3$, з різними δ і v_0/c у плазмі. Крім того, у статті чисельно проаналізовано вплив усіх фізичних параметрів на поширення одиночних хвиль DEA, що відповідають рівнянням KdV та mKdV. Результати цього дослідження можуть допомогти прояснити основні характеристики нелінійних або нелінійних біжучих хвиль, що поширюються як у лабораторній, так і в космічній плазмі, а також у астрофізичному плазмовому середовищі.

Ключові слова: *пило-електронно-акустична одиночна хвиля; рівняння KdV і mKdV; техніка відновного збурення; q-неекстенсивні іони; релятивістська плазма*