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TWO-FLUID SCENARIO FOR DARK ENERGY COSMOLOGICAL MODEL IN FIVE DIMENSIONAL KALUZA-KLEIN SPACE TIME

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In this work, we investigate the evolution of the dark energy equation of state parameter in a five-dimensional Kaluza-Klein homogeneous and isotropic cosmological model filled with a barotropic fluid and a dark fluid. We adopt a special form of the deceleration parameter, $q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^\alpha}$ as proposed by Singha and Debnath [28], which facilitates a smooth transition from early-time deceleration to late-time acceleration. Using this form, we solve the Einstein field equations and analyze the dynamics of the universe under both non-interacting and interacting two-fluid scenarios. The physical and geometrical implications of the model are examined in detail. Key cosmological quantities such as the dark energy density ρ_D, pressure p_D, and density parameter Ω_D are studied for various spatial curvatures—open, closed, and flat geometries. The solutions obtained are physically viable and in good agreement with current observational data, including those from Type Ia supernovae, the cosmic microwave background, and large-scale structure surveys. Additionally, we evaluate the jerk parameter to assess the model's deviation from the standard ΛCDM cosmology. The model demonstrates compatibility with the observed late-time accelerated expansion and provides a unified framework that accommodates a wide range of cosmic behaviors through appropriate parameter choices.

Keywords: *Kaluza-Klein space time; Two-fluid model; Dark energy; Special form of deceleration parameter; Jerk parameter*

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1. INTRODUCTION

The recent advances in the cosmological observation decided that the universe not only expanding but also accelerating has been confirmed by high red shift type Ia Supernovae experiment (SNe.Ia) [1-3], cosmic background radiation (CMBR) [4], large scale structure observation [5], Sloan Digital Sky Survey (SDSS) [6, 7]. According to contemporary cosmology, this is created by a mystery type of energy known as dark energy, which has a positive energy density and a negative pressure. From the observational evidences First Year Wilkinson Microwave Anisotropy Probe (WMAP) [8], and Chandra X-ray observatory [9] combination found that the baryonic matter occupies 4.9%, the dark matter occupies 26.8% and the dark energy occupies 68.3%of the total energy of the universe [10].

It is still unknown exactly what the physical conditions at very early stages of the creation of our universe. The investigation of five dimensional space-time is important because it is thought that the universe may have once existed in a higher dimensional epoch during its early stages of the evolution of the universe. Kaluza [11] and Klein [12] introduced a five dimension that is a model that sought to unify the fundamental forces of gravity and electromagnetism. In a certain sense the Kaluza-Klein theory resembles ordinary gravity, except that it is inscribed in five dimensions instead of four. This theory has been regarded as a candidate of fundamental theory due to the possible work of unifying the fundamental principle. Five dimensional Kaluza Klein spatially homogeneous and isotropic cosmological models are generally considered as good approximation of the present and early stages of the universe. Numerous eminent authors have studied the five-dimensional Kaluza-Klein space-time within the context of general theory of relativity. Chodos et al. [13], Appelquist et al. [14] studied, the present universe is four-dimensional stage may have been preceded by a higher-dimensional phase, which turns into four-dimensional in the sense that any additional dimensions collapse to an undetected Planckian length scale as a result of dynamical contraction. Das et al. [15] studied interacting and non-interacting two-fluid scenario in the anisotropic five-dimensional Bianchi type-I universe within the framework of Lyra geometry. Ray et al. [16] investigated string cosmological model in five dimensional Kaluza-Klein space time. Many prominent authors has been investigated the evolution of the dark energy parameter under two-fluid scenario. Tiwari et al. [17] investigated the equation of state (EoS) parameter for dark energy (DE) in the spatially homogeneous and anisotropic Bianchi type-III spacetime filled with a barotropic fluid and dark energy by considering a variable deceleration parameter. Goswami et al. [18] have studied a Bianchi type-I cosmological model of universe filled with barotropic and dark energy (DE) type fluids. Mishra et al. [19] have studied the stability of the dark energy cosmological models with combination of matter fields and dark energy in an anisotropic space time. Ray et al. [20] discussed about an interacting and non-interacting two-fluid dark energy models in five dimensional Kaluza-Klein universe. Two-fluids cosmological models with matter and radiating source in $(2 + 1)$ dimensional Saez-Ballester scalar-tensor theory of gravitation has been

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investigated by Kumar et al. [21]. Hatka et al. [22] have studied Bianchi type I cosmological model under two-fluids scenario in scale covariant theory of gravitation. Trivedi et al. [23] has explored the features of the five-dimensional Bianchi type-I cosmological universe filled with barotropic fluid and dark energy within the framework of Saez-Ballester theory of gravitation.

Motivated from the above literature. in this paper we have analyzed the evolution of dark energy parameter by considering five dimensional Kaluza-Klein homogeneous and isotropic cosmological filled with two-fluid model by considering a special form of deceleration parameter. In Sect 1, we discussed the introduction of Kaluza-Klein cosmology. The Kaluza-Klein models and its field equations are presented in Sect 2. In Sects 3 and 4 we discussed the interacting and non-interacting two fluid model. Sect 5 we discuss the jerk parameter in our derived models. In Sect 6 we discussed the physical interpretation of our model. Finally, conclusions and summarized are given in the last sect 7.

2. THE METRIC AND FIELD EQUATIONS

The FRW type homogeneous and isotropic 5D Kaluza-Klein space-time has been considered and it is as follows

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right] \tag{1}$$

where $a(t)$ is a scale factor considered to be a function of cosmic time t and $k = -1, 0, +1$ is the curvature parameter for open, flat and closed universe respectively.

The Einstein's field eqn. (with $8\pi G = 1$ and $c = 1$) can be written as

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{2}$$

where T_{ij} is the two fluid energy momentum tensor consisting of dark fluid and barotropic fluid.

The energy conservation law for two fluid is given by

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + p) = 0 \tag{3}$$

where $p = p_m + p_D$ and $\rho = \rho_m + \rho_D$. Here ρ_m and p_m are the energy density and pressure of the perfect fluid and ρ_D and p_D are the energy density and pressure of dark fluid respectively.

The Einstein field eqn (2) with (3) for the metric (1) we may write

$$6\left(\frac{\dot{a^2}}{a^2} + \frac{k}{a^2}\right) = (\rho_m + \rho_D) \tag{4}$$

and

$$3\frac{\ddot{a}}{a} + 3\left(\frac{\dot{a^2}}{a^2} + \frac{k}{a^2}\right) = -(p_m + p_D) \tag{5}$$

Here the over dot indicate a derivatives with respect to cosmic time t.

The EoS parameter (ω) which is considered as an important quantity in describing the dynamics of the universe, is given by

$$p_m = (\omega_m - 1)\rho_m \tag{6}$$

$$p_D = (\omega_D - 1)\rho_D \tag{7}$$

In the following sections we deal with two cases (I) non-interacting two fluid model and (II) interacting two fluid model

3. CASE-I: NON-INTERACTING TWO FLUID MODEL

In this section, the fluids do not interact with each other. The conservation equation for the dark and barotropic fluid separatly as

$$\dot{\rho_m} + 4\frac{\dot{a}}{a}(\rho_m + p_m) = 0 \tag{8}$$

$$\dot{\rho_D} + 4\frac{\dot{a}}{a}(\rho_D + p_D) = 0 \tag{9}$$

Integrating (8) we obtain

$$\rho_m = \rho_0 a^{-4\omega_m} \tag{10}$$

where ρ_0 is an integrating constant.

Now using (10) in (4) and (5) we obtain ρ_D and p_D in terms of scale factor $a(t)$.

$$\rho_D = 6\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0 a^{-4\omega_m} \tag{11}$$

$$p_D = -3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0(\omega_m - 1)a^{-4\omega_m} \tag{12}$$

Now we assume the special form of deceleration parameter which is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{1 + a^\alpha} \tag{13}$$

where α is an arbitrary constant.

After integrating (13) we get

$$a(t) = (e^{m\alpha t} - 1)^{\frac{1}{\alpha}} \tag{14}$$

where m is an integration constant.

By using the scale factor $a(t)$ in (11) and (12) we obtain

$$\rho_D = 6[\frac{m^2 e^{2m\alpha t}}{(e^{m\alpha t} - 1)^2} + \frac{k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}}] - \rho_0(e^{m\alpha t} - 1)^{\frac{-4\omega_m}{\alpha}} \tag{15}$$

and

$$p_D = -3[\frac{m^2 e^{m\alpha t}(2e^{m\alpha t} - \alpha)}{(e^{m\alpha t} - 1)^2} + \frac{k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}}] - \rho_0(\omega_m - 1)(e^{m\alpha t} - 1)^{\frac{-4\omega_m}{\alpha}} \tag{16}$$

By using (15) and (16) in (7) we obtain

$$\omega_D = -\left[\frac{\frac{3m^2 e^{m\alpha t}(2e^{m\alpha t} - \alpha)}{(e^{m\alpha t} - 1)^2} + \frac{3k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}} + \rho_0(\omega_m - 1)(e^{m\alpha t} - 1)^{\frac{-4\omega_m}{\alpha}}}{\frac{6m^2 e^{2m\alpha t}}{(e^{m\alpha t} - 1)^2} + \frac{6k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}} - \rho_0(e^{m\alpha t} - 1)^{\frac{-4\omega_m}{\alpha}}}\right] + 1 \tag{17}$$

The expressions for the matter energy density Ω_m and DE density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{6H^2} = \frac{\rho_0(e^{m\alpha t} - 1)^{\frac{-4\omega_m}{\alpha} + 2}}{6m^2 e^{2m\alpha t}} \tag{18}$$

and

$$\Omega_D = \frac{\rho_D}{6H^2} = 1 + \frac{k(e^{m\alpha t} - 1)^2}{6m^2 e^{2m\alpha t}(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}} - \frac{\rho_0(e^{m\alpha t} - 1)^{\frac{-4\omega_m}{\alpha} + 2}}{6m^2 e^{2m\alpha t}} \tag{19}$$

From eqns. (18) and (19) we obtain

$$\Omega = \Omega_m + \Omega_D = 1 + \frac{k(e^{m\alpha t} - 1)^2}{6m^2 e^{2m\alpha t}(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}} \tag{20}$$

From eqns. (13) and (14), the deceleration parameter q (Recently used Katore et al. [24]) as

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{e^{m\alpha t}} \tag{21}$$

4. CASE-II: INTERACTING TWO FLUID MODEL

In this section we consider the interaction between dark fluid and barotropic fluid. The conservation equation for the dark and barotropic fluid are given by

$$\dot{\rho_m} + 4\frac{\dot{a}}{a}(\rho_m + p_m) = Q \tag{22}$$

$$\dot{\rho_D} + 4\frac{\dot{a}}{a}(\rho_D + p_D) = -Q \tag{23}$$

where the quantity Q represents the interaction between the matter and DE component. We consider $Q > 0$, as it shows that the energy transferred from DE to dark matter. Let us consider ((from ref. Gou et al. [25], Amendola et al. [26])

$$Q = 4H\sigma\rho_m \tag{24}$$

where σ is an coupling constant.

Using (24) in (22) and after integrating we obtain

$$\rho_m = \rho_0 a^{-4(\omega_m - \sigma)} \tag{25}$$

where ρ_0 is an integrating constant.

Now using (25) in (4) and (5) we obtain ρ_D and p_D in terms of scale factor $a(t)$.

$$\rho_D = 6\left(\frac{\dot{a^2}}{a^2} + \frac{k}{a^2}\right) - \rho_0 a^{-4(\omega_m - \sigma)} \tag{26}$$

$$p_D = -3\left(\frac{\dot{a}}{a} + \frac{\dot{a^2}}{a^2} + \frac{k}{a^2}\right) - \rho_0(\omega_m - 1)a^{-4(\omega_m - \sigma)} \tag{27}$$

Putting the value of $a(t)$ from (14) in eqns. (26) and (27) we obtain

$$\rho_D = 6[\frac{m^2 e^{2m\alpha t}}{(e^{m\alpha t} - 1)^2} + \frac{k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}}] - \rho_0(e^{m\alpha t} - 1)^{\frac{-4(\omega_m - \sigma)}{\alpha}} \tag{28}$$

and

$$p_D = -3[\frac{m^2 e^{m\alpha t}(2e^{m\alpha t} - \alpha)}{(e^{m\alpha t} - 1)^2} + \frac{k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}}] - \rho_0(\omega_m - 1)(e^{m\alpha t} - 1)^{\frac{-4(\omega_m - \sigma)}{\alpha}} \tag{29}$$

Using (28) and (29) in (7) we obtain

$$\omega_D = -\left[\frac{\frac{3m^2 e^{m\alpha t}(2e^{m\alpha t} - \alpha)}{(e^{m\alpha t} - 1)^2} + \frac{3k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}}] + \rho_0(\omega_m - 1)(e^{m\alpha t} - 1)^{\frac{-4(\omega_m - \sigma)}{\alpha}}}{\frac{6m^2 e^{2m\alpha t}}{(e^{m\alpha t} - 1)^2} + \frac{6k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}} - \rho_0(e^{m\alpha t} - 1)^{\frac{-4(\omega_m - \sigma)}{\alpha}}}\right] + 1 \tag{30}$$

The expressions for the matter energy density Ω_m and DE density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{6H^2} = \frac{\rho_0(e^{m\alpha t} - 1)^{\frac{-4(\omega_m - \sigma)}{\alpha} + 2}}{6m^2 e^{2m\alpha t}} \tag{31}$$

and

$$\Omega_D = \frac{\rho_D}{6H^2} = 1 + \frac{k(e^{m\alpha t} - 1)^2}{6m^2 e^{2m\alpha t}(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}} - \frac{\rho_0(e^{m\alpha t} - 1)^{\frac{-4(\omega_m - \sigma)}{\alpha} + 2}}{6m^2 e^{2m\alpha t}} \tag{32}$$

From eqns. (31) and (32) we obtain the total energy density parameter Ω as

$$\Omega = \Omega_m + \Omega_D = 1 + \frac{k(e^{m\alpha t} - 1)^2}{6m^2 e^{2m\alpha t}(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}} \tag{33}$$

5. THE JERK PARAMETER (J)

In cosmology, the jerk parameter (j) is a dimensionless quantity that characterizes the rate of change of the universe's expansion acceleration, specifically the third derivative of the scale factor with respect to time, normalized by the hubble parameter cubed. The jerk parameter j is a suitable method to describe models close to ΛCDM model. A deceleration-to-acceleration transition occurs for models with a positive value of j and negative value of q. Flat ΛCDM models have a constant jerk $j = 1$. The jerk parameter in cosmology is defined as the dimensionless third derivative of the scale factor with respect to cosmic time t [27].

The Jerk Parameter in cosmology is defined as the dimensionless third derivative of the scale factor with respect to cosmic time t. It is defined as

$$j(t) = \frac{1}{H^3}\frac{\ddot{a}}{a} \tag{34}$$

where overhead dot denote derivatives with respect to cosmic time.

The jerk parameter (j) appears in the fourth term of a Taylor expansion of the scale factor around a_0.

$$\frac{a(t)}{a_0} = 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2 (t - t_0)^2 + \frac{1}{6} j_0 H_0^3 (t)^3 + 0\left[(t - t_0)^4\right] \tag{35}$$

In equation (34) can be written as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} \tag{36}$$

where H is the Hubble parameter, q is the deceleration parameter and overhead dot denotes the derivatives with respect to cosmic time t.

For the derived model, the jerk parameter (j) can be written as

$$j(t) = 1 - \frac{3\alpha}{e^{m\alpha t}} + \frac{\alpha^2(e^{m\alpha t} + 3)}{e^{2m\alpha t}} \tag{37}$$

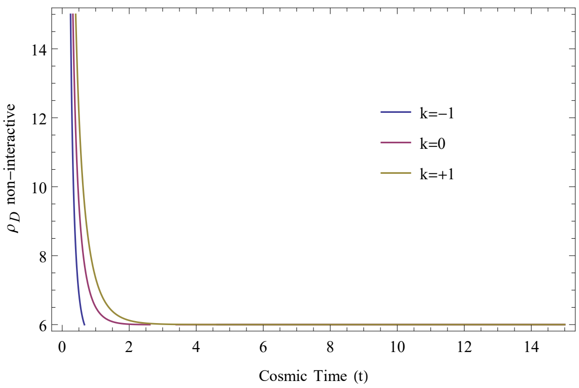

Figure 1. Plot of ρ_D vs. t (non-interactive) with $m = 1$, $\rho_0 = 1$, $\alpha = 3$, $\omega_m = 0.5$

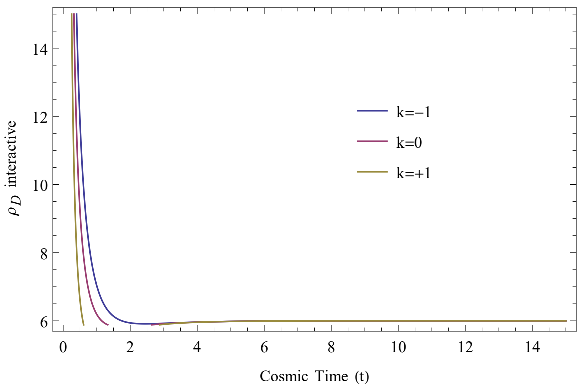

Figure 2. Plot of ρ_D vs. t (interactive) with $m = 1$, $\rho_0 = 1$, $\alpha = 3$, $\omega_m = 0.5$

6. RESULTS AND DISCUSSION:

We have plotted the various figures using these calculations for suitable values of the constants. We also used all the three values of $k = -1, 0, 1$ while plotting the figures.

In figures 1 and 2 we have shown the variation of dark energy density ρ_D versus cosmic time t for both non-interacting and interacting cases from equations (15) and (28). It is observed that for both cases the graphs of ρ_D is decreasing function of cosmic time t in all the three open, closed and flat universe. It indicates that the dark energy model start with infinite density and when cosmic time increases the energy density tends to a finite value.

Figs. 3 and 4 represent the variation of pressure for DE for both cases. We observed that at $t \to 0$ the pressure p_D is negative for open ($k = -1$) and flat ($k = 0$) universe and positive for closed ($k = 1$) universe. Finally, pressure p_D is zero all the three open, closed and flat universe for late time cosmic evolution.

The behavior of EoS for DE with respect to cosmic time t is shown in figs. 5 and 6 which correspond to the eqns. (17) and (30) for both non-interacting and interacting cases respectively. We observed that from both the figures the EoS parameter ω_D is a decreasing function of time t. At $t \to \infty$, the EoS parameter ω_D tends to zero. So, the model indicating matter dominated era of the universe .

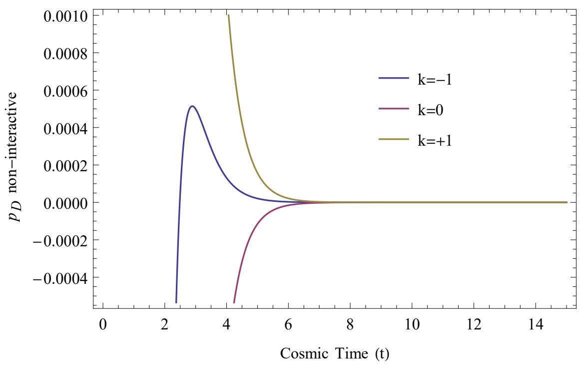

Figure 3. Plot of p_D vs. t (non-interactive) with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$

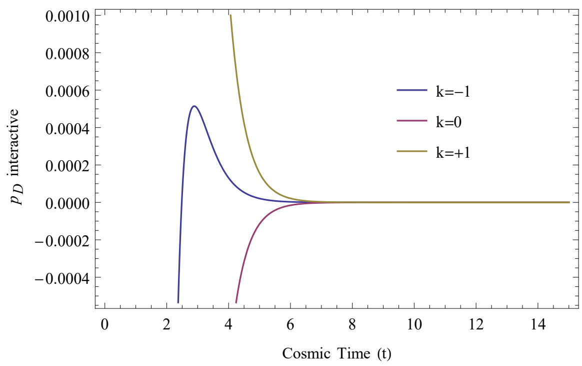

Figure 4. Plot of p_D vs. t (interactive) with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$

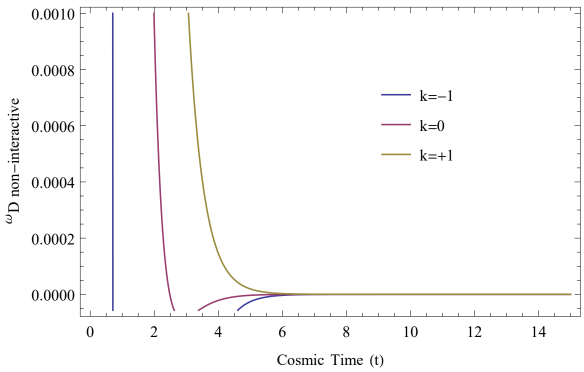

Figure 5. Plot of ω_D vs. t (non-interactive) with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$

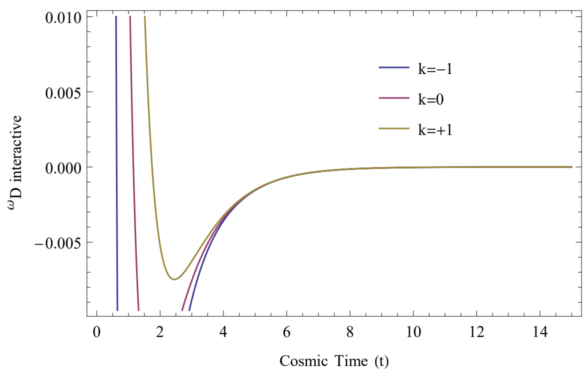

Figure 6. Plot of ω_D vs. t (interactive) with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$

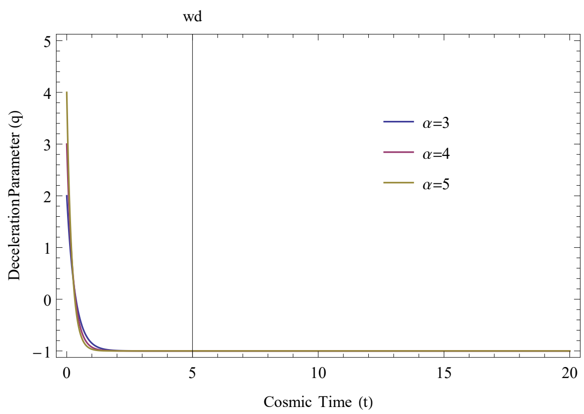

Figure 7. Plot of q vs. t for $m = 1$, $\alpha = 3, 4, 5$.

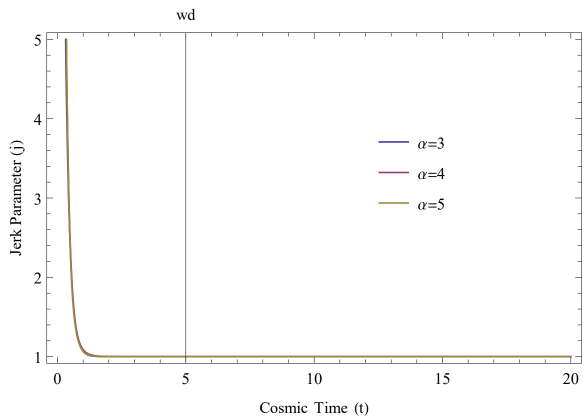

Figure 8. Plot of j vs. t for $m = 1$, $\alpha = 3, 4, 5$.

From eqns. (20) and (33) the total energy density parameter Ω for both non-interacting and interacting cases are same. From the right hand side of these two equations it is clear that for $k = 0$ (i.e for flat universe), $\Omega = 1$ and $k = 1$ (i.e for closed universe), $\Omega > 1$ and $k = -1$ (i.e for open universe), $\Omega < 1$. We also observed that, at late time, Ω approaches to 1 for all the three values of k, which is shows that the universe will acquire a flat structure. These results are fully consistent with the present day's observations.

From the equation (21), we observed that the deceleration parameter q is a decreasing function of cosmic time t . Initially at $t = 0$, the deceleration parameter q is positive which changes sign from positive to negative. As time approaches infinity, deceleration parameter q tend to -1 which indicates that the proposed model universe has a transition from decelerating to accelerating phase. It can be confirmed from the graph of figure 7, which plots the deceleration parameter q against cosmic time t. This graphical representation clearly illustrates the model's expanding behavior over time. Thus our model represents an interesting cosmological model of the universe.

A decelerate phase to accelerate phase transition of the universe occurs for models with a positive value of jerk and negative value of deceleration parameter. In figures 7 and 8 we can observed that deceleration parameter (q) is negative and jerk parameter (j) is positive so that we do have a transition of the model from decelerated to accelerated phase at late time cosmic evolution.

The cosmological model constructed here exhibits the following properties:

- The scale factor
$$a(t) = \left(e^{m\alpha t} - 1\right)^{1/\alpha}$$
evolves from zero at $t = 0$ and grows exponentially at late times, reflecting early decelerated and late-time accelerated expansion.

- The deceleration parameter q evolves from a positive value (decelerating phase) at early times to $q \to -1$ (de Sitter expansion) at late times, for all $\alpha > 0$.

- The total density parameter approaches unity, $\Omega \to 1$ as $t \to \infty$, consistent with observations of a spatially flat universe.

- For suitable choices of parameters, the equation of state parameter ω_D for the dark fluid can evolve through quintessence ($\omega_D > -1$), cosmological constant ($\omega_D = -1$), or phantom ($\omega_D < -1$) regimes.

- Interaction between the fluids alters the evolution of the matter density, allowing a more flexible fit to observational data through the coupling constant σ.

7. CONCLUSION

In this work, we have investigated a homogeneous and isotropic five-dimensional Kaluza-Klein cosmological model incorporating a two-fluid system—comprising a barotropic fluid and a dark fluid—within the framework of open, closed, and flat universes. The Einstein field equations were solved by adopting a hybrid scale factor that leads to a time-dependent deceleration parameter of the form $q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^\alpha}$ which smoothly describes a transition from early-time decelerated expansion to late-time accelerated expansion.

We analyzed both non-interacting and interacting scenarios for the two-fluid system and examined the dynamical behavior of key cosmological quantities including the energy density ρ_D, pressure p_D, and density parameter Ω_D of the dark fluid. In all cases, the solutions are physically realistic and consistent with current observational data from Type Ia Supernovae, cosmic microwave background (CMB), and large-scale structure surveys.

The total density parameter approaches unity at late times, indicating an asymptotically flat universe. Furthermore, the equation of state parameter for the dark fluid can exhibit quintessence, cosmological constant, or phantom-like behavior depending on the choice of parameters. The model remains valid for open, closed, and flat geometries, adding to its robustness. Additionally, the jerk parameter was derived and discussed to explore the model's deviation from the standard ΛCDM paradigm.

Overall, the proposed model successfully captures essential features of cosmic evolution and offers a flexible framework for understanding the late-time acceleration of the universe.

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ДВОРІДИННИЙ СЦЕНАРІЙ ДЛЯ КОСМОЛОГІЧНОЇ МОДЕЛІ ТЕМНОЇ ЕНЕРГІЇ У П'ЯТИВИМІРНОМУ ПРОСТОРІ-ЧАСІ КАЛУЦИ-КЛЕЙНА

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У цій роботі ми досліджуємо еволюцію параметра рівняння стану темної енергії у п'ятивимірній однорідній та ізотропній космологічній моделі Калуци-Клейна, заповненій баротропною та темною рідинами. Ми використовуємо спеціальну форму параметра уповільнення, $q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^{\alpha}}$, як запропоновано Сінгхою та Дебнатхом [28], що сприяє плавному переходу від уповільнення на ранніх етапах до прискорення на пізніх етапах. Використовуючи цю форму, ми розв'язуємо рівняння поля Ейнштейна та аналізуємо динаміку Всесвіту як за невзаємодіючих, так і за взаємодіючих дворідинних сценаріїв. Детально розглядаються фізичні та геометричні наслідки моделі. Ключові космологічні величини, такі як густина темної енергії ρ_D, тиск p_D та параметр густини Ω_D, досліджуються для різних просторових кривин — відкритої, закритої та плоскої геометрій. Отримані рішення є фізично життєздатними та добре узгоджуються з сучасними даними спостережень, включаючи дані наднових типу Ia, космічного мікрохвильового фону та оглядів великомасштабних структур. Крім того, ми оцінюємо параметр ривка, щоб оцінити відхилення моделі від стандартної космології ΛCDM. Модель демонструє сумісність зі спостережуваним прискореним розширенням наприкінці часу та забезпечує єдину структуру, яка враховує широкий діапазон космічної поведінки завдяки відповідному вибору параметрів.

Ключові слова: *простір-час Калуци-Клейна; дворідинна модель; темна енергія; спеціальна форма параметра уповільнення; параметр ривка*