East European Journal of Physics. 1. 318-326 (2025)
DOI:10.26565/2312-4334-2025-1-38 ISSN 2312-4334

BLOOD FLOW ANALYSIS THROUGH AN INCLINED ARTERY HAVING MULTIPLE STENOSIS WITH VARIABLE NANOFLUID VISCOSITY USING SINGLE WALL CARBON NANOTUBE (SWCNT)

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Received December 16, 2024; revised January 25, 2025; accepted February 3, 2025

The steady flow of viscous fluid flow through an inclined tube of the non-uniform cross-section including multiple stenoses has been investigated under the influence of a single-wall carbon Nanotube (SWCNT). We linearized the flow equations and determined the flow resistance and wall shear stress expressions assuming mild stenoses. Studies have examined how parameters affect flow variables. It is found that the resistance of the flow increases with stenoses height. It is also interesting to notice that the wall shear stress decreases with the increase of the height of stenoses. It is also observed that the resistance to the flow ($\bar{\lambda}$) increases with inclination (α), source and sink parameter (β), Grashof number (B_r), dynamic viscosity (μ) and flux (q). The velocity profiles are presented in the form of streamlines.
Keywords: *Stenosis; Resistance to the flow; Carbon Nanotube; Wall shear stress*
PACS: 47.15.-x

1. INTRODUCTION

The narrowing of the atrial lumen, the inner open space or cavity of an artery, due to fatty deposits is one of the biggest health risks today. This can cause hypertension, myocardial infarction, etc., so stenosis, or abnormal and unnatural growth, disrupts normal blood flow. Hydrodynamical factors like wall shear stress, flow resistance, etc. can cause and progress this pathological condition. Knowledge of the flow condition in a stenosed tube may help understand and avoid vascular disorders.

Thus, several authors have investigated mathematical models for constricted duct flows (Young [1], LEE and Fung [2], Shukla et al, [3], Radhakrishnamacharya and Srinivasa Rao [4]).

All these mathematical investigations have described blood as a Newtonian fluid. Furthermore, when the diameters of the channel or tube are small and the rate of shearing is low, blood exhibits non-Newtonian behavior. The amount of red blood cells (RBCs) in erythrocytes influences this behavior. Numerous analytical research 5-9 have been conducted out to analyze the mathematical impacts of stenosis on arterial blood flow characteristics, flow resistance, and shear stress on the wall. Liu et al. [5] established a computational model for pulsating blood flow through stenotic and tapered arteries. Prasad K.M. and Yasa P.R[6] have developed a mathematical model of a micro polar fluid flow in a tapering stenosed artery having permeable walls.

However, all these studies focused on the impact of individual stenosis, assuming a uniform cross-section of the tube. However, it is recognized that numerous blood vessels exhibit gradual changes in cross-section through their length and may present multiple stenoses at junctions and bends (Schneck et al. [7]). Maruthi Prasad and Radhakrishnamacharya [8] examined blood circulation in an artery characterized by multiple stenoses and a non-uniform cross-section, treating blood as a Herschel-Bulkley fluid.

Nano-fluids have generated significant interest from researchers because of their increased thermal conductivity, a concept first introduced by Choi [9]. Nadeem and Noreen Sher Akbar [10] investigated the flow of a micro-polar fluid containing nanoparticles in the small intestine. Maruthi Prasad and Prabhakar Reddy [11] studied the thermal effects of two immiscible fluids through a permeable stenosed artery having Nano-fluid in the core region and Newtonian fluid in the peripheral region. Many researchers have focused on Nano fluids because of their importance in the biomedical field [12-15]. The presence of nanoparticles suspended in the base fluid is insufficient to improve thermal conductivity, as this enhancement is contingent upon the particles' shape and size. Murshed et al. [16] demonstrated that Carbon Nanotubes (CNTs) exhibit thermal conductivity six times superior to other materials. Iijima and Ichihashi [17] discovered that Carbon Nanotubes are in the form of long and thin cylinders of Carbon. They have a wide range of applications in engineering and Science because of their chemical and physical properties. Carbon Nanotubes are used in medicine, gene, and drug delivery systems. The key features of CNT are three types 1) single-wall carbon nanotubes, ii) double-wall carbon nanotubes, and iii) multi-wall carbon nanotubes.

Nano-electronics is one of the most prospective submissions of single-walled nanotubes for their excellent conductivity. They are potential nanomaterials for anisotropic strengthening of thin composite films for balloon chattered

Cite as: S.N. Ahmed, K.M. Prasad, R. Sridevi, East Eur. J. Phys. 1, 318 (2025), https://doi.org/10.26565/2312-4334-2025-1-38

fabrication. Many authors have studied carbon Nano fluids. [18,19]. Many researchers have penned their studies presenting their applications in modern technology. Kim & Yoosuk presented their study of CNT in device applications (Kim & Kuljanishvili, [20]). Thermal outcomes relating to SWCNT and MWCNT were also presented by Majeed, Aaqib Majeed et al., [21]) and Wang, Mansir (Wang et al., [22]).

It is known that numerous ducts within physiological systems are not aligned horizontally but exhibit a certain inclination to the axis. Recently Maruthi Prasad and Sreekala [23] have studied the thermal effects on peristaltic transport in an inclined circular elastic tube.

Keeping this motivation and purpose a mathematical model is formulated to analyze blood flow through an inclined tube with a non-uniform cross-sectional and multiple stenoses, affected by a single wall carbon nanotube, while treating blood as a viscous fluid. Assuming mild stenoses, closed-form solutions have been derived. Resistance to flow and wall shear stress expressions have been developed, and the influence of different parameters on these fluid flow parameters has been examined.

2. MATHEMATICAL FORMATION

Considered is a steady, incompressible blood flow across a non-uniform cross-sectional tube with multiple stenoses. A cylindrical polar coordinate system (z; r) ensures that the tube's center line and z-axis coincide. Presumably, the tube is inclined at an angle of α concerning the parallel (see Fig. 1). The stenosis should be moderate and symmetrical in its development along the axis. The tube's radius is considered as: (Maruthi Prasad and Radhakrishnamacharya [2008] [8]

$$h = R(z) = \begin{cases} R_0 : \ 0 \le z \le d_1, \\ R_0 - \frac{\delta_1}{2}\left(1 + cos\frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right) : \ d_1 \le z \le d_1 + L_1, \\ R_0 : \ d_1 + L_1 \le z \le B_1 - \frac{L_2}{2}, \\ R_0 - \frac{\delta_2}{2}\left(1 + cos\frac{2\pi}{L_2}(z - B_1)\right) : \ B_1 - \frac{L_2}{2} \le z \le B_1, \\ R^*(z) - \frac{\delta_2}{2}\left(1 + cos\frac{2\pi}{L_2}(z - B_1)\right) : \ B_1 \le z \le B_1 + \frac{L_2}{2} \\ R^*(z) : \ B_1 + \frac{L_2}{2} \le z \le B \end{cases} \tag{1}$$

where $R^*(z) = \exp[\beta B^2 (z - B_1)^2]$.
Here δ_1, δ_2 and L_1, L_2 are the maximum heights and lengths of the two stenoses respectively.

Consequently, the fluid flow's governing equations are as follows: The following are the governing equations for a vertical artery's variable viscous Nano fluid that control the conservation of mass, momentum, and temperature:

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \tag{2}$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}\right) = -\frac{\partial \bar{p}}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(2\bar{r}\mu_{nf}\frac{\partial \bar{u}}{\partial \bar{r}}\right) + \frac{\partial}{\partial \bar{z}}\left(\mu_{nf}\left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}}\right)\right) - 2\mu_{nf}\frac{\bar{u}}{\bar{r}^2} - \frac{Cos\alpha}{F} \tag{3}$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}}\right) = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\mu_{nf}\left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}}\right)\right) + \frac{\partial}{\partial \bar{z}}\left(2\mu_{nf}\frac{\partial \bar{w}}{\partial \bar{z}}\right) + g(\rho\gamma)_{nf}(\bar{T} - \bar{T}_0) - \frac{Sin\alpha}{F} \tag{4}$$

$$\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}\right) = \frac{K_{nf}}{(\rho_{C_P})_{nf}}\left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial z^2}\right) + \frac{Q_0}{(\rho_{C_P})_{nf}} \tag{5}$$

In the given equations, $\bar{u}, \bar{v}$ & $\bar{w}$ represent the velocity components, $\bar{T}$ denotes the fluid's temperature, and Q_0 represents the constant heat generation or absorption. In the suggested model for Nano fluids, μ_{nf} represents the variable viscosity of the fluid, γ_{nf} denotes thermal expansion coefficient, K_{nf} denotes thermal conductivity, ρ_{nf} denotes density, $(\rho_{C_p})_{nf}$ is the heat capacitance with the thermo-physical properties) [12] .

$$\mu_{nf} = \frac{\mu_0 e^{-\sigma\theta}}{(1-\emptyset)^{2.5}}, \alpha_{nf} = \frac{K_{nf}}{(\rho_{C_P})_{nf}}, \rho_{nf} = (1-\emptyset)\rho_f + \emptyset_{\rho SWCNT} \tag{6}$$

$$(\rho_{C_p})_{nf} = (1-\emptyset)(\rho_{C_p})_{nf} + \emptyset\,(\rho_{C_p})_{SWCNT}, (\gamma\rho)_{nf} = (1-\emptyset)(\rho\gamma)_f + \emptyset\,(\rho\gamma)_{SWCNT} \tag{7}$$

$$\frac{K_{nf}}{K_f} = \frac{(1-\emptyset) + 2\emptyset\frac{k_{SWCNT}}{k_{SWCNT} - k_F} In\frac{k_{SWCNT} + k_f}{2k}}{(1-\emptyset) + 2\emptyset\frac{k_f}{k_{SWCNT} - k_F} In\frac{k_{SWCNT} + k_f}{2K_f}} \tag{8}$$

Whereas for single wall carbon nanotubes, ρ_{SWCNT} is the density, $(\rho_{cp})_{SWCNT}$ is the heat capacitance, γ_{SWCNT} is the thermal expansion coefficient, k_{SWCNT} is the thermal conductivity and $\emptyset$ is the volume fraction. For the base fluid, μ_f is the viscosity, ρ_f is the density, $\left(\rho_{cp}\right)_f$ is the heat capacitance, γ_f is the thermal expansion coefficient, and k_f is the thermal conductivity.

The non-dimensional variables are

$$r = \frac{\bar{r}}{e_0}\,, w = \frac{\bar{w}}{u_0}, u = \frac{L\bar{u}}{u_0\bar{\delta}}\,, p = \frac{{e_0}^2\bar{p}}{u_0 s\, l\, \mu_0}, \beta = \frac{Q_0 {e_0}^2}{T_0 K_f}\,, R_{en} = \frac{e_0 u_0\, \rho f}{\mu_0}$$

$$G_r = \frac{g\gamma_f \rho_f {e_0}^2 T_0}{u_0 \mu_0}, \theta = \frac{T - T_0}{T_0}, z = \frac{\bar{z}}{sl}\,, F = \frac{F}{\mu u \lambda}, \tau_{rz} = \frac{\tau_{rz}}{\mu\left(\frac{u}{R_0}\right)}, \bar{\delta} = \frac{\delta}{R_o}, \bar{R} = \frac{R(z)}{R_0}$$

G_r constitutes the Grashof number, R_{en} constitutes Reynold's number, β gives the non-dimensional heat source or sink parameter concerning the fluid and u_0 is the average velocity. Using mild stenosis approximation $\frac{\bar{\delta}}{e_0} \ll 1$ and using the condition, $\varepsilon = \frac{e_0}{sl} \ll O(1)$.

The constitutive equations (2) to (5) become

$$\frac{\partial p}{\partial r} = -\frac{cos\alpha}{F} \tag{9}$$

$$\frac{\partial p}{\partial z} + \frac{sin\alpha}{F} = \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\mu_1}{\mu_0}\left(r.\frac{\partial w}{\partial r}\right)\right] + \frac{(\rho_r)_{nf}}{(\rho_r)_f} G_r \theta \tag{10}$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \beta \frac{k_f}{k_{nf}} = 0 \tag{11}$$

The Nano fluid viscosity may be defined as

$$\frac{\mu_{nf}}{\mu_0} = \frac{1}{e^{\omega\theta}(1-\varphi)^{2.5}} \text{ and } e^{\omega\theta} = 1 + \omega\theta, \omega \ll 1$$

The problem's geometrical structure and dimensionless boundary conditions are presented below.

$$h = R(z) = \begin{cases} 1 : \ 0 \le z \le d_1, \\ 1 - \frac{\delta_1}{2}\left(1 + cos\frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right) : \ d_1 \le z \le d_1 + L_1, \\ 1 : \ d_1 + L_1 \le z \le B_1 - \frac{L_2}{2}, \\ 1 - \frac{\delta_2}{2}\left(1 + cos\frac{2\pi}{L_2}(z - B_1)\right) : \ B_1 - \frac{L_2}{2} \le z \le B_1, \\ R^*(z) - \frac{\delta_2}{2}\left(1 + cos\frac{2\pi}{L_2}(z - B_1)\right) : \ B_1 \le z \le B_1 + \frac{L_2}{2} \\ R^*(z) : \ B_1 + \frac{L_2}{2} \le z \le B \end{cases} \tag{12}$$

Where $\frac{R^*(z)}{R_0} = \exp[\beta B^2 (z - B_1)^2]$.

$$w = 0, \theta = 0 at r = h(z) \tag{13}$$

$$\frac{\partial w}{\partial r} = 0, \frac{\partial \theta}{\partial r} = 0\ at r = 0 \quad (14)$$

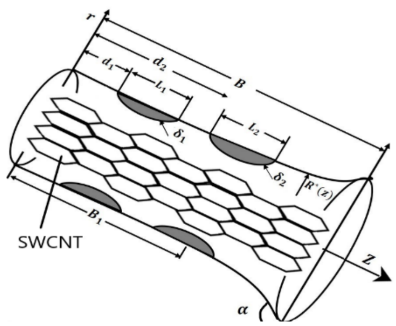

Figure 1. Schematic diagram of the inclined tube with multiple stenoses under the influence of SWCNT

The precise solutions of Eq. (10) & (11) using (13) to (14) are

$$\theta = \frac{\beta}{4}\frac{k_f}{k_{nf}}[h^2 - r^2] \tag{15}$$

$$w = \frac{(r^2-h^2)}{4\mu}\left[\frac{dp}{dz}+\frac{sin\alpha}{F}\right]-\frac{A\beta k}{8\mu}G_r\left[\frac{3h^4-r^4}{8}+\frac{h^2r^2}{2}\right] \quad (16)$$

The non -dimensional flux

$$q = \int_0^h 2rwdr. \quad (17)$$

Substituting Eq. (16) in Eq. (20), an expression is deduced for the pressure gradient as follows

$$\frac{dp}{dz} = \frac{-8\mu q}{h^4} - \frac{sin\alpha}{F} + \frac{h^2}{6}\ A\ G_r\beta\ k \quad (18)$$

Where $A = \frac{(\rho_r)_{nf}}{(\rho_r)_f}, k = \frac{k_f}{k_{nf}}, \frac{\mu_1}{\mu_0} = \mu$

The pressure drop per wave length is

$$\Delta p = -\int_0^1 \frac{dp}{dz}dz \quad (19)$$

Substituting Eq. (18) in Eq. (19), we get

$$\Delta p = -\int_0^1\left[\frac{-8\mu q}{h^4} - \frac{sin\alpha}{F} + \frac{h^2}{6}\ A\ G_r\beta\ k\right]dz \quad (20)$$

The flow resistance

$$\lambda = \frac{\Delta p}{q} = -\frac{1}{q}\int_0^1\left[\frac{-8\mu q}{h^4} - \frac{sin\alpha}{F} + \frac{h^2}{6}\ A\ G_r\beta\ k\right]dz \quad (21)$$

The normalized flow resistance is

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = \frac{\int_0^1\left[\frac{-8\mu q}{h^4} - \frac{sin\alpha}{F} + \frac{h^2}{6} A\ G_r\beta\ k\right]dz}{\int_0^1\left[\frac{-8\mu q}{1} - \frac{sin\alpha}{F} + \frac{1}{6} A\ G_r\beta\ k\right]dz}. \quad (22)$$

The shear stress on the wall is

$$\tau_h = \frac{-h}{2}\left(\frac{\partial P}{\partial z}\right) \quad (23)$$

3. RESULT AND ANALYSIS

In this section, we analyze numerical measures of essential parameters to assess the dependability and accuracy of our exact solutions [Maruthi Prasad et al.33]

$$\frac{R^*(z)}{R_0} = \exp[\beta B^2(z-B_1)^2], \text{and } d_1 = 0.2, L_1 = 0.2, B_1 = 0.8, B = 1, d_2 = 0.6\ ,$$

which are illustrated through graphs. The graphs depict wall shear stress, and flow resistance as functions of the heat source or sink parameter (β), the inclination (α), and the heights of the stenosis (δ_1 *and* δ_2). We consider these graphs for cases involving pure blood and single-walled carbon nanotubes (SWCNT) at volume fractions of $\emptyset = 0.02$ and $\emptyset = 0.04$, while keeping certain parameters constant.

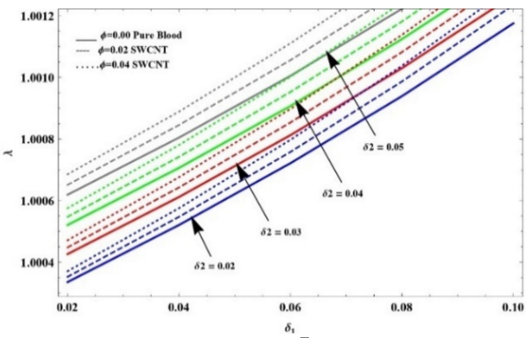

Figure 2. Effect of δ_1 on $\bar{\lambda}$, with δ_2 varying
($k_f = 0.2, k_s = 0.4, G_r = 0.2, q = 0.8, \beta = 0.01, \omega = 0.3, \mu = 0.01$)

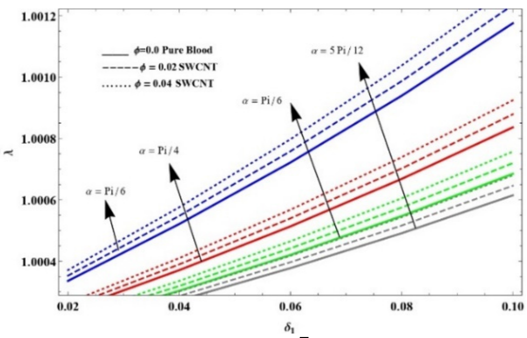

Figure 3. Effect of δ_1 on $\bar{\lambda}$, with α varying
($k_f = 0.2, k_s = 0.4, G_r = 0.2, q = 0.8, \beta = 0.01, \alpha = \frac{\pi}{6}, \omega = 0.3, \mu = 0.01$)

From Figures 2-6, it is observed that when the heights of the primary and secondary stenosis (δ_1 *and* δ_2). increase, the flow resistance also increases. As the height of the stenosis increases, it disturbs the flow pattern, the velocity of the fluid decreases, and the flow resistance increases. Interestingly, when comparing the SWCNT case to the pure blood instance, the flow resistance provides better results.

It is also noticed from Fig. 2-6, that the flow resistance ($\bar{\lambda}$) increases with inclination (α), source and sink parameter (β), dynamic viscosity (μ) and flux (q). It is interesting to note from the Fig. 5 that as the viscosity increases the resistance to the flow also increases, this resistance to more in SWCNT than the pure blood in the diseased artery.

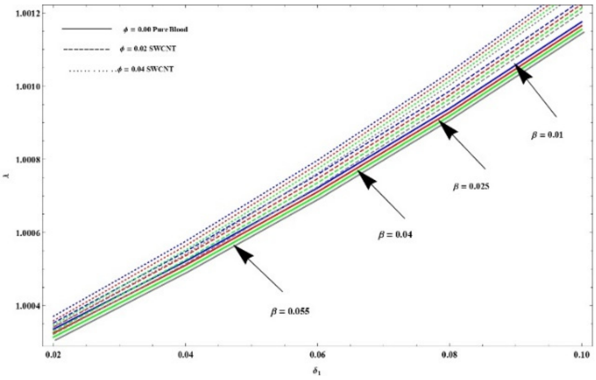

Figure 4. Effect of δ_1 on $\bar{\lambda}$, with β varying
$(k_f = 0.2, k_s = 0.4,, G_r = 0.2, q = 0.8, \omega = 0.3, \alpha = \frac{\pi}{6}, \mu = 0.01)$

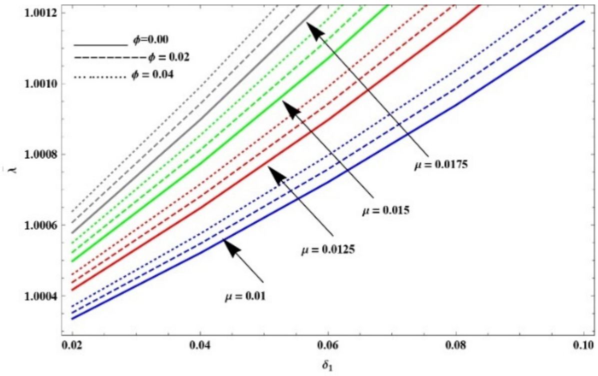

Figure 5. Effect of δ_1 on $\bar{\lambda}$, with μ varying
$(k_f = 0.2, k_s = 0.4, G_r = 0.2, q = 0.8\ \omega = 0.3, \beta = 0.01, \alpha = \frac{\pi}{6})$

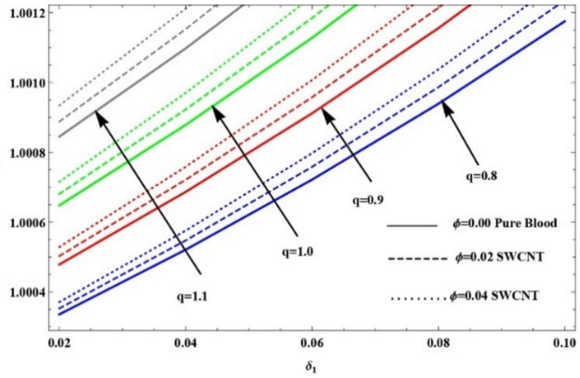

Figure 6. Effect of δ_1 on $\bar{\lambda}$, with q varying
$(k_f = 0.2, k_s = 0.4,, G_r = 0.2, q = 0.8, \omega = 0.3, \alpha = \frac{\pi}{6}, \beta = 0.01, \mu = 0.01)$

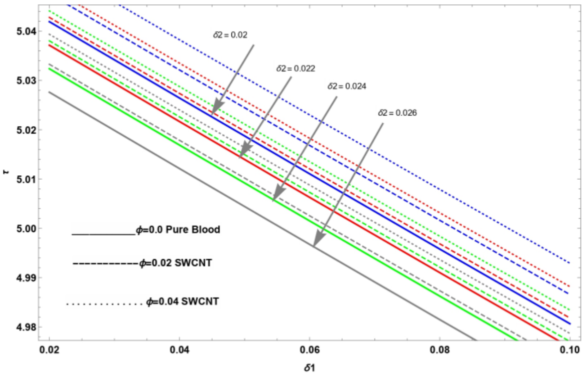

Figure 7. Effect of δ_2 on τ_h , with δ_2 varying
$(k_f = 0.2, k_s = 0.4, G_r = 0.2, q = 0.8, \omega = 0.3, \alpha = \frac{\pi}{6}, \mu = 0.01)$

The graphs of shear stress on the wall along axial displacements are plotted in Fig. 7-11 for different cases. It is observed from these figures that the wall shear stress (τ) decreases with the heights of first and second stenosis $(\delta_1\ and\ \delta_2)$ i.e wall shear stress having stenosis (diseased artery) gives higher results for the pure blood case $(\emptyset = 0)$ compared to the cases involving SWCNTs $(\emptyset = 0.02, 0.04)$.

Figures 7-11 represents the graphs of shear stress on the wall as a function of axial displacements for various cases. Analysis of the figures indicates that wall shear stress (τ) decreases with increasing heights of the first and second stenosis $(\delta_1\ and\ \delta_2)$. Specifically, wall shear stress in the presence of stenosis (diseased artery) provides higher values for the pure blood case $(\emptyset = 0)$ compared to the SWCNT cases$(\emptyset = 0.02,\ 0.04)$.

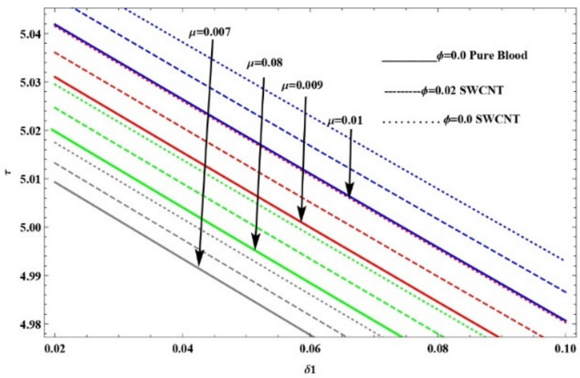

Figure 8. Effect of δ_2 on τ_h , with μ varying
$(k_f = 0.2, k_s = 0.4, G_r = 0.2, q = 0.8, \omega = 0.3, \beta = 0.01, \alpha = \frac{\pi}{6})$

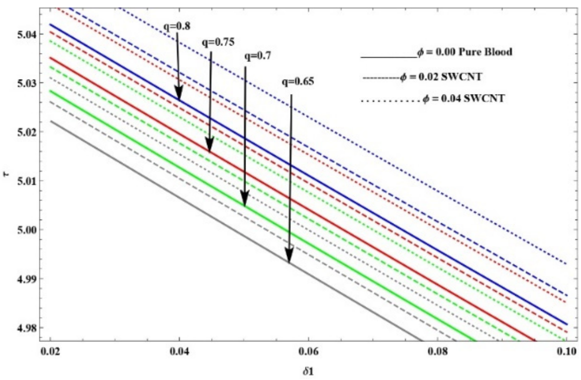

Figure 9. Effect of δ_2 on τ_h , with q varying
$(k_f = 0.2, k_s = 0.4, G_r = 0.2, \omega = 0.3, \alpha = \frac{\pi}{6}, \beta = 0.01, \mu = 0.01)$

These results are observed in the region 0 to 0.1 only. When we apply SWCNT the wall shear stress reduces comparing with without SWCNT this indicates the thermal properties will reduce the frictional force and make the blood move freely. By controlling these parameters, blood flow can be optimized in the systems like drug delivery systems.

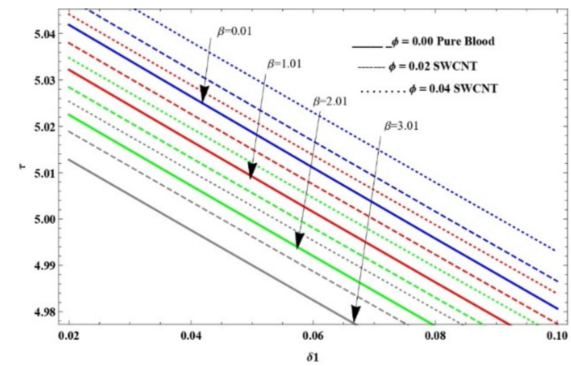

Figure 10. Effect of δ_2 on τ_h , with β varying
$(k_f = 0.2, k_s = 0.4, G_r = 0.2, \omega = 0.3, \alpha = \frac{\pi}{6}, \mu = 0.01)$

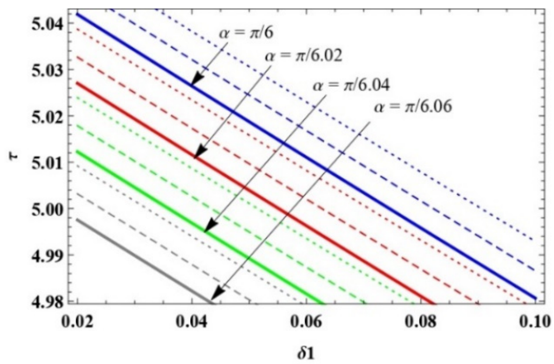

Figure 11. Effect of δ_2 on τ_h , with q varying
$(k_f = 0.2, k_s = 0.4, G_r = 0.2, \omega = 0.3,, \beta = 0.01, \mu = 0.01)$

It is also observed from the figures the wall shear stress decreases with the source and sink parameter (β), Viscosity Parameter (μ), and flux (q). Generally, the wall shear stress will damage the artery system. This analysis will help doctors, particularly in circulatory systems, and microfluidic devices, where the optimized shear stress prevents harm to the tissues and ensuring efficient fluid flow

Fig. 12-15 shows the streamlines for various values of Nanoparticle volume fraction $(\emptyset)$, Source and sink parameter (β), Grashof number $G_{r,}$ and viscosity (μ). Here we discussed 3 different cases i.e. $\emptyset = 0, 0.02, 0.04$. It is observed from Fig. 12, the velocity of blood flow increases with an increase in the concentration of Nanoparticles as compared with the case of pure blood $\emptyset = 0$ i.e. the streamlines come closer in $\emptyset = 0.02, 0.04$ than $\emptyset = 0$. The streamlines for Grashof number G_r, Source, and sink parameter (β) are shown in Fig.13-14. It is interesting to note that, as G_r, β increases the streamline comes closer i.e. the middle part of the tube becomes wider so the velocity of the blood flow increases.

Fig. 15. shows the effect of viscosity parameter (μ) on the arteries. When the viscosity (μ) goes on increasing the velocity profiles are decreasing (i.e) the flow patterns have come very near.

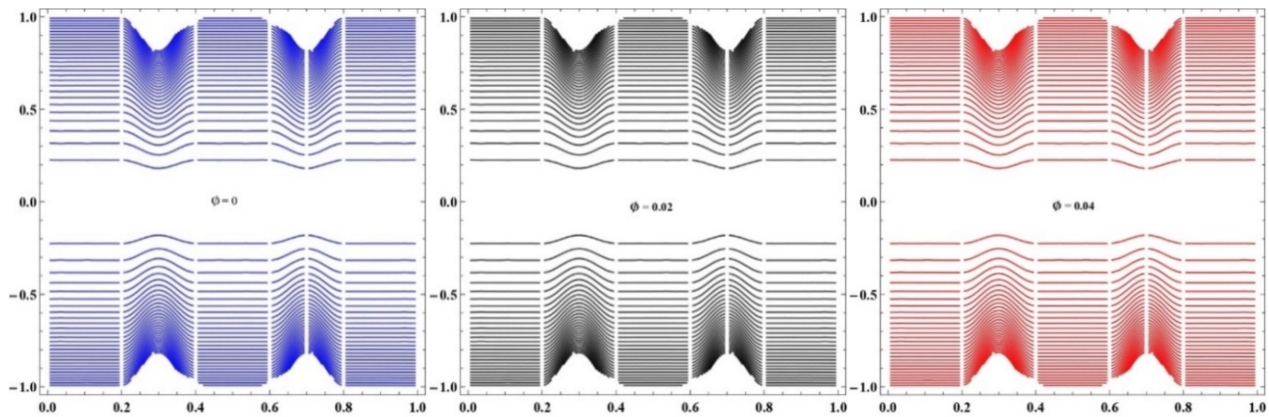

Figure 12. Streamlines for $\emptyset = 0.0, 0.02\ and\ 0.04$

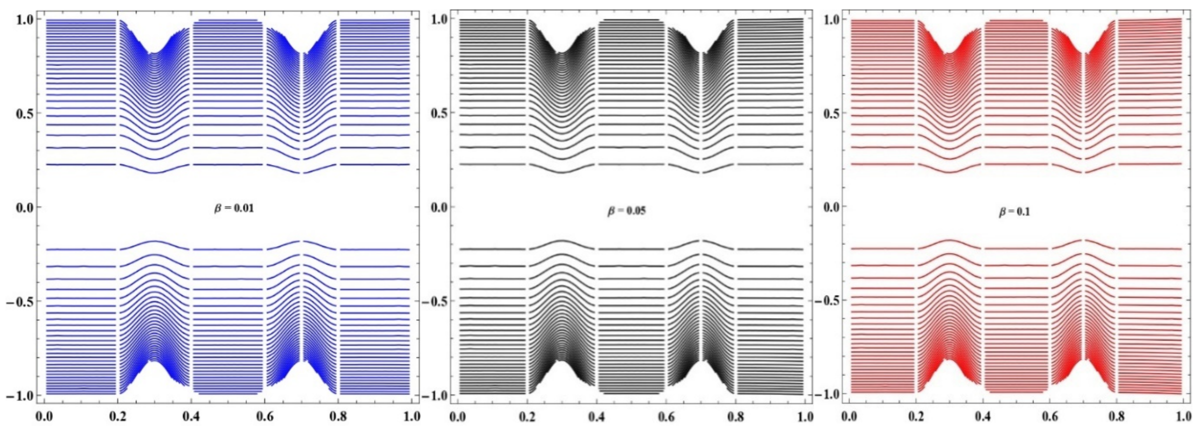

Figure 13. Streamlines for $\beta = 0.01, 0.05\ and\ 0.1$

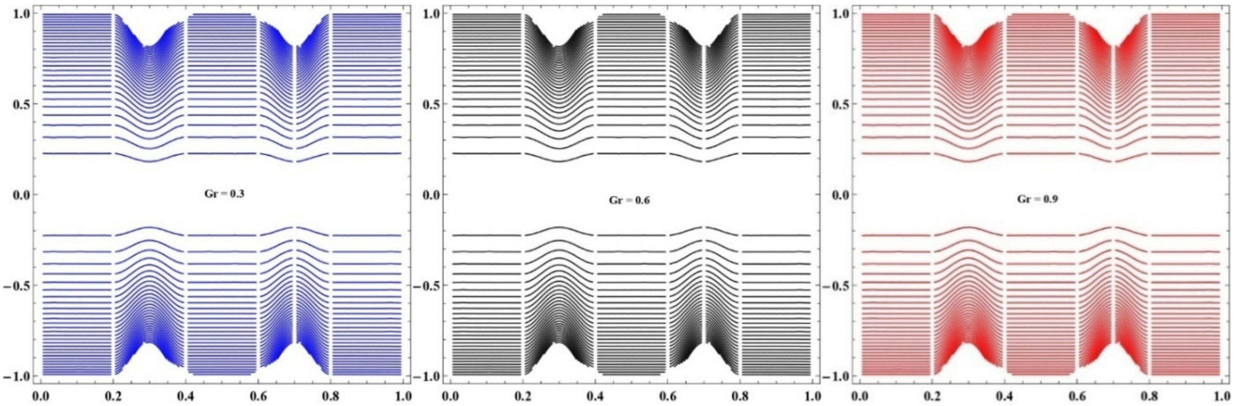

Figure 14. Streamlines for $Gr = 0.3, 0.6$ *and* 0.9

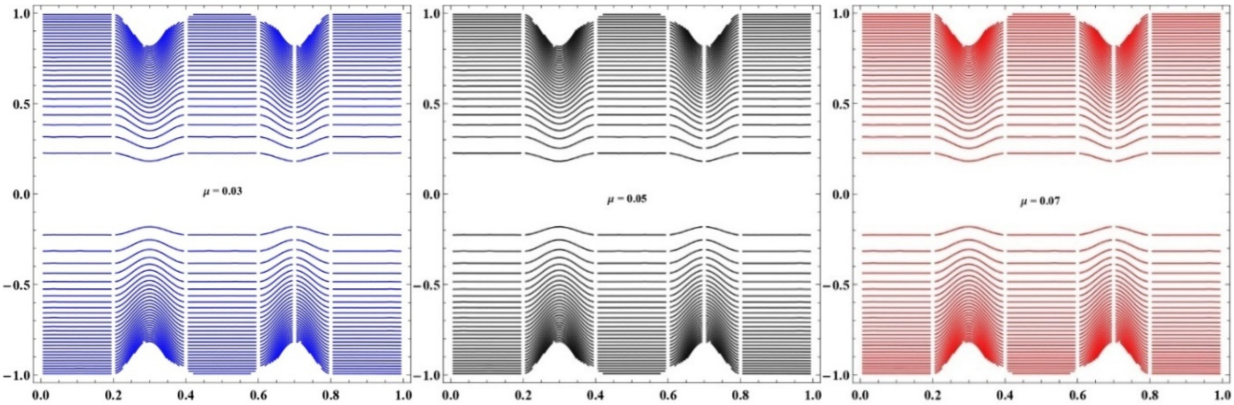

Figure 15. Streamlines for $\mu = 0.03, 0.05$ *and* 0.07

Table 1. Thermo-physical Properties of blood and SWCNT

Physical Properties	Blood	SWCNT
$C_p((J/K_g)K)$	3594	425
$\rho\ (Kg/m^3)$	1063	2600
$K(W/mK)$	0.492	6600
$\gamma X10^{-5}(1/K)$	0.18	1.5

Table 2. Variations of $\bar{\lambda}$ for δ1 & δ2

		(Resistance)$\bar{\lambda}$		
δ1	δ2	$\boldsymbol{\emptyset = 0.00}$	$\boldsymbol{\emptyset = 0.02}$	$\boldsymbol{\emptyset = 0.04}$
0.02	0.02	1.00034	1.00035	1.00037
0.04		1.00052	1.00055	1.00058
0.06		1.00072	1.00076	1.0008
0.08		1.00094	1.00099	1.00104
0.1		1.00118	1.00123	1.0013
0.02	0.04	1.00043	1.00045	1.00047
0.04		1.00061	1.00064	1.00068
0.06		1.00081	1.00085	1.0009
0.08		1.00103	1.00108	1.00114
0.1		1.00127	1.00133	1.0014
0.02	0.06	1.00052	1.00055	1.00058
0.04		1.00071	1.00074	1.00078
0.06		1.00091	1.00095	1.001
0.08		1.00112	1.00118	1.00124
0.1		1.00136	1.00143	1.0015
0.02	0.08	1.00062	1.00065	1.00069
0.04		1.0008	1.00085	1.00089
0.06		1.00101	1.00106	1.00111
0.08		1.00122	1.00128	1.00135
0.1		1.00146	1.00153	1.00161

Table 3. Variations of τ_h for $\delta 1$ & $\delta 2$

		τ		
δ1	δ2	$\emptyset = 0.00$	$\emptyset = 0.02$	$\emptyset = 0.04$
0.02	0.02	5.0419	5.04753	5.05358
0.04		5.02642	5.03212	5.03823
0.06		5.01105	5.01681	5.023
0.08		4.9958	5.00163	5.00789
0.1		4.98067	4.98658	4.99293
0.02	0.04	5.03713	5.04277	5.04883
0.04		5.02165	5.02736	5.03349
0.06		5.00628	5.01205	5.01825
0.08		4.99103	4.99687	5.00315
0.1		4.9759	4.98182	4.98818
0.02	0.06	5.03236	5.03802	5.04409
0.04		5.01689	5.0226	5.02874
0.06		5.00151	5.0073	5.01351
0.08		4.98626	4.99211	4.9984
0.1		4.97113	4.97707	4.98344
0.02	0.08	5.02759	5.03326	5.03935
0.04		5.01212	5.01785	5.024
0.06		4.99675	5.00254	5.00877
0.08		4.98149	4.98736	4.99366
0.1		4.96636	4.97231	4.9787

4. CONCLUSIONS

Effects of SWCNT characteristics on blood flow inclined artery with two symmetrical stenoses have been considered and solved analytically. This analysis will help the doctors particularly in circulatory system, microfluidic devices, where the optimized shear stress is preventing harm to the tissues and ensuring the efficient fluid flow. Based on the Mathematical analysis it is observed that:

- The resistance to the flow λ enhances with the stenoses height and with the addition of SWCNT compared to pure blood.
- The sink and source parameter of heat enhances the temperature &it gives better heat dissipation of the considered base fluid with the addition of SWCNT.
- The Wall shear stresses of arteries decrease with SWCNT stating that SWCNT base fluid give higher results compared to other base fluids.
- The various impacts of heights of multiple stenoses decrease for different Values of Nano fluid viscosity as correlate to constant Nano Fluid viscosity
- With the help of Stream Lines, blood velocity increases with an increase in the concentration of Nanoparticles as correlate with the case of pure blood ($\emptyset = \mathbf{0}$) i.e. by implementing the SWCNT we can reduce the stenosis height or decreases the resistance to the flow

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АНАЛІЗ КРОВОТОКУ ЧЕРЕЗ НАХИЛЕНУ АРТЕРІЮ, ЯКА МАЄ ЧИСЛЕННИЙ СТЕНОЗ ЗІ ЗМІННОЮ В'ЯЗКІСТЮ НАНОРІДИНИ З ВИКОРИСТАННЯМ ОДНОСТІННОЇ ВУГЛЕЦЕВОЇ НАНОТРУБКИ (SWCNT)

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Під впливом одностінної вуглецевої нанотрубки (ОСВНТ) досліджено стійкий потік потоку в'язкої рідини через похилу трубу неоднорідного поперечного перерізу з множинними стенозами. Ми лінеаризували рівняння течії та визначили вирази опору течії та напруги зсуву стінки, припускаючи легкі стенози. Дослідження показали, як параметри впливають на змінні потоку. Встановлено, що опір потоку зростає з висотою стенозу. Також цікаво відзначити, що напруга зсуву стінки зменшується зі збільшенням ступеня стенозу. Також спостерігається, що опір потоку (λ^{-}) збільшується з нахилом (α), параметром джерела та стоку (β), числом Грасгофа (Br), динамічною в'язкістю (μ) і потоком (q). Профілі швидкостей представлені у вигляді ліній струму.

Ключові слова: *стеноз; стійкість до течії; вуглецева нанотрубка; напруга зсуву стінки*