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ANISOTROPIC COSMOLOGICAL MODEL IN A MODIFIED THEORY OF GRAVITATION

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In this research, spatially homogeneous and anisotropic LRS Bianchi type-I cosmological model in $f(R,T)$ theory is discussed by choosing the specific form as $f(R,T) = R + \mu e^{-\gamma R} + \lambda T$, here R is the Ricci scalar, T is the trace of the energy-momentum tensor, μ, γ, and λ are constants. In this research the functional form consists of an exponential function which is more generalised than linear, quadratic and other polynomials. The solutions of the field equations are derived by considering the following two conditions (i) the scale factor $(a(t))$ is considered as a hybrid expansion law. By assumption of this scale factor, we can obtain the deceleration parameter as a function of the time dependent variable (ii) $\sigma \propto \theta$ (the proportionality of shear scalar with expansion scalar). For the obtained model, the physical and geometrical properties like as Hubble parameter (H), expansion scalar (θ), volume (V), pressure (p), the energy density (ρ), equation of state (ω) parameter, state-finder parameter (r, s), deceleration parameter (q), jerk parameter (j) are discussed. The graphical behavior of all the parameters of the model is examined with respect to redshift (z) by taking two different values of $\mu = -2.985, -2.902$. In the discussion of all energy conditions, it is noticed that DEC is satisfied for both the values of μ, whereas NEC is satisfied in past $(z > 0)$, present $(z = 0)$, and violated in future $(z < 0)$ for $\mu = -2.985, -2.902$. For both values of μ, the SEC is violated. The violation of SEC represents the accelerating expansion of the cosmos. The obtained results in the model match with recent observational data.

Keywords: *LRS Bianchi Type-I; $f(R,T)$ Theory; Exponential Functional Form; Perfect Fluid*

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1. INTRODUCTION

The dynamics of macro-objects such as galaxies, clusters, stars, and planets are described by gravity, the concept of gravity aids in visualizing the stretching and twisting of space-time. The formulation of gravity in curvature has modified our perspective of the cosmos, which comprises two fundamental components: dark energy and dark matter [1, 2, 3]. Dark energy (DE) and dark matter (DM) are two important components and which plays a main role in expanding universe. The rapid behaviour of the universe is thought to be due to a mysterious force that has a negative pressure called DE. Analytical research arises in this space to determine whether DE is the only candidate for the universe's acceleration or other universe resources can accelerate. Such uncertainties have created a hunt for an alternative perspective on general relativity (GR) [4].

In general, GR has forecasted the production of singularities, which is a significant issue in cosmology's big bang theory. Without a singularity, models could predict how much pressure would build up during the decelerating and accelerating phases of the cosmos expansion. The foundation of GR is extraordinary, and has long been regarded as the mathematical sound theory of gravity [5, 6]. The GR cannot adequately explain DE and DM, which leads researchers to divert their attention towards its modification. These modifications have been proposed in two different ways. Firstly, Einstein's field equations have been modified [7, 8, 9, 10]. Secondly, modified theories of gravity, scalar-tensor theories etc., came into existence to change the left-hand side part of the Einstein equation. Different modifications to the GR give different theories of gravitation. Various modified theories such as $f(R)$ gravity, $f(T)$ gravity, $f(G)$ gravity and $f(R,G)$ gravity, where R, T and G are scalar curvature, torsion scalar and Gauss-Bonnet scalar, have emerged. Among all the modified theories of gravity, the $f(R)$ theory is the first and most popular gravitational field theory derived from Einstein's Hilbert action, and $f(R)$ is a function of the Ricci scalar. The $f(R)$ theory reveals many aspects that help to explain the DE problem and also this theory has received the most incredible attention in the previous decade. Chiba et al. [11] have discussed the modified theories of gravity have been shown to be equivalent to scalar-tensor theories of gravity that are incompatible with solar system tests of general relativity, as long as the scalar field propagates over solar system scales. The unification of early-time inflation and late-time acceleration has been established using viable $f(R)$ gravity models

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[12]. Within the framework of $f(R)$ gravity, Starobinsky [13] have introduced the DE model. For instance, Buchdahl [14] has looked into modified gravity concepts. Also, the inflationary epoch and dark matter are discussed in this theory [15, 16]. Harko *et al.* [17] have suggest a current improved theory of gravity, in this theory the gravitational Lagrangian is given by an arbitrary Ricci scalar R and trace of the stress-energy tensor T. The $f(R,T)$ gravity field equations are obtained from action principle. In this research, we are considering the following functional form given by

$$f(R,T) = f_1(R) + f_2(T). \tag{1}$$

Also, we take specific choice for functional form as

$$f(R,T) = f_1(R) + f_2(T) = R + \mu\, e^{-\gamma R} + \lambda T, \tag{2}$$

where μ, γ and λ are constants.

By taking the functional form $f_1(R) = R + me^{-sR}$, Paul et al. [18] have discussed the flat FRW cosmological model in $f(R)$ theory, and also Sahoo et al. [19] have discussed the flat FRW cosmological model in the presence of $f(R,T)$ theory with the functional form $f(R,T) = R + me^{-sR} + \lambda T$. The authors have shown that this functional form exhibits the current accelerating expansion of the universe. By using $f(R,T)$ theory, Moraes and Sahoo [19] have studied traversable wormholes with the function $f(R,T) = R + \gamma e^{\chi T}$ and by using the same functional form Moraes et al. [20] have obtained FRW cosmological model in the same theory. Noureen and Zubair [21], Moraes et al. [22] and Yousaf et al. [23] have investigated various aspects of the $f(R,T)$ theory.

In today's cosmos, Bianchi space time is essential for examining and understanding the early phases of the universe's evolution. Stephani et al. [24] have a complete list of all solutions of Einstein field equations for all Bianchi type cosmological models (i.e., from I to IX) with perfect fluid. Many authors have been concentrating on this topic in recent days. Sobhanbabu et al. [25] have investigated Kantowski-Sachs interacting and non-interacting Barrow holographic dark energy (BHDE) models in the Saez-Ballester (SB) theory of gravitation. Luciano [26] has investigated SB gravity in the kantowski-sachs universe: a new reconstruction paradigm for BHDE. The authors in the references [27, 28, 29] have worked on Bianchi universe with various theories. Gudekli and Caliskan [30] and Sahoo and Siva kumar [31] have studied the perfect fluid LRS Bianchi type -I universe model in $f(R,T)$. Sahoo and Raghavender [32], Kumar and Archana [33], and Tiwari *et al.* [34] have discussed the LRS Bianchi type-I cosmological model in $f(R,T)$ theory of gravity by choosing various functional forms. Sobhanbabu et al. [35] have analyzed anisotropic BHDE models in the scalar-tensor theory of gravitation. Very recently, Gupta et al. [36] have investigated observational constraints for BHDE with Hubble and Granda-Oliveros cut-off in the modified theory of gravity. Myrzakulov et al. [37] have discussed the probing DE properties with BHDE model in the f (Q, C) gravity.

This paper organized as follows: In section 2, the formulation of field equations for $f(R,T)$ theory is introduced, the metric is given and solutions to the field equations are derived in section 3, in section 4, we have the discussed the physical and geometric properties of the model obtained, and provides the conclusion in last section.

2. METRIC AND FIELD EQUATIONS OF THE MODEL

Now, we consider the LRS Bianchi type-I homogeneous and anisotropic metric of the form

$$ds^2 = -M^2dx^2 - N^2(dy^2 + dz^2) + dt^2, \tag{3}$$

where M and N are metric potentials and functions of cosmic time t only and (x, y, z) are the co-moving coordinates.

Sharif and Zubair [38], Sahoo and Reddy [39], Tiwari et al. [40], Bishi et al. [41], Shamir and Raza [42], and Rodrigues et al. [43] are some of the authors who have worked with the above metric in various theories of gravity.

The field equations for the metric (3) with help of (2) and (1) are obtained as follows:

$$\left.\begin{aligned} \frac{2\ddot{N}}{N} + \frac{\dot{N}^2}{N^2} = \frac{(8\pi + \frac{3\lambda}{2})p}{1-\mu\gamma e^{-\gamma R}} - \frac{\lambda\,\rho}{2(1-\mu\gamma e^{-\gamma R})} \\ -\frac{(\mu e^{-\gamma R})\,(1+\gamma R)}{2(1-\mu\gamma e^{-\gamma R})} + \frac{\mu\gamma^2 e^{-\gamma R}\dot{R}}{1-\mu\gamma e^{-\gamma R}}\left(\frac{2\dot{N}}{N}\right) \\ + \frac{\mu\gamma^2 e^{-\gamma R}\ddot{R}}{1-\mu\gamma e^{-\gamma R}} - \frac{\mu\gamma^3 e^{-\gamma R}\dot{R}^2}{1-\mu\gamma e^{-\gamma R}} \end{aligned}\right\}, \tag{4}$$

$$\left.\begin{aligned} \frac{\ddot{N}}{N} + \frac{\ddot{M}}{M} + \frac{\dot{M}\dot{N}}{MN} = \frac{(8\pi + \frac{3\lambda}{2})p}{1-\mu\gamma e^{-\gamma R}} - \frac{\lambda\,\rho}{2(1-\mu\gamma e^{-\gamma R})} \\ -\frac{(\mu e^{-\gamma R})\,(1+\gamma R)}{2(1-\mu\gamma e^{-\gamma R})} + \frac{\mu\gamma^2 e^{-\gamma R}\dot{R}}{1-\mu\gamma e^{-\gamma R}}\left(\frac{\dot{M}}{M} + \frac{\dot{N}}{N}\right) \\ + \frac{\mu\gamma^2 e^{-\gamma R}\ddot{R}}{1-\mu\gamma e^{-\gamma R}} - \frac{\mu\gamma^3 e^{-\gamma R}\dot{R}^2}{1-\mu\gamma e^{-\gamma R}} \end{aligned}\right\}, \tag{5}$$

$$\left.\begin{aligned}\frac{\dot{M}\dot{N}}{MN}+\frac{\dot{N}^2}{N^2}=\frac{(\frac{\lambda}{2})p}{1-\mu\gamma e^{-\gamma R}}-\frac{(8\pi+\frac{3\lambda}{2})\rho}{1-\mu\gamma e^{-\gamma R}}\\ -\frac{(\mu e^{-\gamma R})\,(1+\gamma R)}{2(1-\mu\gamma e^{-\gamma R})}+\frac{\mu\gamma^2 e^{-\gamma R}\dot{R}}{1-\mu\gamma e^{-\gamma R}}\left(\frac{\dot{M}}{M}+\frac{2\dot{N}}{N}\right)\\ +\frac{\mu\gamma^2 e^{-\gamma R}\dot{R}}{1-\mu\gamma e^{-\gamma R}}\end{aligned}\right\}. \tag{6}$$

Here dot denotes the derivative with respect to cosmic time t.

3. SOLUTIONS OF THE FIELD EQUATIONS

The three field equations (4) to (6) contains four unknowns namely p, ρ, M, N, also these field equations are highly non-linear, so in order to find deterministic solution we need to assume two physically valid conditions,

(i) The shear scalar σ of the model is proportional to the expansion scalar θ of the model, it builds a relation between the metric potentials as given below (Thorne [44], Collins *et al.*, [45]).

$$M = N^n, \tag{7}$$

where n is an arbitrary constant. If $n = 1$, the model is isotropic model otherwise it is anisotropic.

(ii) Secondly, we take a cosmological scale factor in the form of the hybrid expansion law (Saha *et al.*, [46])

$$a(t) = e^{lt}t^m, \tag{8}$$

here $l > 0$, $m > 0$ are constants. Any other values of l and m will have new directions to explore cosmology in the context of the hybrid expansion, and this scale factor is first proposed by Akarsu *et al.* [47]. The average scale factor $a(t)$ is defined as

$$a(t) = v^{\frac{1}{3}} = (MN^2)^{\frac{1}{3}}. \tag{9}$$

From equations (7), (8) and (9) we get metric potentials as

$$M = (e^{lt}\ t^m)^{\frac{3n}{n+2}}\ \ and\ \ N = (e^{lt}\ t^m)^{\frac{3}{n+2}}. \tag{10}$$

Now the metric (3) with the help of equation (10) gives LRS Bianchi type - I homogeneous and anisotropic cosmological model as

$$ds^2 = -(e^{lt}\ t^m)^{\frac{6n}{n+2}}\ dx^2 - (e^{lt}\ t^m)^{\frac{6}{n+2}}\ (dy^2 + dz^2) + dt^2. \tag{11}$$

4. COSMOLOGICAL PARAMETERS OF THE MODEL

Equation (11) gives the LRS Bianchi type - I homogeneous and anisotropic cosmological model with a perfect fluid matter source in the $f(R,T)$ theory of gravity, along with the physical and geometrical parameters are discussed. This model is useful for discussing the early stages of the evolution of the universe.
With the help of equations (10) and (8), we get the Hubble parameter (H) of the model as

$$H = l + \frac{m}{t}.$$

The expansion scalar (θ) of the model is obtained as

$$\theta = 3\Big(l + \frac{m}{t}\Big).$$

The shear scalar (σ) of the model is given by

$$\sigma^2 = \frac{3(lt+m)^2(n-1)^2}{t^2(n+2)^2}; n \neq -2.$$

The average anisotropic parameter (A_h) for the model is given by

$$A_h = \frac{2(n-1)^2}{(n+2)^2}; n \neq -2.$$

In the present model $A_h = 0$ for $n = 1$ and $A_h \neq 0$ for $n \neq 1$ indicates isotropic and anisotropic nature of the model respectively. The spatial volume (V) of the model is obtained as

$$V = a^3 = e^{3lt}t^{3m}.$$

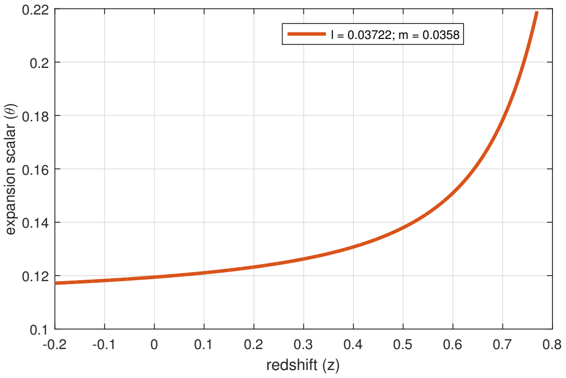

Figure 1. Plot of expansion scalar (θ) versus redshift(z).

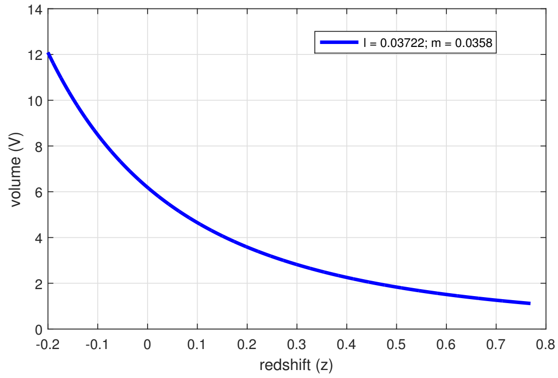

Figure 2. Plot of volume (V) versus redshift(z).

In order to study the behavior of physical parameters, we have plotted them in terms of cosmological redshift. We used the relation between the redshift and the average scale factor $a(t)$ as $1 + z = \frac{a_0}{a}$. We consider the present value of average scale factor a_0 which has been normalized to one.

The graphical behavior of expansion scalar (θ) and volume (V) corresponding to redshift (z) is shown in the Figures 1 and 2. Figure 1 shows that expansion scalar is a positive increasing function of redshift (z). Similarly, the volume is a positive increasing function of redshift (z) which is depicted in Figure 2.

Deceleration parameter (q): The deceleration parameter describes whether the current model is accelerating or not. Recent observational data reveal that the positive sign of q indicates a decelerating mode, whereas the negative sign of q indicates an accelerating phase and q shows expansion at a constant rate if $q = 0$. From equation (12) the deceleration parameter of the model is obtained as

$$q = -1 + \frac{m}{(lt + m)^2}.$$

The behavior of the deceleration parameter (q) with respect to redshift (z) is shown in Figure 3. From Figure 3, it is observed that q exhibits a transition from decelerating phase to accelerating phase at transition redshift (z_t). The transition redshift point (z_t) in the obtained model is observed at $z_t = 0.5$ for $l = 0.3722, m = 0.0358$ which match with recent observational data. The value of q in our model is observed to be negative at present, i. e., $q = -0.8814$, which results the current universe is accelerating and expanding.

Jerk parameter (j): The cosmic jerk parameter $j(t)$ is essential for describing DE and ΛCDM models. When $j = 1$, the model is referred to as a ΛCDM model. The jerk parameter of the model is given by

$$j = \frac{\ddot{a}}{aH^3} = \frac{m^3 + (3lt - 3)m^2 + (3t^2l^2 - 3lt + 2)m + t^3l^3}{(lt + m)^3}.$$

The graphical behavior of jerk parameter (j) corresponding to redshift (z) is shown in Figure 4. For all values of l, m and z, the jerk parameter is positive and reaches the ΛCDM model ($j = 1$).

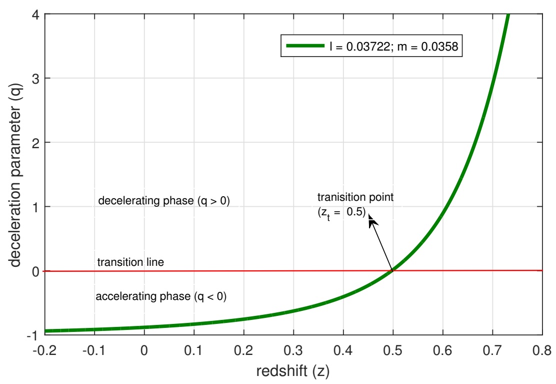

Figure 3. Plot of deceleration parameter (q) versus redshift(z).

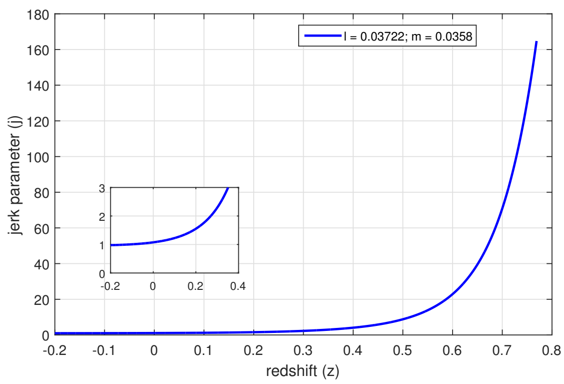

Figure 4. Plot of jerk parameter (j) versus redshift(z).

Pressure and Energy Density: In relativity, pressure, and energy density are two crucial parameters, and they play an essential role in establishing the DE and DM. Another cosmic acceleration known as inflation is predicted to have started in the cosmos before the radiation-dominated epoch in the twenty-first century. Inflation solves many fundamental problems. It gave a lot of evidences for an inflationary phase and DE. By solving the system of equations (4) - (6) with help of equation (10) for the current model pressure and energy density expressions are given as follows:

$$p = \frac{1}{4}\left(\frac{\chi + \phi - 2\psi + \xi_4 - 2\xi_5 - 2\xi_6 + \xi_7}{\xi_1 - \xi_2} + \frac{\chi + \phi + 2\psi + 4\xi_3 - 7\xi_4 - 2\xi_5 - 2\xi_6 - 3\xi_7}{\xi_1 + \xi_2}\right), \tag{12}$$

and

$$\rho = \frac{1}{4}\left(\frac{\chi + \phi - 2\psi - \xi_4 - \xi_5 + 2\xi_6 + 2\xi_7}{\xi_1 - \xi_2} - \frac{\chi + \phi + 2\psi + 4\xi_3 - 7\xi_4 - 2\xi_5 - 2\xi_6 - 3\xi_7}{\xi_1 + \xi_2}\right), \tag{13}$$

$$\chi = \frac{27m^2 + (54lt - 6n - 12)\,m + 27l^2t^2}{t^2(n+2)^2}$$

$$\phi = \frac{1}{t^2(n+2)^2}\left(n^2 + n + 1\right)9m^2 + \Big((18lt - 3)\,n^2 + (18lt - 9)\,n + 18lt - 6\Big)m$$

$$\psi = \frac{1}{(n+2)^2t^2}\Big(9(lt + m)^2(2n + 1)\Big)$$

$$\xi_1 = \frac{16\pi + 3\lambda}{2(1 - \mu\gamma e^{-\gamma\eta})}$$

$$\xi_2 = \frac{\lambda}{2(1 - \mu\gamma e^{-\gamma\eta})}$$

$$\xi_3 = \frac{\mu e^{-\gamma\eta}(1 + \gamma\eta)}{2(1 - \mu\gamma e^{-\gamma\eta})}$$

$$\xi_4 = \frac{3\mu\gamma^2 e^{-\gamma\eta}(lt + m)}{(1 - \mu\gamma e^{-\gamma\eta})(n+2)t}\left(\frac{36m}{t^3(n+2)^2}\right)\left(\left(lt + m - \frac{1}{3}\right)n^2 + \left(2lt + 2m - \frac{4}{3}\right)n + 3lt + 3m - \frac{4}{3}\right)$$

$$\xi_5 = \frac{\mu\gamma^2 e^{-\gamma\eta}}{(1 - \mu\gamma e^{-\gamma\eta})}\left(\frac{72m}{t^4(n+2)^2}\right)\left(\left(lt + m - \frac{3m}{2}\right)n^2 + 3lt + (2lt + 3m - 2)n + \frac{9m}{2} - 2\right)$$

$$\xi_6 = (-\mu\gamma^3 e^{-\gamma\eta})\left(\left(\frac{36m}{t^3(n+2)^2}\right)\left(\left(lt + m - \frac{1}{3}\right)n^2 + 3lt + \left(2lt + 2m - \frac{4}{3}\right)n + 3m - \frac{4}{3}\right)\right)^2,$$

$$\xi_7 = \frac{3n\mu\gamma^2 e^{-\gamma\eta}(lt + m)}{(1 - \mu\gamma e^{-\gamma\eta})(n+2)t}\left(\frac{36m}{t^3(n+2)^2}\right)\left(\left(lt + m - \frac{1}{3}\right)n^2 + \left(2lt + 2m - \frac{4}{3}\right)n + 3lt + 3m - \frac{4}{3}\right),$$

here

$$\eta = \frac{1}{t^2(n+2)^2}\left((n^2 + 2n + 3)18m^2 + \Big((6lt - 1)6n^2 + (3lt - 1)24n + 108lt - 24\Big)m + 18l^2t^2(n^2 + 2n + 3)\right),$$

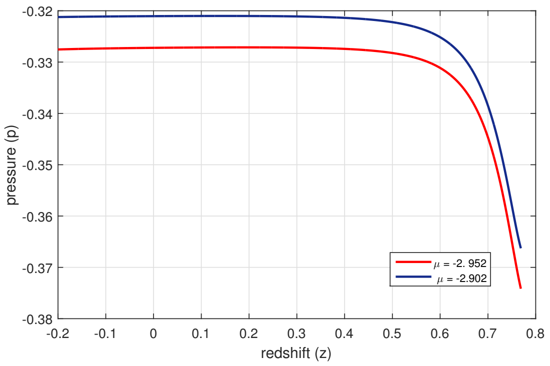

Figure 5. Plot of pressure (p) versus redshift(z) for $l = 0.03722$, $m = 0.0358$, $\gamma = 2.1535$, $\lambda = 7.5562$ and $n = 5$

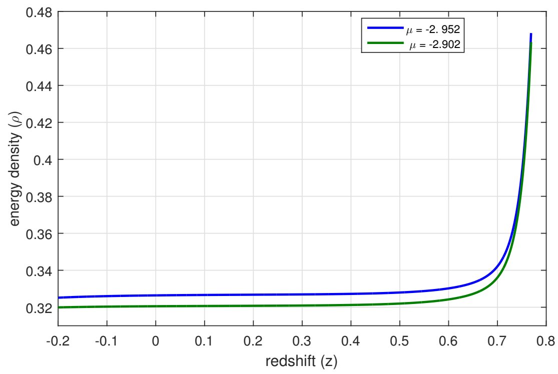

Figure 6. Plot of energy density (ρ) versus redshift(z) for $l = 0.03722$, $m = 0.0358$, $\gamma = 2.1535$, $\lambda = 7.5562$ and $n = 5$

The pressure (p) and energy density (ρ) are plotted against redshift (z) for two different values $\mu = -2.958, -2.902$, which is shown in the Figures 5 and 6. Figure 5, indicates that pressure (p) is negative, increasing function of redshift (z) for both the values of μ. Similarly, the energy density (ρ) is a positive and decreasing function of redshift (z) for both the values of $\mu = -2.985$ and $\mu = -2.902$, which is shown in Figure 6.

Diagnostic Parameters: Many astrophysicists have explored dark energy models to explain the behavior of the universe, even then some geometric parameters cannot classify DE models. As a result, a device that provides reliable information about the universe's accelerating expansion is necessary. Due to this reason, a new pair of state-finder parameters, named $\{r, s\}$ are introduced by Sahni *et al.* (2003) [48]. These parameters represent well-known DE regions of the universe as follows: $(r, s) = (1,0)$ and $(1,1)$ corresponds to the ΛCDM and CDM limits, respectively. $s < 0$ and $r > 1$ represent the Chaplygin gas, whereas $s > 0$ and $r < 1$ describe the DE regions of the phantom and quintessence eras.

For the current model, the statefinder parameters are obtained as

$$r = \frac{\dddot{a}}{aH^3} = \frac{m^3 + (3lt - 3)m^2 + (3t^2l^2 - 3lt + 2)m + t^3l^3}{(lt + m)^3},$$

$$s = \frac{r - 1}{3(q - \frac{1}{2})} = \frac{2m\left(lt + m - \frac{2}{3}\right)}{(3t^2l^2 + 6lmt + 3m^2 - 2m)(lt + m)}.$$

From Figure 7 it is observed that initially it lies in the chaplygin gas region $(r > 1,\ s < 0)$ and moves towards the ΛCDM model $(r = 1, s = 0)$, finally it approaches the quintessence region $(r < 1, s > 0)$ at late-times.

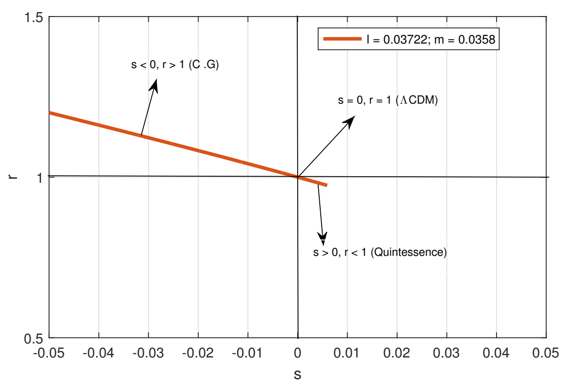

Figure 7. Plot of statefinder parameters (r, s) versus redshift(z).

Equation of State Parameter: The EoS parameter, illustrated by the ratio of pressure and energy density (i.e., $\omega = \frac{p}{\rho}$), is one of the most vital aspects in describing the dark energy phenomenon $\omega = 1$, $\omega = \frac{1}{3}$ and $\omega = 0$, for stiff fluid, dust (radiation matter-dominated) and decelerating phase respectively and also $\omega = -1$ indicates ΛCDM, $-1 < \omega < \frac{-1}{3}$ represents the quintessence region and $\omega < -1$ indicates the phantom region. The equation of state (ω) parameter is

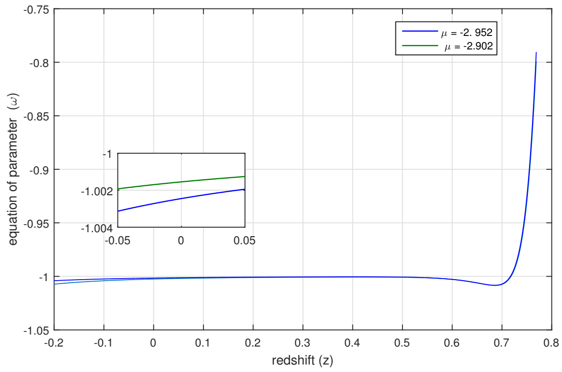

Figure 8. Plot of equation of state (ω) parameter versus redshift(z) for $l = 0.03722$, $m = 0.0358$, $\gamma = 2.1535$ and $\lambda = 7.5562$, $n = 5$.

plotted against redshift (z) and is shown in Figure 8. From Figure 8, it is observed that it exhibits quintom-like behavior, i.e., initially, it lies in the quintessence region ($\omega > -1$) and later on reaches the phantom region ($\omega < -1$) by crossing the phantom divide line ($\omega = -1$).

Energy Conditions: To understand the behavior of light-like, or spacelike, timelike curve compatibility, energy conditions are usually necessary. Null energy conditions (NEC), weak energy conditions (WEC), dominant energy conditions (DEC), and strong energy conditions (SEC) are the four types of energy conditions used in current cosmology. Energy conditions are also significant in theoretical applications, such as verifying the positive mass theorem using DEC and discovering the second law of black hole thermodynamics with NEC. Capozziello et al. [49] used the contraction of time-like and null vectors for the energy-momentum, Ricci, and Einstein tensors to explain energy conditions. Capozziello et al. [50] have explained the energy conditions in modified gravities in a very clear way.

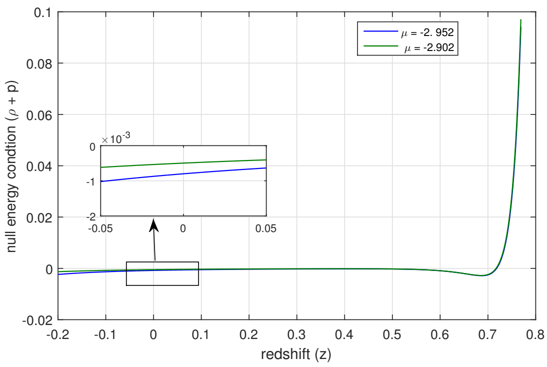

Figure 9. Plot of null energy condition ($\rho + p$) versus redshift(z) for $l = 0.03722$, $m = 0.0358$, $\gamma = 2.1535$, $\lambda = 7.5562$ and $n = 5$.

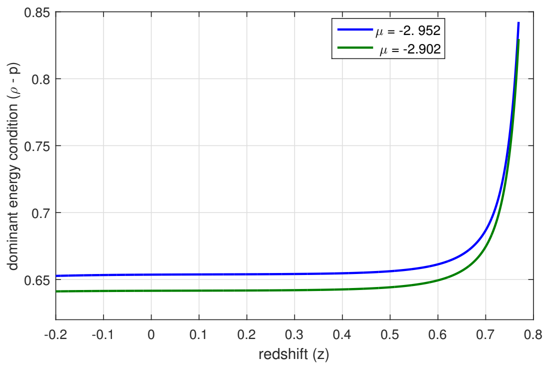

Figure 10. Plot of dominant energy condtion ($\rho - p$) versus redshift(z) for $l = 0.03722$, $m = 0.0358$, $\gamma = 2.1535$, $\lambda = 7.5562$ and $n = 5$.

Many authors have investigated these energy conditions in the context of modified gravity and discovered intriguing results (Santos et al. [51, 52]). The energy conditions can be used to impose constraints on these functions consistent with those observed in the cosmological arena. These energy conditions have been recently discussed in the $f(T)$ (Liu and Reboucas [53]) and $f(R, T)$ (Sharif and Zubair [54]) theories. The energy conditions are derived from a well-known purely geometric relationship known as the [55] Raychaudhuri equation in conjunction with the gravitational attractiveness feature. An anomaly and a singularity are crucial in understanding the universe's evolution.

The behavior of the null energy condition (NEC) corresponding to redshift(z) is shown in the Figure 9. It is observed that for both values of μ, the NEC is positive at present and past whereas it is negative in future. At present $z = 0$ the NEC is satisfied and in future NEC is violated. Violation of NEC leads to the formation of big rip. Same phenomenon we may observe in EoS parameter.

The dominant energy condition and strong energy condition are plotted with respect to redshift (z) and these are shown in the Figures 10, 11 for both the values of μ. The strong energy condition is violated throughout the universe's expansion for both the values of μ. The violation of strong energy condition leads to the accelerating expansion of the cosmos.

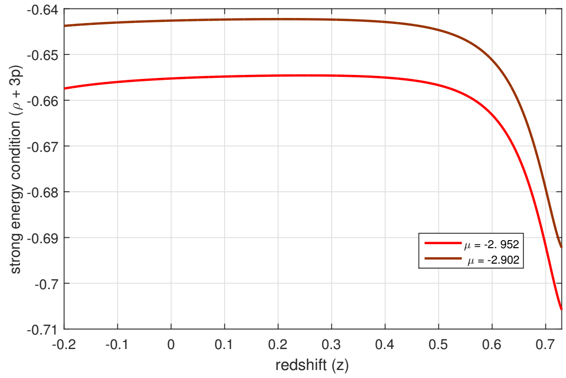

Figure 11. Plot of strong energy condition ($\rho + 3p$) versus redshift(z) for $l = 0.03722$, $m = 0.0358$, $\gamma = 2.1535$, $\lambda = 7.5562$ and $n = 5$.

5. CONCLUSIONS

In this paper, we have discussed the $f(R,T)$ gravity by considering the spatially homogeneous and anisotropic LRS Bianchi type-I cosmological model with $f(R,T) = R + \mu\, e^{-\gamma R} + \lambda\, T$ as a functional form by assuming perfect fluid as a matter source. We have obtained the solutions from the field equations by using two conditions i.e., hybrid expansion law ($a(t) = e^{lt}t^m$) and another condition is $\sigma \propto \theta$ (which leads to the relationship between the metric potentials). The graphical behavior of the geometrical parameters like Hubble parameter (H), expansion scalar (θ) and volume (V), deceleration parameter (q), state-finder parameters (r, s) and jerk parameter (j) are done with respect to redshift (z), by considering the parameters $l = 0.03722$ and $m = 0.0358$. Meanwhile, the graphical behavior of physical parameters like pressure (p), energy density (ρ), EoS parameter (ω) and energy conditions with respect to redshift (z) is done by considering two different values of $\mu = -2.958, -2.902$ and observed the following interesting results.

- The behavior of the expansion scalar (θ) with respect to redshift (z) is a positive function which represents the universe's accelerating expansion.

- From figure 2, it is observed that volume is a positive and increasing function of redshift (z), and at late-times it approaches to large value.

- The deceleration parameter (q) exhibits transition from decelerating phase ($q > 0$) to accelerating phase ($q < 0$) at $z_t = 0.5$, which match with the observational data. The theoretical value of q obtained in the present model is $q = -0.8814$ for $z = 0$ and at late-times ($z < 0$) the value of q approaches to -1 (de-Sitter model). The jerk parameter is positive throughout the evolution of the universe. Akarsu *et al.* (2014) [47] assessed the jerk parameter value as $j = 1$ for the ΛCDM model.

- Figures 5 and 6 represent the behavior of energy density ρ and pressure p against redshift (z) for different values of $\mu = -2.958, -2.902$. It is observed that ρ is positive for both the values of μ throughout the universe's evolution and approaches zero as $z \to -1$. The pressure (p) stays in the negative region for both the values of μ throughout the universe's expansion. The negative value of pressure may be responsible for cosmic acceleration.

- The graphical behavior of EoS parameter (ω) initially it is in quintessence region ($\omega > -1$) and reaches the phantom region ($\omega > -1$) by passing the phantom divide line ($\omega = -1$) for both the values of μ (i.e., it exhibits quintom like behavior).

- The behavior of the null energy condition (NEC) corresponding to redshift(z) is shown in the figure 9. From figure 9 it is observed that for both values of μ, the NEC is positive at present and past whereas it is negative in future. At present $z = 0$ the NEC is satisfied and in future NEC is violated. Violation of NEC leads to the formation of big rip and the same phenomenon we have observe in EoS parameter.

- Similarly the study of dominant energy condition ($\rho - p$) and strong energy condition ($\rho + 3p$) corresponding to redshift (z) were illustrated in the figures 10 and 11. Clearly it is observed that the dominant energy condition is satisfied for both the values of μ while the strong energy condition is violated throughout the evolution of cosmos. The violation of strong energy condition leads to the accelerating expansion of the universe. Finally in the obtained model, the study on the LRS Bianchi type -I space-time with the form $f(R,T) = R + \mu\, e^{-\gamma R} + \lambda T$ is carried out. The behavior of various cosmological parameters illustrated here represents accelerating expansion of universe. The interesting point in this article is without including any exotic fluid the present model represents accelerating and expanding.

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АНІЗОТРОПНА КОСМОЛОГІЧНА МОДЕЛЬ У МОДИФІКОВАНІЙ ТЕОРІЇ ГРАВІТАЦІЇ

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У цьому дослідженні обговорюється просторово однорідна та анізотропна космологічна модель LRS Bianchi типу I в теорії $f(R,T)$ шляхом вибору конкретної форми як $f(R,T) = R + \mu e^{-\gamma R} + \lambda T$, тут R — скаляр Річчі, T — слід тензора енергії-імпульсу, μ, γ і λ є константами. У цьому дослідженні функціональна форма складається з експоненціальної функції, яка є більш узагальненою, ніж лінійні, квадратичні та інші поліноми. Розв'язки рівнянь поля виводяться з урахуванням наступних двох умов (i) масштабний коефіцієнт $(a(t))$ розглядається як гібридний закон розширення. Припускаючи цей масштабний коефіцієнт, ми можемо отримати параметр уповільнення як функцію змінної, що залежить від часу (ii) $\sigma \propto \theta$ (пропорційність скаляра зсуву зі скаляром розширення). Для отриманої моделі обговорюються такі фізичні та геометричні властивості, як параметр Хаббла (H), скаляр розширення (θ), об'єм (V), тиск (p), густина енергії (ρ), параметр рівняння стану (ω), параметр пошуку стану (r, s), параметр уповільнення (q), параметр ривка (j). Графічну поведінку всіх параметрів моделі досліджено щодо червоного зсуву (z), взявши два різних значення $\mu = -2,985, -2,902$. Під час обговорення всіх енергетичних умов помічено, що DEC задовольняється для обох значень μ, тоді як NEC задовольняється в минулому $(z > 0)$, теперішньому $(z = 0)$ і порушується в майбутньому $(z < 0)$ для $\mu = -2,985, -2,902$. Для обох значень μ SEC порушується. Порушення SEC являє собою прискорене розширення космосу. Отримані результати в моделі збігаються з останніми даними спостережень.

Ключові слова: *LRS Бьянчі-I; $f(R,T)$ теорія; експоненціальна функціональна форма; ідеальна рідина*