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FLAT FRIEDMANN-LEMAÎTRE-ROBERTSON-WALKER COSMOLOGICAL MODEL WITH TIME-DEPENDENT COSMOLOGICAL CONSTANT IN BRANS-DICKE THEORY OF GRAVITY

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Recently, there has been much interest in investigating outstanding problems of cosmology with modified theories of gravity. The Brans-Dicke theory of gravity is one such theory developed by Brans and Dicke absorbing Mach's principle into the General Theory of Relativity. In Brans-Dicke theory, gravity couples with a time-dependent scalar field ϕ through a coupling parameter ω. This theory reduces to the General Theory of Relativity if the scalar field ϕ is constant and the coupling parameter $\omega \to \infty$. In this paper, we consider a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe with a time-dependent cosmological constant in Brans-Dicke theory of gravity. Exact solutions of the field equations are obtained by using a power law relation between the scale factor and the Brans-Dicke scalar field ϕ and by taking the Hubble parameter H to be a hyperbolic function of the cosmic time t. We study the cosmological dynamics of our model by graphically representing some important cosmological parameters such as the deceleration parameter, energy density parameter, equation of state parameter, jerk parameter, snap parameter, lerk parameter etc. The statefinder diagnostic pair of the model is also obtained and the validity of the four energy conditions, viz. the Strong energy condition (SEC), Weak energy condition (WEC), Dominant energy condition (DEC) and Null energy condition (NEC), is examined. We find that the universe corresponding to our model is expanding throughout its evolution and exhibits late time cosmic acceleration, which is in agreement with the current observational data.

Keywords: *Brans-Dicke theory; Friedmann-Lemaître-Robertson-Walker universe; Cosmological constant; Hubble parameter; Deceleration parameter*

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1. INTRODUCTION

Recent cosmological and astrophysical observations such as SNIa, CMB (Cosmic Microwave Background), LSS (Large Scale Structure), WMAP (Wilkinson Microwave Anisotropy Probe), SDSS (Sloan Digital Sky Survey) etc. [1]-[10] strongly suggest that our universe is currently undergoing a phase of accelerated expansion. Within the framework of General Relativity, an exotic component with large negative pressure, dubbed dark energy, is considered to be responsible for this expansion. There is also no dearth of candidate for dark energy, the cosmological constant Λ being the simplest and the most natural one which fits the observations well. Another possibility in explaining the observed cosmic acceleration is that at large scales the gravity model of General Relativity breaks down and an action more general than the Einstein-Hilbert action describes the gravitational field.

In recent years, a number of modified gravity theories are considered in literature, and one such theory is the Brans-Dicke theory [11] in which gravity couples with a time-dependent scalar field ϕ through a dimensionless coupling constant ω. The scalar field ϕ plays the role of the inverse of Newton's gravitational constant G, and for a constant ϕ, the Brans-Dicke theory reduces to the General Theory of Relativity where G plays the role of coupling between the gravity of space-time and matter in it. The Brans-Dicke theory has passed solar system experimental tests [12], CMB data [13] and Planck data [14]. Many cosmological models are also constructed by several authors by utilizing this theory in different contexts. Very recently, Song *et al*. [15] have studied alternative dynamics in loop quantum Brans-Dicke cosmology, Tripathy *et al*. [16] have studied a bouncing scenario in the framework of generalised Brans-Dicke theory, Sharif and Majid [17] constructed anisotropic spherical solutions from some known isotropic solutions in the background of self-interacting Brans-Dicke theory and Hatkar *et al*. [18] have explored viscous holographic dark energy in the context of Brans-Dicke theory. Yadav [19] has investigated power-law variation of the scalar field ϕ with the scale factor a of FRW universe filled with dark matter and Tsallis type holographic dark energy. Yadav *et al*. [20] have investigated a Bianchi type-I transitioning universe with hybrid scalar field, Mishra and Dua [21] have examined the dynamics of flat FLRW model of universe with time varying cosmological constant $\Lambda(t)$ and Santhi *et al*. [22] have analyzed some Bianchi type viscous holographic dark energy models in Brans-Dicke theory of gravity. Various authors have also constructed hyperbolic cosmological models in different contexts.

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Chand *et al*., [23] investigated the flat, open and closed Friedmann-Robertson-Walker model within the framework of the Brans-Dicke theory of gravity. The authors have constructed cosmological models for a hyperbolic scale factor and a hybrid scale factor. They have obatined the negative pressure throughout the cosmic evolution for a closed universe in case of the hyperbolic scale factor model whereas in case of hybrid scale factor model, pressure is negative throughout the evolution for both flat and closed universe. Also a negative Λ is obtained throughout the cosmic evolution for an open universe in the hyperbolic scale factor model, and in case of the hybrid scale factor model negative Λ is obtained in the early phases of evolution for open, flat and closed universe. Also their model initially starts from Chaplygin gas region and approaches to ΛCDM model at late times. Esmaeili [24] has constructed two cosmological models in $f(R,T)$ theory using the scale factor a in a hyperbolic form, and a power law form as a fraction of exponential function. Esmaeili and Mishra [25] have constructed a Bianchi type VI_h cosmological model in the framework of $f(R,T)$ theory, using a hyperbolic scale factor. Mishra *et al*. [26] have presented a few cosmological models in the $f(R,T)$ theory. They have also studied two cosmological models: (i) a hyperbolic scale factor model and (ii) a model with specific form of the Hubble parameter. Mishra *et al*. [27] have also presented and analyzed a Bianchi type I cosmological model in the $f(R,T)$ gravity theory with an anisotropic variable parameter in the form of hyperbolic function. Esmaeili [24], Esmaeili and Mishra [25] and Mishra *et al*. [27] have found the SEC to violate throughout the cosmic evolution.

In this work we study the cosmological dynamics of a flat FLRW universe filled with a perfect fluid in Brans-Dicke theory of gravity with a time dependent cosmological constant by considering the Hubble parameter H to be a hyperbolic function of cosmic time t. This paper is organised as follows: In section 2, we derive the Brans-Dicke field equations with time-dependent cosmological cosntant Λ corresponding to a flat Friedmann-Lemaître-Robertson-Walker metric. In section 3, we obtain cosmological solutions of Brans-Dicke field equations by assuming the Hubble parameter to be a hyperbolic function of the cosmic time t, and by using a power law relation between the Brans-Dicke scalar field ϕ and the scale factor a. In section 4, we study the physical and kinematical properties of the model by graphically representing some parameters of cosmological importance. In section 5, we study the evolution of some cosmographic parameters. In section 6, the statefinder diagnostic pair is obtained. In section 7, we examine the validity of the energy conditions. We conclude the paper in section 8 with a brief discussion.

2. METRIC AND THE FIELD EQUATIONS

We consider the Brans-Dicke action in the form

$$S = \int [(R - 2\Lambda)\phi + 16\pi\mathcal{L} + \frac{\omega}{\phi}\phi_{,i}\phi^{,i}]\sqrt{-g}d^4x \tag{1}$$

where R is the Ricci scalar, Λ is the time-dependent cosmological constant, ϕ is a scalar field, $\mathcal{L}$ is the matter-Lagrangian density, ω is the dimensionless Brans-Dicke coupling constant, g is the determinant of the metric tensor g_{ij} and coma $(,)$ represents the ordinary derivative.
Taking variations of the action (1) with respect to g^{ij} and ϕ, the Brans-Dicke field equations are obtained as

$$R_{ij} - \frac{R}{2}g_{ij} + \Lambda g_{ij} = -\frac{8\pi T_{ij}}{\phi} - \frac{\omega}{\phi^2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \frac{1}{\phi}(\phi_{,i;j} - g_{ij}\Box\phi) \tag{2}$$

and

$$\Box\phi = \frac{8\pi T + 2\Lambda\phi}{(3+2\omega)} \tag{3}$$

where R_{ij} is the Ricci tensor, T_{ij} is the energy-momentum tensor, $\Box$ is the D'Alembert operator, $T = g^{ij}T_{ij}$ is the trace of the energy-momentum tensor and semicolon $(;)$ represents the covariant derivative.
We consider the universe to be filled with a perfect fluid of density ρ and pressure p. The energy-momentum tensor T_{ij} for a perfect cosmic fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{4}$$

where u_i is the four velocity.
The line-element for a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe is given by

$$ds^2 = dt^2 - a^2(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2) \tag{5}$$

where $a = a(t)$ is the scale factor.
For the line element (5), the Brans-Dicke field equations (2) and (3) give

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} - \Lambda = -\frac{8\pi p}{\phi} \tag{6}$$

$$3\frac{\dot{a}^2}{a^2} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} - \Lambda = \frac{8\pi\rho}{\phi} \tag{7}$$

$$(3+2\omega)\left(\frac{\ddot{\phi}}{\phi}+3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}\right)-2\Lambda=\frac{8\pi(\rho-3p)}{\phi} \tag{8}$$

where $\dot{a}$ is the derivative of the scale factor a with respect to cosmic time t.

3. COSMOLOGICAL SOLUTIONS OF THE BRANS-DICKE FIELD EQUATIONS

We have three field equations with five unknowns a, ϕ, Λ, p and ρ. In order to determine an exact solution of the field equations, we need two more equations relating the unknowns.
We consider a power law relation between the scalar field ϕ and the scale factor a as

$$\phi=\phi_0 a^m \tag{9}$$

where ϕ_0 is the proportionality constant and m is an arbitrary constant.
Also, we consider the Hubble parameter H to be a hyperbolic function of cosmic time t and take

$$H(t)=\alpha\beta\coth(2\alpha t) \tag{10}$$

where $\alpha>0$, $\beta>0$ are constants. The Hubble parameter H is defined as $H=\frac{\dot{a}}{a}$. Therefore, from (10), we obtain the scale factor a as

$$a(t)=a_0\left\{\sinh(2\alpha t)\right\}^{\frac{\beta}{2}} \tag{11}$$

where a_0 is the present value of the scale factor a.
Using relations (9) and (11) in (6)-(8), we obtain

$$\Lambda(t)=2\alpha^2\beta(m\omega-3)\left\{\mathrm{cosech}(2\alpha t)\right\}^2+\alpha^2\beta^2\left(6-\frac{m^2\omega}{2}-3m\omega\right)\left\{\coth(2\alpha t)\right\}^2 \tag{12}$$

$$\begin{aligned}p(t)=&\frac{\phi_0 a_0{}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[2\alpha^2\beta(m+m\omega-1)\{\mathrm{cosech}(2\alpha t)\}^2\right]\\&+\frac{\phi_0 a_0{}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[\alpha^2\beta^2(3-2m-3m\omega-m^2-m^2\omega)\{\coth(2\alpha t)\}^2\right]\end{aligned} \tag{13}$$

$$\begin{aligned}\rho(t)=&\frac{\phi_0 a_0{}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[2\alpha^2\beta(3-m\omega)\{\mathrm{cosech}(2\alpha t)\}^2\right]\\&+\frac{\phi_0 a_0{}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[3\alpha^2\beta^2(m+m\omega-1)\{\coth(2\alpha t)\}^2\right]\end{aligned} \tag{14}$$

From (9), we obtain the expression for the scalar field ϕ as

$$\phi=\phi_0\left[a_0\{\sinh(2\alpha t)\}\right]^{\frac{m\beta}{2}} \tag{15}$$

4. PHYSICAL AND KINEMATICAL PROPERTIES OF THE MODEL

The spatial volume parameter $V(t)$ of the model is obtained as

$$V(t)=\{a(t)\}^3=a_0{}^3\{\sinh(2\alpha t)\}^{\frac{3\beta}{2}} \tag{16}$$

The deceleration parameter q , which indicates whether the cosmic expansion is uniform, decelerating or accelerating, is defined by $q(t)=-\frac{1}{aH^2}\frac{d^2a}{dt^2}=-1-\frac{\dot{H}}{H^2}$.
For our model,

$$q(t)=\frac{2}{\beta}\{\mathrm{sech}(2\alpha t)\}^2-1 \tag{17}$$

The expansion scalar $\theta(t)$, defined by $\theta=3H$, is obtained as

$$\theta(t)=3\alpha\beta\coth(2\alpha t) \tag{18}$$

Using the relations (13) and (14), we obtain the equation of state (EoS) parameter $\eta(t)=\frac{p(t)}{\rho(t)}$ as

$$\eta(t)=\frac{2\alpha^2\beta(m+m\omega-1)\{\mathrm{cosech}(2\alpha t)\}^2+\alpha^2\beta^2(3-2m-3m\omega-m^2-m^2\omega)\{\coth(2\alpha t)\}^2}{2\alpha^2\beta(3-m\omega)\{\mathrm{cosech}(2\alpha t)\}^2+3\alpha^2\beta^2(m+m\omega-1)\{\coth(2\alpha t)\}^2} \tag{19}$$

Now, in order to study the physical and kinematical properties of the constructed model, we represent the evolution of various parameters graphically by using the expressions obtained above in figure 1 to figure 10, which enables us to have a better understaning about the evolving universe corresponding to our model.

We plot the scale factor a, spatial volume V and scalar field ϕ against the cosmic time t for $\alpha = 0.107$, $\beta = 0.603$ in figures 1, 2 and 3 respectively. These figures show that the scale factor a, spatial volume V and scalar field ϕ relates to the cosmic time t with direct proportionality, thereby showing the increasing behaviour throughout the universe's evolution, hinting about the evolution of the observable universe at an accelerated rate at late times.

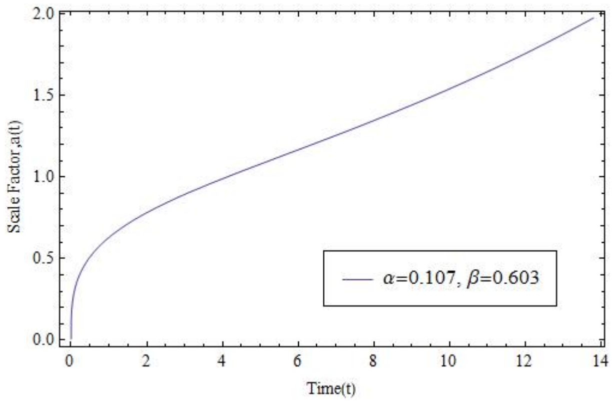

Figure 1. Evolution of the scale factor a vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$, $a_0 = 1$

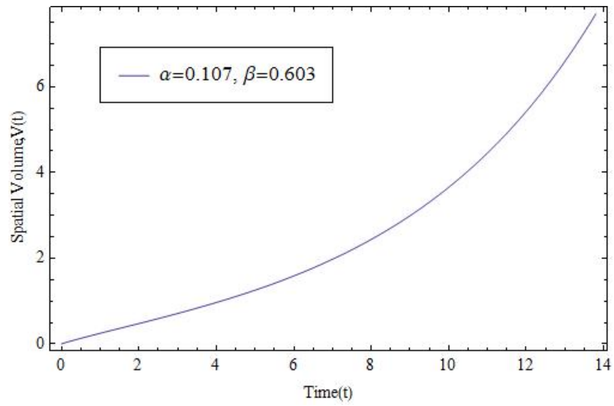

Figure 2. Evolution of the spatial volume V vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$, $a_0 = 1$

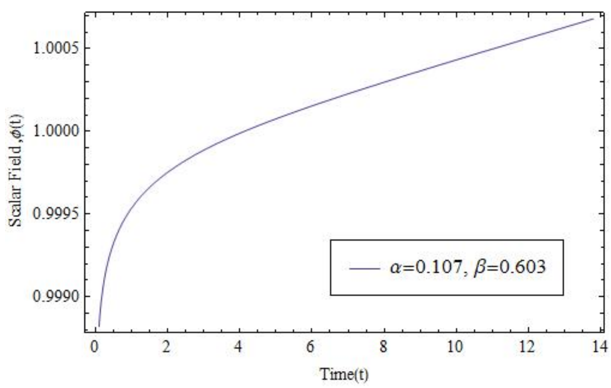

Figure 3. Evolution of the scalar field ϕ vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$, $a_0 = \phi_0 = 1$, $m = 0.001$

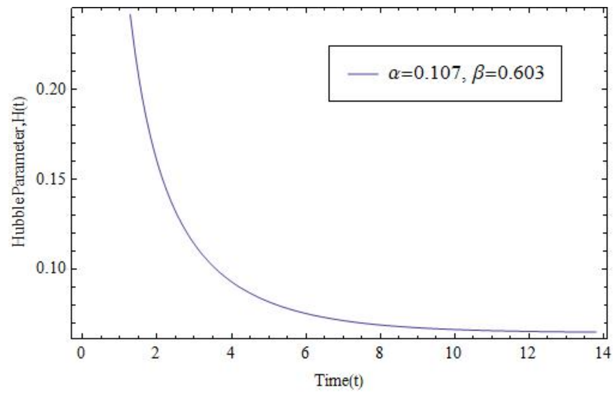

Figure 4. Evolution of the Hubble parameter H vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$

Figure 4 displays the variation of Hubble parameter H against the cosmic time t and figure 5 displays the expansion scalar plots against the cosmic time t. The Hubble parameter H and expansion scalar θ, which provides information regarding the expansion rate of the evolving universe, relates to the cosmic time t with an inverse proportionality like relation, which in turn results in the continuous decreasing nature and fading away of the Hubble parameter H and expansion scalar θ, signifying the late time phases of accelerated cosmic expansion.

Evolution of the deceleration parameter q against the cosmic time t is displayed in the figure 6. It undergoes a transition from an early phase with positive values to attain negative ones at a later phase ($t \sim 5.64$) of cosmic evolution. The transition is an evident hint of the decelerating phase of evolution of the universe turning into an accelerating one at a later phase. At late times, q tends to -1, which indicates that the evolving universe undergoes the phase of accelerated expansion at late times.

Figure 7 displays the graphical representation of the cosmological constant Λ against the cosmic time t. It is seen that, Λ is negative in the early phases of cosmic evolution, which later attains the positive values.

Figure 8 shows the variation of the pressure of the cosmic fluid with the increasing cosmic time t. A decreasing behaviour of $p(t)$ is seen transitioning from positive values at early phases to negative values at a later phase of the cosmic

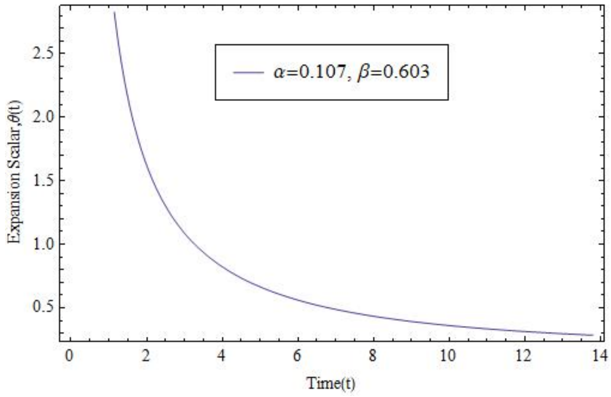

Figure 5. Evolution of the expansion scalar θ vs cosmic time t for $\alpha = 0.107, \beta = 0.603$

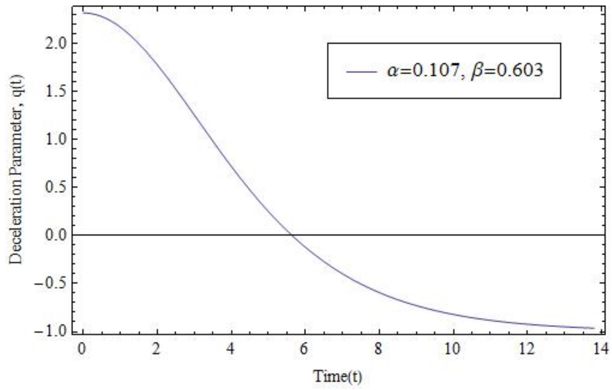

Figure 6. Evolution of the deceleration parameter q vs cosmic time t for $\alpha = 0.107, \beta = 0.603$

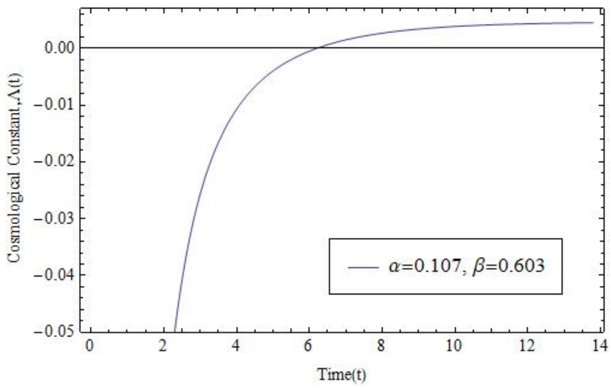

Figure 7. Evolution of the cosmological constant Λ vs cosmic time t for $\alpha = 0.107, \beta = 0.603, m = 0.001, \omega = 1628$

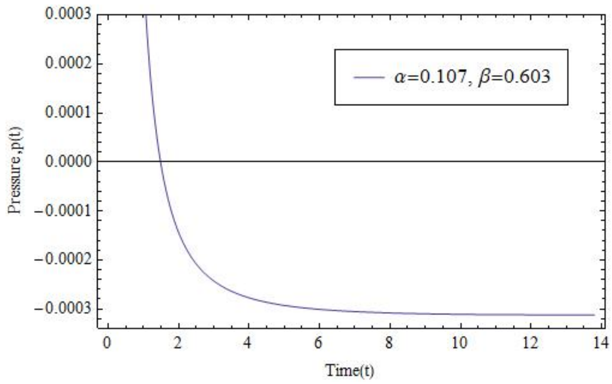

Figure 8. Evolution of the pressure p vs cosmic time t for $\alpha = 0.107, \beta = 0.603, a_0 = \phi_0 = 1, m = 0.001, \omega = 1628$

evolution. Thus the graph of $p(t)$ versus cosmic time t is indicating the early era of radiation domination transitioning to the era of dark energy domination at future phases of cosmic evolution, passing through the era of matter domination ($p = 0$ at $t \sim 1.49$). The negative pressure indicates the presence of some exotic component in the universe, dubbed dark energy, which could possibly be responsible for the late time cosmic acceleration.

In figure 9, it is seen that the energy density ρ is inversely proportional to the cosmic time t. $\rho(t)$ decreases gradually but remain positive throughout the evolution of the universe which eventually tends to zero at late times, hinting that the universe will keep expanding forever.

Figure 10 illustrates the evolution of the EoS parameter η which decreases as the universe evolves. It is positive in the early universe, signifying the radiation dominating era of the early universe, which later crosses the fixed point $\eta = 0$ at $t \sim 1.49$ signifying the matter dominating era, and attains the negative values signifying the dark energy dominating era. Upto a certain period of time, our model lies in the region of quintessence phase ($-1 < \eta < -\frac{1}{3}$), within $t \sim 5.22 - t \sim 24.55$, later $\eta(t)$ attains the value -1 at $t \sim 22.8$ till the future phases of evolution indicating the ΛCDM behaviour of the model at future phases of cosmic evolution. At the current epoch, $\eta \sim -0.96$ for $\alpha = 0.107$, $\beta = 0.603$.

5. EVOLUTION OF THE COSMOGRAPHIC PARAMETERS

The cosmographic parameters enable us in exploring the phenomena of cosmic evolution in a model independent manner. Hence we study the evolution of the cosmographic parameters viz., the jerk parameter $j(t)$, snap parameter $s(t)$ and lerk parameter $l(t)$ which are defined as [28]
$j(t) = \frac{1}{aH^3}\frac{d^3a}{dt^3}$, $s(t) = \frac{1}{aH^4}\frac{d^4a}{dt^4}$ and $l(t) = \frac{1}{aH^5}\frac{d^5a}{dt^5}$.

These parameters obtained from the Taylor series expansion of the scale factor $a(t)$ containing the third, fourth and fifth order cosmic time derivative of the scale factor $a(t)$ are dimensionless and are useful in understanding the cosmic evolution in a better way.

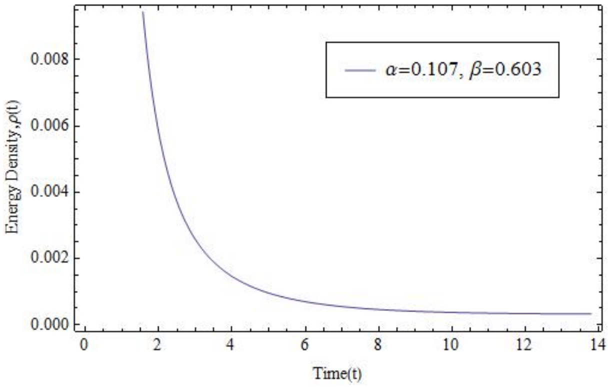

Figure 9. Evolution of the energy density ρ vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$, $a_0 = \phi_0 = 1$, $m = 0.001$, $\omega = 1628$

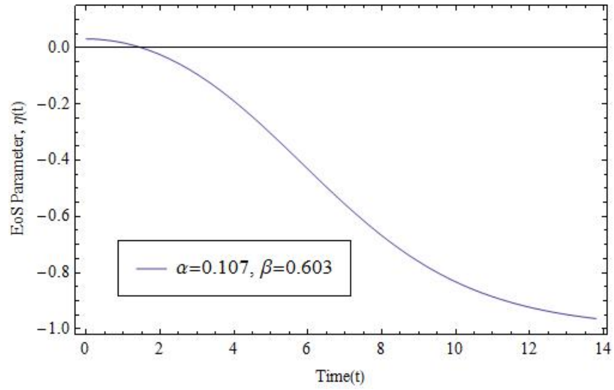

Figure 10. Evolution of the Eos parameter η vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$, $m = 0.001$, $\omega = 1628$

For our model, these parameters are obtained as

$$j(t) = \left(\frac{-6\beta + 8}{\beta^2}\right)\{\text{sech}(2\alpha t)\}^2 + 1 \tag{20}$$

$$s(t) = \left(\frac{-12\beta^2 + 44\beta - 48}{\beta^3}\right)\{\text{sech}(2\alpha t)\}^2 + \left(\frac{-12\beta + 16}{\beta^3}\right)\{\text{sech}(2\alpha t)\}^2\{\tanh(2\alpha t)\}^2 + 1 \tag{21}$$

$$l(t) = \left(\frac{-20\beta^3 + 140\beta^2 - 400\beta + 384}{\beta^4}\right)\{\text{sech}(2\alpha t)\}^2 + \left(\frac{-60\beta^2 + 240\beta - 256}{\beta^4}\right)\{\text{sech}(2\alpha t)\}^2\{\tanh(2\alpha t)\}^2 + 1 \tag{22}$$

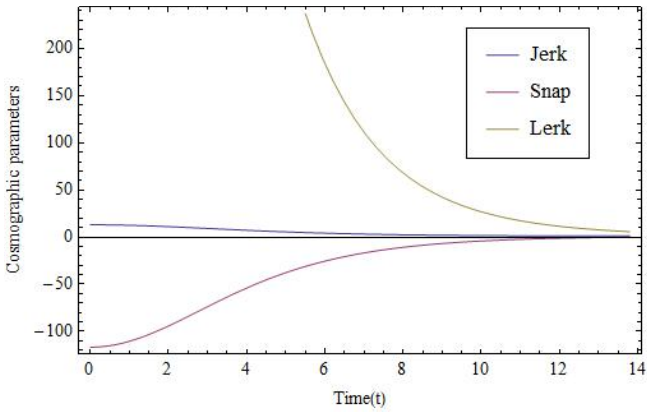

Figure 11. Evolution of the jerk (j), snap (s) and lerk (l) parameters vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$

From the graphical representation of the cosmographic parameters in figure 11, we observe that the universe corresponding to our model is expanding throughout its evolution and it exhibits the late time cosmic acceleration. The jerk, snap and lerk parameters tend to 1 at late times, agreeing with the current observational data.

6. STATEFINDER DIAGONSTIC

The dimensionless geometric pair $\{r, s\}$, known as the statefinder pair [29], are defined as $r(t) = \frac{1}{aH^3}\frac{d^3a}{dt^3}$ and $s(t) = \frac{2(1-r)}{3(1-2q)}$. These are useful in distinguishing various dark energy models including quintessence, Chaplygin gas, braneworld and other interacting models of dark energy successfully in a model independent manner. The pair helps in characterizing the dark energy properties. In particular, the ΛCDM and SCDM model behaviours of convergence or divergence can be found out with the help of the fixed position of the pair $\{r, s\}$. The fixed point $\{r = 1, s = 0\}$ resembles the ΛCDM behaviour of the model and the fixed point $\{r = 1, s = 1\}$ resembles the SCDM behaviour of the model, while the regions $\{r < 1, s > 0\}$ and $\{r > 1, s < 0\}$ respectively represents the phase of quintessence and Chaplygin gas like behaviour of the model.

As our model indicates the presence of some exotic component in the universe with negative pressure, so it will be interesting to study the behaviour of this unknown component and its resemblance with various dark energy candidates proposed in the literature.

For the present model, the pair $\{r, s\}$ is obtained in terms of cosmic time t as

$$r(t) = \left(\frac{-6\beta + 8}{\beta^2}\right)\{\text{sech}(2\alpha t)\}^2 + 1 \tag{23}$$

$$s(t) = \frac{4}{3}\frac{3\beta - 4}{3\beta^2\{\cosh(2\alpha t)\}^2 - 4\beta} \tag{24}$$

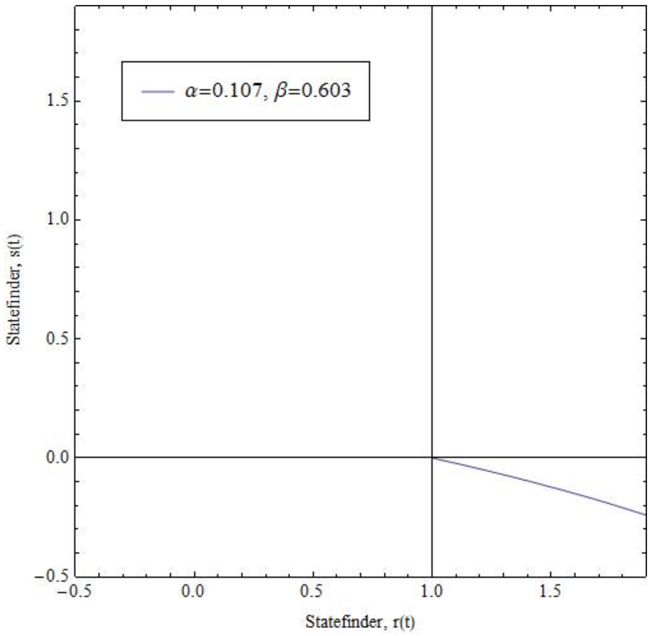

Figure 12. Plot of the statefinder pair $\{r, s\}$ vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$

Figure 12 shows the statefinder pair $\{r, s\}$ plot in the evolving universe. It shows that our model begins with Chaplygin gas behaviour $\{r > 1, s < 0\}$ and ends up with ΛCDM behaviour $\{r = 1, s = 0\}$.

7. ENERGY CONDITIONS:

The four energy conditions viz, the Strong Energy Condition (SEC), Weak Energy Condition (WEC), Dominant Energy Condition (DEC) and Null Energy Condition (NEC) are simply some constraints on some of the linear combinations of the energy density ρ and the pressure p. These four conditions are satisfied by all the normal matter in the universe, because of the postive energy density and the positive pressure of the normal matter. For that reason, violation of any of the energy conditions implies the presence of some non-normal matter in the universe [30]. The validity of SEC is the implication of decelerating universe, independent of whether the universe is open, flat, or closed. The validity of WEC is the implication of the ever positive and non-increasing nature of the energy density. The DEC provides an upper bound on the energy density and the rate of cosmic expansion. The validity of NEC is the implication of a weak upper bound on the Hubble parameter and inverse proportionality of the energy density and the size of the universe.

These energy conditions are given by:

SEC : $\rho + 3p \geq 0, \rho + p \geq 0$
WEC : $\rho + p \geq 0, \rho \geq 0$
DEC : $\rho + p \geq 0, \rho - p \geq 0, \rho \geq 0$
NEC : $\rho + p \geq 0$

For our model,

$$(\rho+3p)(t) = \frac{\phi_0 {a_0}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[2\alpha^2\beta(3m+2m\omega)\left\{\mathrm{cosech}(2\alpha t)\right\}^2\right] + \frac{\phi_0 {a_0}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[3\alpha^2\beta^2(2-m-2m\omega-m^2-m^2\omega)\left\{\coth(2\alpha t)\right\}^2\right] \tag{25}$$

$$(\rho+p)(t) = \frac{\phi_0 {a_0}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[2\alpha^2\beta(2+m)\left\{\mathrm{cosech}(2\alpha t)\right\}^2\right] + \frac{\phi_0 {a_0}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[\alpha^2\beta^2(m-m^2-m^2\omega)\left\{\coth(2\alpha t)\right\}^2\right] \tag{26}$$

$$(\rho-p)(t) = \frac{\phi_0 {a_0}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[2\alpha^2\beta(4-m-2m\omega)\left\{\mathrm{cosech}(2\alpha t)\right\}^2\right] + \frac{\phi_0 {a_0}^m}{8\pi}\left\{\sinh(2\alpha t)\right\}^{\frac{m\beta}{2}}\left[\alpha^2\beta^2(-6+5m+6m\omega+m^2+m^2\omega)\left\{\coth(2\alpha t)\right\}^2\right] \tag{27}$$

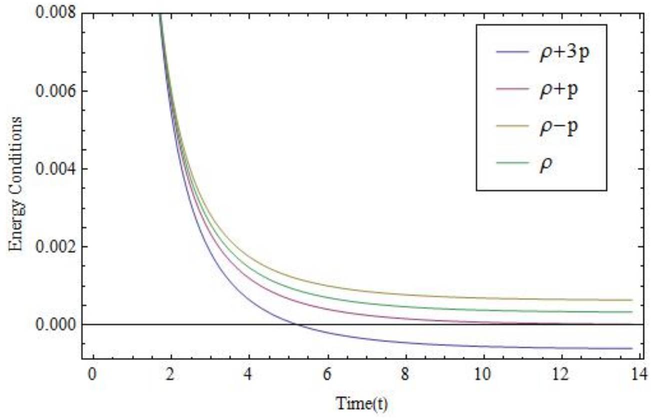

Figure 13. Evolution of the energy conditions vs cosmic time t for $\alpha = 0.107$, $\beta = 0.603$, $a_0 = \phi_0 = 1$, $m = 0.001$, $\omega = 1628$

Figure 13 illustrates the validity of the energy conditions. We observe that for $\alpha = 0.107, \beta = 0.603$, all of the four energy conditions are satisfied in the early universe. But in the long run ($t \sim 5.22$), SEC is violated. Violation of the SEC indicates the accelerating expansion of the universe.

8. CONCLUDING REMARKS

In this paper, we investigate a flat FLRW universe filled with a perfect fluid within the framework of Brans-Dicke theory of gravity with a time dependent cosmological constant. For the purpose of obtaining an exact solution of the field equations, so as to construct a cosmological model, the extra conditions taken into consideration are: (i) the scale factor a has a power law relation with the scalar field ϕ and (ii) the Hubble parameter H is a hyperbolic function of the cosmic time t. We examine the physical and kinematical properties of the constructed model by studying the evolution of some important parameters such as the scale factor a, scalar field ϕ, spatial volume parameter V, Hubble parameter H, expansion scalar θ, deceleration parameter q, cosmological constant Λ, pressure p, energy density ρ, EoS parameter η, jerk parameter j, snap parameter s, lerk parameter l, and by examining the statefinder pair $\{r, s\}$ and validity of the four energy conditions.

We observe that the universe corresponding to our model is expanding throughout its evolution and it exhibits the late time cosmic acceleration.
The Hubble parameter, the expansion scalar and the energy density decrease as the universe evolves but remain positive throughout the evolution of the universe.
The pressure is initially positive but attains negative values later in the evolving universe. The negative pressure indicates the presence of some exotic component in the universe which could be responsible for the late time cosmic acceleration and hence can be considered to be the so called dark energy.
The cosmological constant shows negative behaviour in the early universe which transits into positive one at a later phase of the cosmic evolution.
The EoS parameter attains the value -1 at late times, hinting that our model behaves like ΛCDM model in the late phases of the cosmic evolution.
Further, the jerk, snap and lerk parameters tend to 1 at late times which asserts the current observational data.
The statefinder pair $\{r, s\}$ identifies the constructed model's behaviour resembleing the ΛCDM at late times.

Violation of the SEC indicates presence of some non-normal matter in the universe which could possibly be the reason for the accelerating expansion of the universe at late times.

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ПЛОСКА КОСМОЛОГІЧНА МОДЕЛЬ ФРІДМАНА-ЛЕМЕТРА-РОБЕРТСОНА-УОКЕРА ІЗ ЗАЛЕЖНОЮ ВІД ЧАСУ КОСМОЛОГІЧНОЮ КОНСТАНТОЮ В ТЕОРІЇ ГРАВІТАЦІЇ БРАНСА-ДІККЕ

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Останнім часом виник великий інтерес до дослідження видатних проблем космології за допомогою модифікованих теорій гравітації. Теорія гравітації Бранса-Дікке є однією з таких теорій, розроблених Брансом і Дікке, які ввібрали принцип Маха в Загальну теорію відносності. У теорії Бранса-Дікке гравітація пов'язана з залежним від часу скалярним полем ϕ через параметр зв'язку ω. Ця теорія зводиться до загальної теорії відносності, якщо скалярне поле ϕ є постійним, а параметр зв'язку $\omega \rightarrow \infty$. У цій статті ми розглядаємо плоский Всесвіт Фрідмана-Леметра-Робертсона-Уокера (FLRW) із залежною від часу космологічною константою в теорії гравітації Бранса-Дікке. Точні розв'язки рівнянь поля отримані за допомогою степеневого співвідношення між масштабним фактором і скалярним полем Бранса-Дікке ϕ і за допомогою параметра Хаббла H як гіперболічної функції космічного часу t . Ми вивчаємо космологічну динаміку нашої моделі шляхом графічного представлення деяких важливих космологічних параметрів, таких як параметр уповільнення, параметр щільності енергії, параметр рівняння стану, параметр ривка, параметр миттєвого примикання, параметр lerk тощо. Також отримано діагностичну пару моделі вимірювача стану і справедливість чотирьох енергетичних умов, а саме. Досліджується сильний енергетичний стан (SEC), слабкий енергетичний стан (WEC), домінуючий енергетичний стан (DEC) і нульовий енергетичний стан (NEC). Ми виявили, що Всесвіт, який відповідає нашій моделі, розширюється протягом своєї еволюції та демонструє пізнє космічне прискорення, що узгоджується з поточними даними спостережень.

Ключові слова: *теорія Бренса-Дікке; Всесвіт Фрідмана-Леметра-Робертсона-Уокера; космологічна стала; параметр Хаббла; параметр уповільнення*