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A NEW FOURTH-ORDER COMPRESSION-DEPENDENT EQUATION OF STATE

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This study introduces a new first- to fourth-order exponential equation of state (EOS) to enhance accuracy across varying compression levels. The proposed exponential EOS is compared to the widely used fourth-order Birch-Murnaghan EOS, and it not only matches but surpasses precision, especially at high compression. This comparison serves as a clear benchmark for the readers to understand the superiority of the new model. The findings of this study are crucial, as they reveal that the fourth-order exponential EOS provides an unmatched accuracy level at higher compression, notably for materials like HCP-iron and sodium halides. The Birch-Murnaghan EOS, though effective at low compression, deviates from experimental values at higher levels. Additionally, the study examines the Shanker EOS, in which M. Kumar et al. [Physica B: Condensed Matter, 239(3-4), 337-344 (1997)] suggest the requirement to improve at high compression and improve by fitting parameters that vary from material to material. This limitation is removed by developing the fourth-order exponential EOS, which is more versatile, offering reliable results across both low- and high-compression scenarios in high-pressure physics.
Keywords: *Equation of state; Compression; Fourth-order exponential equation of state; Birch-Murnaghan fourth-order equation of state*
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1. Introduction

Equations of state (EOS) play a crucial role in understanding how materials respond to changes in pressure and volume, particularly within materials science, condensed matter physics, geophysics, and planetary science. Our study aims to develop and compare different EOS models to better understand their effectiveness in high-pressure scenarios. These relationships are essential for delving into high-pressure physics, technological materials, and planetary interiors, as they allow scientists to predict and model how materials might behave under extreme conditions, such as compression, expansion, or phase changes [1-3].

Traditionally, the Birch-Murnaghan equation of state (BM-EOS) has been the go-to formula for characterizing the compressional behavior of solids. However, as experimental techniques have advanced and allowed for a broader range of pressure exploration, it has become evident that traditional EOS models like the BM-EOS have significant limitations. Calculated values deviate from experimental results at high pressures, particularly under exceptionally high compression [4-6].

A novel approach has been proposed to address the limitations of traditional EOS models: the fourth-order exponential equation of state. This innovative formulation, incorporating higher-order adjustments and an exponential element, aims to significantly enhance the accuracy and applicability of EOS under extreme conditions. It offers a more precise and versatile depiction of the pressure-volume relationship, marking a significant advancement in the field and providing researchers with a powerful tool for their studies [7-10].

Our upcoming study aims to develop exponential equations of state (EOS) from first to fourth order and assess their efficacy in calculating pressure under high compression. Our primary goal is to compare the effectiveness of the novel fourth-order exponential EOS with the widely used Birch-Murnaghan EOS and available experimental data. We seek to identify their respective advantages and limitations by comprehensively analyzing each model's mathematical framework, accuracy, parameter sensitivity, and applicability in high-pressure scenarios [11-12].

Our research is poised to have a significant impact, with potential applications in fields like shockwave physics and planetary science, as well as studying materials under ultrahigh pressure. By delving into the theoretical foundations of the fourth-order exponential EOS and the Birch-Murnaghan EOS, we aim to evaluate their performance across various materials and pressure ranges. Our findings could significantly enhance the precision of high-pressure material modelling and open up new avenues for exploration, thereby underlining the importance of our research in these fields [13-15]. This potential impact should make you feel the significance of our study in these fields.

Furthermore, we will explore the work of Kumar et al. [16], who proposed modifying the Shanker EOS [17] by introducing fitting parameters, as well as the efforts of Srivastava-Pandey [20], who increased accuracy by considering third-order compression and the anharmonic effect of solids. However, limitations in accuracy were identified at high compression. In our study, we have derived first-to-fourth-order exponential equations of state, observing an improvement in accuracy by increasing the order of compression in the equation of state. For the validity of the work, we have taken the sample ε-Fe, NaF, NaCl, NaBr, and NaI and tested up to the pressure range of 3595 kbar.

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2. THEORY AND METHODOLOGY

The general equation by which we can derive the equation of state by the Gruneisen model is expressed as [17]:

$$P\left(\frac{V}{V_0}\right)^{4/3} = -\frac{B_0}{f_0 V_0}\int_{V_0}^{V} f\ dV\ ,$$

$$P = -\left(\frac{V}{V_0}\right)^{-4/3} \frac{B_0}{f_0 V_0}\int_{V_0}^{V} f\ dV\ . \quad (1)$$

The equation (1) is the fundamental equation for deriving an EOS [17]. This formulation applies to all EOS for various types of solids and different potential functions. Shanker et al. [17] demonstrated that equation (1) produces the Born-Mie EOS [18] and the Brenan-Stacey EOS [19].
Let a potential function be given by

$$f = \frac{V_0}{V}\exp\alpha\left(1-\frac{V}{V_0}\right).$$

Let $y = 1-\frac{V}{V_0}$ then

$$f = (1-y)^{-1}\exp(\alpha y)\ ,\ f = (1+y+y^2+y^3+.....................)\exp(\alpha y)\ . \quad (2)$$

At $V = V_0$, $f = f_0 = 1$ and $P = 0$, Then equation (1) becomes:

$$P = B_0\left(\frac{V}{V_0}\right)^{-4/3}\int_0^y (1+y+y^2+y^3+...................)e^{\alpha y}dy\ . \quad (3)$$

2.1. First-order exponential equation of state:

For first order equation of state equation (3) can be written as:

$$P = B_0\left(\frac{V}{V_0}\right)^{-4/3}\int_0^y (1+y)e^{\alpha y}dy\ . \quad (4)$$

On the integration of equation (4), we get the expression for pressure as a function of compression:

$$P = B_0\left(\frac{V}{V_0}\right)^{-4/3}\left[\frac{(\alpha y+\alpha-1)e^{\alpha y}-(\alpha-1)}{\alpha^2}\right]. \quad (5)$$

Equation (5) represents first-order which is newly derived.

At $V = V_0$ then $P = 0$. Hence, equation (4) satisfies the condition of the equation of state and is valid according to the Stacey criterion [19].

The bulk modulus of the solid is given by:

$$B_T = -V\frac{dP}{dV} \quad (6)$$

Using equations (5) and (6), we can find bulk modulus corresponding to equation (5) as:

$$B_T = B_0\left(\frac{V}{V_0}\right)^{-1/3}(1+y)\exp(\alpha y)+\frac{4}{3}P\ . \quad (7)$$

The first-order pressure derivative of bulk modulus $\left(\frac{dB_T}{dP}\right)$ Corresponding to equation (6) is given by:

$$B_T^{'} = \left(1-\frac{4P}{3B_T}\right)\left(\frac{1}{3}+\frac{V}{V_0}\left\{\alpha+\frac{1}{(1+y)}\right\}\right)+\frac{4}{3} \quad (8)$$

The value of α is determined to form B_0' , the zero-pressure value of $\frac{dB_T}{dP}$ at $V = V_0$. Using equation (8), we get:

$$\alpha = \frac{3B_0^{'} - 8}{3}. \tag{9}$$

2.2. Second-order exponential equation of state

For second order equation of state equation (3) can be written as:

$$P = B_0 \left(\frac{V}{V_0}\right)^{-4/3} \int_0^y (1 + y + y^2) e^{\alpha y} dy. \tag{10}$$

On integration equation (10), give Shanker equation of state [17]:

$$P = B_0 \left(\frac{V}{V_0}\right)^{-4/3} \left[\frac{\left\{\alpha^2 y^2 + \left(\alpha^2 - 2\alpha\right) y + \alpha^2 - \alpha + 2\right\} e^{\alpha y} - (\alpha^2 - \alpha + 2)}{\alpha^3}\right]. \tag{11}$$

Where $\alpha = \frac{3B_0^{'} - 8}{3}$. At $V = V_0$ then $P = 0$. Hence, equation (11) satisfies the condition of the equation of state and is valid according to the Stacey criterion. Equation (11) represents a second-order exponential equation of state (Shanker EOS) [17].

2.3. Third-order exponential equation of state

For third order equation of state equation (3) can be written as:

$$P = B_0 \left(\frac{V}{V_0}\right)^{-4/3} \int_0^y (1 + y + y^2 + y^3) e^{\alpha y} dy. \tag{12}$$

On integration equation (12), we can find Srivastava-Pandey EOS as [20]:

$$P = \frac{B_0}{\alpha^4} \left(\frac{V}{V_0}\right)^{-4/3} \left[\left\{\alpha^3 (1 + y + y^2 + y^3) + \alpha^2 (-3y^2 - 2y - 1) + \alpha(6y + 2) - 6\right\} e^{\alpha y} - (\alpha^3 - \alpha^2 + 2\alpha - 6)\right]. \tag{13}$$

Where $\alpha = \frac{3B_0^{'} - 8}{3}$. At $V = V_0$ then $P = 0$. Hence, equation (13) satisfies the condition of the equation of state and is valid according to the Stacey criterion. Equation (13) represents a third-order exponential equation of state (Srivastava-Pandey EOS) [20].

2.4. Fourth-order exponential equation of state

Due to complex integration, Shanker and Srivastava refused to solve the fourth-order equation of state, but due to including fourth-order compression, we include translators and vibrational, rotational and anharmonicity properties of solids; therefore, its accuracy increases than others. Now, further expand the equation (3) up to the fourth order to develop a new fourth-order equation of state. Thus, equation (3) can be written as:

$$P = B_0 \left(\frac{V}{V_0}\right)^{-4/3} \int_0^y (1 + y + y^2 + y^3 + y^4) e^{\alpha y} dy. \tag{14}$$

On integration equation (10), we can find EOS as follows:

$$P = \left[\frac{B_0}{\alpha^5} \left(\frac{V}{V_0}\right)^{-4/3}\right] \times \left[\left\{\begin{matrix} \alpha^4 y^4 + (\alpha^4 - 4\alpha^3) y^3 + (\alpha^4 - 3\alpha^3 + 12\alpha^2) y^2 + \\ (\alpha^4 - 2\alpha^3 + 6\alpha^2 - 24\alpha) y + \\ (\alpha^4 - \alpha^3 + 2\alpha^2 - 6\alpha + 24) \end{matrix}\right\} e^{\alpha y} - (\alpha^4 - \alpha^3 + 2\alpha^2 - 6\alpha + 24)\right]. \tag{15}$$

Where $\alpha = \frac{3B_0^{'} - 8}{3}$. At $V = V_0$ then $P = 0$. Hence, equation (15) satisfies the condition of the equation of state and is valid according to the Stacey criterion. Equation (15) represents the fourth-order exponential equation of state.

2.5. Birch-Murnaghan fourth order equation of state

The fourth-order Birch-Murnaghan equation of state extends the original model to include higher-order pressure-volume terms, improving accuracy for describing material behaviour under extreme compression. The fourth order Birch-Murnaghan EOS can be expressed as [21]:

$$P=\frac{3}{2}B_0\left[x^{-7/3}-x^{-5/3}\right]\left[\begin{array}{l}1+\frac{3}{4}\left(B_0^{'}-4\right)\left(x^{-2/3}-1\right)+\\ \frac{1}{24}\left\{9B_0^{'2}-63B_0^{'}-9B_0\left(\frac{1}{9B_0}\left(9B_0^{'2}-63B_0^{'}+143\right)\right)(x^{-2/3}-1)^2\right\}\end{array}\right]. \quad (16)$$

Where $x=\frac{V}{V_0}$.

Equations (5), (11), (13), (15), and (16) calculate the pressure of solids at different compressions.

In a past study, obtaining a fourth-order equation was described as impossible because it does not follow the general condition of the equation of state, and researchers derive the third-order compression-dependent equation. This statement was wrong. In this study, a fourth-order compression-dependent equation is derived, following the condition of the equation of state. By including fourth-order compression, the accuracy of the equation of state increases [22].

Table 1. Input parameters used in the present work.

Solids	K_0 (kbar)	K_0'
ε-Fe	1750 [23]	5.3 [23]
NaF	465 [24]	5.28 [24]
NaCl	240 [24]	5.39 [24]
NaBr	199 [24]	5.46 [24]
NaI	151[24]	5.59 [24]

Table 2. Calculated values of pressure P(kbar) for (A) first-order exponential EOS, (B) second-order exponential EOS (Shanker EOS), (C) third-order exponential EOS (Srivastava-Pandey EOS), (D) fourth-order exponential EOS (present study) (E) fourth order Birch-Murnaghan EOS (B-M EOS) and with experimental data for ε-Fe.

V/V_0	1st order EOS	2nd order EOS	3rd order EOS	4th order EOS	4th Order B-M EOS	Experimental [25]
1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.877	322.4	324.1	324.2	324.3	325.4	299.6
0.862	381.0	383.4	383.7	383.7	385.4	355.6
0.847	445.0	448.6	449.0	449.0	451.5	417.5
0.832	515.2	520.1	520.7	520.8	524.4	486.2
0.817	592.0	598.7	599.7	599.8	604.9	562.2
0.802	676.0	685.1	686.4	686.7	693.7	646.4
0.788	761.7	773.4	775.3	775.6	785.0	739.9
0.773	861.9	877.1	879.7	880.2	892.9	843.8
0.758	971.7	991.1	994.7	995.5	1012.3	959.2
0.743	1092.0	1116.6	1121.5	1122.5	1144.6	1087.6
0.728	1223.8	1254.7	1261.3	1262.7	1291.5	1230.5
0.713	1368.3	1406.9	1415.5	1417.5	1454.8	1390.0
0.698	1526.9	1574.5	1585.7	1590.5	1636.4	1568.0
0.684	1688.7	1746.5	1760.7	1792.4	1824.6	1767.1
0.669	1878.6	1949.1	1967.3	1998.2	2048.8	1989.9
0.654	2087.2	2172.8	2195.9	2229.5	2299.4	2239.8
0.639	2316.6	2419.9	2449.2	2496.8	2580.1	2520.6
0.624	2568.9	2693.3	2729.9	2798.3	2894.9	2836.5
0.609	2846.8	2995.8	3041.5	3168.3	3248.8	3192.6
0.594	3153.0	3330.9	3387.7	3496.8	3647.5	3595.0

Table 3. Calculated values of pressure P(kbar) for (A) first-order exponential EOS, (B) second-order exponential EOS (Shanker EOS), (C) third-order exponential EOS (Srivastava-Pandey EOS), (D) fourth-order exponential EOS (present study) (E) fourth order Birch-Murnaghan EOS (B-M EOS) and with experimental data for NaF.

V/V_0	1st order EOS	2nd order EOS	3rd order EOS	4th order EOS	4th Order B-M EOS	Experimental [26,27]
1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.980	9.7	9.8	9.8	9.8	9.8	10.0
0.962	19.8	19.8	19.8	19.8	19.8	20.0
0.946	29.7	29.7	29.7	29.7	29.8	30.0
0.932	39.4	39.5	39.5	39.5	39.5	40.0
0.868	94.7	95.3	95.3	95.3	95.7	94.0
0.832	136.6	137.9	138.1	138.1	139.1	140.0
0.804	176.1	178.5	178.8	178.9	180.7	180.0
0.782	212.2	215.7	216.2	216.3	219.2	210.0
0.778	219.3	223.0	223.6	223.8	226.8	224.0

Table 4. Calculated values of pressure P(kbar) for (A) first-order exponential EOS, (B) second-order exponential EOS (Shanker EOS), (C) third-order exponential EOS (Srivastava-Pandey EOS), (D) fourth- order exponential EOS (present study) (E) fourth order Birch-Murnaghan EOS (B-M EOS) and with experimental data for NaCl.

V/V_0	1st order EOS	2nd order EOS	3rd order EOS	4th order EOS	4th Order B-M EOS	Experimental [26,27]
1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.963	10.1	10.1	10.1	10.1	10.1	10.0
0.933	20.2	20.2	20.2	20.2	20.2	20.0
0.907	30.5	30.6	30.6	30.6	30.6	30.0
0.883	41.5	41.7	41.8	41.8	41.9	40.0
0.760	132.8	135.4	135.9	136.0	138.1	135.0
0.702	206.6	212.9	214.4	214.8	220.6	200.0
0.697	214.3	221.1	222.7	223.1	229.5	220.0
0.675	251.3	260.4	262.7	263.3	272.5	250.0
0.658	283.5	294.9	298.0	298.9	310.9	290.0

Table 5. Calculated values of pressure P(kbar) for (A) first-order exponential EOS, (B) second-order exponential EOS (Shanker EOS), (C) third-order exponential EOS (Srivastava-Pandey EOS), (D) fourth- order exponential EOS (present study) (E) fourth order Birch-Murnaghan EOS (B-M EOS) and with experimental data for NaBr.

V/V_0	1st order EOS	2nd order EOS	3rd order EOS	4th order EOS	4th Order B-M EOS	Experimental [25,26]
1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.956	10.1	10.1	10.1	10.1	10.1	10.0
0.921	20.6	20.6	20.6	20.6	20.6	20.0
0.891	31.1	31.3	31.3	31.3	31.4	30.0
0.866	41.8	42.1	42.1	42.1	42.3	40.0
0.746	124.2	127.0	127.5	127.6	129.7	130.0
0.725	146.0	149.8	150.6	150.8	153.9	160.0
0.663	230.1	239.2	241.6	242.2	251.1	240.0
0.633	284.5	297.8	301.6	302.8	317.0	290.0
0.616	320.4	336.7	341.7	343.2	361.7	340.0

Table 6. Calculated values of pressure P(kbar) for (A) first-order exponential EOS, (B) second-order exponential EOS (Shanker EOS), (C) third-order exponential EOS (Srivastava-Pandey EOS), (D) fourth-order exponential EOS (present study) (E) fourth order Birch-Murnaghan EOS (B-M EOS) and with experimental data for NaI.

V/V_0	1st order EOS	2nd order EOS	3rd order EOS	4th order EOS	4th Order B-M EOS	Experimental [26,27]
1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.942	10.6	10.6	10.6	10.6	10.6	10.0
0.899	21.5	21.6	21.6	21.6	21.6	20.0
0.865	32.4	32.6	32.7	32.7	32.8	30.0
0.836	43.7	44.1	44.2	44.2	44.4	40.0
0.694	142.9	147.6	148.7	149.0	152.6	150.0
0.648	199.7	208.4	210.7	211.4	219.2	210.0
0.641	210.0	219.4	222.1	222.9	231.5	230.0
0.609	263.3	277.4	281.7	283.1	297.6	280.0
0.599	282.5	298.4	303.4	305.1	322.0	310.0

3. RESULT AND DISCUSSION

In the past, researchers have noted that the accuracy of the equation of state improves with an increase in the order of compression. Birch-Murnaghan's fourth-order equation of state was a significant development in this regard. However, our study proposes a novel approach—a first—to the fourth-order exponential equation of state. This new equation promises to provide more precise insight into the results as we compare it with Birch-Murnaghan's equation and the available experimental values [24-26].

Our research is built on a foundation of thorough calculations. We have utilised equations (5), (11), (13), (15), and (16) to calculate the pressure at different compressions, which are listed in Tables 2-6, along with the associated experimental values and references. The input values employed in the calculation can also be found in Table 1, with corresponding references. To enhance clarity, we have plotted a graph showing the relationship between pressure at different compressions, as illustrated in Fig 1-5.

The HCP-iron is known for its exceptional strength and high binding energy per nucleon, indicated by its remarkably high bulk modulus at zero pressure. Regarding low compression, various equations of state (EOS) and the Birch-Murnaghan fourth-order equation tend to produce results that exceed experimental values. However, at higher compression, the fourth-order exponential EOS yields results that closely match experimental values, while the Birch-Murnaghan equation produces results that surpass experimental data. Essentially, the accuracy of the fourth-order exponential equation of state is improved at high compression, while the accuracy of the Birch-Murnaghan equation decreases. At low compression, all EOS produce similar results.

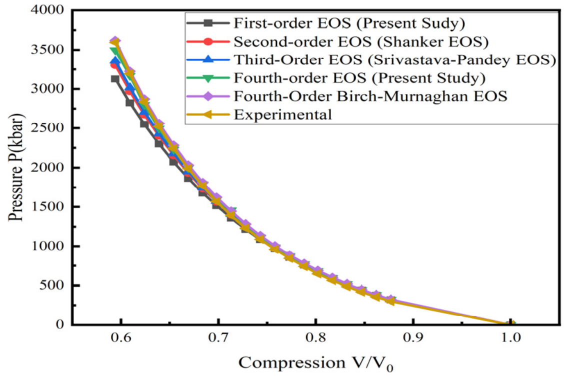

Figure 1. Variation of pressure with increasing compression of ε-Fe

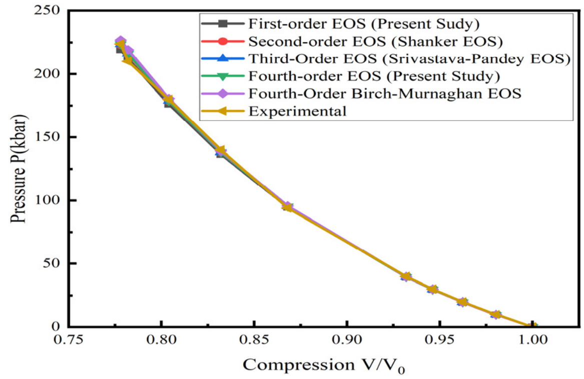

Figure 2. Variation of pressure with growing compression of NaF

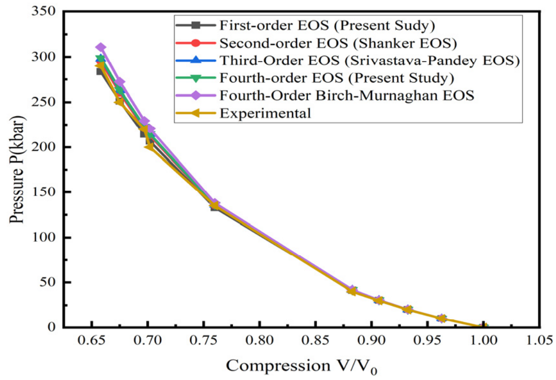

Figure 3. Variation of pressure with increasing compression of NaCl

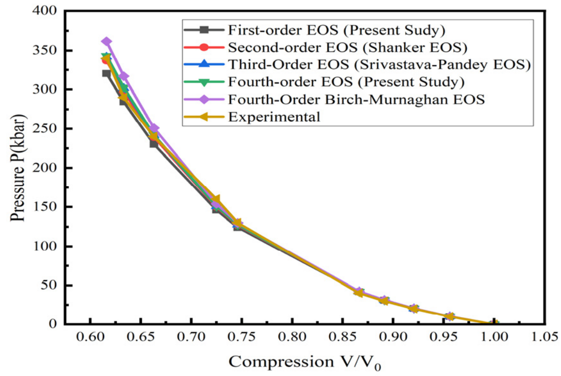

Figure 4. Variation of pressure with increasing compression of NaBr

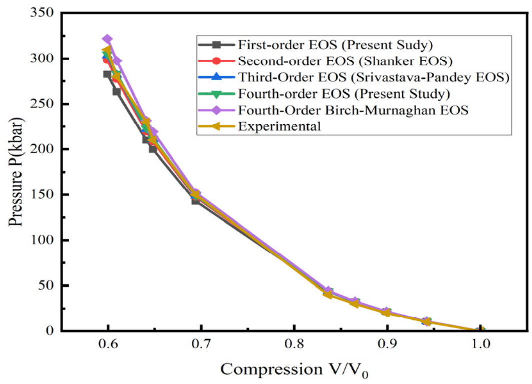

Figure 5. Variation of pressure with increasing compression of NaI

In the case of sodium halides, all EOS and Birch-Murnaghan EOS provide similar results. Still, when compression increases, deviation in Birch-Murnaghan EOS increases, whereas the fourth-order exponential equation of state goes towards better accuracy, as seen in Table 3-6 and Figure 2-5. From the above discussion, we conclude that first- and second-order exponential equations can be used to calculate compression-dependent pressure at low compression. Still, at high compression, the fourth-order exponential equation of state is better than other orders and Birch-Murnaghan EOS.

The Birch-Murnaghan equation of state (EOS) is suitable only for HCP-iron. However, at higher compression, it starts to deviate from experimental data. On the other hand, the fourth-order exponential EOS closely matches experimental results at higher compression, indicating its superior accuracy compared to other EOS models. It performs similarly at low compression but shows increased accuracy at high compression. Therefore, the fourth-order exponential equation of state can be utilised effectively at both low and high compression, highlighting its significance in high-pressure solid-state physics.

The second-order exponential equation of state is also known as the Shanker EOS. In a study by M. Kumar et al. [16], it was suggested that the accuracy of the Shanker EOS at high compression be enhanced by introducing a fitting parameter. The value of the fitting parameter varies for different solids. The Srivastava-Pandey EOS [20] overcomes this limitation by incorporating higher-order compression and anharmonic vibrations of solids. However, accuracy is significantly improved by including fourth-order compression. The fourth-order EOS includes symmetric and asymmetric vibrations at high compressions, significantly increasing accuracy.

4. CONCLUSION

The study presents a first- to fourth-order exponential equation of state (EOS) to enhance the accuracy of pressure calculations under varying compressions, comparing it with the Birch-Murnaghan equation. At low compression, all EOS, including Birch-Murnaghan, produce similar results. However, the fourth-order exponential EOS shows better accuracy at higher compression, while the Birch-Murnaghan EOS deviates from experimental values. The fourth-order exponential EOS proves more reliable for HCP-iron and sodium halides, especially at high compressions. The study concludes that the fourth-order exponential EOS is suitable for both low and high compressions, offering improvements over existing models, such as Shanker and Srivastava-Pandey EOS, by incorporating higher-order compression and anharmonic vibrations for greater accuracy in high-pressure physics.

Ethical Approval:
The authors confidently declare that the manuscript is their original work and has not been published elsewhere.

Competing interests:
The authors of this paper explicitly confirm that they have no financial interests or personal relationships, such as employment, consultancies, stock ownership, honoraria, paid expert testimony, patent applications/registrations, and grants or other funding that could potentially influence the work presented in this report.

Author's Contribution:
All authors were engaged in developing the research outline, with Abhay P. Srivastava taking charge of all calculations and the initial draft of the manuscript. Meanwhile, Professor B. K. Pandey played a pivotal role by offering resources and providing guidance throughout the project.

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You can rest assured that the study's conclusions are backed by information in the references and readily accessible to the public.

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НОВЕ РІВНЯННЯ СТАНУ ЧЕТВЕРТОГО ПОРЯДКУ, ЗАЛЕЖНОГО ВІД СТИСНЕННЯ

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У цьому дослідженні представлено нове експоненціальне рівняння стану першого-четвертого порядку (EOS) для підвищення точності на різних рівнях стиснення. Запропонований експоненціальний EOS порівнюється з широко використовуваним EOS Берча-Мурнагана четвертого порядку, і він не тільки відповідає, але й перевершує точність, особливо при високому стисненні. Це порівняння служить чітким орієнтиром для читачів, щоб зрозуміти перевагу нової моделі. Висновки цього дослідження мають вирішальне значення, оскільки вони показують, що експоненціальний EOS четвертого порядку забезпечує неперевершений рівень точності при більш високому стисненні, особливо для таких матеріалів, як HCP-залізо та галогеніди натрію. EOS Берча-Мурнагана, хоча ефективний при низькому стисненні, відхиляється від експериментальних значень на вищих рівнях. Крім того, дослідження вивчає Shanker EOS, в якому M. Kumar et al. [Physica B: Condensed Matter, 239(3-4), 337-344 (1997)] висувають вимогу щодо покращення при високому стисненні та покращення шляхом підгонки параметрів, які відрізняються від матеріалу до матеріалу. Це обмеження усувається завдяки розробці експоненціального EOS четвертого порядку, який є більш універсальним, пропонуючи надійні результати в сценаріях як низького, так і високого стиснення у фізиці високого тиску.

Ключові слова: *рівняння стану; стиснення; показникове рівняння стану четвертого порядку; рівняння стану Берча-Мурнагана четвертого порядку*