East European Journal of Physics. 1. 44–58 (2025)

DOI: 10.26565/2312-4334-2025-1-04 ISSN 2312-4334

THERMODYNAMICS OF HOMOGENEOUS AND ISOTROPIC UNIVERSE FOR VARIOUS DARK ENERGY CONDITIONS

Neeru Goyal[a], Anil Kumar Yadav[b], Tensubam Alexander Singh[c], Aditya Sharma Ghrera[a], Asem Jotin Meitei[d,e], Kangujam Priyokumar Singh[e*]

[a]*Department of Applied Sciences, The NorthCap University, Gurugram - 122017, India*

[b]*Department of Physics, United College of Engineering and Research,Greater Noida - 201310, India*

[c]*Department of Mathematical Sciences, Bodoland University, Kokrajhar-783370, Assam, India*

[d]*Department of Mathematics, Pravabati College, Mayang Imphal-795132, Manipur, India*

[e]*Department of Mathematics, Manipur University, Canchipur, Imphal-795003, Manipur, India*

[*]*Corresponding Author e-mail: pk_mathematics@yahoo.co.in*

Received October 10, 2024; revised January 01, 2025; accepted January 30, 2025

The thermodynamic properties of homogeneous and isotropic universe for various dark energy conditions with decaying cosmological term $\Lambda(t)$ are investigated. To obtain the explicit solution of Einstein's field equations, we have considered a linearly varying deceleration parameter in the form of $q = -\alpha t + m - 1$ with α and m as scalar constants. We have constrained the model parameters H_0 and m as 68.495 km/s/Mpc and 1.591 respectively by bounding the derived model with combined pantheon compilation of SN Ia and H(z) data sets. Furthermore, we have studied the time varying dark energy states for two different assumptions i) $\Lambda = \Lambda_1 t^{-2}$ and ii) $\Lambda \propto [R(t)]^{-2n}$. For a specific assumption, our models indicate a dark energy like behaviour in in open, flat and closed space - time geometry. The temperature and entropy density of the model remain positive for both the cases i) $\Lambda = \Lambda_1 t^{-2}$ and ii) $\Lambda \propto [R(t)]^{-2n}$.. Some physical properties of the universe are also discussed.

Keywords: *FRW Model; Homogeneous; Thermodynamics; Pantheon; Dark energy*

PACS: 98.80.-K; 98.80Jk.

1. INTRODUCTION

Most of the studies in the recent years suggest that the understanding of the fate of the accelerating expansion of the universe in view of Type Ia Supernova [1, 2, 3, 4] is a very challenging and interesting field for the present research in Cosmology. Various studies have been done to explain this special discovery out of which [5, 6] can be mentioned. It is an interesting component that considered as dark energy possessing a negative pressure and is recommended to understand the accelerating expansion of the universe. The current simplest candidate for a standard model of cosmology and a good understanding with most observations [7, 8] is the ΛCDM model which is required due to two major problems as fine-tuning or why so small and coincidence [9, 10, 11]. There are mainly three different methods to express the dark energy problem i.e., dynamical dark energy [5], the anthropic principle [9] and interacting dark energy [12, 13]. Out of which, dynamical dark energy has an hypothetical form called as Quintessence which is reported as a scalar field minimally connected to gravity which can fall to the late time inflation accelerating cosmological expansion for some particular form of potential, Also, due to these particular potentials it lighten the cosmological coincidence model [11]. In this paper, we have intended to focus on this quintessence and phantom phase models since the role of thermodynamics in cosmology remain essential to study the transverse of irreversible energy flow from gravitational field to matter creation that can helps to transform space-time into matter as suggested by [14]. Also, the irreversible matter creation, the big-bang initial singularity remain unstable. This dissipative process of the Einstein field equation leads to the possibility of cosmological model from empty space to creation of matter and entropy. Gravitational entropy remain meaningful as associated with the entropy which is necessary to produce matter. This extend signifies the possibility of impact fullness of third law of thermodynamics. As the source of dark energy of the current phase of universe can modify the horizon entropy, so its thermodynamics in both cosmological as well as gravitational set ups have more impaction [15, 16, 17, 18, 21, 23, 24]. Also, it seems that the properties of such modifications to the thermodynamics are in line with non-extensive thermodynamics of space-time and the current universe. Here, we investigated the mutual relationship between the thermodynamic laws with the Einstein field equations (EFE) with the concept of Einstein theory of gravitation. In order to study the model here we apply the thermodynamical laws of Apparent Horizon of FRW universe as FRW metric is an exact solution of EFE of general relativity, which describes an isotropic homogeneous and expanding universe. The solution of this model can proved its generic properties that are different from dynamical FL Model which are specific solutions for scale factor $R(t)$ that assumes the only contribution to stress energy, cold matter, radiation and cosmological constant.

Cite as: N. Goyal, A.K. Yadav, T.A. Singh, A.S. Ghrera, A.J. Meitei, K.P. Singh, East Eur. J. Phys. 1, 44 (2025), https://doi.org/10.26565/2312-4334-2025-1-04

It is worthwhile to mention that there was a major breakthrough after results of Supernovae type Ia project [25]. A new type of fluid with negative pressure, called dark energy, leads cosmic acceleration of the universe at present epoch. In the recent past, many cosmological modes have been investigated to describe the nature of dark energy [26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37]. The most suitable candidate of dark energy is a time varying cosmological constant term $\Lambda(t)$ but it suffers few problems on the theoretical scale namely fine tuning and cosmic coincidence problems. In order to alleviate cosmological constant problems, one can refer the following Refs. [38, 39, 40, 41, 42, 43, 44]. Up to now, the dark energy and its physical nature is still mysterious and unclear, and we only know its some phenomenological properties such as the dark energy is a cosmic fluid with equation of state parameter, numerically equivalent to -1 and it violated strong energy condition. Also, the dark energy is homogeneously permeated in the universe and its clustering property is smaller than dark matter. Some applications of time dependent Λ term are given in the Refs. [45, 46, 47, 48]. The idea of that during the evolution of the universe the energy density of the vacuum decouples into the particles thus the value of cosmological term decreases with age of the universe. As the result one has the creation of particles although the typical rate of the creation is very small. Now, it has been established that the universe is in accelerating phase at present epoch. This acceleration of the universe is usually described by inclusion of dark energy density along with the matter energy density in the Einstein's field equation [38, 39, 49, 52, 51, 52, 53, 37]. The observational estimates suggest that the universe is filled with dark matter with null pressure and dark energy with negative pressure. However, the nature of dark matter and dark energy is still mysterious. In the recent past, some cosmological models have been investigated to explore the problems associated with dark energy and its possible solutions [54, 55, 56, 57, 58]. We also note that some important properties of dark energy in light of the early JWST observations are explored in Ref. [59]. Nunes et al. [60] have investigated the dark sector interactions from the full-shape galaxy power spectrum and described its new features in context of accelerating universe. The soundness of dark energy properties and its applications are given in ref. [61]. Motivated by above investigations, in this paper, we confine ourself to investigate some thermodynamic properties of homogeneous and isotropic universe for various dark energy conditions. The study reveals that the derived models might be a suitable model to describe the dynamics and fate of the universe at present epoch.

The structure of our paper is as follows: In Section 2, we have presented thermodynamical behaviour and entropy of the model where we have expressed the entropy production rate, apparent Horizon and Cui-Kim temperature of the apparent horizon. In Section 3, we derived some basis of Einstein's gravity with detail solutions of FRW metric. Sections 4 and 5 deal with two independent models: i) $\Lambda = \Lambda_1 t^{-2}$ and ii) $\Lambda = \Lambda_2 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}$ and its physical properties respectively. In Section 6, we summarize our findings in details.

2. THERMODYNAMICAL BEHAVIOUR AND ENTROPY

Thermodynamical study has been an important tool to incept a gravitational theory as pivotal event. Black hole thermodynamics and recent Conformal field theory correspondence shows a strong corelation between gravity and thermodynamics, also it has a great significance on recent observations. From the concept of thermodynamics, the interaction between first and second law of thermodynamics with volume V [14] can be expressed as

$$\tau ds = d(\rho V) + \rho dV, \tag{1}$$

where τ and S represents the temperature and entropy respectively. The above equation can be written as

$$\tau dS = d(p + \rho)V - V dp, \tag{2}$$

to define a perfect fluid as a thermodynamic system an integrability condition is required which can be written as

$$dp = \left(\frac{p+\rho}{\tau}\right) d\tau. \tag{3}$$

Using equations (2) and (3), we have the differential equation

$$dS = \frac{1}{\tau} d(p+\rho)V - (p+\rho)V \frac{d\tau}{\tau^2}. \tag{4}$$

Rewriting above equation

$$dS = d\left[\frac{(p+\rho)V}{\tau}\right]. \tag{5}$$

Therefore, the entropy can be defined as

$$S = \left[\frac{(p+\rho)V}{\tau}\right]. \tag{6}$$

The well known relation between pressure and energy density is read as

$$p = \gamma\rho \tag{7}$$

where $p = \gamma\rho$ and the parameter "γ" stands for equation of state parameter.
Let the entropy density is

$$S_1 = \frac{S}{V} = \frac{p+\rho}{\tau} = \frac{(1+\gamma)\rho}{\tau}. \tag{8}$$

Consider the apparent horizon of the universe for the assumed model appeared at r_A where

$$r_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}. \tag{9}$$

Then the entropy density in terms of temperature with the help of first law of thermodynamics can be expressed as

$$d(\rho V) + \gamma\rho dV = (1+\gamma)\tau d\left(\frac{\rho V}{\tau}\right), \tag{10}$$

which on integration yields

$$\tau = \rho^{\frac{\gamma}{1+\gamma}}. \tag{11}$$

From equation (3), we obtain

$$S_1 = (1+\gamma)\rho^{\frac{\gamma}{1+\gamma}}. \tag{12}$$

Now the Cui-Kim temperature of the apparent horizon can be obtained as

$$\tilde{T} = \frac{1}{2\pi r_A}. \tag{13}$$

Here, the above equation (6) represents the entropy which does not depends on any individual fluids and is only depends on the isotropic pressure and total matter density of the fluid. Many authors have investigated on thermodynamical aspects of cosmological model using different theories with different fluid contents. Samant et. al [16] have investigated on the validity of second law of thermodynamics using Kaluza-Klein metric with Bulk viscosity in the context of $f(R,T)$ theory and found that the second law of thermodynamics doesn't hold for the assumed model. As we know that the actions of thermodynamic parameters is directly related to the energy density of the universe. Recently, Shekh et.al., [17] investigated thermodynamical aspects of relativistic hydrodynamics in $f(R,G)$ gravity for accelerated spatially homogeneous and isotropic FRW cosmological model with a non-perfect (un-magnetized) fluid in the framework of $f(R,G)$ gravity model by defining entropy density with the condition stated above. Jamil et.al [18] have investigated on Horava-Lifshitz cosmology for thermodynamical validity in different types of universe and found that the model remain valid for closed and flat universe but conditionally valid for open universe which matches to the result obtained during our study.

3. EINSTEIN FIELD EQUATION AND THEIR SOLUTION

The Einstein's field equation can be written as follows

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}, \tag{14}$$

where $G_{\mu\nu}$ the Einstein tensor, Λ is the cosmological constant which can be regarded as dark energy of the model introduced by Einstein and $T_{\mu\nu}$ is the energy-momentum tensor.

To study the nature of the model universe, it is quiet necessary to consider a metric by which the Einstein field equations can be evaluated and further solutions can be evaluated. Let us consider FRW metric with a maximally symmetric spatial section as

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{15}$$

where $R(t)$ the cosmic scale factor and the spatial curvature index $k = -1, 0, +1$ corresponds to spatially open, flat and closed universe respectively.
Now, consider the fluid representation for the energy-momentum tensor which can be written as follows

$$T_{\mu\nu} = [\rho, p, p, p]. \tag{16}$$

In a co-moving coordinate system, the Einstein field equation (13) with the use of equations (14) and (15), we have

$$3H^2 + \frac{3k}{R^2} = \rho + \Lambda, \tag{17}$$

$$3H^2 + \frac{k}{R^2} + 2\dot{H} = -p + \Lambda, \tag{18}$$

where H stands for well-known Hubble parameter.

The above system of equation consists of two equation and four unknowns. To make the system consistent two additional constraints required. So first we have considered the well-known relation between pressure and energy density as described in Eq. (7). The parameter γ in Eq. (7) takes a vital role to model the universe for its different values. For $\gamma = 0, \frac{1}{3}$ and -1 the model represents a dust, radiating and vacuum energy of the fluid. Similarly, for $\gamma < 0$ is considered as an accelerating expansion of the universe in the context of dark energy. Moreover, for different range of γ such as quintessence phase, $-1 < \gamma < 0$, phantom phase $\gamma < -1$ and for cosmological constant cold dark matter ΛCDM universe, we have $\gamma = -1$. Still, it is worthwhile to note that there is no clear understanding on equation of state parameter of dark energy yet.

Furthermore, we also considered a linearly varying deceleration parameter [19, 20, 21, 22] as

$$q = -\alpha t + m - 1 \tag{19}$$

where α and m are scalar constants and $q = -\frac{R\ddot{R}}{\dot{R}^2}$ the deceleration parameter which helps to predict whether the model is accelerating or decelerating in nature.

It is worthwhile to note that the concept of linearly varying deceleration parameter was given in Ref. [19] and later on its observational analysis are presented in Refs. [20, 22]. In this paper, we confine ourself to describe the thermodynamics of the universe for various dark energy conditions on the basis of linearly varying deceleration parameter.

The proposed form of deceleration parameter yields

$$R = a_1 \left(\frac{t}{2m - \alpha t}\right)^{\frac{1}{m}}, \quad \alpha > 0, m > 0. \tag{20}$$

where, $a_1 = k_1{}^{\frac{1}{m}}$ is an arbitrary constant while k_1 denotes an integrating constant.
The Hubble parameter can be calculated by using equation (8) as

$$H = -\frac{2}{t(\alpha t - 2m)}. \tag{21}$$

Hence the apparent horizon for the model is appeared at

$$r_A = \left(\frac{4}{t^2(\alpha t - 2m)^2} + \frac{k}{a_1^2}\left(\frac{\alpha t}{2m - \alpha t}\right)^{-\frac{2}{m}}\right). \tag{22}$$

The energy density and pressure can be calculated as follows

$$\rho = \frac{12}{t^2(\alpha t - 2m)^2} + \frac{3k}{a_1^2}\left(\frac{\alpha t}{2m - \alpha t}\right)^{-\frac{2}{m}} - \Lambda, \tag{23}$$

$$p = \frac{8(m - \alpha t) - 12}{t^2(\alpha t - 2m)^2} - \frac{k}{a_1^2}\left(\frac{\alpha t}{2m - \alpha t}\right) + \Lambda. \tag{24}$$

Now, the equation of state parameter can be calculated on using (18)

$$\gamma = \frac{8(m - \alpha t) + \Lambda t^2(\alpha t - 2m)^2 - \frac{k}{a_1^2}\left(\frac{\alpha t}{2m - \alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t - 2m)^2 - 12}{12 - \Lambda t^2(\alpha t - 2m)^2 + \frac{k}{a_1^2}\left(\frac{\alpha t}{2m - \alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t - 2m)^2}. \tag{25}$$

3.1. Observational confrontation

The red-shift z is read as

$$z = -1 + \frac{R_0}{R} \tag{26}$$

where, R_0 denotes the present value of scale factor and it is taken as 1.
Therefore, Hubble's parameter in term of z is obtained as

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt} \tag{27}$$

Eqs. (20), (26) and (27) lead to

$$H = \frac{H_0}{(1+a_1^m)(1+z)^m}\left[1 + a_1(1+z)^m\right]^2 \tag{28}$$

where, H_0 reads the present value of Hubble parameter.

Solving Eqs. (20) and (26), the time - redshift relationship is obtained as

$$t(z) = \frac{2m}{\alpha + [a_1(1+z)]^m} \tag{29}$$

Thus, the deceleration parameter in terms of redshift z is computed as

$$q = -1 + m - \frac{2m\alpha}{\alpha + [a_1(1+z)]^m} \tag{30}$$

It is worthwhile to note that the observational analysis of the linearly varying q in terms of time has been carried out in Refs. [20, 22]. In particular, Akarsu et al. [20] have investigated the kinematics and fate of the universe by confronting observational data for the linearly time-varying deceleration parameter model and its comparison with standard ΛCDM model while Pacif [22] has investigated dark energy cosmological model by considering the linearly time-varying deceleration parameter and constrained the model parameter with observational data. In this paper, we use the recent 1048 pantheon compilation of SN Ia data points [62] and 57 $H(z)$ data sets [63, 64, 65] while in Pacif [22], 580 data points of SN Ia is used. Furthermore, we also estimate H_0 along with model parameters m and a_1 whereas the value of $H_0 = 67.8\ km/s/Mpc$ is taken as prior in Pacif [22].

The χ^2 estimator is read as

$$\chi^2 = \sum_{i=1}^{N}\left[\frac{E_{th}(z_i) - E_{obs}(z_i)}{\sigma_i}\right]^2. \tag{31}$$

where $E_{th}(z_i)$ and $E_{obs}(z_i)$ denote the theoretical and observed values respectively, and σ_i^2 denotes standard deviation of each $E_{obs}(z_i)$. N is the number of data points.
The joint χ^2 estimator is read as

$$\chi^2_{joint} = \chi^2_{OHD} + \chi^2_{Pantheon} \tag{32}$$

The left panel of Fig. 1 depicts the two-dimensional contours in the $H_0 - m$ plane at 1σ, 2σ and 3σ confidence regions while the right panel of Fig.1 shows the two-dimensional contours in the $H_0 - a_1$ plane at 1σ, 2σ and 3σ confidence regions by bounding our model with joint 57 $H(z)$ data sets and pantheon compilation of SN Ia data. We constrained the model parameters H_0, m and a_1 as 68.495 km/s/Mpc, 1.591 and 1.462 respectively. The values of parameters m and a_1 differ slightly from Pacif [22]. That is why, we choose $m = 1.591$ and $a_1 = 1.462$ for graphical analysis of various parameters of the derived model. Fig. 2 depicts the dynamics of deceleration parameter q versus redshift z for $m = 1.591$ and $a_1 = 1.462$. From Fig. 2, we observe that the derived model exhibits a model of transitioning universe from early decelerating phase to current accelerating phase. Furthermore, we obtain the transition redshift and present value of deceleration parameter as $z_t = 0.73$ and $q_0 = -0.535$ respectively. It is worthwhile to note that an useful approach to compare the linearly varying deceleration parameter models with standard ΛCDM model are given in Ref. [20]. The present value of deceleration parameter and transition redshift are reported as $q_0 = -0.556 \pm 0.046$ and $z_t = 0.682 \pm 0.082$ respectively [20]. Moreover, Akarsu et al. [20] have obtained $z_t = 0.733^{+0.148}_{-0.095}$ for linearly varying deceleration parameter with redshift (LVDPz) which is very close to the transition redshift of this paper.

4. MODEL WITH $\Lambda \propto t^{-2}$ $i.e.$ $\Lambda = \Lambda_1 t^{-2}$

This case gives the results of the physical parameter of the model as

$$\rho = \frac{12}{t^2(\alpha t - 2m)^2} + \frac{3k}{a_1^2}\left(\frac{t}{2m - \alpha t}\right)^{-\frac{2}{m}} - \Lambda_1 t^{-2}, \tag{33}$$

$$p = \frac{8(m - \alpha t) - 12}{t^2(\alpha t - 2m)^2} - \frac{k}{a_1^2}\left(\frac{t}{2m - \alpha t}\right) + \Lambda_1 t^{-2}, \tag{34}$$

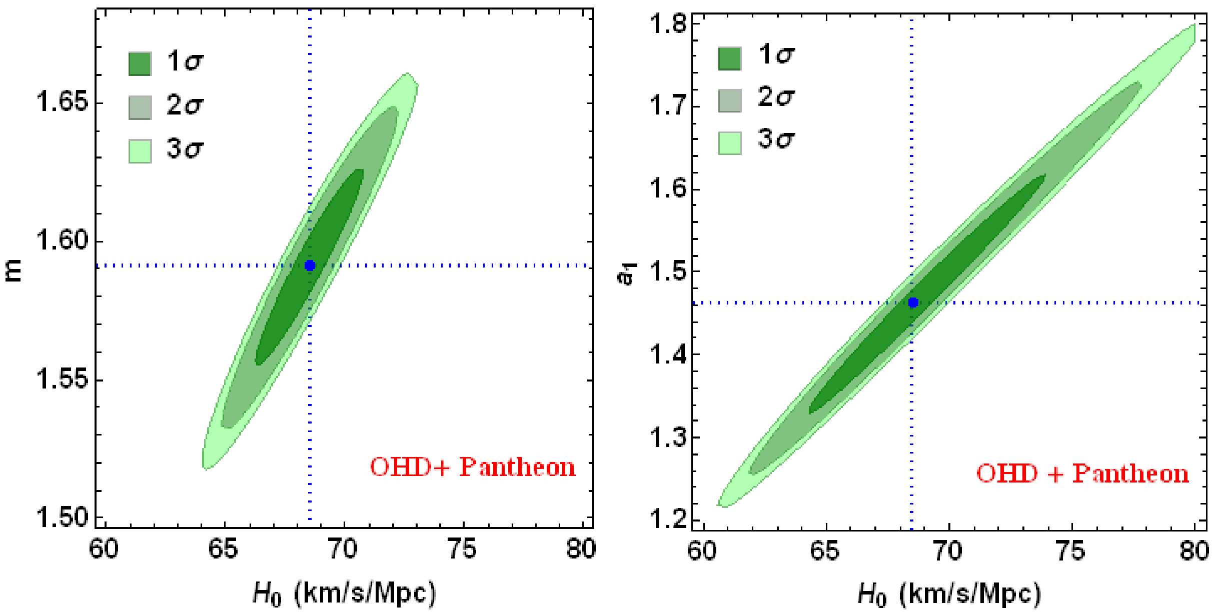

Figure 1. The left panel of Fig. 1 depicts the two-dimensional contours in the $H_0 - m$ plane at 1σ, 2σ and 3σ confidence regions while the right panel of Fig.1 shows the two-dimensional contours in the $H_0 - a_1$ plane at 1σ, 2σ and 3σ confidence regions by bounding our model with OHD + Pantheon compilation of SN Ia data.

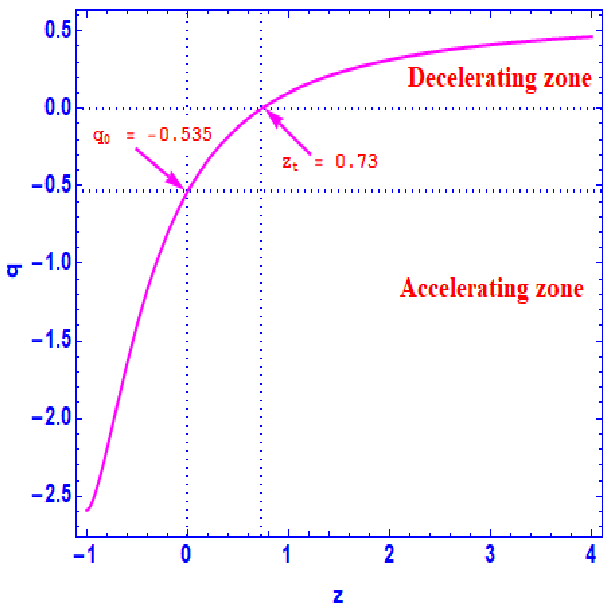

Figure 2. The deceleration parameter q versus z for $m = 1.59$ and $a_1 = 1.462$.

$$\gamma = \frac{8(m-\alpha t) + \Lambda_1 t^2(\alpha t - 2m)^2 - \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t-2m)^2 - 12}{12 - \Lambda_1 t^2(\alpha t-2m)^2 + \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t-2m)^2}. \tag{35}$$

Temperature

$$\tau = \left(\frac{12}{t^2(\alpha t-2m)^2} + \frac{3k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} - \Lambda_1 t^{-2}\right)^{\beta}, \tag{36}$$

where

$$\beta = \frac{\frac{8(m-\alpha t)+\Lambda_1 t^2(\alpha t-2m)^2 - \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t-2m)^2-12}{12-\Lambda_1 t^2(\alpha t-2m)^2+\frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t-2m)^2}}{1+\frac{8(m-\alpha t)+\Lambda_1 t^2(\alpha t-2m)^2 - \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t-2m)^2-12}{12-\Lambda_1 t^2(\alpha t-2m)^2+\frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t-2m)^2}}. \tag{37}$$

The entropy density

$$S_1 = (1+\gamma)\left(\frac{12}{t^2(\alpha t-2m)^2} + \frac{3k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} - \Lambda_1 t^{-2}\right)^{1-\beta}. \tag{38}$$

Moreover, using Eq. (29) in Eqs. (33) - (35), one may compute the energy density ρ and pressure p in terms of redshift z as following

$$\rho(z) = \frac{3k\left[(a_1(z+1))^{-m}\right]^{-2/m}}{a_1^2} + \frac{3(a_1(z+1))^{-2m}\left[\alpha+(a_1(z+1))^m\right]^4}{4m^4} - \frac{\Lambda_1\left[\alpha+(a_1(z+1))^m\right]^2}{4m^2} \tag{39}$$

$$p(z) = \frac{(a_1(z+1))^{-2m}\left({a_1}^2\left(\alpha+(a_1(z+1))^m\right)^2\left(-6\alpha(a_1(z+1))^m + (m(\Lambda_1 m+2)-3)(a_1(z+1))^{2m} - \alpha^2(2m+3)\right)-\zeta\right)}{4{a_1}^2 m^4} \tag{40}$$

where $\zeta = 4\alpha k m^4 (a_1(z+1))^m$.

Fig.3 and Fig. 4 show the behavior of the universe for energy density ρ and pressure $p$with respect to redshift z respectively. From the above graphical representations, we observe that the the energy density and pressure decrease as $z \to 0$. The variation of EOS parameter $\gamma = \frac{p}{\rho}$ with respect to time has been graphed in Fig. 5. We observe that open, flat and closed model of the universe, the EOS parameter varies as with negative sign, therefore, the derived model in open, flat and closed space - time geometry depicts dark energy EOS parameter like behaviour for $\Lambda = \Lambda_1 t^{-2}$. Furthermore, from Eq. (35), it is clear that the entropy density S_1 is decreasing function of time and at $t \to \infty$, $S_1 \to 0$, which indicates that the second law of thermodynamics remains impact-less on this model of universe.

In general theory of relativity, the energy conditions have significant role to describes the Hawking's Penrose singularity [23] whereas to verify the positive mass theorem, the dominant energy condition (DEC) is required to validate [24]. It consists of a couple of constraints which characterise the nature of the obscurity of lightlike, timelike or spacelike curves. Furthermore, to identify the second law of black hole thermodynamics, null energy condition plays a major role [66]. The four different types of energy conditions are Null energy condition (NEC), Weak energy condition (WEC), Strong energy condition (SEC) and Dominant energy condition (DEC) are respectively (i) $\rho + p \geq 0$ (ii) $\rho + p \geq 0, \rho \geq 0$ (iii) $\rho + 3p \geq 0$ (iv) $\rho > |p|$ [67]. The graphical representation of the energy conditions for this case are presented in Fig. 6. In the derived model, WEC, NEC and DEC are validated whereas it violates SEC as expected in dark energy models. Moreover, the SEC is violated for the derived model of the universe which indicates that the universe is accelerating in nature, which supports the results of Shekh et al. [17].

5. MODEL WITH $\Lambda \propto [R(t)]^{-2n}$ $i.e.$ $\Lambda = \Lambda_2\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}$

The different parameters of this model are obtained as

$$\rho = \frac{12}{t^2(\alpha t-2m)^2} + \frac{3k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} - \Lambda_2\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}, \tag{41}$$

$$p = \frac{8(m-\alpha t)-12}{t^2(\alpha t-2m)^2} - \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} + \Lambda_2\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}, \tag{42}$$

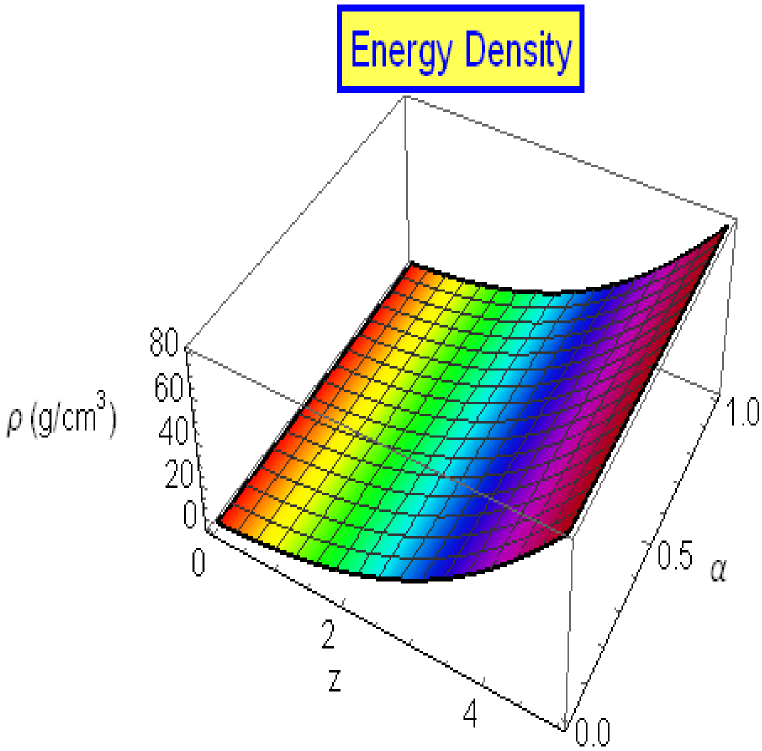

Figure 3. The energy density ρ versus redshift z for $m = 1.591$ and $a_1 = 1.462$ for model $\Lambda = \Lambda_1 t^{-2}$.

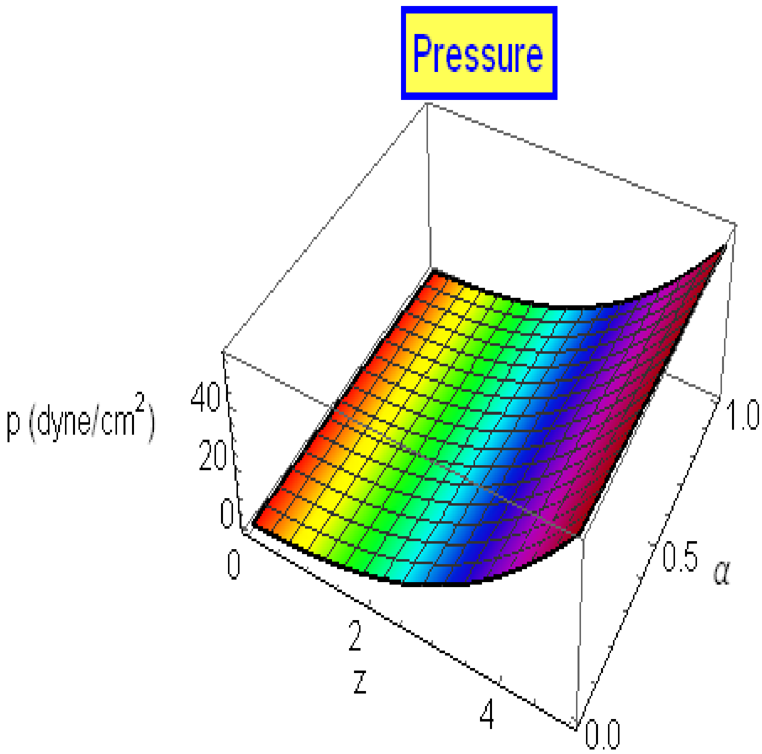

Figure 4. The pressure p versus redshift z for $m = 1.591$ and $a_1 = 1.462$ for model $\Lambda = \Lambda_1 t^{-2}$.

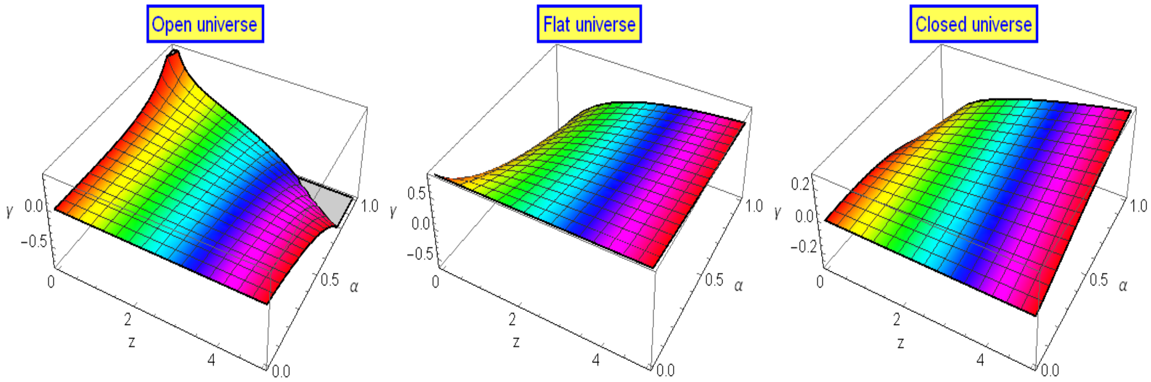

Figure 5. EOS parameter γ versus redshift z for model $\Lambda = \Lambda_1 t^{-2}$ for $m = 1.591$ and $a_1 = 1.462$.

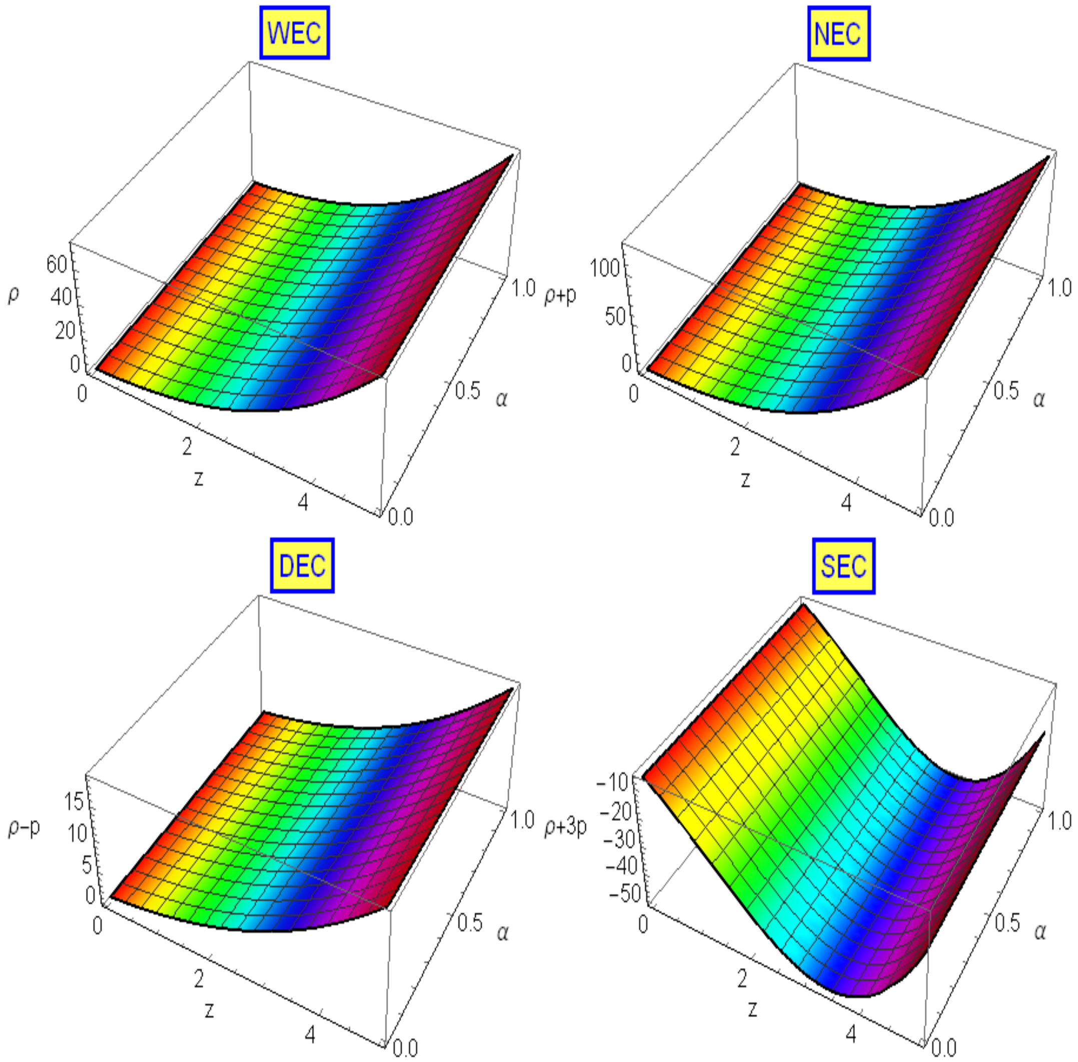

Figure 6. Energy conditions: i) WEC $\rho \geq 0$ ii) NEC $\rho + p \geq 0$ iii) DEC $\rho - p \geq 0$ and iv) SEC $\rho - 3p \leq 0$ for model $\Lambda = \Lambda_1 t^{-2}$.

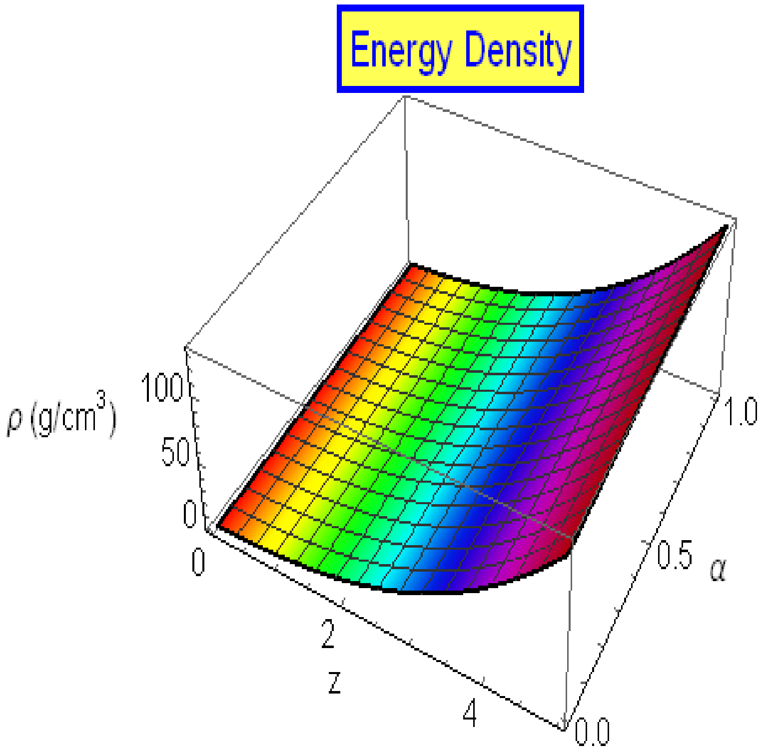

Figure 7. The energy density ρ versus redshift z for m = 1.591 and a_1 = 1.462. for model $\Lambda = \Lambda_2\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}$.

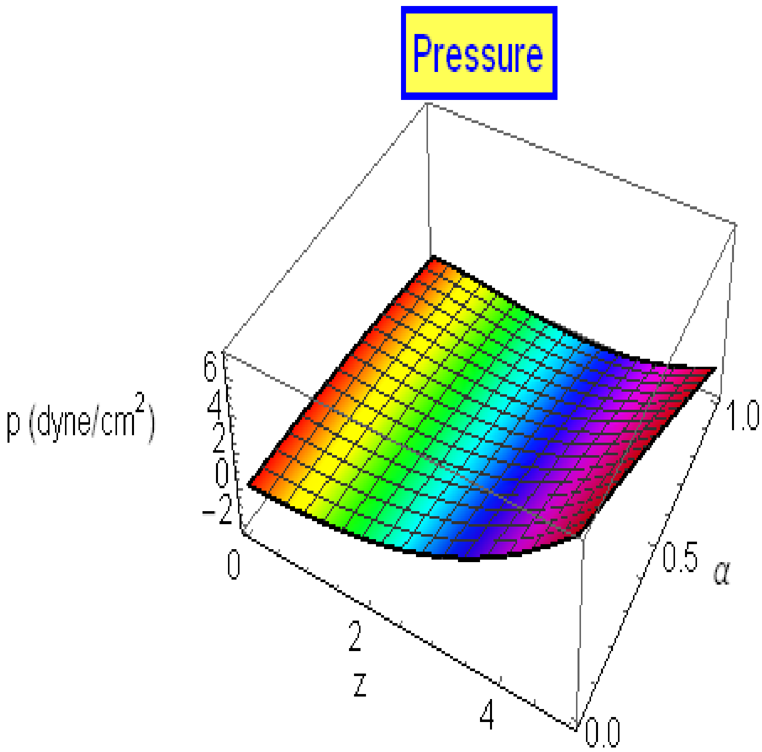

Figure 8. The pressure p versus redshift z for m = 1.591 and a_1 = 1.462. for model $\Lambda = \Lambda_2\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}$.

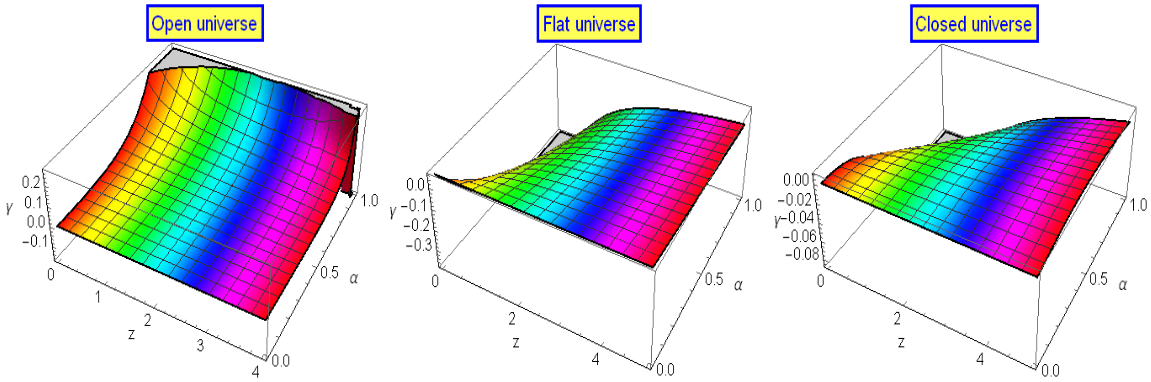

Figure 9. EOS parameter γ versus redshift z for model $\Lambda = \Lambda_2 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}$ for $m = 1.591$ and $a_1 = 1.462$.

$$\gamma = \frac{8(m-\alpha t) + \Lambda_2 \alpha^2 t^4 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2(n+m)}{m}} - \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t - 2m)^2 - 12}{12 - \Lambda_2 \alpha^2 t^4 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2(n+m)}{m}} + \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{\frac{2}{m}} t^2(\alpha t - 2m)^2}. \tag{43}$$

The temperature is given by

$$\tau = \left(\frac{12}{t^2(\alpha t - 2m)^2} + \frac{3k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} - \Lambda_2 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}\right)^{\eta}, \tag{44}$$

where

$$\eta = \left(\frac{\frac{8(m-\alpha t)+\Lambda_2\alpha^2 t^4\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2(n+m)}{m}} - \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t-2m)^2-12}{12-\Lambda_2\alpha^2 t^4\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2(n+m)}{m}} + \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{\frac{2}{m}} t^2(\alpha t-2m)^2}}{1+\frac{8(m-\alpha t)+\Lambda_2\alpha^2 t^4\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2(n+m)}{m}} - \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t-2m)^2-12}{12-\Lambda_2\alpha^2 t^4\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2(n+m)}{m}} + \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{\frac{2}{m}} t^2(\alpha t-2m)^2}}\right). \tag{45}$$

And the entropy density obtained as

$$S_1 = \left(1 + \frac{8(m-\alpha t) + \Lambda_2 \alpha^2 t^4 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2(n+m)}{m}} - \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} t^2(\alpha t - 2m)^2 - 12}{12 - \Lambda_2 \alpha^2 t^4 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2(n+m)}{m}} + \frac{k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{\frac{2}{m}} t^2(\alpha t - 2m)^2}\right)$$
$$\left(\frac{12}{t^2(\alpha t - 2m)^2} + \frac{3k}{a_1^2}\left(\frac{t}{2m-\alpha t}\right)^{-\frac{2}{m}} - \Lambda_2 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}\right)^{1-\eta}. \tag{46}$$

Moreover, using Eq. (29) in Eqs. (41) and (42), we obtain an expression of the energy density ρ and pressure p in terms of redshift z as following

$$\rho(z) = \frac{3k\left[\alpha(a_1(z+1))^{-m}\right]^{-2/m}}{a_1{}^2} + \frac{3(a_1(z+1))^{-2m}\left[\alpha + (a_1(z+1))^m\right]^4}{4m^4} - \Lambda_2\left[\alpha(a_1(z+1))^{-m}\right]^{-\frac{2n}{m}} \tag{47}$$

$$p(z) = \frac{(a_1(z+1))^{-2m}\left(\alpha + (a_1(z+1))^m\right)^3\left(2m\left((a_1(z+1))^m - \alpha\right) - 3\left(\alpha + (a_1(z+1))^m\right)\right)}{4m^4} - \frac{\alpha k (a_1(z+1))^{-m}}{a_1{}^2} +$$
$$\Lambda_2\left(\alpha(a_1(z+1))^{-m}\right)^{-\frac{2n}{m}} \tag{48}$$

The expressions for energy density ρ, pressure p in terms of time are obtained in Eqs. (41) and (42) respectively while Eqs. (47) and (48) exhibit ρ and p in terms of redshift z. The graphical behaviours of energy density, pressure and EOS parameter versus redshift z are shown in Fig. 7, Fig. 8 and Fig. 9 respectively. As in the earlier case here also we can see the same results for energy density and EOS parameter for model $\Lambda = \Lambda_2\left(\frac{\alpha t}{2m-\alpha t}\right)^{-\frac{2n}{m}}$ for $m = 1.591$ and $a_1 = 1.462$. Moreover, we also obtain here the similar results for energy conditions which indicates that the assumption made

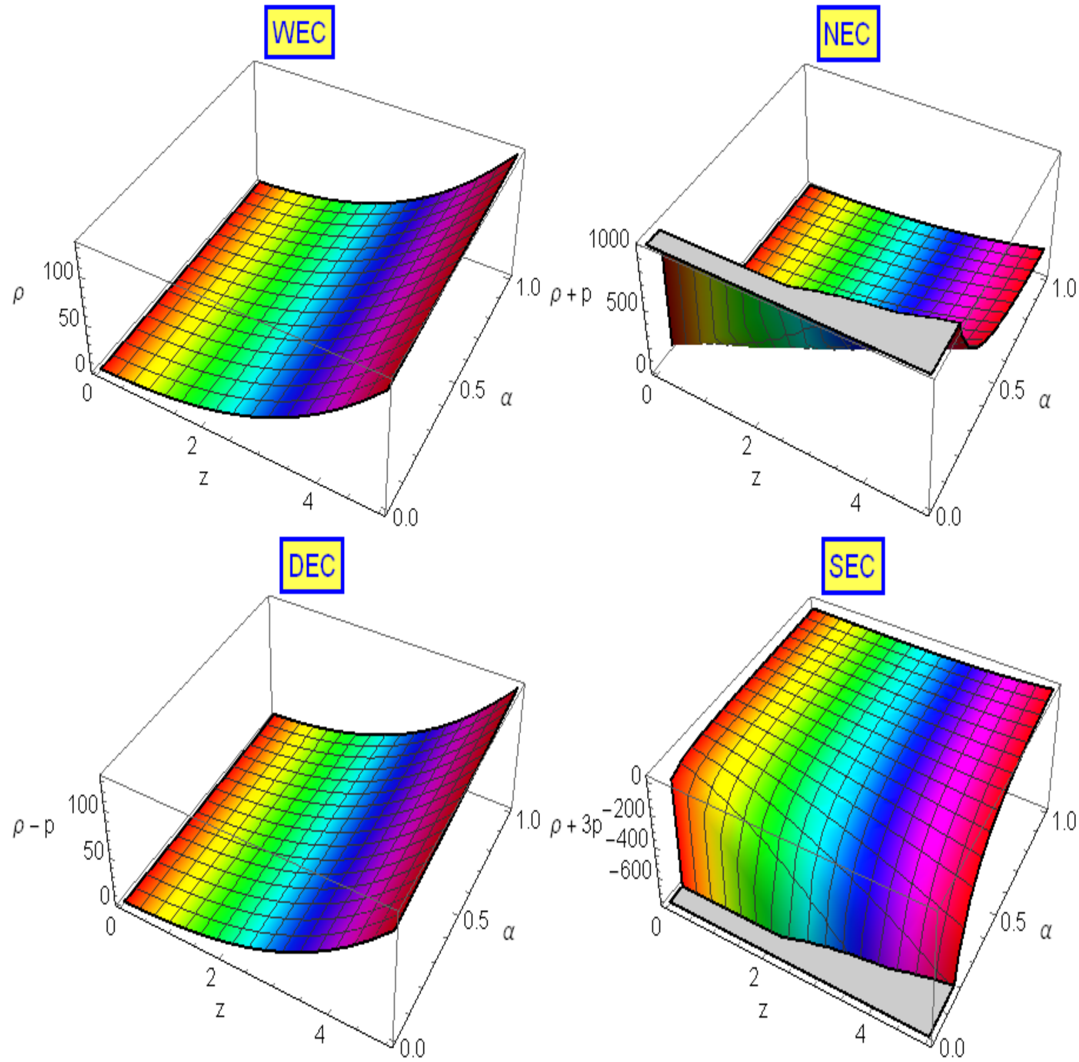

Figure 10. Energy conditions: i) WEC $\rho \geq 0$ ii) NEC $\rho + p \geq 0$ iii) DEC $\rho - p \geq 0$ and iv) SEC $\rho + 3p \leq 0$ for model $\Lambda = \Lambda_2 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}$.

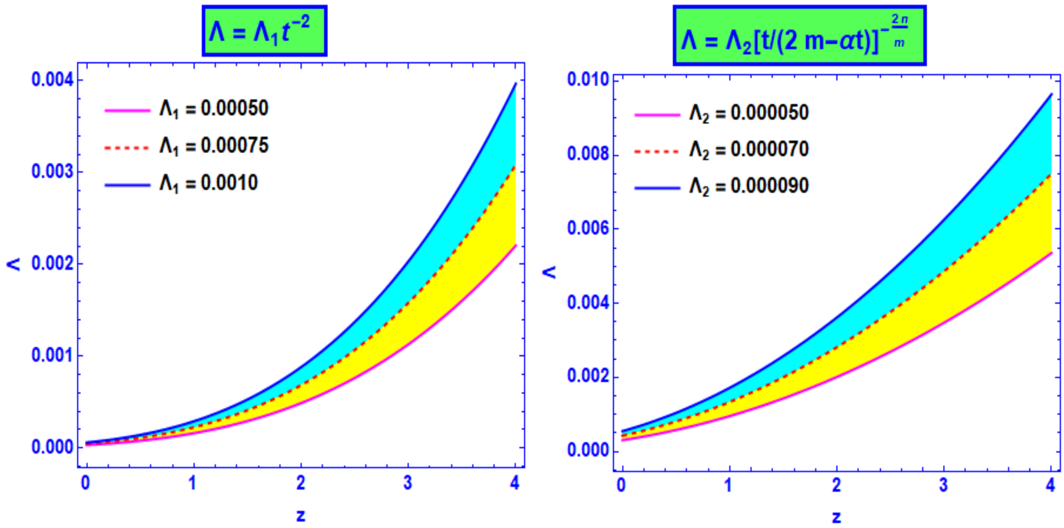

Figure 11. The behaviour of cosmological constant Λ with redshift z for models i) $\Lambda = \Lambda_1 t^{-2}$ (left panel) and ii) $\Lambda = \Lambda_2 \left(\frac{t}{2m-\alpha t}\right)^{-\frac{2n}{m}}$ (right panel) for $m = 1.591$ and $a_1 = 1.462$.

for dark energy in this case remain valid in energy condition aspects. The Fig. 10 describes the graphical representations of energy conditions. We observe that the derived model of the universe validates WEC, NEC and DEC while it violates SEC. The violation of SEC favors the accelerating expansion of the universe. It is worthwhile to note that the entropy density of the model remain positive and it approaches to a small positive value at present epoch.

6. CONCLUSIONS

The present study deals with the phenomenon early stage of the universe and the dynamics of the universe at present epoch with in the framework of isotropic and homogeneous space - time. It is also well established that the mathematical formulation of different cosmological models through the laws of physics becomes an essential component in understanding the nature of the universe. Hence, in this present work, we have investigated FRW cosmological model in the context of Einstein's theory of gravitation. We have studied time varying dark energy states of two different assumptions: i) $\Lambda = \Lambda_1 t^{-2}$ and ii) $\Lambda \propto [R(t)]^{-2n}$. This study reveals that the universe was expanding with positive value of deceleration parameter at early time and it enters into accelerating phase at $z_t = 0.73$. Moreover, we also observed that the Hubble parameter approaches to infinite when time approaches to zero, this indicates that, the universe describes a power law inflation. The temperature and entropy density of the model remain positive for both the cases. In view of energy conditions, the assumptions yields identical results. Our study suggests that the SEC violates for our model which indicates the domination of dark energy type fluid at present epoch. Finally, we conclude that the second law of thermodynamics remain impact-less in both the assumptions. Furthermore, the study suggests that the models of the universe presented in this paper is finite with increasing rate of expansion that leads accelerating phenomenon of the universe at present epoch. The nature of cosmological constant as function of redshift in the derived models have been graphed in Fig. 11. From Fig. 11, we observe that the cosmological constant Λ decrease with redshift z and finally it approaches a small positive value at present epoch ($z = 0$) which is supported by type Ia supernova observations [2, 3, 4].

Declaration of competing interest

In this paper the authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported.

Acknowledgments

The authors wish to place on record their sincere thanks to the editor and reviewer(s) for illuminating suggestions that have significantly improved the manuscript in terms of research quality.

ORCID

Neeru Goyal, https://orcid.org/0009-0007-0283-0584; **Anil Kumar Yadav**, https://orcid.org/0000-0002-5174-5542; **Tensubam Alexander Singh**, https://orcid.org/0000-0003-2968-9350; **Aditya Sharma Ghrera**, https://orcid.org/0000-0002-2715-5390; **Asem Jotin Meitei**, https://orcid.org/0000-0003-3384-5264; **Kangujam Priyokumar Singh**, https://orcid.org/0000-0002-8784-4091

REFERENCES

[1] J.A.S. Lima, A.S.M. Germano, and L.R.W. Abramo, Phys. Rev. D, **53**, 4287 (1996). https://doi.org/10.1103/PhysRevD.53.4287

[2] S. Permutter, et al., Nature, **391**, 51 (1998). https://doi.org/10.1038/34124

[3] A.G. Riess, et al., Astron. J. **116**, 1009 (1998). https://doi.org/10.1086/300499

[4] S. Perlmutter et al., Astrophys. J. **517**, 565 (1999). https://doi.org/10.1086/307221

[5] E.J. Copeland, M. Sami, and S. Tsujikawa, Int. J. of Mod. Phys. D, **15**, 1753 (2006). https://doi.org/10.1142/S021827180600942X

[6] B. Ratra, M.S. Vogeley, Publ. Astron. Soc. Pacific, **120**(865), 235 (2008). https://doi.org/10.1086/529495

[7] J.A. Frieman, M.S. Turner, and D. Huterer, Annu. Rev. Astron. Astrophys. **46**, 385 (2008). https://doi.org/10.1146/annurev.astro.46.060407.145243

[8] H.K. Jassal, J.S. Bagla, and T. Padmanabhan, Mon. Not. R. Astron. Soc. **405**, 2639 (2010). https://doi.org/10.1111/j.1365-2966.2010.16647.x

[9] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989). https://doi.org/10.1103/RevModPhys.61.1

[10] I. Zlatev, L. Wang, P. J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999). https://doi.org/10.1103/PhysRevLett.82.896

[11] Y. Chen, Z.H. Zhu, J. S. Alcaniz, and Y. Gong, Astrophys. J. **711**, 439 (2010). https://doi.org/10.1088/0004-637X/711/1/439

[12] L. Amendola, Phys. Rev. D, **62**, 043511 (2000). https://doi.org/10.1103/PhysRevD.62.043511

[13] G. Caldera-Cabral, R. Maartens, and L.A. Ureña-López, Phys. Rev. D, **79**, 063518 (2009). https://doi.org/10.1103/PhysRevD.79.063518

[14] I. Prigogine, J. Geheniau, E. Gunzig, and P. Nardone, Proc. Natl. Acad. Sci. **85**, 7428 (1988).

[15] H. Moradpour, N. Sadeghnezhad, S. Ghaffari, and A. Jahan, Adv. High Energy Phys. **2017**, 9687976 (2017). https://doi.org/10.1155/2017/9687976

[16] G.C. Samanta, R. Myrzakulov, and P. Shah, Zeitschrift für Naturforsch. A, **72**, 365 (2017). https://doi.org/10.1515/zna-2016-0472

[17] S.H. Shekh, S. Arora, V.R. Chirde, and P.K. Sahoo, Int. J. Geom. Methods Mod. Phys. **17**, 2050048 (2020). https://doi.org/10.1142/S0219887820500486

[18] M. Jamil, D. Momeni, M. Raza, and R. Myrzakulov, Eur. Phys. J. C, **72**, 1999 (2012). https://doi.org/10.1140/epjc/s10052-012-1999-9

[19] Ö. Akarsu, and T. Dereli, Int. J. Theor. Phys. **51**, 612 (2012). https://doi.org/10.1007/s10773-011-0941-5

[20] Ö. Akarsu, T. Dereli, S. Kumar, and L. Xu, Euro. Phys. J Plus, **129**, 22 (2014). https://doi.org/10.1140/epjp/i2014-14022-6

[21] R.N. Patra, A.K. Sethi, B. Nayak, and R.R. Swain, New Astron. **66**, 74 (2019). https://doi.org/10.1016/j.newast.2018.08.001

[22] S.K.J. Pacif, Euro. Phys. J. Plus, **135**, 792 (2020). https://doi.org/10.1140/epjp/s13360-020-00769-y

[23] S.W. Hawking, and G.F.R. Ellis, *The Large Scale Structure of Space-time*, (Press Syndicate of the University of Cambridge, New York, 1973).

[24] R. Schoen, and S.T. Yau, Commun. Math. Phys. **79**, 231 (1981). https://doi.org/10.1007/BF01942062

[25] S. Perlmutter et al., Astrophys. J. **517**, 565 (1999). https://doi.org/10.1086/307221

[26] B. Wang, E. Abdalla, F. Atrio-Barandela, and D. Pavon, Rep. Prog. Phys. **79**, 096901 (2016). https://doi.org/10.1088/0034-4885/79/9/096901

[27] P.J.E. Peebles, and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003). https://doi.org/10.1103/RevModPhys.75.559

[28] T. Padmanabhan, Phys. Rep. **380**, 235 (2003). https://doi.org/10.1016/S0370-1573(03)00120-0

[29] E.J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D, **15**, 1753 (2006). http://dx.doi.org/10.1142/S021827180600942X

[30] M. Li, X.D. Li, S. Wang, and Y. Wang, Front. Phys. **8**, 828 (2013). https://doi.org/10.1007/s11467-013-0300-5

[31] S. Weinberg, *Sources and Detection of Dark Matter and Dark Energy in the universe: The cosmological constant problem*, (Springer, 2001), pp. 18–26.

[32] V. Sahni, Classical Quantum Gravity, **19**, 3435 (2002). https://doi.org/10.1088/0264-9381/19/13/304

[33] J. Garriga, and A. Vilenkin, Phys. Rev. D, **64**, 023517 (2001). https://doi.org/10.1103/PhysRevD.64.023517

[34] J.A. Frieman, M.S. Turner, and D. Huterer, Annu. Rev. Astron. Astrophys. **46**, 385 (2008). https://doi.org/10.1146/annurev.astro.46.060407.145243

[35] S. Nojiri, and S.D. Odintsov, Phys. Rep. **505**, 59 (2011). https://doi.org/10.1016/j.physrep.2011.04.001

[36] R. Caldwell, and M. Kamionkowski, Annu. Rev. Astron. Astrophys. **59**, 397 (2009). https://doi.org/10.1146/annurev-nucl-010709-151330

[37] Ö Akarsu, and C.B. Killinc, Gen. Relativ. Gravit. **42**, 763 (2010). https://doi.org/10.1007/s10714-009-0878-7

[38] S. Kumar, and A.K. Yadav, Mod. Phys. Lett. A, **26**, 647 (2011). https://doi.org/10.1142/S0217732311035018

[39] A.K. Yadav, Astrophys. Space Sci. **335**, 565 (2011). https://doi.org/10.1007/s10509-011-0745-3

[40] A.K. Yadav, and L. Yadav, Int. J. Theor. Phys. **50**, 218, (2011). https://doi.org/10.1007/s10773-010-0510-3

[41] S. Nojiri, and S.D. Odintsov, Phys. Rev. D, **70**, 103522 (2004). https://doi.org/10.1103/PhysRevD.70.103522

[42] S. Nojiri, S.D. Odintsov, and S. Tsujikawa, Phys. Rev. D, **71**, 063004 (2005). https://doi.org/10.1103/PhysRevD.71.063004

[43] S. Nojiri, and S.D. Odintsov, Gen. Relativ. Gravit. **38**, 1285 (2006). https://doi.org/10.1007/s10714-006-0301-6

[44] S. Nojiri, and S.D. Odintsov, Phys. Rev. D, **72**, 023003 (2005). https://doi.org/10.1103/PhysRevD.72.023003

[45] J.C. Carvalho, J.A.S. Lima, and I. Waga, Phys. Rev. D, **46**, 2404 (1992). https://doi.org/10.1103/PhysRevD.46.2404

[46] J.M. Salim, and I. Waga, Class. Quant. Grav. **10**, 1767 (1993). https://doi.org/10.1088/0264-9381/10/9/018

[47] M.S. Berman, Phys. Rev. D, **43**, 1075 (1991). https://doi.org/10.1103/PhysRevD.43.1075

[48] A.K. Yadav, Int. J. Theor. Phys. **49**, 1140 (2010). https://doi.org/10.1007/s10773-010-0295-4

[49] A.K. Yadav, Astrophys. and Space Sc. **361**, 1 (2016).

[50] G.K. Goswami, A.K. Yadav, and B. Mishra, Mod. Phys. Lett. A, **35**, 2050224 (2020). https://doi.org/10.1142/S0217732320502247

[51] H. Amirhashchi, A.K. Yadav, N. Ahmad, and V. Yadav, Phys. Dark Uni. **36**, 101043 (2022). https://doi.org/10.1016/j.dark.2022.101043

[52] G.K. Goswami, M. Mishra, A.K. Yadav, and A. Pradhan, Mod. Phys. Lett. A, **33**, 2050086 (2020). https://doi.org/10.1142/S0217732320500868

[53] S. Kumar, and C.P. Singh, Gen. Relativ. Grav. **43**, 1427 (2011). https://doi.org/10.1007/s10714-010-1125-y

[54] E. Abdalla, *et al.*, J. High Energy Astrophys. **34**, 49 (2022). https://doi.org/10.1016/j.jheap.2022.04.002

[55] E.D. Valentino, *et al.*, Astropart. Phys. **131**, 102605 (2021). https://doi.org/10.1016/j.astropartphys.2021.102605

[56] E.D. Valentino, *et al.*, Astropart. Phys. **131**, 102607 (2021). https://doi.org/10.1016/j.astropartphys.2021.102607

[57] E.D. Valentino, *et al.*, Astropart. Phys. **131**, 102604 (2021). https://doi.org/10.1016/j.astropartphys.2021.102604

[58] E.D. Valentino, *et al.*, Astropart. Phys. **131**, 102606 (2021). https://doi.org/10.1016/j.astropartphys.2021.102606

[59] A.S. Adil, U. Mukhopadhyay, A.A. Sen, and S. Vagnozzi, JCAP **2023(10)**, 072 (2023). https://doi.org/10.1088/1475-7516/2023/10/072

[60] R.C Nunes, S. Vagnozzi, S. Kumar, E.Di Valentino, and O. Mena, Phys. Rev. D, **105**, 123506 (2022). https://doi.org/10.1103/PhysRevD.105.123506

[61] E.D. Valentino, S. Gariazzo, O. Mena, and S. Vagnozzi, JCAP **2020(7)**, 045 (2020). https://doi.org/10.1088/1475-7516/2020/07/045

[62] D.M. Scolnic, *et al.*, Astrophys. J. **859**, 101 (2018). https://doi.org/10.3847/1538-4357/aab9bb

[63] G.S. Sharov, and V.O. Vasiliev, Mathematical Modelling and Geometry **6**, 1 (2018). https://mmg.tversu.ru/images/publications/2018-611.pdf

[64] P. Biswas, P. Roy, and R. Biswas, https://doi.org/10.48550/arXiv.1908.00408.

[65] A.K. Yadav, *et al.*, J. High Energy Astrophys. **43**, 114 (2024). https://doi.org/10.1016/j.jheap.2024.06.012

[66] S.M. Carroll, *Spacetime and geometry: An introduction to general relativity*, (Carlifonia, USA, 2004)

[67] M. Visser, "Lorentzian Wormholes: From Einstein to Hawking," in: *AIP Series in Computational and Applied Mathematical Physics*, (American Institute of Physics, 1995).

ТЕРМОДИНАМІКА ОДНОРІДНОГО ТА ІЗОТРОПНОГО ВСЕСВІТУ ДЛЯ РІЗНИХ УМОВ ТЕМНОЇ ЕНЕРГІЇ

Ніру Гоял[a], Аніл Кумар Ядав[b], Тенсубам Олександр Сінгх[c], Адітя Шарма Грера[a], Асем Джотін Мейтей[d,e], Кангуджам Прійокумар Сінгх[d*]

[a]Департамент прикладних наук, Університет НортКап, Гуруграм - 122017, Індія

[b]Департамент фізики, Об'єднаний коледж інженерії та досліджень, Велика Нойда - 201310, Інді

[c]Департамент математичних наук, Університет Бодоленд, Кокрайхар-783370, Ассам, Індія

[d]Департамент математики, коледж Правабаті, Маянг Імпхал-795132, Маніпур, Індія

[e]Департамент математики, Маніпурський університет, Канчіпур, Імфал-795003, Маніпур, Індія

Досліджено термодинамічні властивості однорідного та ізотропного Всесвіту для різних умов темної енергії з спадаючим космологічним членом $\Lambda(t)$. Щоб отримати явний розв'язок рівнянь поля Ейнштейна, ми розглянули лінійно змінний параметр уповільнення у формі $q = -\alpha t + m - 1$ з α і m як скалярними константами. Ми обмежили параметри моделі H_0 і m як $68,495$ км/с/Мпк і $1,591$ відповідно, обмеживши похідну модель комбінованою компіляцією пантеону наборів даних SN Ia і H(z). Крім того, ми досліджували стани темної енергії, що змінюються в часі, для двох різних припущень: i) $\Lambda = \Lambda_1 t^{-2}$ та ii) $\Lambda \propto [R(t)]^{-2n}$. Для конкретного припущення наші моделі вказують на поведінку, подібну до темної енергії, у відкритому, плоскому та закритому просторі – геометрія часу. Температура та щільність ентропії моделі залишаються додатними для обох випадків: i) $\Lambda = \Lambda_1 t^{-2}$ і ii) $\Lambda \propto [R(t)]^{-2n}$.. Також обговорюються деякі фізичні властивості Всесвіту.

Ключові слова: *модель FRW; однорідний; термодинаміка; пантеон; темна енергія*