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FLRW COSMOLOGICAL MODEL IN F(R,T) GRAVITY

Aroonkumar Beesham*
Mangosuthu University of Technology, PO Box 12363, Jacobs 4026, South Africa
**Corresponding Author e-mail: abeesham@yahoo.com*

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In this paper, the Friedmann-Lemaitre-Robertson-Walker cosmological models with a perfect fluid in the f(R,T) theory of gravity are re-discussed. There are several ways to generate solutions. One way is to assume a barotropic equation of state. The other is to use a deceleration parameter that varies linearly with time. An existing solution in the literature is reviewed, where solutions are obtained by assuming, in addition to a barotropic equation of state, a linear varying deceleration parameter. It is pointed out such an assumption leads to an over-determination of the solution. Hence, the feasibility of the solutions is a necessary condition to be satisfied. Only one of the assumptions of an equation of state or of a linearly varying deceleration parameter is sufficient to generate solutions. The proper solutions are given and discussed.

Keywords: *f(R,T) gravity; FLRW models; Linear varying deceleration parameter; Cosmological solutions; Feasibility of solutions*

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1. INTRODUCTION

Recent observations from the anisotropy of the Cosmic Microwave Background (CMB) [1], supernova type Ia (SNeIa) [2], large scale structure [3], baryon acoustic oscillations [4] and weak lensing [5] indicate the phenomenon of the accelerated expansion of the universe at late times. At early times the universe was decelerating, and there was a transition from deceleration to acceleration. There are bacsically two ways to try to explain this. One is that, in general relativity, the matter of the universe contains an exotic component dubbed dark energy which causes a gravitationally repulsive force. Several candidates have been proposed in this direction [6]-[10]. The other way is a modification of general relativity resulting in modified gravity theories which change the Einstein-Hilbert Lagrangian, such as $f(R)$ gravity [11].

Recently, Harko et al. [12] generalized $f(R)$ gravity by introducing an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor. The dependence upon T (in addition to R in the Lagrangian) may be due to quantum effects (conformal anomaly) or to an exotic imperfect fluid. As a result of coupling between matter and geometry, the motion of test particles is non-geodetic, and an extra acceleration is always present. In $f(R,T)$ gravity, where $f(R,T)$ is an arbitrary function of R and T, cosmic acceleration may result not only from the geometrical contribution to the total cosmic energy density, but from the matter content. This theory can be applied to explore several issues of current interest and may lead to some major differences. Houndjo [13] developed the cosmological reconstruction of $f(R,T)$ gravity for $f(R,T) = f_1(R) + f_2(T)$ and discussed the transition of the deceleration matter dominated era to the acceleration one.

Various aspects of the theory have been explored by literally hundreds of authors since Harko et al [12] introduced that theory. We cite a few of the key articles and also recent papers that have a relation to the work that we do in this article. All these articles contain additional references. Akarsu and Dereli [14] studied accelerating universes with a linearly varying deceleration parameter (LVDP). An LVDP in higher dimensions with strange quark matter and domain walls was investigated by Caglar [15]. Bishi et al [16] have applied a quadratic deceleration parameter to $f(R,T)$ gravity, finding bouncing cosmologies. Sofuoglu et al [17] have applied a cubic deceleration parameter to f(R,T) gravity, finding a big-bang singularity at the beginning, and a big rip one in the future.

The LVDP, as well as other variations of it have attracted a lot of interest. Alkaound et al [18] have studied an LVDP in Lyra's geometry, focussing on observational constraints, and future singularities, such as the big rip. Perturbation theory has been used [19] to study the big rip singularity with a LVDP. Ramesh and Umadevi [20] have studied Friedmann-Lemaitre-Robertson Walker (FLRW) solutions in f(R,T) gravity, in which they obtained solutions by assuming, in addition to a barotropic equation of state, a LVDP. In this study, we review this solution, and point out that, firstly, both those assumptions lead to an over-determination of the solution. Only one of them is sufficient to generate a solution. Secondly, each of the assumptions leads to a different solution. The assumption of an equation of state leads to the equivalent solutions in general relativity. Only the second assumption of a linearly varying deceleration parameter alone leads to a solution that exhibits a transition from deceleration to acceleration. Thirdly, there appear to be several errors in the paper, which we correct here.

This paper is organised as follows. In section 2, we give a brief introduction to f(R,T) gravity. Section 3 provides details of the solution by Ramesh and Umadevi [20]. In section 4, we provide the updated solution and in section 5 we give the conclusion.

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2. REVIEW OF F(R,T) GRAVITY

The action for $f(R,T)$ gravity is:

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\, d^4x + \int L_{\rm m}\sqrt{-g}\, d^4x\,, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R, and of the trace T of the energy-momentum tensor of the matter, T_{ab}. L_m is the matter Lagrangian density, and the energy-momentum tensor of matter is defined as:

$$T_{ab} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{ab}}\,, \tag{2}$$

and the trace of T_{ab} by $T = g^{ab}T_{ab}$. By assuming that the Lagrangian density $L_{\rm m}$ of matter depends only on the metric tensor components g_{ab}, and not on its derivatives, we obtain:

$$T_{ab} = g_{ab}L_m - 2\frac{\partial L_m}{\partial g^{ab}}\,. \tag{3}$$

By varying the action S of the gravitational field with respect to the metric tensor components g^{ab} we get:

$$\delta S = \frac{1}{16\pi}\int\left[f_R(R,T)\,\delta R + f_T(R,T)\frac{\delta T}{\delta g^{ab}}\delta g^{ab} - \frac{1}{2}g_{ab}f(R,T)\,\delta g^{ab} + 16\pi\frac{1}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_{\rm m}\right)}{\delta g^{ab}}\right]\sqrt{-g}d^4x\,, \tag{4}$$

where we have denoted $\partial f(R,T)/\partial R$ by $f_R(R,T)$ and $\partial f(R,T)/\partial T$ by $f_T(R,T)$. For the variation of the Ricci scalar, we obtain

$$\delta R = \delta\left(g^{ab}R_{ab}\right) = R_{ab}\delta g^{ab} + g^{ab}\left(\nabla_d\delta\Gamma^d_{ab} - \nabla_b\delta\Gamma^d_{ad}\right)\,, \tag{5}$$

where ∇_d is the covariant derivative with respect to the symmetric connection Γ associated with the metric g. The variation of the Christoffel symbols yields

$$\delta\Gamma^d_{ab} = \frac{1}{2}g^{de}\left(\nabla_a\delta g_{be} + \nabla_b\delta g_{ea} - \nabla_e\delta g_{ab}\right)\,, \tag{6}$$

and the variation of the Ricci scalar provides the expression

$$\delta R = R_{ab}\delta g^{ab} + g_{ab}\Box\delta g^{ab} - \nabla_a\nabla_b\delta g^{ab}\,. \tag{7}$$

Therefore, for the variation of the action of the gravitational field we obtain

$$\begin{aligned}\delta S \;=\; \frac{1}{16\pi}\int\Bigg[& f_R(R,T)\,R_{ab}\delta g^{ab} + f_R(R,T)\,g_{ab}\Box\delta g^{ab} - f_R(R,T)\,\nabla_a\nabla_b\delta g^{ab} \\ &+ f_T(R,T)\frac{\delta\left(g^{de}T_{de}\right)}{\delta g^{ab}}\delta g^{ab} - \frac{1}{2}g_{ab}f(R,T)\,\delta g^{ab} + 16\pi\frac{1}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{ab}}\Bigg]\sqrt{-g}d^4x\,.\end{aligned} \tag{8}$$

where $\Box = \nabla^d\nabla_d$. We define the variation of T with respect to the metric tensor as

$$\frac{\delta\left(g^{ef}T_{ef}\right)}{\delta g^{ab}} = T_{ab} + \Theta_{ab}\,, \tag{9}$$

where

$$\Theta_{ab} \equiv g^{de}\frac{\delta T_{de}}{\delta g^{ab}}\,. \tag{10}$$

After partially integrating the second and third terms in Eq. (8), we obtain the field equations of the $f(R,T)$ gravity model as

$$f_R(R,T)\,R_{ab} - \frac{1}{2}f(R,T)\,g_{ab} + \left(g_{ab}\Box - \nabla_a\nabla_b\right)f_R(R,T) = 8\pi T_{ab} - f_T(R,T)\,T_{ab} - f_T(R,T)\,\Theta_{ab}\,. \tag{11}$$

Note that when $f(R,T) \equiv f(R)$, from Eqs. (11) we obtain the field equations of $f(R)$ gravity.

By contracting, Eq. (11) gives the following relation between the Ricci scalar R and the trace T of the stress-energy tensor,

$$f_R(R,T)\,R + 3\Box f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)\,T - f_T(R,T)\,\Theta\,, \tag{12}$$

where we have denoted $\Theta = \Theta_{\mu}^{\mu}$.

By eliminating the term $\Box f_R(R,T)$ between Eqs. (11) and (12), the gravitational field equations can be written in the form

$$f_R(R,T)\left(R_{ab}-\frac{1}{3}Rg_{ab}\right)+\frac{1}{6}f(R,T)g_{ab} = 8\pi\left(T_{ab}-\frac{1}{3}Tg_{ab}\right)-f_T(R,T)\left(T_{ab}-\frac{1}{3}Tg_{ab}\right) -f_T(R,T)\left(\Theta_{ab}-\frac{1}{3}\Theta g_{ab}\right)+\nabla_a\nabla_b f_R(R,T)\ . \quad (13)$$

Taking into account the covariant divergence of Eq. (11), with the use of the following mathematical identity [21]

$$\nabla^a\left[f_R(R,T)R_{ab}-\frac{1}{2}f(R,T)g_{ab}+(g_{ab}\Box-\nabla_a\nabla_b)f_R(R,T)\right]\equiv 0\,, \quad (14)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and of the trace of the energy-momentum tensor T, we obtain for the divergence of the stress-energy tensor T_{ab}, the equation

$$\nabla^a T_{ab}=\frac{f_T(R,T)}{8\pi}\left[(T_{ab}+\Theta_{ab})\nabla^a\ln f_T(R,T)+\nabla^a\Theta_{ab}\right]\ . \quad (15)$$

Next we consider the calculation of the tensor Θ_{ab}, once the matter Lagrangian is known. From Eq. (3) we obtain first

$$\frac{\delta T_{de}}{\delta g^{ab}} = \frac{\delta g_{de}}{\delta g^{ab}}L_m+g_{de}\frac{\partial L_{\mathrm{m}}}{\partial g^{ab}}-2\frac{\partial^2 L_m}{\partial g^{ab}\partial g^{de}} = \frac{\delta g_{de}}{\delta g^{ab}}L_{\mathrm{m}}+\frac{1}{2}g_{de}g_{ab}L_m-\frac{1}{2}g_{de}T_{ab}-2\frac{\partial^2 L_m}{\partial g^{ab}\partial g^{de}}\ . \quad (16)$$

From the condition $g_{ad}g^{db}=\delta_a^b$, we have

$$\frac{\delta g_{de}}{\delta g^{ab}}=-g_{df}g_{eh}\delta_{ab}^{fh}\,, \quad (17)$$

where $\delta_{ab}^{fh}=\delta g^{fh}/\delta g^{ab}$ is the generalized Kronecker symbol. Therefore for Θ_{ab} we find

$$\Theta_{ab}=-2T_{ab}+g_{ab}L_{\mathrm{m}}-2g^{de}\frac{\partial^2 L_m}{\partial g^{ab}\partial g^{de}}\ . \quad (18)$$

We take the matter Lagrangian to be given by $L_{\mathrm{m}}=p$. Now, there is degeneracy in the choice of the matter Lagrangian in the sense that this choice does not make any difference to the resulting field equations in general relativity. Hence one could also choose $L_m=-\rho$, where ρ is the energy density. [22]. We now indicate briefly how this Lagrangian leads to the energy momentum tensor (Hawking and Ellis [23] give an excellent derivation of this). The fluid current four-vector is defined as $j^a=\rho u^a$, where u^a is the fluid four-velocity. Now it is assumed that this is conserved, i.e., ${j^a}_{;a}=0$. Taking the Lagrangian to be $L_m=-\rho$, and varying so that the action is stationary, we get the momentum equation:

$$(\rho+p)\dot{u^a}=-p_{;b}(g^{ba}+u^a u^b) \quad (19)$$

where $\rho=\mu(1+\epsilon)$, μ is the density, ϵ is the internal energy and the pressure p is given by $p=\mu^2(d\epsilon/d\mu)$. So $\dot{u}^a$ is the acceleration.

We now turn to the form of the energy momentum tensor. The conservation of current may be expressed as:

$${j^a}_{;a}=\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^a}(\sqrt{-g}j^a) \quad (20)$$

or

$$2\mu\delta\mu=(j^a j^b-j^d j_d g^{ab})\delta g_{ab} \quad (21)$$

Now, in general, the Lagrangian L is a scalar function of some fields Ψ^a. The equations of motion can be obtained by the requirement that the action:

$$I=\int L dv \quad (22)$$

be invariant under a variation of the fields in some suitable region. The variation of the fields can be written as an integrand in Δg_{ab} only. Then the integral $\partial I/\partial u$ is:

$$\frac{1}{2}\int(T^{ab}\delta g_{ab})dv \quad (23)$$

where T^{ab} are the components of a symmetric tensor which is taken to be the energy momentum tensor of the fields. Thus, from equations (21)-(23), we get:

$$T^{ab} = \left[\mu(1+\epsilon) + \mu^2 \frac{d\epsilon}{d\mu}\right] u^a u^b + \mu^2 \frac{d\epsilon}{d\mu} g^{ab} \tag{24}$$

or, finally

$$T_{ab} = (\rho + p) u_a u_b + p g_{ab} \tag{25}$$

The four-velocity u_a satisfies the conditions $u_a u^a = -1$ and $u^a \nabla_b u_a = 0$. Then, with the use of Eq. (18), we obtain for the variation of the energy momentum of a perfect fluid the expression

$$\Theta_{ab} = -2T_{ab} + p g_{ab} \,. \tag{26}$$

As in the case of [20], we take $f(R,T) = R + 2\mathcal{F}(T)$, where $\mathcal{F}(T) = \lambda T$. The gravitational field equations immediately follow from Eq. (11), and are given by

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab} - 2\mathcal{F}'(T)\, T_{ab} - 2\mathcal{F}'(T)\Theta_{ab} + \mathcal{F}(T) g_{ab} \,, \tag{27}$$

where the prime denotes a derivative with respect to the argument.

For the perfect fluid (25), $\Theta_{ab} = -2T_{ab} + p g_{ab}$, and the field equations become

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab} + 2\mathcal{F}'(T)\, T_{ab} - 2p\mathcal{F}'(T) g_{ab} + \mathcal{F}(T) g_{ab} \,. \tag{28}$$

The above equation for $\mathcal{F}(T) = \lambda T$, i.e., $f(R,T) = R + 2\lambda T$, where the trace $T = -\rho + 3p$ finally simplifies as follows:

$$R_{ab} - \frac{1}{2} R g_{ab} = (8\pi + 2\lambda) T_{ab} + \lambda (p - \rho) g_{ab}. \tag{29}$$

3. BRIEF OUTLINE OF THE RAMESH/UMADEVI PAPER

In this section, we briefly outline the paper [20]. The FLRW metric was given by

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{30}$$

where a is the scale factor, (r, θ, ϕ) are the usual spherical coordinates, and k represents the geometrical curvature of the universe, i.e., $k = 0$ implies a flat universe, $k = +1$ is a closed universe, and $k = -1$ is an open universe. For the FLRW metric (30), and the energy–momentum tensor (25), the field equations (29) in $f(R,T)$ gravity have been given as [20]:

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}^2}{a^2}\right) + \frac{k}{a^2} = p(8\pi + 7\lambda) - \lambda p, \tag{31}$$

$$3\left(\frac{\dot{a}^2}{a^2}\right) + 3\frac{k}{a^2} = p(8\pi + 3\lambda) + 5\lambda p. \tag{32}$$

Ramesh and Umadevi then make two assumptions to derive their solutions, viz.,

1. A barotropic equation of state (EoS) of the form

$$p = \epsilon\rho, \ \epsilon = constant \tag{33}$$

where the constant $\epsilon = -1$ to describe DE, $\epsilon = 0$ for pressure-free matter (dust), $\epsilon = 1/3$ for radiation.

2. A linear varying deceleration parameter (LVDP) q of the form [14]

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -nt + m - 1, \tag{34}$$

where $n \geq 0$ and $m \geq 0$ are constants.

Solving (34), the solutions were given as the three different forms for the scale factor:

$$a = a_0\, exp\left[\frac{2}{\sqrt{m^2 - 2c_1 n}} \text{arctanh}\left(\frac{nt - m}{\sqrt{m^2 - 2c_1 n}}\right)\right] \quad \text{for } n > 0 \text{ and } m \geq 0, \tag{35}$$

$$a = a_0 (mt + c_2)^{1/m} \quad \text{for } n = 0 \text{ and } m > 0, \tag{36}$$

$$a = a_0 e^{c_1 t} \quad \text{for } n = 0 \text{ and } m = 0, \tag{37}$$

where a_0, a_1, a_2, a_3, c_1, c_2 and c_3 are constants of integration.

It is then stated that by taking $a_0 = 0$ in equation (35), the following solution is obtained:

$$a(t) = exp\left[\frac{2}{m}\text{arctanh}\left(\frac{n}{m}t - 1\right)\right]. \tag{38}$$

The Hubble parameter was given as:

$$H = \frac{\dot{a}}{a} = -\frac{2}{t(nt - 2m)}. \tag{39}$$

and the energy density as:

$$\rho = \frac{1}{[8\lambda(3\epsilon + 1) + 16\epsilon\lambda]}\left[\frac{24(nt - m + 1)}{(nt^2 - 2mt)^2}\right] \tag{40}$$

Since they assumed the barotropic equation of state (33) where ϵ is a constant, the pressure is just $p = \epsilon\rho$:

$$p = \frac{\epsilon}{[8\lambda(3\epsilon + 1) + 16\epsilon\lambda]}\left[\frac{24(nt - m + 1)}{(nt^2 - 2mt)^2}\right] \tag{41}$$

4. REVIEW OF THE SOLUTION IN PREVIOUS SECTION 3

In this section, we first go through the paper [20] as discussed in the previous section, correcting the equations.

- Equations (31) and (32) should read as follows:

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}^2}{a^2}\right) + \frac{k}{a^2} = \lambda\rho - (8\pi + 3\lambda)p, \tag{42}$$

$$3\left(\frac{\dot{a}^2}{a^2}\right) + 3\frac{k}{a^2} = (8\pi + 3\lambda)\rho - \lambda p. \tag{43}$$

- Then in the solutions (35), the constants $a_1,\ a_2,\ a_3$ are missing.
- They state that by taking $a_0 = 0$ in equation (35), a solution is obtained. However, if one takes $a_0 = 0$ in equation (35), then one gets $a = 0$.
- In the paragraph just before the conclusion, it is claimed that the energy density ρ us always positive iirrespective of the curvature of the space. However, this is only true if the constants n, m, λ, ϵ are such as to allow positivity - they have to ensure that both numerator and denominator in the equation for the energy density (40) are both positive, or both are negative.
- In their solutions (40) and (41), the "8λ" in the denominators should read "8π".
- We notice that equations (42) and (43) are two equations in the three unknowns a, ρ and p. Hence only one extra condition is necessary to solve these equations. However, in their paper, Ramesh and Umadevi [20] have chosen two conditions, viz., (33) and (34). We now show that any one of them is sufficient to generate solutions, but that only the second condition allows for the transition from an early decelerated universe to a late accelerated one.

Let us start with the first condition of a barotropic equation of state (33), where ϵ is a constant. In this case, equations (42) and (43) can be written as:

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}^2}{a^2}\right) + \frac{k}{a^2} = \lambda\rho - (8\pi + 3\lambda)\epsilon\rho, \tag{44}$$

$$3\left(\frac{\dot{a}^2}{a^2}\right) + 3\frac{k}{a^2} = (8\pi + 3\lambda)\rho - \lambda\epsilon\rho. \tag{45}$$

Without loss of generality, we now focus on the case $k = 0$, which can easily be extended to the cases $k = \pm 1$. This assumption of the EoS (33) alone is sufficient to obtain a solution since we then have only two unknowns, viz., a and ρ, and two equations. From equations (44) and (45), we obtain the following equation for ρ:

$$[(8\pi + \lambda)^2 - \lambda^2]\rho = -2\lambda\left(\frac{\ddot{a}}{a}\right) + (24\pi + 8\lambda)\left(\frac{\dot{a}^2}{a^2}\right) \tag{46}$$

It is possible to also write a similar equation for the pressure p alone by eliminating the energy density ρ from equations (42) and (43).The two resulting equations will be sufficient to obtain solutions. The general solutions to these equations are quite complicated, involving hypergeometric functions, so we do not list them here. This is quite unlike general relativity.

We now focus on the second condition of a LVDP (34) alone. This equation alone is enough to find a solution without the additional need for an equation of state. We first note that in the solutions given by [20], viz., (35), (36) and (37), the constants a_1, a_2, a_3 as well as c_3 do not appear. The corrected solutions as given by Akarsu and Dereli [14] to equation (34) are:

$$a = a_1\, exp\left[\frac{2}{\sqrt{m^2 - 2c_1 n}}\text{arctanh}\left(\frac{nt - m}{\sqrt{m^2 - 2c_1 k}}\right)\right] \quad \text{for } n > 0 \text{ and } m \geq 0, \tag{47}$$

$$a = a_2(mt + c_2)^{\frac{1}{m}} \quad \text{for } n = 0 \text{ and } m > 0, \tag{48}$$

$$a = a_3 e^{c_3 t} \quad \text{for } n = 0 \text{ and } m = 0, \tag{49}$$

where a_1, a_2, a_3, c_1, c_2 and c_3 are constants of integration. In these solutions, we see the constants a_1, a_2, a_3 as well as c_3, and also that there is no a_0. The last two of the above solutions are for constant q, which have been dealt with previously. The new solution (47), was found by [14]. Only the solution for $k > 0$ and $m > 0$ is discussed further, and the integration constant c_1 has been set equal to . This sets the initial time of the universe as $t_i = 0$. If we need early deceleration and late-time acceleration, we have to choose $n > 0$ and $m > 0$ for compatibility with the observed universe. The condition $n > 0$ corresponds to increasing acceleration ($\dot{q} = -n < 0$). In order to get early deceleration, the condition $m > 0$ must hold, and it can even be $m > 1$. Hence equation (47) is reduced to:

$$a = a_1\, exp\left[\frac{2}{m}\text{arctanh}\left(\frac{n}{m}t - 1\right)\right]. \tag{50}$$

We now have to find the energy density and pressure from equations (42) and (43). Let us first find the Hubble parameter $H = \dot{a}/a$. From equation (50), we find the Hubble parameter as:

$$H \equiv \frac{\dot{a}}{a} = -\frac{2}{t(nt - 2m)}. \tag{51}$$

From the above two equations, we find

$$\frac{\ddot{a}}{a} = \frac{4nt - 4m + 4}{2mt - nt^2} \tag{52}$$

From equations (42) and (43), we can derive an expression for the energy density ρ (for $k = 0$):

$$(64\pi^2 + 16\pi\lambda)\rho = (24\pi + 8\lambda)H^2 - 2\lambda(\dot{H} + H^2) \tag{53}$$

and then using equations (50) and (51), we find that:

$$\rho = \frac{96\pi + 24\lambda - 8\lambda nt + 8\lambda m}{(64\pi^2 + 16\pi\lambda)(nt - 2m)^2 t^2} \tag{54}$$

This solution for ρ is a generalisation of the one given by Akarsu and Dereli [14] for the case $k = 0$, and it reduces to that when $\lambda = 0$ (note the system of units we are using corresponds to that used in [20] in which they put only the gravitational constant $G = 1$. In ref [14], the condition $8\pi G = 1$ is used). The pressure may also be determined similarly, as well as for the cases $k = \pm 1$.

Now we determine the equation for the pressure using the LVDP. Again, from equations (50) and (51), we get (for $k = 0$):

$$p = -\frac{64\pi(kt - m + \frac{3}{2}) + 24\lambda - 24m\lambda + 24n\lambda t}{(64\pi^2 + 48\pi\lambda + 8\lambda^2)(nt^2 - 2mt)^2}. \tag{55}$$

The equation of state $\omega = p/\rho$ is given by:

$$\omega = \frac{(64\pi(nt - m + \frac{3}{2}) + 24\lambda - 24m\lambda + 24n\lambda t)(64\pi^2 + 16\pi\lambda)}{(64\pi^2 + 48\pi\lambda + 8\lambda^2)(96\pi + 24\lambda - 8\lambda nt + 8\lambda m)} \tag{56}$$

It can be seen clearly that the pressure (55) is not just a multiple of the energy density (54), as can also be seen from the equation of state (56). If we put $\lambda = 0$, then we recover general relativity, and the corresponding equations as in [14]. They have plotted all these parameters in general relativity, and shown that with a LVDP, it is possible to obtain a transition from deceleration to acceleration. In addition, they have shown that for the values $m = 0.097$ and $n = 1.6$, it is possible to satisfy observational constraints.

5. CONCLUSION

In this work, we have discussed the FLRW solutions in $f(R,T)$ using a linear deceleration parameter. We first began by giving a brief review of the $f(R,T)$ theory of gravity. Then we discussed the paper by Ramesh and Umadevi [20]. Various points from that paper were been clarified. The full solutions to the equations for a linearly varying deceleration parameter as proposed by Akarsu and Dareli were provided and discussed next. We note the following:

- The solutions with a LVDP do not have a barotropic equation of state in general.
- In the above sense, either of the assumptions made is not compatible with the other, and each has to be made separately to generate solutions.
- The solutions in f(R,T) theory provide a transition from deceleration to acceleration.
- The kinematical quantities such as the scale factor, Hubble parameter and deceleration parameter have the same behaviour as that discusssed by Akarsu and Dareli [14].
- f(R,T) offers a wider range of possibilities than general relativity.
- Several investigations have been made with slightly different forms of the LVDP such as linear in different forms of time t, the redshift z, or in the scale factor a [24]. These authors found that these models compare just as well, if not better, than the standard ΛCDM model.

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ORCID

Aroonkumar Beesham, https://orcid.org/0000-0001-5350-6396

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КОСМОЛОГІЧНА МОДЕЛЬ FLRW У f(R,T) ГРАВІТАЦІЇ

Арункумар Бішем

Технологічний університет Мангосуту, ПС 12363, Джейкобс, 4026, Південна Африка

У цій статті обговорюються космологічні моделі Фрідмана-Леметра-Робертсона-Уокера з ідеальною рідиною в f(R,T) теорії гравітації. Існує кілька способів створення рішень. Один із способів — припустити баротропне рівняння стану. Інший полягає у використанні параметра уповільнення, який змінюється лінійно з часом. Оглядається існуюче рішення в літературі, де рішення отримані шляхом припущення, на додаток до баротропного рівняння стану, лінійного змінного параметра уповільнення. Зазначається, що таке припущення призводить до надмірної визначеності рішення. Отже, здійсненність рішень є необхідною умовою, яка повинна бути задоволена. Лише одне з припущень рівняння стану або лінійно змінного параметра уповільнення є достатнім для створення рішень. Надаються та обговорюються відповідні рішення.

Ключові слова: *f(R,T) гравітація; моделі FLRW; лінійний змінний параметр уповільнення; космологічні рішення; здійсненність рішень*