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NUMERICAL INVESTIGATION OF JOULE HEATING EFFECT ON MICROPOLAR NANOFLUID FLOW OVER AN INCLINED SURFACE IN PRESENCE OF HEAT SOURCE

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The MHD boundary layer flow of a micropolar nanofluid across an inclined stretching surface in the presence of a heat source is examined in this paper. This study employs permeable inclined surfaces with energy flow as its primary observation with heat radiation and the Dufour impact. Additionally, the impact of Joule heating, viscous dissipation and heat source on the porous media are considered. This study uses similarity transformations to convert nonlinear partial differential equations that governs the flow to ordinary differential equations. The bvp4c computational technique in MATLAB is used to illustrate the numerical findings. Based on the findings we were able to determine that the velocity and angular velocity of the fluid increases with the angle of inclination, the temperature profile increases with the increasing values of Eckert number whereas the concentration profile decreases with Eckert number. These findings are further illustrated through numerical data presented in table and visual representations in figures. These findings will enable engineers and scientists to better control fluid flow, leading to improvements in complex systems that rely on it.
Keywords: *MHD; Micropolar nanofluid; Joule heating; Inclined sheet; Heat source; Porous medium; Radiation*
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INTRODUCTION

The dynamics of fluid flows caused by stretching surfaces have received extensive consideration. It is due of their widespread use in designing and industrial processes. The boundary layer flow toward an inclined surface has drawn interest from scientists because of its modern and building applications, such as material fabricated by expulsion, paper making, hot moving, plastic manufacture, and versatile expulsion. Sakiadis [1] initially studied the boundary layer fluid flow of a viscous incompressible fluid on a continuous moving surface. Gupta et al. [2] utilized a similar solution approach to explore heat and mass transfer within the boundary layer of a stretched sheet subjected to blowing or suction. The investigation of micropolar nanofluid flow across an inclined surfaces has been a subject of significant interest in the field of heat transfer and fluid dynamics. Nanofluids, which are a class of engineered fluids containing nanoparticles, have shown great potential in enhancing heat transfer capabilities compared to their base fluids. Moreover, the incorporation of micropolar effects, which account for the rotational motion of fluid particles, adds an additional layer of complexity to the analysis of such flows [3].

An inclined surface is a surface which is neither vertical nor horizontal, but has a slope or angle relative to a reference plane or surface. An inclined surface's geometry may be characterized using the slope, angle of inclination, and surface dimensions. The angle of inclination is the angle formed between an inclined surface and a horizontal plane. Deebani et al. [4] investigate the flow of a 2D micropolar fluid across an inclined linear shrinking/stretching surface under suction, convection, slip, and thermal radiation impact. Roja et al. [5] explored the two-dimensional steady incompressible MHD flow of a micropolar fluid over an inclined permeable surface with natural convection. Suriyakumar et al. [6] observed the combined influences of internal heat generation and suction on mixed convection nanofluid flow across an inclined surface. Meanwhile, Ziaei Rad et al. [7] derived similarity solutions for nanofluid flow within the boundary layer of an inclined surface. Selva Rani et al. [8] conducted an investigation on the convective heat transfer properties of nanofluids flowing over an inclined plate, incorporating the influences of thermal radiation and a variable surface temperature. Rafique et al. [9, 10] studied about the boundary layer flow of micropolar nanofluid over linearly inclined stretching surface under the influence of a magnetic field. Eid et al. [11] conducted a mathematical study on the energy transfer dynamics of micropolar magnetic viscous nanofluid flow over an inclined permeable surface, considering the effects of Dufour phenomenon and thermal radiation. Waleign et al. [12] developed a mathematical model to investigate how various thermal and physical properties influence the behavior of micropolar nanofluid flow near an inclined surface.

Joule heating occurs due to the interactions between the conductor's atomic structure and the flowing electric current. As charged particles collide, some of their kinetic energy converts to heat, causing the conductor's temperature to rise. Several studies have explored the impact of Joule heating on fluid dynamics and heat transfer under various situations, indicating its major effect on magnetohydrodynamic (MHD) flows. Yadav and Sharma [13] examined the impact of Joule heating on magnetohydrodynamic flow induced by an exponentially moving stretching sheet, embedded in porous media. Srinivasacharya et al. [14] conducted a study examining the combined impacts of Hall current and Joule heating on the

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viscous fluid flow passing over an exponentially stretching sheet. Jayanthi et al. [15] studies MHD nanofluid flow via a stretched vertical surface as impacted by Joule heating, chemical reaction, viscosity dissipation, thermal radiation, and activation energy. Prasad et al. [16] look at the combined effects of Hall current and thermal diffusion on the unsteady MHD free convective rotating flow of nanofluids in a porous media. The study examines flow past a moving vertical semi-infinite flat plate in the presence of a heat source and a chemical reaction.

Motivated by the above works, the objectives of this study is to investigate the Joule heating and heat source effects on magnetohydrodynamic (MHD) micropolar viscous nanofluid flow past over a permeable inclined surface. This is an addition of Joule Heating and Heat source effects to the problem discussed by Eid et al. [11]. The equations that govern the flow are transformed using similarity transformations and solved numerically with MATLAB's bvp4c solver. The impact of key dimensionless parameters on velocity, angular velocity, temperature, solute concentration, and nanoparticle concentration are visualized through graphical representations.

MATHEMATICAL FORMULATION

A two-dimensional (2-D) boundary layer flow of micropolar nanofluid is studied as it moves over an inclined surface that is extending linearly at an angle Ω . The surface and free stream velocities are considered to be $u_w\left(x\right)=bx$ and $u_\infty\left(x\right)=0$, respectively, where 'b' is a constant and x is the coordinate along the surface. A transverse magnetic field is placed perpendicularly to the flow direction with negligible induced magnetic field effects. The micropolar nanofluid contains constantly distributed micropolar finite size particles and nanoparticles, allowing for spinning effects and extra space for particles to move before colliding. The analysis includes the impacts of Brownian motion, Dufour, radiation, Joule heating, heat source, and thermophoresis. The temperature and nanoparticle fraction at the wall are held constant at T_w and C_w , while the ambient values for nanofluid mass and temperature fractions (C_∞ and T_∞) are achieved as the distance from the wall (y) approaches infinity, as shown in Figure 1.

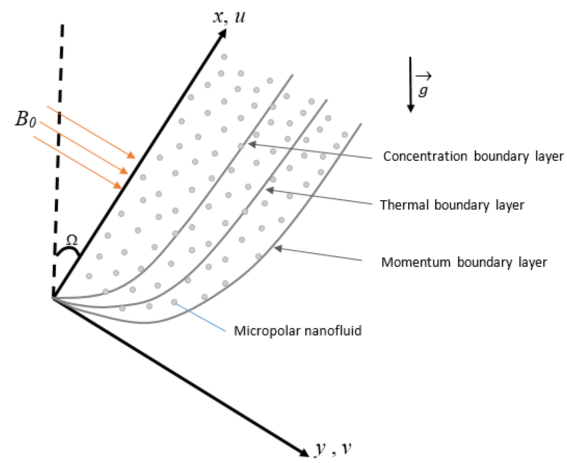

Figure 1. Flow geometry of the problem.

The governing equations of the flow are given as [10, 11]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial N}{\partial y}+g\left[\beta_T\left(T-T_\infty\right)+\beta_C\left(C-C_\infty\right)\right]\cos\Omega-\frac{\sigma B_0^2}{\rho}u-\frac{\upsilon}{K_p'}u \tag{2}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{\kappa}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right)+\left(\frac{\mu+\kappa}{\rho C_P}\right)\left(\frac{\partial u}{\partial y}\right)^2+\frac{D_M K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2}+\frac{Q}{\rho C_P}\left(T-T_\infty\right)+\frac{\sigma B^2 u^2}{\rho C_P} \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2} \tag{5}$$

The boundary conditions are

$$\left.\begin{array}{l} u = u_w = bx, v = v_w, N = -m\dfrac{\partial u}{\partial y},\ T = T_w,\ C = C_w \ \text{ at } \ y = 0 \\ u \to 0,\quad N \to 0,\quad T \to T_\infty,\quad C \to C_\infty \quad \text{as} \quad y \to \infty \end{array}\right\} \tag{6}$$

By using the Rosseland [17] approximation for radiation, radiation heat flux is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

Where σ^* denotes the Stefan–Boltzmann constant and k^* represents mean absorption coefficient. Considering that the variations in temperature throughout the flow such that the term T^4 may be stated as a linear function of the temperature, and expanding T^4 in a Taylor series about T_∞ and neglecting the higher order terms that are beyond the first degree in $(T - T_\infty)$, then we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

Using equations (7) and (8), equation (4) can be written as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(1 + \frac{16T_\infty^3\sigma^*}{3k^*\kappa}\right)\frac{\partial^2 T}{\partial y^2} + \tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right) + \left(\frac{\mu+\kappa}{\rho C_P}\right)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{D_M K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2} + \frac{Q'}{\rho C_P}(T - T_\infty) + \frac{\sigma B^2 u^2}{\rho C_P} \tag{9}$$

We introduce the following similarity variable and dimensionless functions:

$$\eta = \sqrt{\frac{b}{\upsilon}}y,\ f(\eta) = \frac{\psi}{x\sqrt{b\upsilon}},\ h(\eta) = \sqrt{\frac{\upsilon}{b}}\frac{N}{bx},\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

Using the relation $u = \dfrac{\partial \psi}{\partial y}$ and $v = -\dfrac{\partial \psi}{\partial x}$ (where ψ is the stream function), we get

$$u = xbf'(\eta) \ \ and \ \ v = -\sqrt{b\upsilon}\,f(\eta).$$

Using the above transformations the equation of continuity (1) is identically satisfied and other equations (2), (3), (5) and (9) reduces to

$$(1+K)f''' - (f')^2 + ff'' + Kh' + (Gr\,\theta + Gc\,\phi)\cos\Omega - \left(M + \frac{1}{Kp}\right)f' = 0 \tag{10}$$

$$\left(1 + \frac{K}{2}\right)h'' + fh' - f'h - K(2h + f'') = 0 \tag{11}$$

$$\frac{1}{\Pr}\left(1 + \frac{4}{3}R\right)\theta'' + f\theta' + N_b\theta'\phi' + N_t(\theta')^2 + (1+K)Ec(f'')^2 + Q\theta + Ec\,M(f')^2 + Df\,\phi'' = 0 \tag{12}$$

$$\phi'' + Le\,f\,\phi' = 0 \tag{13}$$

Where

$$K = \frac{\kappa}{\mu},\ Gr_x = \frac{g\beta_T(T_w - T_\infty)}{b^2 x},\ Gc_x = \frac{g\beta_c(C_w - C_\infty)}{b^2 x},\ M = \frac{\sigma B_0^2}{b\rho},\ Kp = \frac{bK_p'}{\upsilon},\ Pr = \frac{\upsilon}{\alpha},\ R = \frac{16T_\infty^3\sigma^*}{3k^*k},$$

$$N_b = \frac{\tau D_B(C_w - C_\infty)}{\upsilon},\ N_t = \frac{\tau D_T(T_w - T_\infty)}{\upsilon T_\infty},\ Ec = \frac{u_w^2}{C_P(T_w - T_\infty)},\ Q = \frac{Q'}{b\rho C_P},\ Le = \frac{\upsilon}{D_B},\ Df = \frac{D_M K_T(C_w - C_\infty)}{\upsilon C_S C_P(T_w - T_\infty)}.$$

The boundary conditions (6) reduce to

$$\left.\begin{array}{l} f(\eta) = f_w,\ f'(\eta) = 1, h(\eta) = -m\,f''(0), \theta(\eta) = 1,\ \phi(\eta) = 1 \ \ at \ \ \eta = 0 \\ f'(\eta) = 0,\quad h(\eta) = 0,\quad \theta(\eta) = 0,\quad \phi(\eta) = 0 \quad as \quad \eta \to \infty \end{array}\right\} \tag{14}$$

Here, to eliminate x-dependence in the local Grashof number (Gr_x), local modified Grashof number (Gc_x), β_t (thermal expansion coefficient) and β_c (concentration expansion coefficient) are required to be directly proportional to x^1. Therefore, assume that [18, 19].

$$\beta_t = nx^1, \beta_c = n_1 x^1 \tag{15}$$

Where n_1 and n signifies constants. Substituting equation (15) into the quantities Gr_x and Gc_x, consequences become

$$Gr = \frac{g\,n\left(T_w - T_\infty\right)}{b^2}\ ,\ Gc = \frac{g\,n_1\left(C_w - C_\infty\right)}{b^2}$$

The important physical quantities of interest in this problem are the skin friction coefficient (C_f), the Nusselt number (Nu_x) and the Sherwood number (Sh_x) and defined as

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}\ ,\ Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}\ ,\ C_f = \frac{t_w}{\frac{1}{2}u_w^2\rho}$$

The associated expressions for the skin friction coefficient, the reduced Sherwood number, and the reduced Nusselt Number are as follows:

$$C_{fx} = C_f\sqrt{\mathrm{Re}}\ ,\ -\phi'(0) = \frac{Sh_x}{\sqrt{\mathrm{Re}}}\ \ ,\ -\theta'(0) = \frac{Nu_x}{\sqrt{\mathrm{Re}}}$$

where $\mathrm{Re} = \frac{u_w x}{\nu}$ represents the Reynolds Number.

RESULTS AND DISCUSSIONS

The boundary value problem represented by the equations (10) to (13) with the initial boundary conditions given by in equation (14) is solved using bvp4c solver by developing suitable codes in MATLAB. Graphs are used to analyze the effects of various dimensionless parameters on velocity, angular velocity, temperature, solute concentration profile. In order to verify the accuracy of applied numerical method, a comparison of numerical results of present study with previous study is presented in table 1, where we have found an excellent agreement. The graphical representation of velocity profile, micro-rotation profile, temperature profile, concentration profile for various parameters that appears in the equations are depicted in Figure 2 to Figure 11.

Table 1 is created to verify the appropriateness and efficacy of the bvp4c approach. The results obtained are in good agreement with the literature in limiting cases, confirming the method's suitability.

Table 1. Comparison of findings of the Nusselt number $-\theta'$ (0) at $Gc = Gr = M = K = R = Df = m = Kp = 0,\ Le = Pr = 10\ and\ \Omega = 90°$

Nt	Nb	Khan and Pop [20]	Rafique et al. [10]	Eid et al. [11]	Present Study
0.1	0.1	0.9524	0.9524	0.9524	0.9524
0.3	0.3	0.1355	0.1355	0.1355	0.1354
0.5	0.5	0.0179	0.0179	0.0179	0.0177

Figures 2 to 5 demonstrates how increasing values of magnetic parameter (M) influence the fluid velocity, micro-rotation or angular velocity, temperature, and concentricity profile. Figure 2 reveals a significant decrease in the fluid's velocity as the magnetic variable M is increased.

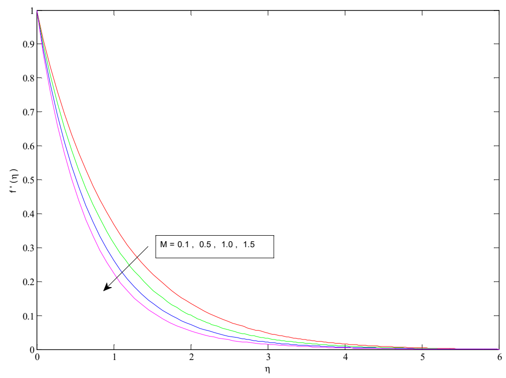

Figure 2. Velocity profile for different M

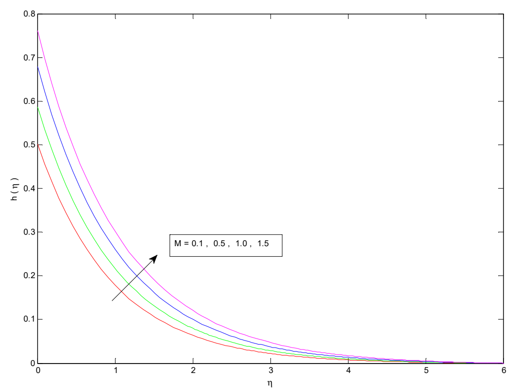

Figure 3. Micro-rotation profile for different M

This phenomenon can be attributed to the Lorentz force, which generates friction and consequently slows down the fluid's movement. Figure 3 shows that the micro-rotation of tiny particles within the fluid exhibits a decreasing trend with the increasing values of *M*.

Figures 4 and 5 shows that temperature *θ(η)* and concentricity *φ(η)* increases with higher values of *M*, as friction generates more heat and mass, leading to increased temperature and concentration.

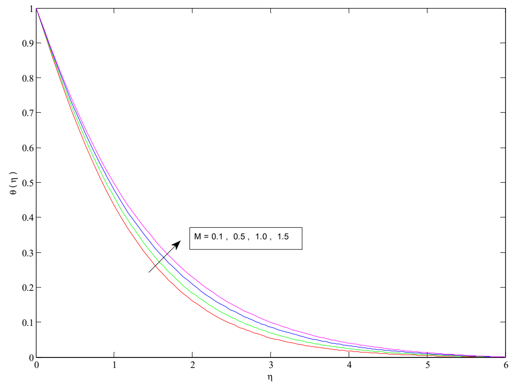

Figure 4. Temperature profile for different *M*

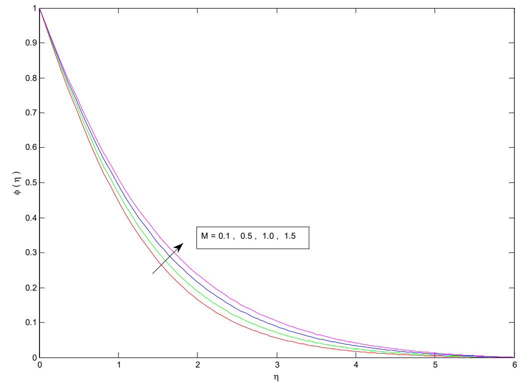

Figure 5. Concentration profile for different *M*

Figures 6 and 7 illustrates the impact of the Eckert number (*Ec*) on the temperature and concentration profiles respectively. Figure 6 reveals a significant relationship between the Eckert number (*Ec*) and temperature. It represents that as Eckert number increases, so does the amount of temperature. Physically, rise in *Ec* increases the fluid friction and fluid particles strike more frequently to each other, consequently they generate the heat energy in the medium. Figure 7 shows that the concentration profile decreases with Eckert number.

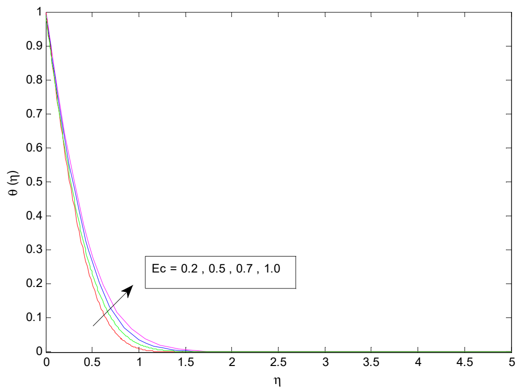

Figure 6. Temperature profile for different *Ec*

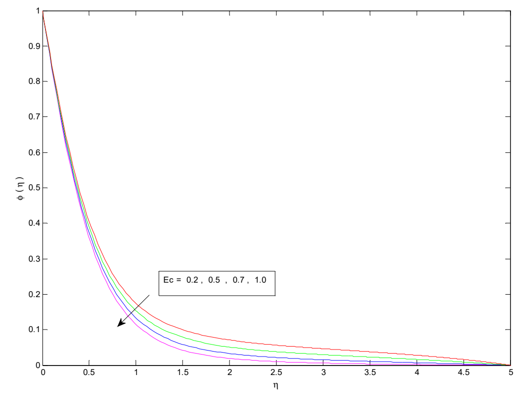

Figure 7. Concentration profile for different *Ec*

Figures 8 and 9 illustrate the impact of the heat source parameter on the temperature and concentration profiles. It is observed that as the heat generation parameter increases, the temperature and concentration profiles increases.

Figures 10 and 11 illustrate the impact of the angle of inclination on the velocity and micro-rotation profiles, which depicts that velocity and angular velocity of the fluid increases as the inclination angle increases. This is due to the increase of buoyancy effect with the inclination angle.

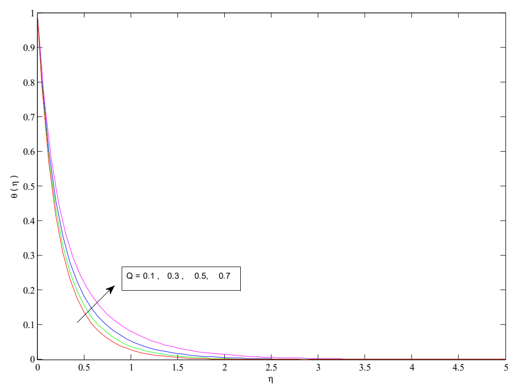

Figure 8. Temperature profile for different *Q*

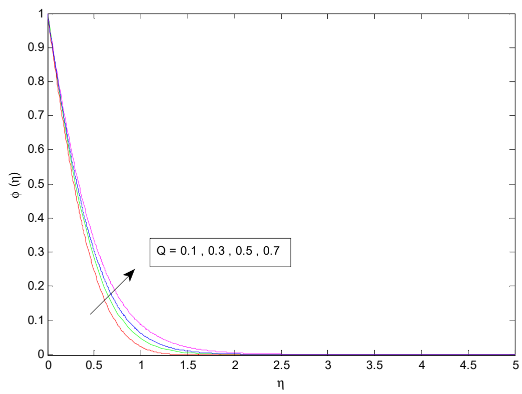

Figure 9. Concentration profile for different *Q*

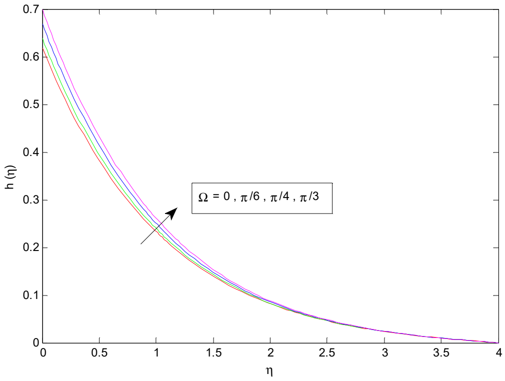

Figure 10. Velocity profile for different Ω

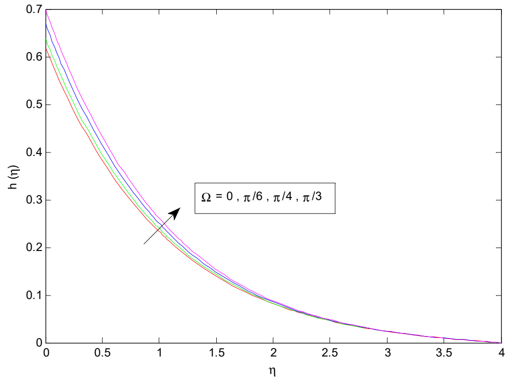

Figure 11. Micro-rotation profile for different Ω

CONCLUSION

In this study, an analysis has been carried out on the effect of magnetic parameter, heat source, Eckert number and angle of inclination on MHD micropolar nanofluid flow past an inclined plate in presence of viscous dissipation, thermal radiation, Dufour effect, Joule heating where it is observed that the parameters have significant influence on the velocity, temperature, concentration and micro-rotation profile. The resulting ordinary differential equations are solved numerically using the bvp4c method. To ensure the accuracy of the computational results obtained in this study, they are compared with findings from previous research. Additionally, the results of the current analysis are presented in a visual format through various graphs, facilitating a clear understanding of the outcomes. Following are some conclusions drawn from the above analysis.

- The velocity and angular velocity of the fluid increases with the angle of inclination.
- The temperature profile increases with the increasing of Eckert number whereas the concentration profile decreases as the Eckert number increases.
- The fluid velocity deceases with the Magnetic Parameter M, whereas micro-rotation, temperature and Nanoparticle concentricity increases with M.

Future scope: Micropolar nanofluids have several uses, including electronic chips, thermal energy retention, polymeric polymers, semiconductor wafers, industrial, and biomedical industries. As a result, in the future, the current analysis will be expanded to include the effects of variable thermal conductivity and viscosity in presence of non-uniform heat source and sink.

Nomenclature

u,v	Velocity components along x and y direction	T_w	Wall temperature
μ	Coefficient of dynamic viscosity	T_∞	Ambient temperature
κ	Coefficient of Vortex viscosity.	Pr	Prandtl Number.
ρ	Free stream density	Ec	Eckert Number.
υ	Coefficient of kinematic viscosity	K	Material parameter.
N	Angular velocity or micro-rotation	Le	Lewis number
γ	Viscosity of spin gradient.	ψ	Stream function
b	Constant.	θ	Dimensionless temperature
D_M	Chemical molecular diffusivity	ϕ	Dimensionless concentration
D_B	Brownian diffusion coefficient,	C_f	The local skin friction coefficient.
$\vec{g}$	Acceleration due to gravity.	Nu	Nusselt number.
β_t	Thermal expansion coefficient	S_h	Sherwood number.
β_c	Concentration expansion coefficient	N_t	Thermophoresis parameter
f_w	Suction (injection)	N_b	Brownian motion parameter
σ^*	Stefan–Boltzmann constant	Gr	Local Grashof number
k^*	Mean absorption coefficient	Gc	Local modified Grashof number
C_P	Specific heat at constant pressure	Ω	Angle of inclination
u_w	Surface velocity	Kp	Porosity parameter
C	Concentricity	R	Radiation parameter
C_w	Species concentration at the surface	Q	Heat Source parameter
C_∞	Fluid concentration outlying the surface	Df	Dufour Number.
T	Temperature	M	Magnetic parameter.
		τ_w	Wall shear stress

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ЧИСЕЛЬНЕ ДОСЛІДЖЕННЯ ВПЛИВУ ДЖОУЛЕВОГО НАГРІВУ НА МІКРОПОЛЯРНИЙ ПОТІК НАНОРІДИНИ ПО НАХИЛІЙ ПОВЕРХНІ ЗА НАЯВНОСТІ ДЖЕРЕЛА ТЕПЛА

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У цій статті досліджено МГД потік мікрополярної нанофлюїду в пограничному шарі через похилу розтягнуту поверхню за наявності джерела тепла. У цьому дослідженні в якості основного спостереження використовуються проникні похилі поверхні з потоком енергії з тепловим випромінюванням і впливом Дюфура. Крім того, розглядається вплив джоулева нагрівання, в'язкої диссипації та джерела тепла на пористі середовища. Це дослідження використовує перетворення подібності для перетворення нелінійних диференціальних рівнянь у частинних похідних, які керують потоком, у звичайні диференціальні рівняння. Для ілюстрації чисельних результатів використовується обчислювальна техніка bvp4c у MATLAB. На підставі отриманих даних ми змогли визначити, що швидкість і кутова швидкість рідини зростає зі збільшенням кута нахилу, температурний профіль зростає зі збільшенням числа Еккерта, тоді як профіль концентрації зменшується зі збільшенням числа Еккерта. Ці висновки додатково ілюструються числовими даними, представленими в таблиці, і візуальними представленнями на малюнках. Ці відкриття дозволять інженерам і вченим краще контролювати потік рідини, що призведе до вдосконалення складних систем, які покладаються на нього.
Ключові слова: *МГД; мікрополярний нанофлюїд; Джоулеве нагрівання; похилий лист; джерело тепла; пористе середовище; випромінювання*