

INFLUENCE OF ELECTROPHYSICAL PARAMETERS OF MAGNETODIELECTRIC LAYER ON A PCP ON ITS ELECTRODYNAMIC CHARACTERISTICS

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Within the framework of the impedance concept, approximate analytical formulas for the distributed surface impedance of the magnetodielectric layer with the inhomogeneous permeability and permittivity located on a perfectly conducting plane (PCP) for the cases of a quadratic law of changes in electrical parameters along the layer thickness are obtained. A comparative analysis of electromagnetic waves reflection coefficient from this structure for various laws of change of the permeability and permittivity is presented.

Keywords: *Magnetodielectric layer; Impedance concept; Surface impedance; Inhomogeneous permeability; Inhomogeneous permittivity; Reflection coefficient*

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INTRODUCTION

One of the ways to expand the limits of change in the electrodynamic characteristics of antenna and waveguide devices in SHF and EHF bands is the deposition of dielectric (ϵ -type), magnetic (μ -type), magnetodielectric (μ, ϵ -type) coatings, in the general case, with variable physical properties, directly on their metal radiating devices (for example, [1]-[12] for ϵ -type, [10]-[14] for μ -type) and waveguide surfaces (for example, [15]-[19] for ϵ -type, [20]-[23] for μ -type, [24]-[31] for μ, ϵ -type), including metamaterial coating [30]-[34]. Modern technologies to produce thin-film coatings make it possible to obtain non-uniform in the direction perpendicular to the ideally conducting plane of the base and inhomogeneous structures [31], [35]-[41]. Note that the nonuniformity of the coating is achieved by using multilayered magnetodielectrics. To calculate the parameters of the devices by setting and solving the corresponding boundary value problem, it is desirable to use the boundary conditions of the impedance type [12], [30], [31], [42]-[48].

An approximate analytical solution of the field equations for an inhomogeneous magnetodielectric layer on a PCP with a linear law of change the permittivity is obtained in [31] by the authors. A similar solution was found in [51] also by the authors for a layer with a linear law of change in magnetic permeability.

In this paper the approximate analytical expressions for the distributed surface impedance of the magnetodielectric layer with the inhomogeneous permeability and permittivity located on a PCP for the cases of a quadratic law of changes in electrical parameters along the layer thickness are presented. If the surface impedance is found, it is not difficult to determine other electrodynamic characteristics of the structure under consideration (for example, reflection (absorption) coefficient [21]-[23] or backscattering cross section [43]). A comparative analysis of reflection characteristics for various laws of change the permeability and permittivity is also presented.

APPROXIMATE BOUNDARY CONDITIONS FOR ELECTROMAGNETIC FIELDS

The one-sided impedance boundary conditions allow to decrease the number of interfacing electrodynamic volumes which should be taken into account for the solution of a problem. Rejecting the need to determine fields inside the adjacent metal-dielectric elements at the problem formulation level is the main benefit of the impedance approach. The Shchukin-Leontovich impedance condition on the connected boundary surface S (see for example [12, 30, 31, 42-48]) can be written in following form:

$$[\mathbf{n}, \mathbf{E}]_S = \bar{Z}_S [\mathbf{n}, [\mathbf{n}, \mathbf{H}]]_S, \quad (1)$$

where \mathbf{E} and \mathbf{H} are the vectors the electrical and magnetic fields with harmonic time dependence (in our case, the time t dependence is $e^{i\omega t}$, $\omega = 2\pi f$ is the circular frequency, f is the frequency, measured in Hz), \mathbf{n} is the normal to impedance surface, directed inside the impedance region, $\bar{Z}_S = \bar{R}_S + i\bar{X}_S = Z_S / Z_0$ is the surface impedance normalized to the resistance of free space $Z_0 = 120\pi$ Ohm.

Thus, only the tangential components of the electromagnetic fields are involved in the boundary condition (1), there are some restrictions on the form of the surface S . It is clear that the condition (1) holds if the radius of surface curvature is much greater than the length of the incident electromagnetic wave.

SURFACE IMPEDANCE OF MAGNETODIELECTRIC LAYER WITH INHOMOGENEOUS PARAMETERS ON PCP

Let us consider a plane layer of a magnetodielectric substance specified in Cartesian coordinate system (x, y, z) for $-\infty < x, y < \infty, -h_d \leq z \leq h_d$ with permeability μ_1 and permittivity ϵ_1 . The layer is placed on PCP at $z = h_d$. Let the plane monochromatic electromagnetic wave with $E_x(z) = E_{0x}e^{-ikz}$ ($k = 2\pi / \lambda$, λ is the wavelength in a free space) be incident from the free half-space $z = -\infty$ on the magnetodielectric layer (Fig. 1).

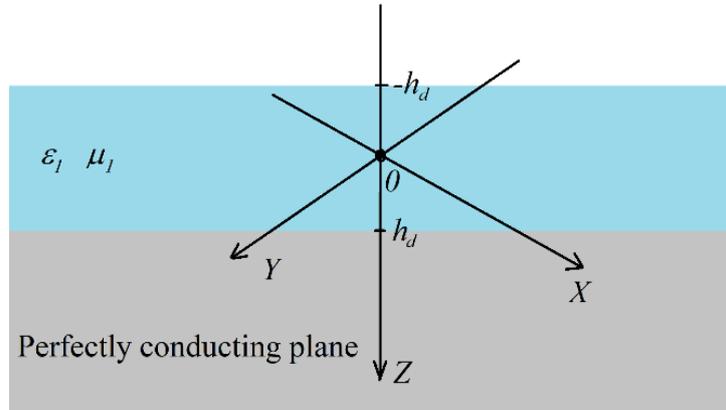


Figure 1. The dipole geometry and accepted designations

Then the distributed surface impedance for this layer determined by the expression (1) can be written as

$$\bar{Z}_S = E_{0x}(-h_d) / H_{0y}(-h_d), \tag{2}$$

where the fields $E_{0x}(-h_d)$ and $H_{0y}(-h_d)$, i.e. $E_x(z)$ and $H_y(z)$ at the plane $z = -h_d$, inside the magnetodielectric layer with material parameters $\mu_1 = \mu_1(z)$, $\epsilon_1 = const$ can be found as solution of the following differential equations

$$\frac{d^2 E_x(z)}{dz^2} - \frac{1}{\mu_1(z)} \frac{d\mu_1(z)}{dz} \frac{dE_x(z)}{dz} + k^2 \mu_1(z) \epsilon_1 E_x(z) = 0, \tag{3a}$$

$$H_y(z) = \frac{i}{k\mu_1(z)} \frac{dE_x(z)}{dz}, \tag{3b}$$

with the boundary conditions on the surfaces $z = \pm h_d$.

For the case $\epsilon_1 = \epsilon_1(z)$, $\mu_1 = const$, the field equations have the form

$$\frac{d^2 E_x(z)}{dz^2} + k^2 \mu_1 \epsilon_1(z) E_x(z) = 0, \tag{4a}$$

$$H_y(z) = \frac{i}{k\mu_1} \frac{dE_x(z)}{dz}, \tag{4b}$$

also, with the boundary conditions on the surfaces $z = \pm h_d$.

The equations (3) and (4) are valid for arbitrary permeability and permittivity functions $\mu_1(z)$ and $\epsilon_1(z)$. The relation (2) for normal incidence of plane wave on the plane magnetodielectric layer is exact. The solutions of the equations (3) and (4) are quite complex, can be obtained analytically for a limited number of the functions $\mu_1(z)$ ($\epsilon_1(z)$), and can be expressed by special functions. If the distribution $\mu_1(z)$ or $\epsilon_1(z)$ is a relatively slow varying function within the layer, then an approximate solution in a class of elementary functions can be obtained.

Let consider the following form of $\mu_1(z)$ and $\epsilon_1(z)$:

$$\mu_1(z) = \mu_1(0)[1 - \mu_r f(z)], \tag{5a}$$

$$\varepsilon_1(z) = \varepsilon_1(0)[1 - \varepsilon_r f(z)], \quad (5b)$$

where the constant $\mu_r = |[\mu_1(0) - \mu_1(-h_d)] / \mu_1(0)|$ ($\varepsilon_r = |[\varepsilon_1(0) - \varepsilon_1(-h_d)] / \varepsilon_1(0)|$) is the relative value of permeability (or permittivity) change in the layer ($\mu_r \ll 1$, $\varepsilon_r \ll 1$), $f(z) = (-z / h_d)^n$, ($n = 1, 2, 3, \dots$) is a predefined function. Then for the linear law of change of function $f(z) = (-z / h_d)$ the approximate analytical solution of the equation system (3) and (4) by the method of expanding the desired function into a series in a small parameter μ_r (ε_r) up to terms of order μ_r^2 (ε_r^2) have the form ([51], [31])

$$\bar{Z}_S^\mu = i\bar{Z}_1^\mu (1 - \mu_r) \frac{\tan(k_{1\mu} h_d)}{1 + \mu_r f_{Lin}(k_{1\mu} h_d) \tan(k_{1\mu} h_d)}, \quad (6a)$$

$$\bar{Z}_S^\varepsilon = i\bar{Z}_1^\varepsilon \frac{\tan(k_{1\varepsilon} h_d)}{1 + \varepsilon_r f_{Lin}(k_{1\varepsilon} h_d) \tan(k_{1\varepsilon} h_d)}, \quad (6b)$$

$$f_{Lin}(k_{1\mu(\varepsilon)} h_d) = \left(\frac{1}{2k_{1\mu(\varepsilon)} h_d} + \frac{i}{2} \right),$$

where h_d is the total thickness of the magnetodielectric layer; $k_{1\mu}^2 = k^2 \mu_1(0) \varepsilon_1$, $\bar{Z}_1^\mu = \sqrt{\mu_1(0) / \varepsilon_1}$; $k_{1\varepsilon}^2 = k^2 \mu_1 \varepsilon_1(0)$ $\bar{Z}_1^\varepsilon = \sqrt{\mu_1 / \varepsilon_1(0)}$ respectively for (6a) and (6b).

We further use the same method for solving equations (3) and (4) for the quadratic law of change permeability and permittivity $f(z) = (z / h_d)^2$. Then the result is:

$$f = 10.0 \bar{Z}_S^\mu = i\bar{Z}_1^\mu (1 - \mu_r) \frac{\tan(k_{1\mu} h_d) + \mu_r f_{Sq1}(k_{1\mu} h_d)}{1 + \mu_r f_{Sq2}(k_{1\mu} h_d) \tan(k_{1\mu} h_d)}, \quad (7a)$$

$$\bar{Z}_S^\varepsilon = i\bar{Z}_1^\varepsilon \frac{\tan(k_{1\varepsilon} h_d) + \varepsilon_r f_{Sq1}(k_{1\varepsilon} h_d)}{1 + \varepsilon_r f_{Sq2}(k_{1\varepsilon} h_d) \tan(k_{1\varepsilon} h_d)}, \quad (7b)$$

$$f_{Sq1}(k_{1\mu(\varepsilon)} h_d) = \frac{1}{k_{1\mu(\varepsilon)} h_d} - \frac{\tan(k_{1\mu(\varepsilon)} h_d)}{(k_{1\mu(\varepsilon)} h_d)^2}, \quad f_{Sq2}(k_{1\mu(\varepsilon)} h_d) = \frac{k_{1\mu(\varepsilon)} h_d}{6}.$$

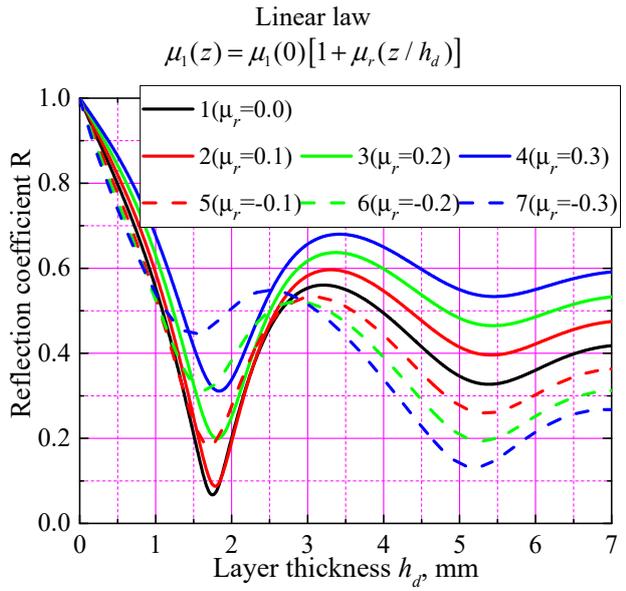
The field reflection coefficient R from the structure under consideration, and, accordingly, the power absorption coefficient A will be determined by the following expressions:

$$R = |1 - \bar{Z}_S^{\mu(\varepsilon)}| / |1 + \bar{Z}_S^{\mu(\varepsilon)}|, \quad A = 1 - R^2. \quad (8)$$

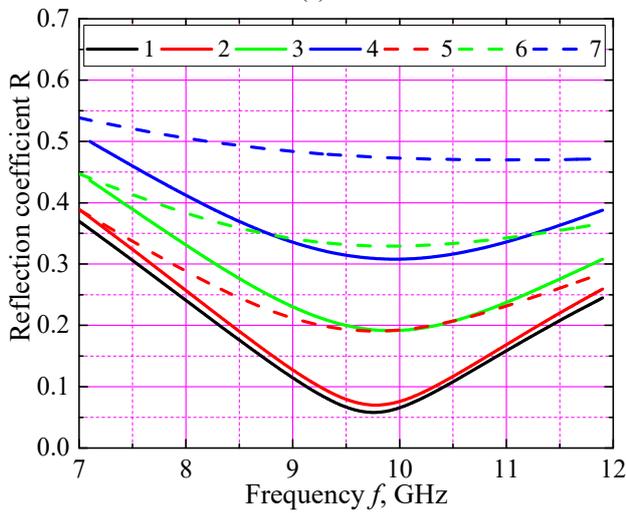
NUMERICAL RESULTS

The material parameters of the magnetodielectric TDK IR-E110 used in these calculations at the frequency band $f = 7 \div 12$ GHz according [25] are $\varepsilon_1 = 8.84 - i0.084$ and $\mu_1 = 2.42 - 0.0825 f[\text{GHz}] - i0.994$. If the layer thickness is equal to the quarter wavelength in the magnetodielectric ($h_d \approx 1.8$ mm at GHz), as seen in the Figs. 2a-5a, the reflection coefficient R has the distinct minimum both for $\mu_1(z)$ and for $\varepsilon_1(z)$ (first resonance). Here and further in the Fig. 2: if $\mu_r, \varepsilon_r > 0$, then $\mu_1(z), \varepsilon_1(z)$ increasing towards the PCP, if $\mu_r, \varepsilon_r < 0$, then $\mu_1(z), \varepsilon_1(z)$ decreasing towards the PCP. The next resonance occurs at the layer thickness equal to three quarters of the wavelength in the magnetodielectric ($h_d \approx 5.4$ mm at $f = 10.0$ GHz). Moreover, these resonances are observed for both considered laws of distribution of material parameters in the layer.

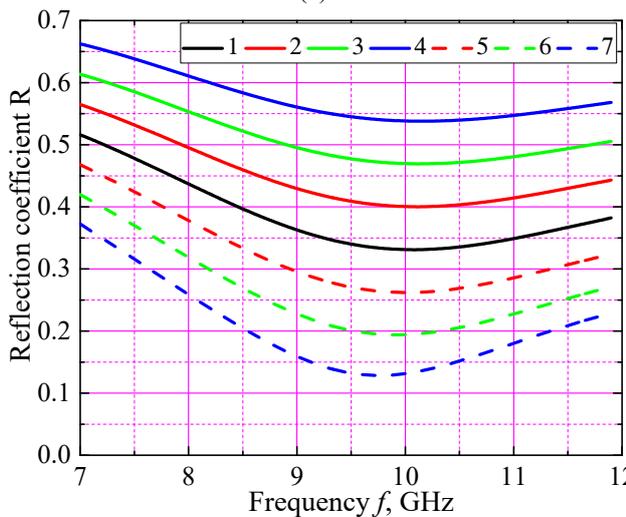
Due to the losses in the substance, we do not have a perfect absorption for the uniform distribution, as the black curves show in all the figures. Thus, a small decrease in the refractive index of the medium at the air-medium interface makes it possible to reduce reflection compared to the case of an increase in this index (Figs. 2a,b, 4a,b – red curves), and even compared to a homogeneous medium (Figs. 4a, b – red and black curves). The latter is very important for creating new structures with strong radio wave absorption. For a relatively thick layer of a medium, see Figs. 2c, 4c, the reflection is smaller in the case of a decrease in the refractive index in the direction of an ideal conductor, since it is more difficult for a wave that has entered the medium to leave it.



(a)

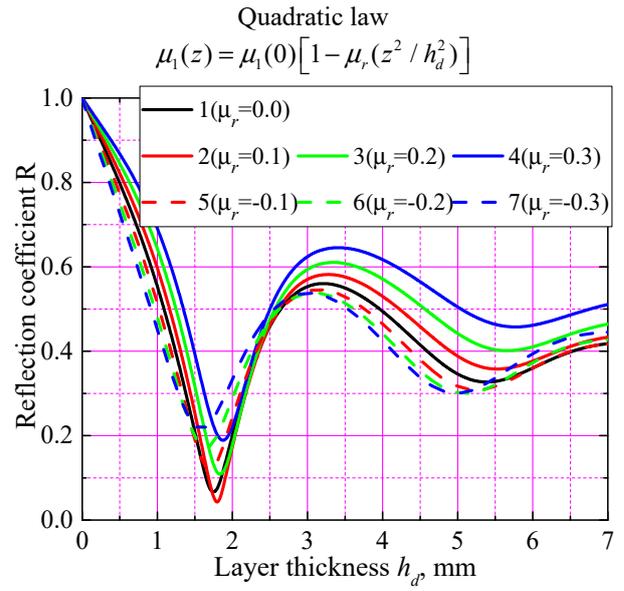


(b)

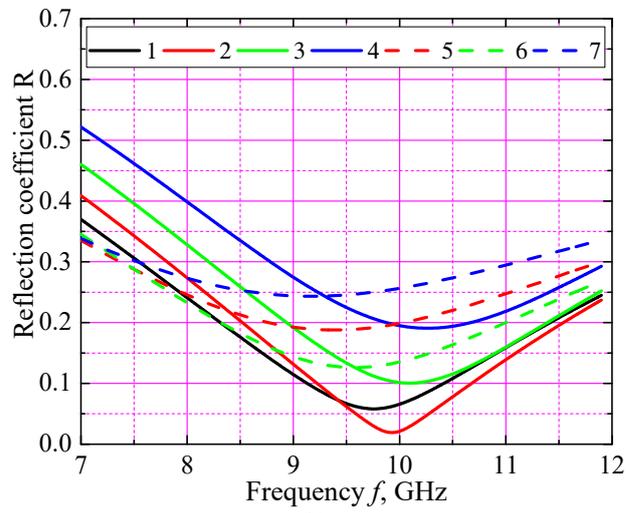


(c)

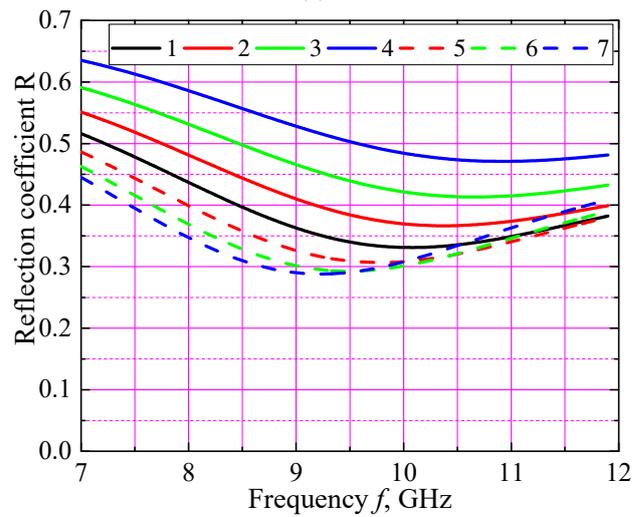
Figure 2. Reflection coefficient R versus layer thickness h_d at $f = 10$ GHz (a); and frequency (b) - $h_d = 1.8$ mm, (c) - $h_d = 5.4$ mm for linear law $\mu_1(z)$.



(a)

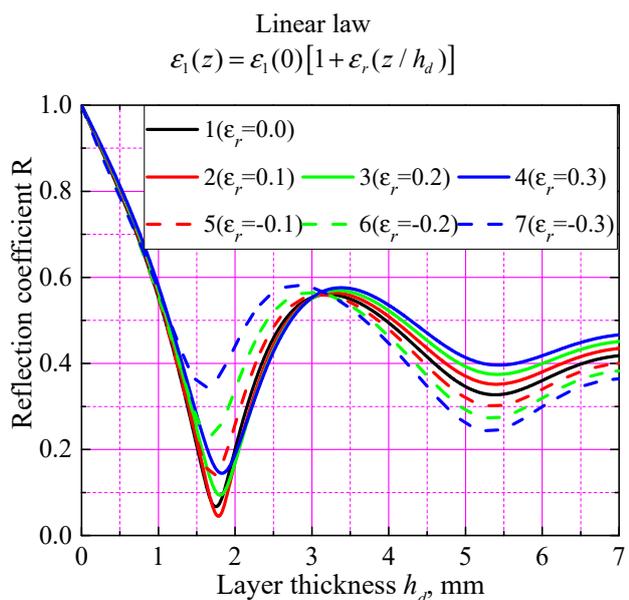


(b)

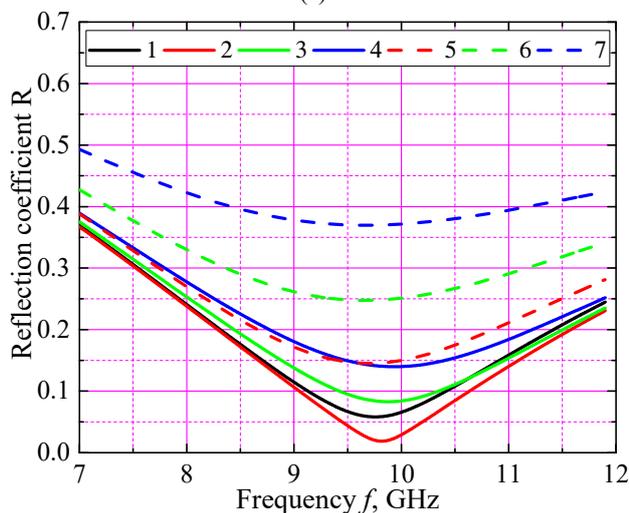


(c)

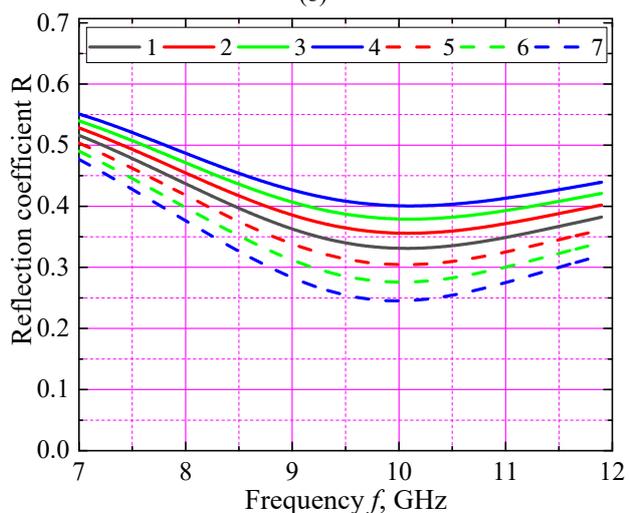
Figure 3. Reflection coefficient R versus layer thickness h_d at $f = 10$ GHz (a); and frequency (b) - $h_d = 1.8$ mm, (c) - $h_d = 5.4$ mm for quadratic law $\mu_1(z)$.



(a)

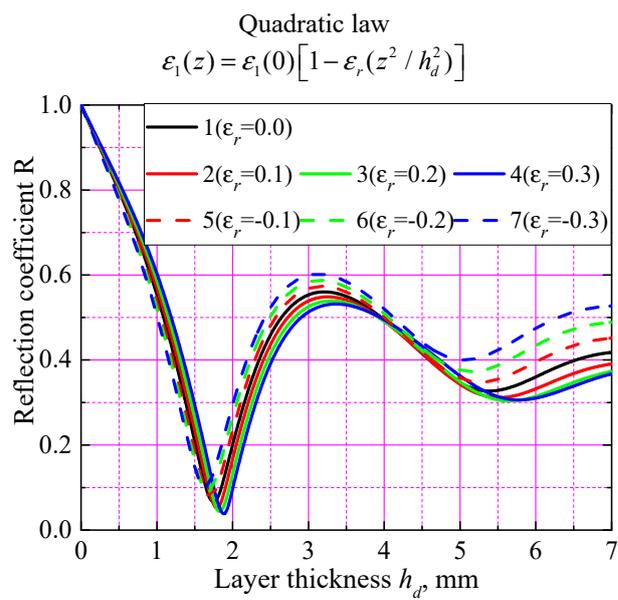


(b)

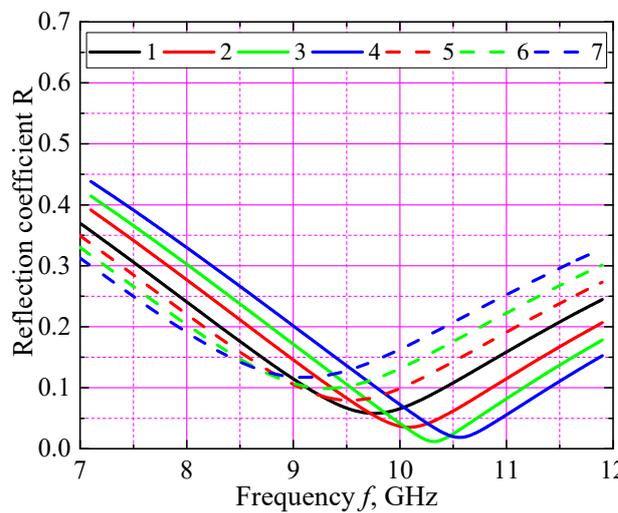


(c)

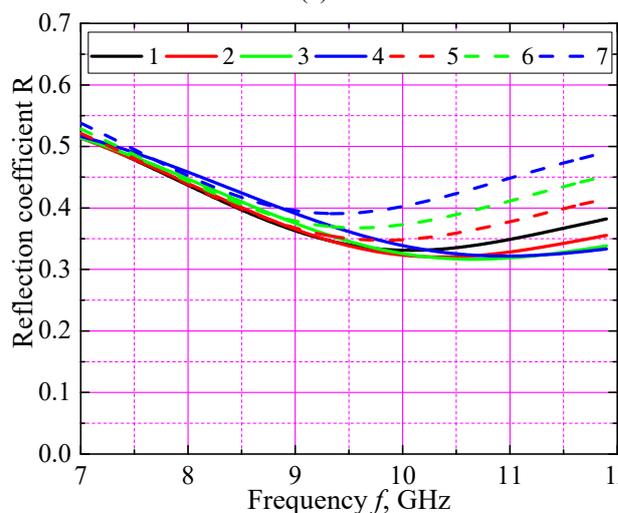
Figure 4. Reflection coefficient R versus layer thickness h_d at $f = 10$ GHz (a); and frequency (b) - $h_d = 1.8$ mm, (c) - $h_d = 5.4$ mm for linear law $\epsilon_1(z)$.



(a)



(b)



(c)

Figure 5. Reflection coefficient R versus layer thickness h_d at $f = 10$ GHz (a); and frequency (b) - $h_d = 1.8$ mm, (c) - $h_d = 5.4$ mm for quadratic law $\epsilon_1(z)$.

CONCLUSION

The work presents an approximate analytical solution of the field equations for finding the distributed surface impedance of a magnetodielectric layer with the inhomogeneous permeability and permittivity located on PCP. In contrast to the known solutions for creating coatings with inhomogeneous parameters, when a multilayer magnetodielectric is used, the solution found is valid for the continuous change in the parameters of the magnetodielectrics inside the layer according to a certain law. The analysis shows that the influence of the magnetodielectric inhomogeneity on the surface impedance can reach tens of percent compared to the case of a uniform layer, which can be considered as additional means of controlling the electrodynamic characteristics of antenna-waveguide devices and creating new absorbing structures. Note that similar results can also be obtained for cylindrical [12] and spherical [30] structures if the corresponding field equations are solved in the cylindrical or spherical coordinate systems. Expressions for the surface impedances of metal conductors coated with the magnetodielectric layer were obtained in [12] for the case of the uniform coating and are in quite satisfactory agreement with the experimental data presented in [3], [7].

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**ВПЛИВ ЕЛЕКТРОФІЗИЧНИХ ПАРАМЕТРІВ МАГНІТОДІЕЛЕКТРИЧНОГО ШАРУ НА ІПП НА ЙОГО
ЕЛЕКТРОДИНАМІЧНІ ХАРАКТЕРИСТИКИ**

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У рамках імпедансної концепції отримані наближені аналітичні формули для розподіленого поверхневого імпедансу магнітодіелектричного шару з неоднорідними магнітною та діелектричною проникностями, розташованого на ідеально провідній площині (ІПП) для випадків квадратичного закону зміни електричних параметрів вздовж товщини шару. Проведено порівняльний аналіз коефіцієнта відбиття електромагнітних хвиль від цієї структури для різних законів зміни магнітної проникності та діелектричної проникності.

Ключові слова: *магнітодіелектричний шар; імпедансна концепція; поверхневий імпеданс; неоднорідна магнітна проникність; неоднорідна діелектрична проникність; коефіцієнт відбиття*