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HEAT GENERATION EFFECT ON 3D MHD FLOW OF CASSON FLUID VIA POROUS STRETCHING/SHRINKING SURFACE WITH VELOCITY SLIP CONDITION

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There are extensive range of applications related to nuclear industry, industrial manufacturing, science and engineering processing, in which the boundary layer hydromagnetic motion of Casson liquids perform vital role. Casson liquid is a useful liquid in the nuclear industry for optimizing the design and operation of nuclear reactors. Researchers have investigated transfer of heat in liquid motions with linear stratification, which is a phenomenon where the temperature varies linearly with height, affecting various fields such as medical equipment, glass fiber production, electronic devices, polymer sheets, paper production, filaments, and medicine. However, the most discussion of heat transfer problems is to get numerical solutions of a comprehensive Casson liquid model with heat generation described by the BVP4 via shooting method. In this study, a new velocity slip boundary condition is applied at the stretching or shrinking surface. These conditions are grounded in the previously established Buongiorno model, providing a more practical and realistic approach compared to previous study. The time independent Gov. Eqs. changed into a set of couple non-linear ODEs with help of suitable similarity conversions. The equations are evaluating via R-K-F by help of MATLAB software programming.
Keywords: *Magnetohydrodynamic; Shrinking/stretching surface; Velocity slip; Heat Generation/absorption; Casson fluid; 3D*
PACS: 04.25.D, 47.50.-d, *43.28.JS, 62.60. +v.

INTRODUCTION

Till date, lots of plentiful fields (such as astrophysics, oceanography) in analytical, experimental and exact solutions are studied to describe the NNF because in view of their real time applications existing in biological lubricants and biomedical flows, industrial processes ("Metal extrusion, drawing of plastics and rubber sheets, coal-oil slurries, blowing, manufacturing, extrusion of polymeric fluids"), polymer and metal extrusion mechanisms and technological applications like coating of wires, oil recovery. Therefore, the upcoming research scholars and scientists are doing towards rheological features of NNF. In 1959, Casson **[1]** introduce Casson liquid as a NN model. The laminar motion of pseudo-plastic NN NFs ("Al_2O_3 + CMC") within the porous circular concentric region was examined by Barnoon and Toghraie **[2]**. Peri P.K. Kameswaran et al. **[3]** developed the SP motion of NN Casson liquid via SS with Soret and Dufour effects. The transfer of heat of Casson viscous gad motion on linear SS was created by Mahabaleswar et al. **[4]**. Duguma et al. **[5]** described the 2D BL motion of incompressible viscous Casson NFs via permeable SS. Himanshu et al. **[6]** exhibited the SP motion of Au-blood liquid via SS. The non-linear mixed convective HMT features of a NN Casson liquid motion via SS was explored by Vishnu Ganesh et al. **[7]**. Shankar Goud et al. [8] studied the streamline BL Casson liquid motion via wedge inspired by magnetic effect. Recently some of scientists respectively, NNF model **[9]**, Eyring-Powell fluid model **[10]**, Casson nanofluid with mixed convection model **[11]**, Maxwell fluid with Cattaneo-Christov model **[12]**, Williamson nanofluid model **[13]**, Walter's nanofluid model **[14]**, and Casson NFs with convective condition **[15]**.

Recently, Adel et al. **[16]** exhibited the behavior of a slippery NFs flowing via permeable SS. The rate of HMT in an MHD viscoelastic NFs via SS with HG was described by Raja Sekhar et al. **[17]**. Ali et al. **[18]** described the motion of a Ree-Eyring HNFs by a stretch motion. Akolade et al. **[19]** created the heat source and generalized Fourier's law on Carreau liquid motion via NLSS. Saleem et al. **[20]** examine and comparison of the effects of momentum fields. The Artificial neural networks are applied in Casson liquid motion past via SS was examination by Srinivasacharya and Shravan Kumar **[21]**. Ouyang et al. **[22]** developed the thermal conductivity and stability by delving into VD via SS with velocity slip. Biswal et al. **[23]** created an exciting and rapidly developing field takes thermal radiation in blood motion. Eid et al. **[24]** presented the MF and ohmic dissipation on NN Casson liquid motion via VSS. Some of the numerical solutions in SS medals **[25-31]**.

The heat generation effect on fluid motion has been expansive motivation research work in heat transfer problems and it is attractive applications in practical, numerical fields and industrial ("such as the storage of nuclear wastes, heaters and coolers of electrical and mechanical devices, thermal insulation, chemical factories etc."). In general, the term "Heat Generation" is occurring high temperature variation between the surface and ambient liquid. Some of the problem of HG on Casson liquid motion is considered to be a constant, space dependent or temperature dependent. The 3D motion and transfer of heat caused by a bidirectional SS with HG was created by Khan et al. **[32]**. Javed and Siddiqui **[33]** presented the numerical computation for mixed convection transfer of heat motion of micropolar NFs. Some of authors **[34-42]** described the numerical computation for HG effect on NNNFs motions via linear or non-linear SS.

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MATHEMATICAL ANALYSIS

The 3D NFs motion via shrinking or SS with VS with MHD is considered. The physical model of the coordinate system is explored in **Fig. 1**.

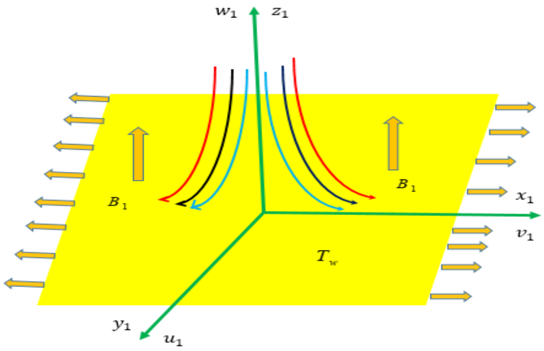

Figure 1. Flow Chart of the Problem

1. The problem created by the SS in $x_1\, y_1$ – surface area with VS.
2. The z_1 directional area is orthogonally to SS $b_1 > 0$ or SHS $b_1 < 0$.
3. The NFs motion occupies area at $z_1 > 0$ and VC of the surface trough x_1 and y_1 directional areas are $u_w(x) = a_1\, x_1$ and $v_w(\mathrm{y}_1) = b_1\, y_1$, respectively.
4. The liquid is EC under influence of UMF B_1 as well as VMF is $w_1 = w_0$, where $w_0 < 0$ then it is called suction and $w_0 > 0$ then it is known as injection.

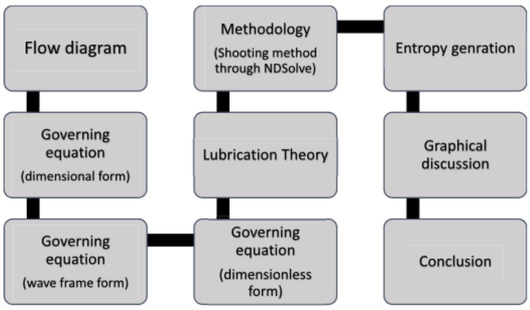

Figure 4. Problem Layout

Under the above considerations, the basic Gov. Eqs are:

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} + \frac{\partial w_1}{\partial z_1} = 0, \tag{1}$$

$$u_1 \frac{\partial u_1}{\partial x_1} + v_1 \frac{\partial u_1}{\partial y_1} + w_1 \frac{\partial u_1}{\partial z_1} = \upsilon_1 \left(1 + \frac{1}{\beta_1}\right) \frac{\partial^2 u_1}{\partial z_1{}^2} - \frac{\sigma_1 B_1^2}{\rho_f 0} u_1 - \frac{\mu_e}{\rho K} \mu_1, \tag{2}$$

$$u_1 \frac{\partial v_1}{\partial x_1} + v_1 \frac{\partial v_1}{\partial y_1} + w_1 \frac{\partial v_1}{\partial z_1} = \upsilon_1 \left(1 + \frac{1}{\beta_1}\right) \frac{\partial^2 v_1}{\partial z_1{}^2} - \frac{\sigma_1 B_1^2}{\rho_f} v_1 - \frac{\mu_e}{\rho K} \mu_1,, \tag{3}$$

$$u_1 \frac{\partial T_1}{\partial x_1} + v_1 \frac{\partial T_1}{\partial y_1} + w_1 \frac{\partial T_1}{\partial z_1} = \alpha_1 \left(\frac{\partial^2 T_1}{\partial z_1{}^2}\right) + \frac{Q_0 (\mathrm{T}_1 - \mathrm{T}_\infty)}{\left(\rho_1 c_p\right)_f}. \tag{4}$$

Corresponding B.Cs. are

$$\left.\begin{array}{l} u_1 = u_w(x_1) = \mathrm{U}_w(x_1) + \mathrm{N}_1\, \mu_1 \dfrac{\partial u_1}{\partial z_1},\;\; v_1 = v_w(x_1) = V_w(x_1) + \mathrm{N}_2\, \mu_1 \dfrac{\partial v_1}{\partial z_1},\;\; w_1 = 0, \\ \left.\begin{array}{l} T_1 = T_w,\;\; -k_1 \dfrac{\partial T_1}{\partial z_1} = h_f (\mathrm{T}_f - \mathrm{T}_1) = 0, \quad at \quad z_1 = o \\ u_1 \to 0,\; v_1 \to 0,\; T_1 \to T_\infty,\; C_1 \to C_\infty, \;\; as \quad z_1 \to \infty \end{array}\right\} \end{array}\right\}. \tag{5}$$

The below dimensionless functions and translated variables are:

$$\left.\begin{array}{l} \eta_1 = z_1\sqrt{\dfrac{a_1}{\upsilon_1}},\ u_1 = a_1 x_1 f'(\eta_1),\ v_1 = a_1 y_1 g'(\eta_1), \\ \mathrm{w}_1 = -\sqrt{a_1\upsilon_1}\left(f(\eta_1)+g(\eta_1)\right),\ \theta(\eta_1) = \dfrac{T_1 - T_\infty}{T_f - T_\infty},\quad \phi(\eta_1) = \dfrac{C_1 - C_\infty}{C_w - C_\infty}. \end{array}\right\}. \tag{6}$$

Utilizing the above dimensions, **Eq. (6)** is identically satisfied and translate **Eqs. (2)- (4)** becomes:

$$f''' = -f''(f+\mathrm{g}) + (f')^2 + (M+P)f' - 1, \tag{7}$$

$$g''' = -g''(f+\mathrm{g}) + (g')^2 + (M+P)g' - 1, \tag{8}$$

$$\theta'' = -\Pr\left((f+g)\theta' - H\theta\right). \tag{9}$$

With subject to the B.Cs. are:

$$\left.\begin{array}{l} f(0)=0,\ \ \mathrm{g}(0)=0,\ \ f'(0)=1+\mathrm{A}f''(0),\ \ \mathrm{g}'(0)=\lambda+\mathrm{Bg}''(0), \\ \theta(0)=1,\ \ \theta'(0)=-Bi(1-\theta(0)), \\ f'(\eta_1)\to 0,\ \ \mathrm{g}'(\eta_1)\to 0,\ \ \theta(\eta_1)\to 0,\ \ \phi(\eta_1)\to 0 \quad \text{as} \quad \eta_1\to\infty \end{array}\right\}. \tag{10}$$

The physical quantities of practical interest are C_{fx_1} and C_{fy_1}, and Nu_{x_1}, it is defined as

$$C_{fx_1} = \frac{\tau_{wx_1}}{\rho_1 U_{w_1}^2},\quad C_{fy} = \frac{\tau_{w_1 y_1}}{\rho_1 V_{w_1}^2},\quad Nu_{x_1} = \frac{x_1 q_{x_1}}{k(\mathrm{T}_w - \mathrm{T}_\infty)},\ \text{and}\ \mathrm{Sh}_{x_1} = \frac{x_1 q_m}{D_B(C_w - C_\infty)}. \tag{11}$$

Defines the SF via x_1, y_1 directional area, HF q_{w_1} and MF q_m from SS are

$$\tau_{wx_1} = \mu_1\left(\frac{\partial u_1}{\partial z_1}\right)_{z_1=0},\quad \tau_{w_1 y_1} = \mu_1\left(\frac{\partial v_1}{\partial z_1}\right)_{z_1=0},\quad q_{w_1} = -k\left(\frac{\partial T_1}{\partial z_1}\right)_{z_1=0},\quad q_{w_1} = -D_B\left(\frac{\partial C_1}{\partial z_1}\right)_{z_1=0}. \tag{12}$$

Substituting the u_1, v_1, T_1 from the **Eq. (11)** onto **Eq. (12)** and using **Eq. (6)**, we getting

$$\mathrm{Re}_{x_1}^{1/2} C_{fx_1} = f''(0),\ \ \mathrm{Re}_{x_1}^{1/2} C_{fy_1} = g''(0),\ \ Nu_{x_1}\mathrm{Re}_{x_1}^{-1/2} = -\theta'(0),\ \ \mathrm{Sh}_{x_1}\mathrm{Re}_{x_1}^{1/2} = -\phi'(0), \tag{13}$$

where $\mathrm{Re}_{x_1} = U_{w_1}(x_1/v_1)$ and $\mathrm{Re}_{y_1} = U_{w_1}(\mathrm{y}_1/v_1)$ are LRN.

RESULTS AND DISCUSSION

To discuss the outstanding variations of velocity of NN motion and $\mathrm{Re}_x^{-1/2} Nu_x$ (“Heat Transfer Rate”) due to relevant physical parameters involved in this study with statistical solutions are explained through their plotted graphs: **2-10**. The present work is considering different cases, like pure fluid, NN liquid, stretching $(\lambda \ge 0)$ and shrinking cases $(\lambda < 0)$.

The physical parameter Pr (“Prandtl number”) on $\theta(\eta_1)$ (“Temperature Profile”) and $\mathrm{Re}_x^{1/2} Nu_x$ (“Heat transfer rate”) as predict **Figs. 2(a)-2(b)** with higher statistical values of $\Pr$ for the cases of Pure liquid $(\beta = 0)$, NN liquid (“Casson liquid”) $(\beta = 0.5)$ and presence of slip parameter on axial direction $(A = B = 0.1)$, absence of $(A = B = 0)$ slip parameter on transverse direction. It is perceived, the $\theta(\eta_1)$ decline the layer in region $0.02 \le \eta_1 \le 1.5$ as well as $\mathrm{Re}_x^{1/2} Nu_x$ with distinct statistical values of $\Pr$. We noticed that the temperature is more in pure fluid when compared with NN liquid, because of thermal conductivity is more in Casson liquid.

Figs. 3(a)-3(b) presented $\theta(\eta_1)$ (“Temperature Profile”), $\mathrm{Re}_x^{1/2} Nu_x$ (“Heat Transfer Rate”) with higher numerical values of H for the cases of Pure liquid $(\beta = 0)$ and non-Newtonian liquid (“Casson Liquid”) $(\beta = 0.5)$ respectively. It is perceived, the $\theta(\eta_1)$ decline the layer in region $0.0015 \le \eta_1 \le 1$ as well as $\mathrm{Re}_x^{1/2} Nu_x$. We noticed that the temperature, heat transfer is more in pure fluid when compared with NN liquid, because of thermal conductivity is more in Casson liquid.

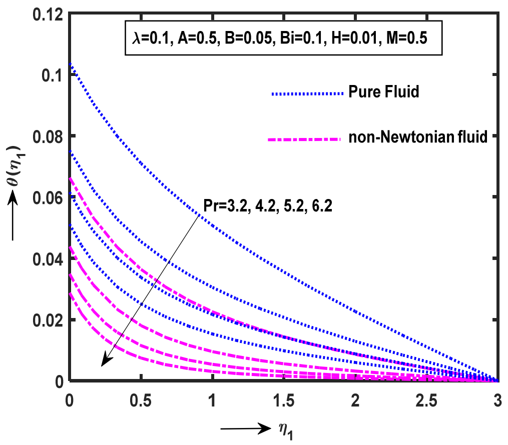

Figure 2(a). Outline of $\Pr$ on $\theta(\eta_1)$

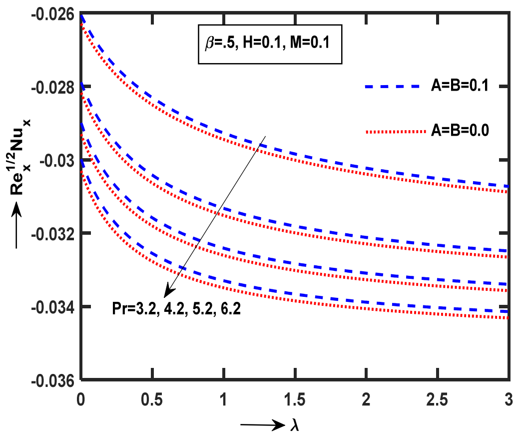

Figure 2(b). Outline of $\Pr$ on $\mathrm{Re}_x^{1/2}\, Nu_x$

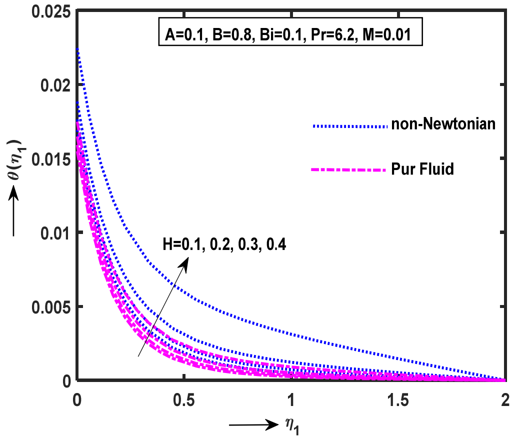

Figure 3(a). Outline of H on $\theta(\eta_1)$

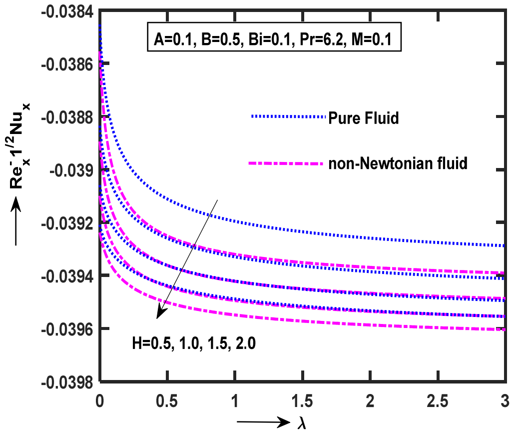

Figure 3(b). Outline of H on $\mathrm{Re}_x^{1/2}\, Nu_x$

The main characteristics of this model is β ("Casson Parameter") on $f(\eta_1)$ ("Axial Direction"), $g(\eta_1)$ ("Transverse Direction") as predict **Fig. 4**. It is perceived, the decline of both axial and transverse direction with distinct numerical values of β. We observe that, the Casson liquid motion is very high motion in axial direction on stretching surface while compare to transverse direction. Because, the plastic dynamic viscosity of Casson liquid motion is very high. Due to this, the Casson liquid is slow motion on surface in axial direction.

The physical parameter M ("Magnetic Field Parameter") on $f(\eta_1)$ ("Axial Direction"), $g(\eta_1)$ ("Transverse Direction") as predict **Fig. 5**. It is perceived, the decline of both axial and transverse direction with high distinct numerical values of M. We observe that, the magnetic field parameter is very high motion in transverse direction on SS while compare to axial direction. Because, the magnetic force applied to Casson liquid which has generate drag force named as "Lorentz force". This force acts Casson liquid in opposite direction to the motion.

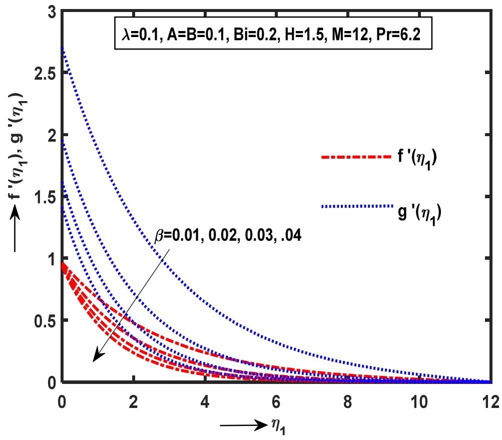

Figure 4. Outline of β on $f'(\eta_1)$, $g'(\eta_1)$

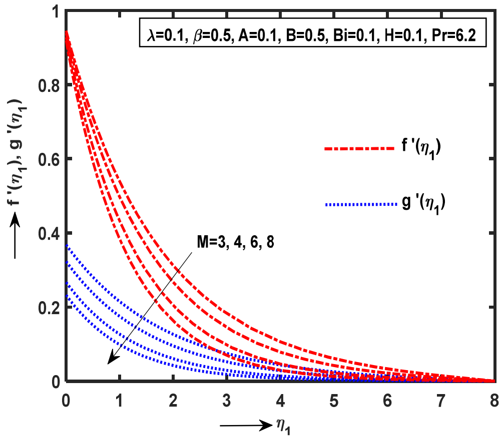

Figure 5. Outline of M on $f'(\eta_1)$, $g'(\eta_1)$

The characteristics of Bi ("Stretching/Shrinking Parameter") on $\theta(\eta_1)$ ("Temperature Profile") and B ("Slip Parameter on Transverse Direction") for Stretching $(Bi \geq 0)$ and shrinking $(Bi < 0)$ cases as depicted **Fig. 6(a)-6(b)**. It is perceived, the Bi improves temperature while opposite direction of B with distinct enlarge statistical values. We observe that, the Casson liquid flow is very high temperature motion in case of $(Bi \geq 0)$ ("Stretching") while compare to $(Bi < 0)$ ("Shrinking"). Because, the viscosity of Casson liquid motion in stretching sheet is very high.

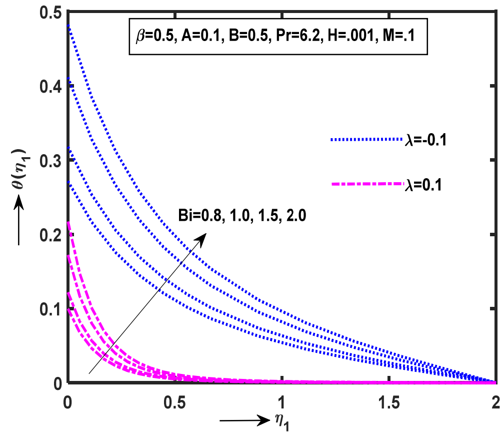

Figure 6(a). Outline of Bi on $\theta(\eta_1)$

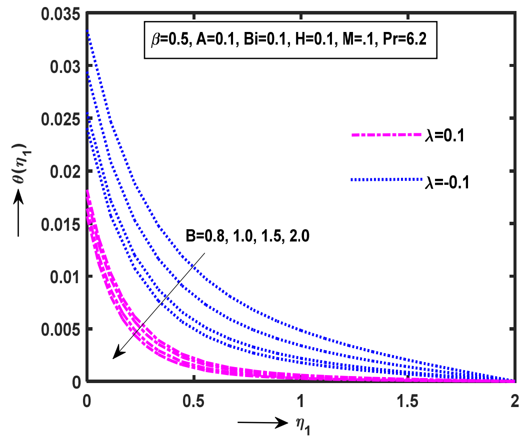

Figure 6(b). Outline of B on $\theta(\eta_1)$

Figs. 7(a)-7(b) illustrate the characteristics of A ("Slip Parameter on Axial Direction") on $\mathrm{Re}_x^{1/2} C_{fx}$ ("Skinfriction" coefficient along axial direction"), $\mathrm{Re}_x^{1/2} C_{fy}$ ("Skinfriction" Coefficient along Transverse Direction") respectively. It is clear that, the A ("Slip Parameter on Axial Direction") declined both axial and transverse directions of Skinfriction coefficient foe higher enlarge statistical values of "A". We noticed that, the Skinfriction is less movement in Casson liquid flow via stretching surface.

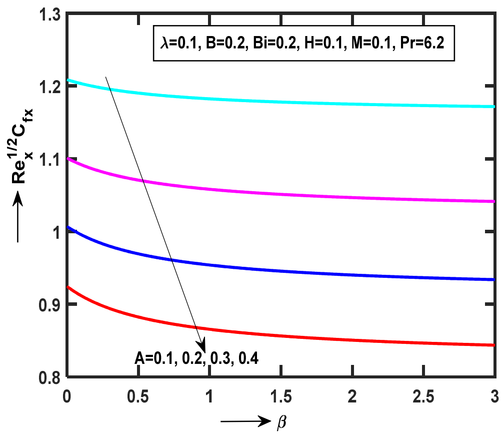

Figure 7(a). Outline of A on $\mathrm{Re}_x^{1/2} C_{fx}$

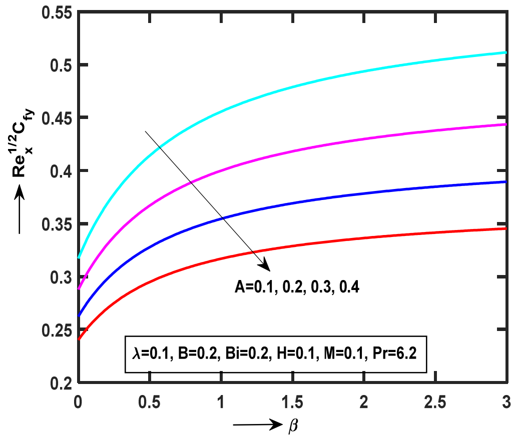

Figure 7(b). Outline of A on $\mathrm{Re}_x^{1/2} C_{fy}$

The Table 1 and 2 presented that, the comparison study of present and previous study for Skinfriction coefficient with various numerical cases of magnetic parameter *M*.

Table. 1 Evaluation of SFC $-f''(0)$ for $A = B = Bi = 0$

M	Present study	Sarah et al. [36]	Nadeem et al. [37]	Gupta and Sharma [38]	Ahmad and Nazar [39]
0.0	1.000000	1.00000	1.0004	1.0003181	1.0042
10	3.316624	3.31662	3.3165	3.3165824	3.3165
100	10.04987	10.04987	10.049	10.049864	10.049

Table. 2 Comparison of SFC $-f''(\infty)$ for $A = B = Bi = 0$

M	Present study	Nadeem et al. [37]
0.0	1.173719	--
10	3.367222224	3.3667
100	10.06646642	10.066

CONCLUSIONS

A statistical analysis has been done for HG effect on 3D MHD motion of Casson liquid via SS with VS Condition. The main contribution of the present investigations is mentioned below:

- The $\mathrm{Re}_x^{1/2}\, Nu_x$ is very less motion in Casson liquid when presence of slip parameter while comparing with absence of slip parameter for higher values of *Pr*.
- The temperature is less transfer in NN liquid motion when compares with pure liquid motion with higher statistical values of *Pr*.
- The HG is high in pure fluid while opposite motion in HTR when compared with Casson liquid for escalate values of *H*.

Nomenclature			
(x_1, y_1)	Cartesian coordinate's	T_w	Constant fluid Temperature of the wall
$u_1,\ v_1,\ w_1$	velocity components along x_1, y_1, z_1 -axis	U_w	Stretching velocity
A	Velocity slip along x-axes $\sqrt{a\gamma_0}N_1$	U_∞	Free stream velocity
B	Velocity slip along y-axes $\sqrt{a\gamma_0}N_2$	***Greek symbols***	
f	Dimensionless stream function	ρ_1	Density
f'	Dimensionless velocity	σ_1	Boltzmann constant
H	Heat Generation Parameter $\left(\frac{Q_0}{a_1(\rho_1 c)_f}\right)$	λ	Constant stretching/shrinking parameter b_1/a_1
M	Magnetic field parameter $=\frac{\sigma_1 B_0^2}{a_1\rho_f}$	υ_1	Kinematic viscosity of the fluid
P	Porous Parameter $=\left(\frac{\mu_e}{\rho v}\right)$	θ	Dimensionless temperature
Pr	Prandtl number $=\left(\frac{\upsilon_1}{\alpha_1}\right)$	α_1	Thermal diffusivity $=k/(\rho c_p)_f$
Re_x	Reynolds number	***Subscripts***	
T_1	Temperature of the fluid	∞	condition at free stream
T_∞	fluid temperature far away from the surface		
Abbreviations			
NFs	Nanofluids	HTR	Heat Transfer Rate
HT	Heat TRansfer	SS	Stretching Sheet
HMT	Heat and Mass Transfer	VD	Viscous Dissipation
HG	Heat generation	SHS	Shrinking Sheet
MHD	Magnetohydrodynamic	3D	Three Dimensional
MF	Magnetic Field	NN	non-Newtonian
BL	Boundary Layer	B.Cs.	Boundary Conditions
SP	Stagnation Point	HNFs	Hybrid Nanofluids
BVP	Boundary Value Problem	RKF	Range Kutta Fehlberg
SFC	Skinfriction Coefficient	NNF	non-Newtonian Nanofluid

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ВПЛИВ ГЕНЕРАЦІЇ ТЕПЛА НА 3D МГД ПОТІК КАССОНОВОЇ РІДИНИ ЧЕРЕЗ ПОРИСТУ ПОВЕРХНЮ ЩО РОЗТЯГУЄТЬСЯ/СКОРОЧУЄЬСЯ З УМОВОЮ ШВИДКІСНОГО КОВЗАННЯ

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Існує широкий спектр застосувань, пов'язаних з ядерною промисловістю, промисловим виробництвом, наукою та інженерною обробкою, у яких гідромагнітний рух прикордонного шару рідин Кассона відіграє життєво важливу роль. Рідина Casson є корисною рідиною в атомній промисловості для оптимізації конструкції та роботи ядерних реакторів. Дослідники досліджували передачу тепла в русі рідини з лінійною стратифікацією, яка є явищем, коли температура змінюється лінійно з висотою, впливаючи на різні галузі, такі як медичне обладнання, виробництво скловолокна, електронні пристрої, полімерні листи, виробництво паперу, ниток і медицина . Проте найбільше обговорення проблем теплообміну полягає в тому, щоб отримати чисельні рішення комплексної рідинної моделі Кассона з утворенням тепла, описаним BVP4 за допомогою методу зйомки. У цьому дослідженні нова гранична умова швидкісного ковзання застосована на поверхні розтягування або звуження. Ці умови ґрунтуються на раніше встановленій моделі Буонгіорно, що забезпечує більш практичний і реалістичний підхід порівняно з попереднім дослідженням. Незалежне від часу Gov. Eqs. змінено на набір пари нелінійних ODE за допомогою відповідних перетворень подібності. Рівняння оцінюються через R-K-F за допомогою програмного забезпечення MATLAB.

Ключові слова: *магнітогідродинаміка; поверхня, що скорочується/розтягується; швидкісне ковзання; генерація/поглинання тепла; Кассонова рідина; 3D*