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EVOLUTION OF SOLITARY WAVE IN A COLLISIONLESS QUANTIZED MAGNETO-PLASMA WITH ION PRESSURE ANISOTROPY

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This paper presents a comprehensive study in a collisionless plasma composed of charged state of heavy positive ion and light positive as wel as negative ion. By deriving the Korteweg-de Vries (KdV) equation and by using its standard solution we analyze the characteristics of the solitary profile under varying parameters. We found that the solution gives both rarefactive and compressive soliton. The compressive structures are formed for the slow mode, while rarefactive solitary structures are formed for the fast mode. Furthermore, with the application of planar dynamical systems bifurcation theory, the phase portraits have been analyzed. This dynamical system analysis allowed us to extract important information on the stability of these structures as represented by the KdV equation.

Keywords: *KdV Equation; Solitary Wave; Quantum Plasma; Dynamical System; Reductive perturbation method; Pressure Anisotropy*

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1. INTRODUCTION

Two key properties of plasma are nonlinearity and dispersion; nonlinearity causes the wave to steepen, while dispersion aims to make the wave broader. A wave that propagates in the plasma without losing its identity even after interacting with other waves is referred to as a solitary wave when the nonlinearity and dispersion are balanced. A confined nonlinear wave with a steady-state shape is a soliton [1]. Washimi and Taniuti made the first theoretical prediction of the presence of ion-acoustic solitary waves of modest but finite amplitudes in plasma in 1966 [2]. The fundamental characteristics of the ion-acoustic wave would be considerably altered in the linear [3–5] and nonlinear regimes [6, 7] in the presence of a second ion species in a plasma. They discovered that two ion acoustic modes, known as the fast and slow ion modes, are supported by their model. These two ion wave modes are confirmed to exist by the experimental studies [4, 5].

The physics of positive and negative-ion quantum plasmas, and in particular multi-ion plasmas, have attracted a lot of attention lately because of their presence in plasma environments ranging from laboratory to astrophysical. [8–11]. The constituents of the degenerate quantum plasma includes electrons, heavy ions with positive charges, and light ions [12]. It is commonly known that negative ions exist in the Halley's comet [13] and the Earth's ionosphere [14]. Positive-negative ion plasmas have also been discovered to exist in a variety of settings, including neutral beam sources [15], low-temperature laboratory studies [16], reactors for plasma processing [17], etc. Numerous authors [11, 18–21] used positively charged heavy and light ions in quantum plasmas to study nonlinear waves.

In the presence of elevated magnetic fields, the plasma ion pressure exhibits anisotropic behavior, and the plasma behaves differently in parallel and perpendicular directions relative to the external magnetic field [22]. So, to consider the effect of ionic pressure anisotropy pressure i.e., the parallel ion pressure ($P_{\|}$) and perpendicular ion pressure ($P_{\perp}$) become very important. Numerous studies have been reported on the impact of pressure anisotropy on the propagation of solitary and shock waves in different plasma regimes [23, 24]. For example, Almas et al. [25] investigated the properties of ion-acoustic solitary waves composed of anisotropic pressure of electron-positron-ion(e-p-i) plasma and found that the characteristics of such waves are more sensitive to parallel ion pressure than perpendicular ion pressure. Khalid et al. [26] also studied the propagation of ion-acoustic electrostatic waves in a magnetized electron-ion plasma with pressure anisotropy. Mahmood et al. [27] studied the properties of non-linear electrostatic structure in anisotropic pressure plasma and found that only the width of the soliton depends on the perpendicular pressure ($P_{\perp}$), however, an increase in the parallel pressure ($P_{\|}$) decreases both the amplitude as well as the width of the soliton. Manesh et al. [28] studied the properties of solitary waves in an anisotropic plasma with lighter and heavier ions and found that the light ion's pressure anisotropy determines the polarity of solitary waves, and it is rarefactive for anisotropic lighter ion whereas compressive for the isotropic lighter ion. Khan et al. [29] studied the properties of soliton and cnoidal wave in an anisotropic superthermal electron-positron-ion plasma and found that the wavelength of the cnoidal wave structure is reduced on increasing the parallel and perpendicular anisotropy of ion. Khalid and Rahman [30] studied the ion pressure anisotropy of the ion acoustic non-linear periodic waves in a magnetized plasma. They reported that the increase of parallel pressure of ions

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decreases the amplitude and width of the ion-acoustic periodic waves and the ion-acoustic waves behave differently than ion-acoustic periodic (cnoidal) waves in anisotropic plasmas.

Apart from classical plasmas, the effect of pressure anisotropy has been widely investigated in dense quantum magnetized plasmas. For example, Bordbar and Karami [31] studied the structural properties of an anisotropic dense neutron star and analyzed the compactness, redshift, etc. of such a dense matter as a function strong magnetic field of the order of 10^{17} Gauss which creates the anisotropy. Patidar and Sharma [32] explored the magneto hydrodynamic (MHD) wave modes in anisotropic relativistic degenerate plasma and found fast and slow wave modes propagating under the combined influence of various forces such as pressure anisotropy, exchange potential, Bohm force, and magnetic field. Irfan et al. [33] observed a strong modification of amplitude and width of weakly nonlinear ion-acoustic waves considering the pressure anisotropy of positive ions and electron trapping effects in a dense quantum magneto-plasma. Moreover, in the non-relativistic and ultra-relativistic regimes, the anisotropic ion pressure also affects the stability of solitary waves.

Phase plane analysis is a useful graphical method for analyzing second-order systems with respect to their initial condition, and it is a useful tool for investigating the qualitative behavior of dynamical systems. Geometrically, a curve or point represents the trajectory of a dynamical system for a particular initial state in a phase plane. Furthermore, we may learn more about the system's stability and the presence of solutions using this technique [34]. The significance of phase plane analysis in understanding the qualitative solutions of plasma systems is commonly acknowledged and used by researchers [35–38]. Recently, in various plasma systems, researchers have examined the bifurcation features of small-amplitude nonlinear waves within the framework of equations such as the Burgers equation [39], ZK equation [40], etc. [41, 42]

The objective of the present paper is to study the solitary wave propagation in collisionless quantum magneto-plasma considering the ionic pressure anisotropy. The Korteweg-de Vries (KdV) equation is derived using the reductive perturbation technique(RPT) to study the solitary wave nature in such plasma. These plasmas are believed to exist in white dwarfs and neutron stars. The results obtained here may be useful for laboratory as well as space astrophysical plasma environments wherein such plasma environments are prevalent.

2. THEORETICAL FORMULATION

We consider a collisionless plasma composed of charged state of heavy positive ion and light positive as wel as negative ion. The normalized set of governing equations is given by [43]:

$$\frac{\partial N_{ln,lp}}{\partial T}+\frac{\partial(N_{ln,lp}V_{ln,lpx})}{\partial x}+\frac{\partial(N_{ln,lp}V_{ln,lpy})}{\partial y}+\frac{\partial(N_{ln,lp}V_{ln,lpz})}{\partial z}=0 \tag{1}$$

$$\frac{\partial V_{lnx}}{\partial T}+\left(V_{lnx}\frac{\partial}{\partial x}+V_{lny}\frac{\partial}{\partial y}+V_{lnz}\frac{\partial}{\partial z}\right)V_{lnx}=\vartheta\frac{\partial\Phi}{\partial x}-P_{ln\parallel}N_{ln}\frac{\partial N_{ln}}{\partial x} \tag{2}$$

$$\frac{\partial V_{lny}}{\partial T}+\left(V_{lnx}\frac{\partial}{\partial x}+V_{lny}\frac{\partial}{\partial y}+V_{lnz}\frac{\partial}{\partial z}\right)V_{lny}=\vartheta\frac{\partial\Phi}{\partial y}+V_{lnz}\Omega_{ln}-P_{ln\perp}\frac{1}{N_{ln}}\frac{\partial N_{ln}}{\partial y} \tag{3}$$

$$\frac{\partial V_{lnz}}{\partial T}+\left(V_{lnx}\frac{\partial}{\partial x}+V_{lny}\frac{\partial}{\partial y}+V_{lnz}\frac{\partial}{\partial z}\right)V_{lnz}=\vartheta\frac{\partial\Phi}{\partial z}+V_{lny}\Omega_{ln}-P_{ln\perp}\frac{1}{N_{ln}}\frac{\partial N_{ln}}{\partial z} \tag{4}$$

$$\frac{\partial V_{lpx}}{\partial T}+\left(V_{lpx}\frac{\partial}{\partial x}+V_{lpy}\frac{\partial}{\partial y}+V_{lpz}\frac{\partial}{\partial z}\right)V_{lpx}=-\frac{\partial\Phi}{\partial x}-P_{lp\parallel}N_{lp}\frac{\partial N_{lp}}{\partial x} \tag{5}$$

$$\frac{\partial V_{lpy}}{\partial T}+\left(V_{lpx}\frac{\partial}{\partial x}+V_{lpy}\frac{\partial}{\partial y}+V_{lpz}\frac{\partial}{\partial z}\right)V_{lpy}=-\frac{\partial\Phi}{\partial y}+V_{lpz}\Omega_{lp}-P_{lp\perp}\frac{1}{N_{lp}}\frac{\partial N_{lp}}{\partial y} \tag{6}$$

$$\frac{\partial V_{lpz}}{\partial T}+\left(V_{lpx}\frac{\partial}{\partial x}+V_{lpy}\frac{\partial}{\partial y}+V_{lpz}\frac{\partial}{\partial z}\right)V_{lpz}=-\frac{\partial\Phi}{\partial z}+V_{lpy}\Omega_{lp}-P_{lp\perp}\frac{1}{N_{lp}}\frac{\partial N_{lp}}{\partial z} \tag{7}$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=N_e\left[1+Z_{hp}\mu_{hp}+\alpha_p-\mu_{ln}\right]+N_{ln}\mu_{\mathrm{ln}}-Z_{hp}\mu_{hp}-N_{lp}-N_p\left[\alpha_e+\mu_{ln}-1-Z_{hp}\mu_{hp}\right] \tag{8}$$

Here $\alpha_p=\alpha_e+\mu_{\mathrm{ln}}-1-Z_{h_p}\mu_{hp}$, $\mu_{hp}=\frac{n_{h_10}}{Z_{lp}n_{lp0}}$, $\alpha_e=1+Z_{h_p}\mu_{hp}+\alpha_p-\mu_{\mathrm{ln}}$, $\mu_{\mathrm{ln}}=\frac{Z_{\mathrm{ln}}}{Z_{lp}n_{lp0}}$, $\Omega_{lp,ln}=\omega_{clp,ln}/\omega_{ph}$, $\vartheta=\frac{m_{\mathrm{lp}}Z_{\mathrm{ln}}}{Z_{\mathrm{lp}}m_{\mathrm{ln}}}$, $\ln\Lambda$ is the logarithm of Coulomb parameter, $A_{ln(lp)}$ is the atomic weight of heavy positive ion, Z_{hp} is the charged state of heavy positive ions, $Z_{lp(ln)}$ is the charged state of light positive (negative) ions, $N_{lp(ln)}$ is the normalized plasma light positive (negative) ion density, ϕ is the normalized electrostatic potential. $P_{lp(ln)\parallel}$ and $P_{lp(ln)\perp}$ are the parallel and perpendicular pressure of light positive (negative) ion. The pressure equations for the anisotropic and adiabatic system are given by Chew–Goldberger–Law popularly known as (CGL) or double adiabatic theory [44–46], according to which $\frac{d}{dt}\left(P_{i\parallel}B^2/n_i^3\right)=0$ and $\frac{d}{dt}\left(P_{i\perp}/n_iB\right)=0$. In the case of electrostatic waves in a magnetized plasma, the ambient magnetic field $B = B_0$ is constant with time, i.e. $\frac{d}{dt}(B) = 0$ where B_0 is the magnetic field at equilibrium. Moreover, the normalized parallel and perpendicular ion pressures obtained from the CGL theory are given as $P_{i\|} = 3P_{i\|0}/n_{i0}\varepsilon_{Fe}$ and $P_{i\perp} = P_{i\perp 0}/n_{i0}\varepsilon_{Fe}$ where $P_{i\|0} = n_{i0}T_{i\|}$ and$P_{i\perp 0} = n_{i0}T_{i\perp}$ are the equilibrium values of parallel and perpendicular pressure functions respectively, and n_{i0} is the unperturbed ion density. [22, 45, 47] The variations in the ambient magnetic field alter the ionic temperatures in parallel and perpendicular directions to the magnetic field, i.e., $T_{i\|} \propto B_0$ and $T_{i\perp} \propto \left(\frac{n_{i0}}{B_0}\right)^2$ respectively [47, 48].

The other plasma parameters are normalized as follows:

$\Phi = \dfrac{\epsilon_{Fe}}{e},\ t = T\omega_p^{-1},\ x = X \times \lambda_{Fe},\ N_j = \dfrac{n_j}{n_{j0}},\ \lambda_{Fe} = \dfrac{C_s}{\omega_s},\ \varepsilon_{Fe} = \left(\frac{\hbar}{2m_e}\right)\left(\frac{3\pi^2}{n_{e0}}\right)^{\frac{2}{3}}$ Where, λ_{Fe} is the Thomas-Fermi length, C_s is the Fermi ion sound velocity, ω_{ph} is the plasma frequency, $m_{lp(ln)}$ is the mass of light negative (light positive) ions

3. DERIVATION OF KDV EQUATION

To derive the evolution equation we employed the reductive perturbation technique. The stretched coordinates [22] used here are given by:

$$\begin{gathered}\xi = \varepsilon^{1/2}\left(I_x x + I_y y + I_z z - MT\right), \tau = \epsilon^{3/2}T,\\ \eta_{ln(lp)\|} = \epsilon^{\frac{1}{2}}\eta_{ln(lp)\|0}, \eta_{ln(lp)\perp} = \epsilon^{\frac{1}{2}}\eta_{ln(lp)\perp 0}\end{gathered} \tag{9}$$

Where M is the phase velocity (Mach number) and ϵ is a small nonzero constant measuring the strength of dispersion. In terms of the expansion parameter ϵ, the physical variables in equations are expanded in a power series as

$$\left.\begin{aligned} N_j &= 1 + \epsilon N_j^{(1)} + \epsilon^2 N_j^{(2)} + \epsilon^3 N_j^{(3)} + \dots\\ V_{ix} &= \epsilon V_{ix}^{(1)} + \epsilon^2 V_{ix}^{(2)} + \epsilon^3 V_{ix}^{(3)} + \dots\\ V_{jy,z} &= \epsilon^{\frac{3}{2}} V_{jy,z}^{(1)} + \epsilon^2 V_{jy,z}^{(2)} + \epsilon^{\frac{5}{2}} V_{jy,z}^{(3)} + \dots\\ \phi &= \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \epsilon^3\phi^{(3)} + \dots \end{aligned}\right\} \tag{10}$$

Substituting the above stretched coordinates from Eq.(9) and the respective expansions from Eq. (10) in the Eqs.(1)-(8), and then collecting the terms appearing in the lowest order of ϵ gives the following relations which gives the phase velocity as

$$M = \pm\sqrt{\frac{b \pm \sqrt{b^2 - 4ac}}{2a}} \tag{11}$$

Where
$$\begin{aligned} a &= \left(\mu_e\alpha_1 - \mu_p\Upsilon_1\right)\\ b &= I_x^2\left(\mu_e\alpha_1\left(P_{\mathrm{ln}\|} + P_{lp\|}\right) - (1 - \vartheta\mu_{\mathrm{ln}}) + \mu_p\Upsilon_1\left(P_{ln\|} + P_{lp\|}\right)\right)\\ c &= I_x^4\left(\mu_e\alpha_1 P_{ln\|}P_{lp\|} + P_{ln\|} - \vartheta\mu_{ln}P_{lp\|} - \mu_p\Upsilon_1 P_{ln\|}P_{lp\|}\right)\end{aligned}$$

The second-order perturbation terms were also obtained by equating the coefficient of the next higher order of ϵ. Then using standard procedure we obtain the KdV equation as

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0 \tag{12}$$

Where

$$A = \frac{q}{p},\quad B = \frac{r}{p}$$

$$\begin{aligned} p &= \left(\frac{\mu_{ln}2\vartheta M I_x^2}{\left(M^2 - P_{ln\|}I_x^2\right)^2} + \frac{2MI_x^2}{\left(M^2 - P_{lp\|}I_x^2\right)^2}\right)\\ q &= \left(2\mu_p\Upsilon_2 - 2\mu_e\alpha_2 - \frac{\mu_{ln}\vartheta^2 M^2 I_x^2\left(3I_x^2 + P_{ln\|}\frac{I_x^4}{M^2}\right)}{\left(M^2 - P_{ln\|}I_x^2\right)^3} + \frac{M^2 I_x^2\left(3I_x^2 + P_{lp\|}\frac{I_x^4}{M^2}\right)}{\left(M^2 - P_{lp\|}I_x^2\right)^3}\right)\\ r &= \left(1 - \left(\left(I_x^2 P_{ln\perp} + 1\right)\frac{(1 - I_x^2)\mu_{ln}\vartheta M^2}{\left(M^2 - P_{ln\|}I_x^2\right)^2\Omega_{ln}^2} + \left(I_x^2 P_{lp\perp} + 1\right)\frac{(1 - I_x^2)M^2}{\left(M^2 - P_{lp\|}I_x^2\right)^2\Omega_{ln}^2}\right)\right)\end{aligned}$$

To obtain the solution of Eq. (12), the authors consider the new variable $\chi = \xi - U\tau$ where χ is the transformed coordinate with respect to a frame moving with velocity U. By taking $\phi^{(1)} = \phi$, Eq.(12) becomes

$$-\frac{U}{A}\frac{d\phi}{d\chi} + \phi\frac{d\phi}{d\chi} + D\frac{d^3\phi}{d\chi^3} = 0 \tag{13}$$

Where $D = \frac{B}{A}$

Now, employing the method used in [49] results the solution of equation(13) as

$$\phi = \phi_m \text{sech}^2\left(\frac{\chi}{W}\right) \tag{14}$$

Where $\phi_m = \frac{3U}{A}$, $W = 2\sqrt{\frac{B}{U}}$ are the amplitude and width of solitary wave respectively.

4. RESULTS AND DISCUSSION

The authors examine the properties and nature and solitary wave propagation under various physical scenarios through the analysis of the exact solution provided by Eq. (14). The data range employed in the current paper is widely established across diverse plasma environments. Specifically, the authors consider plasma density on the order of $10^{26} - 10^{30}$ and a magnetic field on the order of $10^{10} - 10^{12}$ with $Te \sim 1$ keV. This set of parameters corresponds to well-known plasma environments found in various astrophysical contexts.

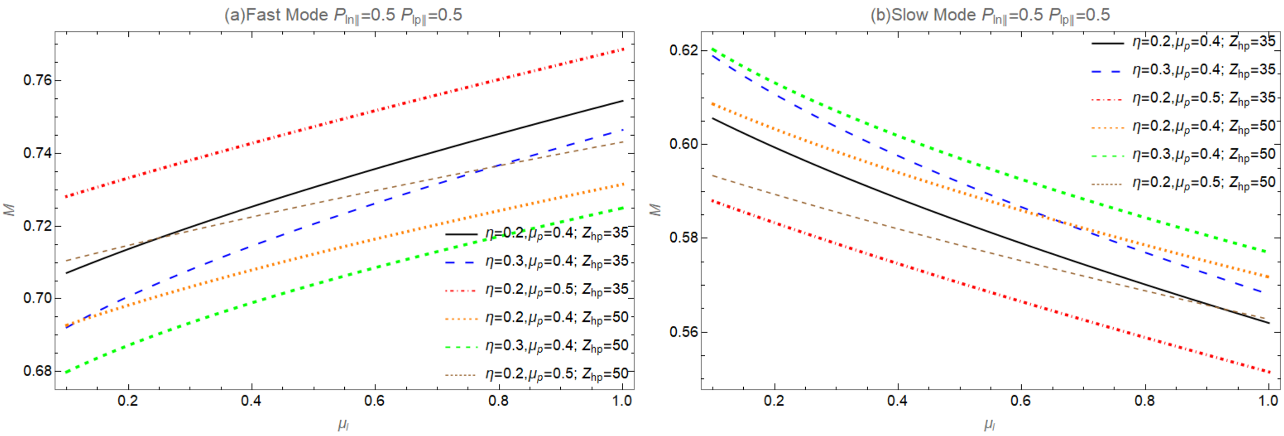

Figure 1. Variations in mach number for different combination of magnetic quantization, positron density and heavy positive ion

The normalized phase velocity, also known as the Mach number, is shown in Figure 1(a) for different combinations of the heavy positive ion, positron density, and magnetic quantization parameter for the fast mode. Figure shows that the velocity is in the subsonic region and that it rises as the density of lighter positive ions increases. The slow mode is plotted for the same set of parameters in Fig. 1(b). While in slow mode, the velocity is subsonic, but it decreases as the density of lighter positive ions increases. By comparing Figs. 1(a) and 1(b), we can observe that the velocity is maximum in the fast mode and lowest in the slow mode for $\eta = 0.2$, $\mu_p = 0.5$, $Z_{hp} = 35$, and $\eta = 0.3$, $\mu_p = 0.4$, $Z_{hp} = 50$, respectively.

Depending on the sign of the nonlinearity coefficient A, the soliton solution given by Eq. (29) may yield either a positive (compressive) or a negative (rarefactive) profile.Fig2 displays solitary profiles for variation of Z_{hp} for fast mode. It is clear that when Z_{hp} increases, the amplitude of the solitary profile decreases. Figures 2(a), (b), and (c) show that when light positive ions are isotropic and light negative ions are anisotropic, the width is at its maximum. For fast mode, the solitary profile is rarefactive. The slow mode is depicted in Fig. 3 for the same combination as in Fig. 2. The solitary profile in this instance is compressive. The fast and slow modes are always precisely opposite to one another. This is

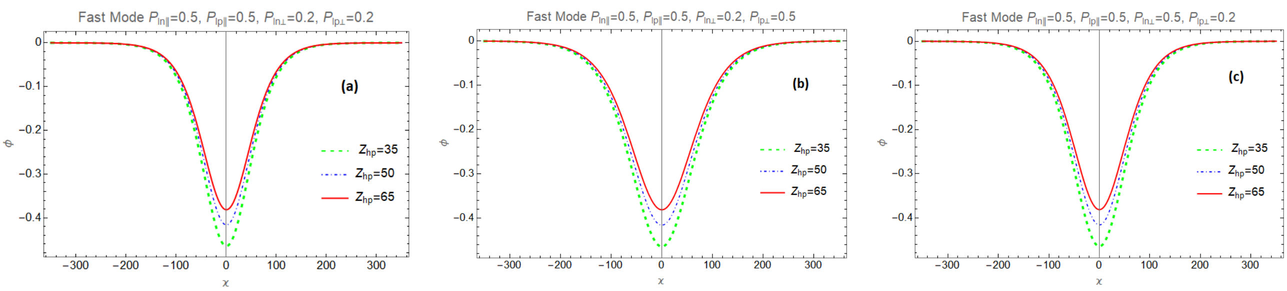

Figure 2. Solitary profile of fast mode with variation of heavy positive ion

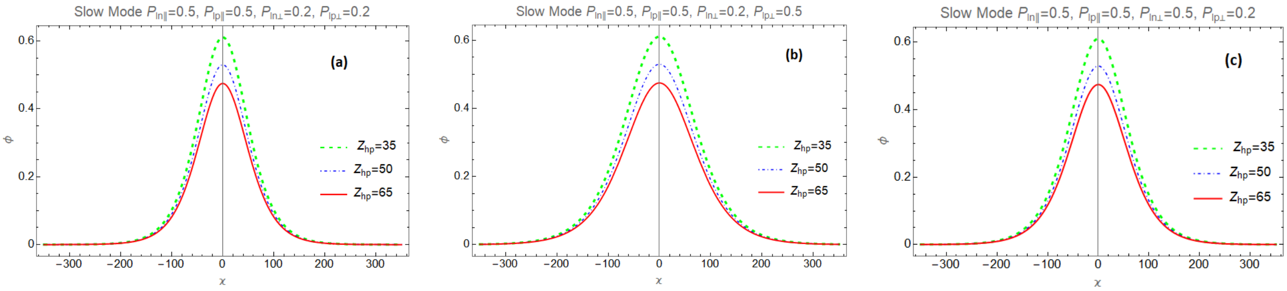

Figure 3. Solitary profile of slow mode with variation of heavy positive ion

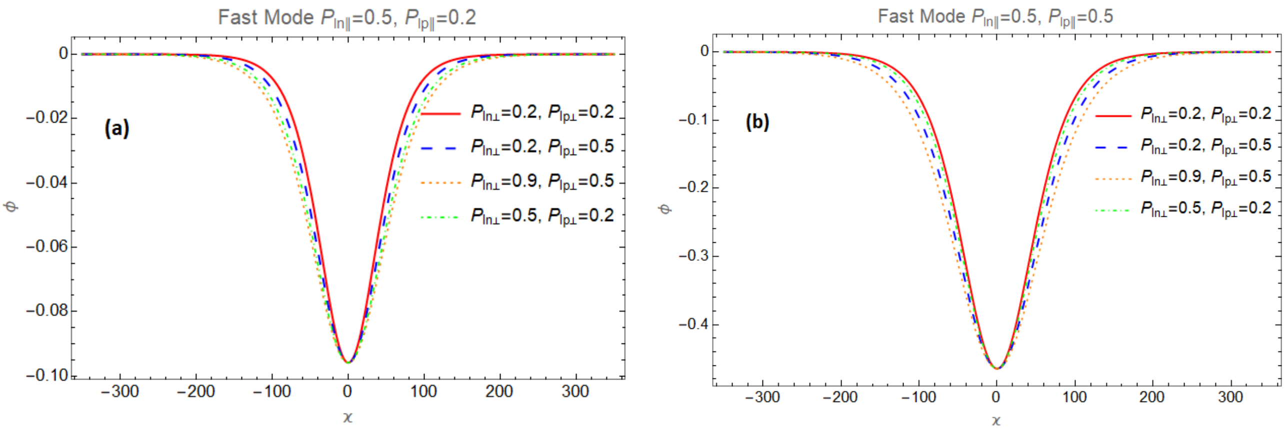

Figure 4. Solitary profile of fast mode with different combination of perpendicular pressure of light positive ion and light negative ion.

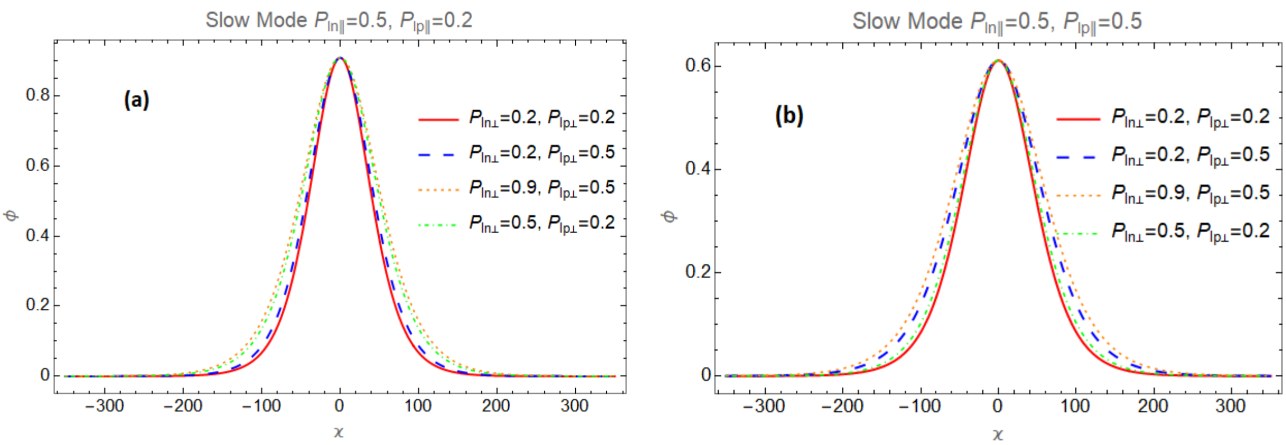

Figure 5. Solitary profile of slow mode with different combination of perpendicular pressure of light positive ion and light negative ion.

an intrinsic property of the plasma that has been investigated both theoretically and experimentaly. If the fast mode is compressive (rarefactive), the slow mode is rarefactive (compressive).

Figures 4 and 5 depict the changes in the solitary profile for different combinations of perpendicular pressure. Perpendicular pressure to the wave affects its width, but its amplitude stays constant. The width reaches its greatest when when both light negative ion and light positive ions are anisotropic for $P_{ln\parallel} > P_{lp\parallel}$ (figs. 4a and 5a), but it is at its highest when light negative ion is anisotropic but light positive ion is isotropic for $P_{ln\parallel} = P_{lp\parallel}$ (figs. 4b and 5b)

5. DYNAMICAL SYSTEM ANALYSIS

Phase plane analysis is a useful tool that is widely used in fusion research to understand the dynamics and stability of plasma systems. A phase plane in plasma physics generally shows the development of two relevant factors, including temperature and plasma density. Achieving controlled and persistent fusion reactions requires maintaining the stability of plasma systems.

Regarding our research and phase plane analysis, we would like to state that the reason for this section of the manuscript is that the phase plane technique entails visually identifying whether limit cycles are present in a differential equation's solutions. Scientists examine plasma state trajectories and equilibrium point locations using phase plane analysis. The way that small deviations from these equilibrium points change over time is how stability is determined. The solutions are interpreted as a family of functions that are graphically depicted as a two-dimensional vector field in the phase plane. At typical points, arrows denoting the derivatives of points with respect to a parameter are drawn. The system's behavior throughout the examined plane regions can be seen by constructing these vectors, which makes limit cycle identification easier. Phase portrait refers to the overall visual representation, and phase path refers to a particular trajectory along a flow line (a path tangent to the vectors).

In order to obtain the dynamical system equation we apply the transformation of the space and variable as $\chi = \xi - U\tau$ and finally obtain the transformed equation as

$$\begin{aligned} \frac{d\phi}{d\chi} &= z \\ \frac{dz}{d\chi} &= \frac{U}{DA}\phi - \frac{1}{2D}\phi^2 \end{aligned} \tag{15}$$

The Hamiltonian of the system (15) is

$$H = \frac{1}{2}z^2 - \frac{U}{2DA}\phi^2 - \frac{1}{6D}\phi^3 \tag{16}$$

The dynamical system described by (15) involves the equilibrium points $P = (0,0)$ and $Q = \left(0, \frac{2U}{A}\right)$.

To derive the eigenvalues, it is essential to have the Jacobian matrix (JM) of the dynamical system. It is evident that system (15) can be expressed as

$$\begin{pmatrix} \phi \\ z \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{U}{DA} - \frac{1}{2D}\phi & 0 \end{pmatrix} \begin{pmatrix} \phi \\ z \end{pmatrix} \tag{17}$$

The expression for the JM is

$$J = \begin{pmatrix} 0 & 1 \\ \frac{U}{DA} - \frac{1}{2D}\phi & 0 \end{pmatrix} \tag{18}$$

The eigenvalues can be found by using the relation $|J - \lambda I| = 0$. Where I indicates the identity matrix. The eigenvalues are provided by $\lambda_{a,b} = \pm\sqrt{\frac{U}{DA} - \frac{1}{2D}\phi}$. At $P = (0,0)$ as the fist equilibrium point, the eigenvalues are $\pm\sqrt{\frac{U}{DA}}$ and at the second equilibrium point $Q = \left(0, \frac{2U}{A}\right)$,the eigenvalues are $\pm\sqrt{-\frac{U}{DA}}$. Utilizing the concept of dynamical systems [] we can see that the first equilibrium point is saddle and the second equilibrium point is centre if $\frac{U}{DA} > 0$. If $\frac{U}{DA} < 0$ the nature of the points will reverse. Note that $P = (0,0)$ must be a saddle point always in order for the results to be physically plausible. The condition for Q to be a saddle point is when the phase speed is negative, which contradicts physically. But Q can be positioned on either the positive or negative axis for centers, which means Q can either be $\left(0, \frac{2U}{A}\right)$ or $\left(0, \frac{-2U}{A}\right)$

It is self-explanatory to see the phase portraits themselves. They display how various parameters affect the structure of orbits and how equilibrium points shift. In figures the limit cycles are seen to travel along the z axis. Figure 6 is plotted for the fast mode with variation Z_{hp}. We see that the centre Q is in the negative z-axis as the nonlinear coefficient A is negative. We can see in figure 8, A is positive in slow mode then Q is positioned in the positive z-axis. The centre Q moves towards to saddle point P with increasing charge of heavy positive ion (Z_{hp}) (figure 6 and 7). Referring to figures 8 and 9 we observe that the centre moves closest to saddle point when the light positive ion is anisotropic and light negative ion is isotropic and when both are anisotropic, the centre moves away from the saddle point.

6. CONCLUSIONS

In this paper we have studied the effect of pressure anisotropy in a collisonless plasma composed of charged state of heavy positive ion and light positive as well as negative ion. Using reductive perturbation technique Kdv equation is

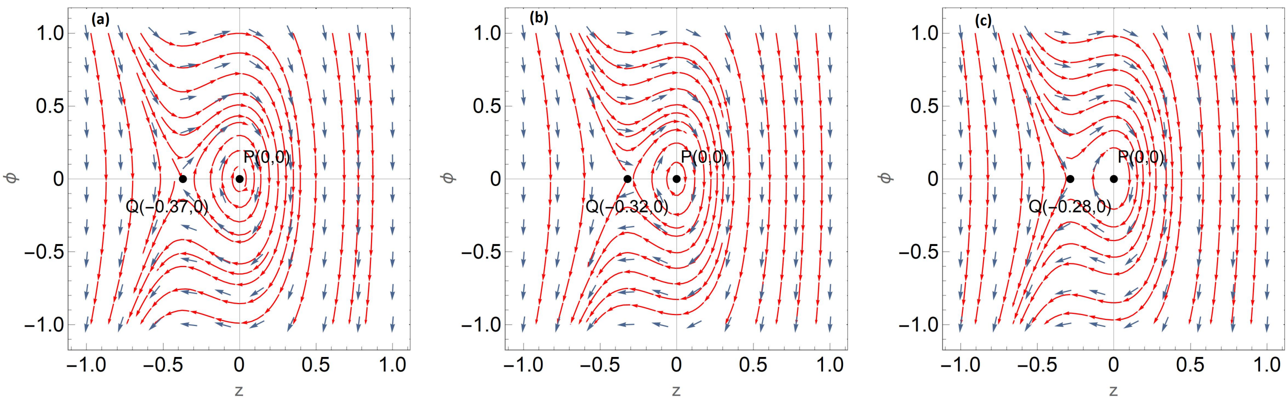

Figure 6. Phase portrait of the system (15) for fast mode when $P_{ln\parallel} = 0.5, P_{lp\parallel} = 0.5, P_{ln\perp} = 0.2, P_{lp\perp} = 0.2$ (i)$Z_{hp} = 35$,(ii)$Z_{hp} = 50$,(iii)$Z_{hp} = 65$

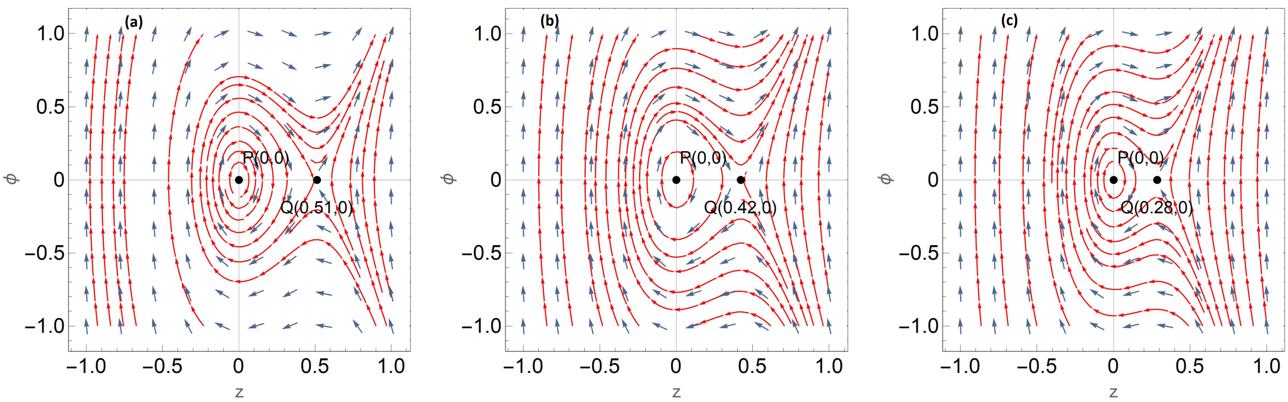

Figure 7. Phase portrait of the system (15) for slow mode when $P_{ln\parallel} = 0.5, P_{lp\parallel} = 0.5, P_{ln\perp} = 0.2, P_{lp\perp} = 0.2$ (i)$Z_{hp} = 35$,(ii)$Z_{hp} = 50$,(iii)$Z_{hp} = 65$

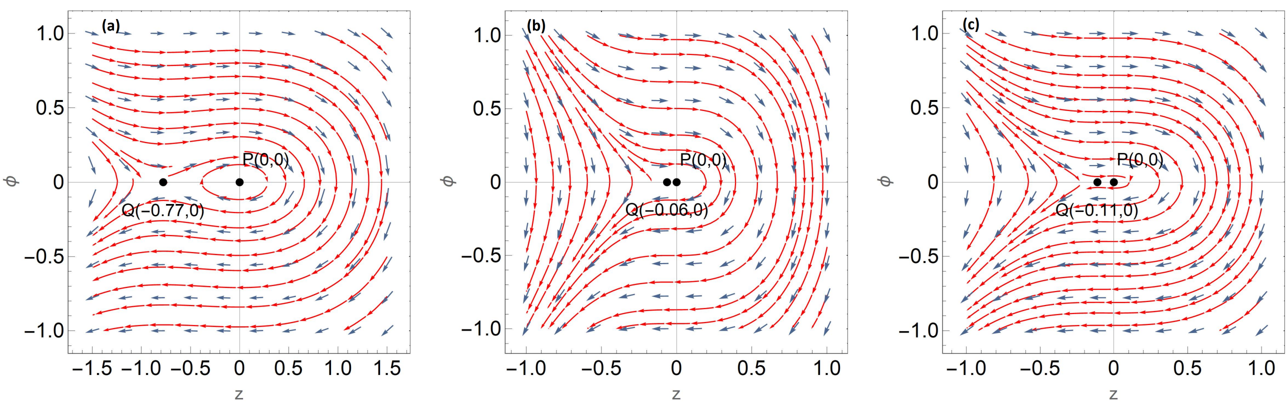

Figure 8. Phase portrait of the system (15) for fast mode when $Z_{hp} = 35, P_{ln\parallel} = 0.5, P_{lp\parallel} = 0.5, P_{lp\perp} = 0.2$ (i)$P_{ln\perp} = 0.5$,(ii)$P_{ln\perp} = 0.2$,(iii)$P_{ln\perp} = 0.9$

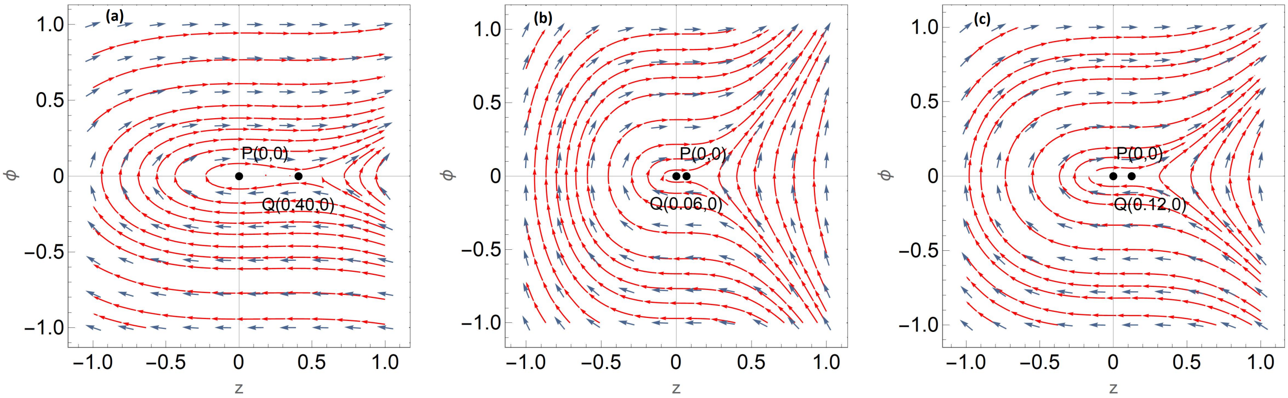

Figure 9. Phase portrait of the system (15) for slow mode when Z_{hp} = 35, $P_{ln\parallel}$ = 0.5, $P_{lp\parallel}$ = 0.5, $P_{lp\perp}$ = 0.2 (i)$P_{ln\perp}$ = 0.5,(ii)$P_{ln\perp}$ = 0.2,(iii)$P_{ln\perp}$ = 0.9

derived and found the analytic solution. For the system under investigation, a biquadratic dispersion relation has been derived, yielding the fast and slow modes.We observed that the phase velocity is within subsonic range in both fast mode and slow mode. We found that compressive structures are formed for the slow mode, while rarefactive solitary structures are formed for the fast mode mode. It was found that the soliton width is influenced by the perpendicular pressure but the amplitude remains the same. However that the amplitude of the solitary profile is influenced by the charge state of heavy positive ion. We then convert our evolutionary equation into a system of two ordinary differential equations (ODEs) in order to do a phase plane analysis. We extract significant information about the stability of stationary structures given by the KdV equation via dynamical system analysis. We have introduced the center and saddle points and graphically displayed the phase portrait of the stated plasma system for various values.

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ЕВОЛЮЦІЯ ОДИНОЧНОЇ ХВИЛІ В КВАНТОВАНІЙ МАГНІТОПЛАЗМІ БЕЗ ЗІТКНЕНЬ З АНІЗОТРОПІЄЮ ІОННОГО ТИСКУ

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Ця стаття представляє комплексне дослідження плазми без зіткнень, що складається із зарядженого стану важкого позитивного іона та легкого позитивного, а також негативного іона. Виводячи рівняння Кортевега-де Фріза (KdV) і використовуючи його стандартне рішення, ми аналізуємо характеристики ізольованого профілю за змінних параметрів. Ми виявили, що розчин дає як розріджений, так і стиснутий солітон. Для повільного режиму формуються стислі структури, а для швидкого – розріджені солітарні структури. Крім того, із застосуванням теорії біфуркацій планарних динамічних систем проаналізовано фазові портрети. Цей аналіз динамічної системи дозволив нам отримати важливу інформацію про стабільність цих структур, представлену рівнянням KdV.

Ключові слова: *рівняння KdV; поодинока хвиля; квантова плазма; анізотропія тиску; аналіз фазової площини; динамічна система*