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DYNAMICS OF STRING COSMOLOGICAL MODEL IN $f(R, L_m)$ THEORY OF GRAVITY

S.D. Katore[a], P.R. Agrawal[b], H.G. Paralikar[a], A.P. Nile[b]
[a]*Department of Mathematics, Sant Gadge Baba Amravati University, Amravati 444602, India*
[b]*Department of Mathematics, Brijlal Biyani Science College, Amravati, 444602, India*
Corresponding Author e-mail: katore777@gmail.com, prachi.gadodia@gmail.com, hparalikar05@gmail.com; ankushnile15@gmail.com
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The present paper examines the FLRW model with cosmic string within the framework of $f(R,L_m)$ gravity, considering two different forms of $f(R,L_m)$ gravity such as $f(R,L_m)=\frac{R}{2}+L_m^\eta+\beta$ and $f(R,L_m)=\Lambda e^{\frac{R}{2\Lambda}+\frac{L_m}{\Lambda}}$, where η,β and Λ are free model parameters. The solutions of the models are obtained using both, the power law and the hybrid expansion law. The resulting physical and dynamical parameters of the models analyzed and presented through graphical representations.
Keywords: $f(R,L_m)$ *gravity; String Cosmological Model; Power law; Hybrid expansion law*
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INTRODUCTION

A potentially effective approach to interpret the latest observational findings (Riess *et al*. (1998); Peebles and Ratra (2003)) regarding the Universe's late-time acceleration and dark matter is to propose that Einstein's theory of general relativity may not hold at large scales. Instead, a broader gravitational framework could better describe these phenomena. Harko *et al.* (2010) introduced an advanced form of matter-curvature coupling theories, referred as $f(R,L_m)$ gravity, where f represents a variable function dependent on the matter Lagrangian L_m and the Ricci scalar R. This theory of gravity can be viewed as the most extensive expansion of all the gravitational theories formulated in Riemann space. The trajectory of test particle within this theory deviates from geodesic paths, resulting in an additional force perpendicular to the four-velocity vector.

Functional expressions for $f(R)$ gravity incorporates logarithmic, exponential and power law models, an extension of framework is known as $f(R,L_m)$ gravity which has been recently emerged as $f(R,L_m)=f_1(R)+f_2(R)G(L_m)$, where f_1,f_2 and G are arbitrary functions of the Ricci scalar and the matter Lagrangian density respectively. The gravitational field equations and the equation of motions for a particular, in which the action of gravitational field has an exponential dependence on the standard general realistic Hilbert Einstein density, $f(R,L_m)=\Lambda e^{\frac{R}{2\Lambda}+\frac{L_m}{\Lambda}}$ are also derived. The Kasner-type static, cylindrically symmetric interior string solutions were investigated in the $f(R,L_m)$ theory of gravity by Harko *et al.* (2015), and the thermodynamic parameter of the string was explicitly obtained. In the article, "cosmology in $f(R,L_m)$ gravity", Jaybhaye *et al.* (2022a) analyzed by utilizing $H(z)$, Pantheon and combining $H(z)+$Pantheon datasets. There, they determined the optimal ranges for model parameters. Additionally, a study was conducted on the variation in cosmological parameters based on the constraints established by these observational datasets. Also, the authors of Jaybhaye *et al.* (2022a) investigated the stability of the obtained model.

Solanki *et al.* (2023) studied $f(R,L_m)$ gravity by considering non-linear models. They obtained the Wormhole solutions by assuming three different cases which are linear barotropic EoS, anisotropic EoS and isotropic EoS, whereas Jaybhaye *et al.* (2022b) discussed about constraints on energy conditions and used cosmographic parameters like mean Hubble parameter, deceleration parameter, jerk parameter and snap parameter. Wang *et al.* (2012) also discussed energy conditions by considering a special model in FRW cosmology and compared with observational astronomical results. Singh *et al.* (2023) studied a constrained cosmological model in $f(R,L_m)$ gravity. Shukla *et al.* (2023) used equation of state parameter and Garg *et al.* (2023) used a linear equation of state parameter to study the expansion of the universe. Lobato *et al.* (2021) investigated Neutron stars with realistic equation of state, Patil *et al.* (2023) analyzed FLRW cosmology with Hybrid scale factor, Pawde *et al.* (2023) studied anisotropic behavior of universe with varying deceleration parameter and Jaybhaye *et al.* (2024) derived bouncing cosmological models in $f(R,L_m)$ gravity.

The cosmic string in $f(R)$ gravity have been studied by Ladke *et al.* (2022) and explored three non-static plane symmetric cosmological models. Carvalho *et al.* (2021) investigated the formation and evolution of cosmic string wakes in $f(R)$ Gravity where they considered a simple model in which baryonic matter flows past a cosmic string. To obtain

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the solution Carvalho *et al.* (2021) used Zel'dovich approximation and also explored the propagation of light in $f(R)$ cosmic string. Also, they compared the results with wakes formed by cosmic string solutions obtained in General Relativity and Scalar Tensor Theories of Gravity. Silva *et al.* (2021) studied comic string in modified theories of gravitation. Also, several authors studied $f(R)$ theory of gravity in different content [Adhav *et al.* 2012; Hatkar *et al.* 2018; Agrawal and Nile 2024; Malik 2024].

Bishi *et al.* (2015) studied Bianchi type V string cosmological model with bulk viscosity in $f(R,T)$ Gravity by considering a special form and linearly varying parameter. Dasunaidu *et al.* (2018) examined the kinematical behavior of five dimensional non static spherically symmetric cosmological models in the presence of a massive string in $f(R,T)$ Gravity. Also, Agrawal and Nile (2024); Thakre *et al.* (2024) studied $f(R,T)$ theory of gravity in different content.

Here in the present study, we have studied two models for the flat FLRW metric in the presence of string motivated by the above discussion. The work has been organized as, basic formation of $f(R,L_m)$, metric and field equations, cosmological model-I, cosmological model-II, common physical parameters for Model-I and Model-II and lastly, result and discussion of the paper have been given.

BASIC FORMATION OF $f(R,L_m)$

The following action governs the gravitational interaction in $f(R,L_m)$ gravity.

$$S=\int f(R,L_m)\sqrt{-g}d^4x\,, \tag{1}$$

where $f(R,L_m)$ represents an arbitrary function of the Ricci scalar R and the matter Lagrangian term L_m.
Now the following field equation can be acquired by varying action (1) for the metric tensor g_{uv},

$$f_R R_{\mu\nu}+(g^{\mu\nu}\nabla_\mu\nabla^\nu-\nabla_\mu\nabla_\nu)f_R-\frac{1}{2}(f-f_{L_m}L_m)g_{\mu\nu}=\frac{1}{2}f_{L_m}T_{\mu\nu}\,, \tag{2}$$

were $f_R\equiv\dfrac{\partial f}{\partial R}$, $f_{L_m}\equiv\dfrac{\partial f}{\partial L_m}$, $g^{\mu\nu}\nabla_\mu\nabla^\nu$ is the D'Alembertian, and $T_{\mu\nu}$ represents the energy-momentum tensor for the perfect fluid, defined by

$$T_{\mu\nu}=\frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}\,. \tag{3}$$

The relation between the trace of energy momentum tensor T, Ricci scalar R and the Lagrangian density of matter L_m obtained by contracting the field equation (2) is

$$Rf_R+3g^{\mu\nu}\nabla_\mu\nabla_\nu f_R-2(f-f_{L_m}L_m)=\frac{1}{2}f_{L_m}T\,. \tag{4}$$

Here, $g^{\mu\nu}\nabla_\mu\nabla_\nu F=\dfrac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu F)$ for any scalar function F

Moreover, one can acquire the following result by taking covariant derivative in equation (2),

$$\nabla^\mu T_{\mu\nu}=2\nabla^\mu\log(f_{L_m})\frac{\partial L_m}{\partial g^{\mu\nu}}\,.$$

The energy momentum tensor for cosmic string is given by (Adhav *et al.* 2011),

$$T^\mu_\nu=\rho u_\mu u^\nu-\lambda x_\mu x^\nu\,, \tag{5}$$

where ρ is rest energy density of strings with particles attached to them, λ is the tension density of strings. Orthonormalization of four velocity vectors u^μ and the x_μ, the direction of anisotropy of strings, obeys the following relation.

$$u_\nu u^\nu=-x_\nu x^\nu=1\,,\qquad u^\nu x_\nu=0$$

Metric and Field Equations in $f(R, L_m)$ Gravity

We consider the flat FLRW metric as

$$ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right]. \tag{6}$$

Here, $a(t)$ is the scale factor that measures the cosmic expansion at time t.
Using equations (5) and (6), the Friedmann equations that describes dynamics of the universe in $f(R, L_m)$ gravity are read as

$$3\frac{\dot{a}^2}{a^2} f_R + \frac{1}{2}\left(f - f_R R - f_{L_m} L_m\right) + 3\frac{\dot{a}}{a}\dot{f}_R = \frac{1}{2} f_{L_m}\rho , \tag{7}$$

$$\frac{\ddot{a}}{a} f_R + 3\frac{\dot{a}^2}{a^2} f_R - \ddot{f}_R - 3\frac{\dot{a}}{a}\dot{f}_R + \frac{1}{2}\left(f_{L_m} L_m - f\right) + 3\frac{\dot{a}}{a}\dot{f}_R = \frac{1}{2} f_{L_m}(\rho - \lambda) . \tag{8}$$

Cosmological $f(R, L_m)$. Model-I

Here we consider the following form of $f(R, L_m)$ model as (Harko *et al.* 2014),

$$f(R, L_m) = \frac{R}{2} + L_m^{\eta} + \beta , \tag{9}$$

where β and η are free model parameters.
For this particular $f(R, L_m)$ model we take $L_m = \rho$ (Harko *et al.* 2014), the Friedmann equation (7) and (8) becomes

$$3\frac{\dot{a}^2}{a^2} = (2\eta - 1)\rho^{\eta} - \beta , \tag{10}$$

$$2\frac{\ddot{a}}{a} = \eta\rho^{\eta-1}\lambda - \rho^{\eta} - \beta . \tag{11}$$

Here we have system of two equations as presented in equations (10) and (11) involving three unknowns a, ρ and λ. As a result in order to solve a system of equations, it becomes essential to consider a specific condition for getting deterministic solution. For that we consider two different laws.

i) Power Law

In the available literature, there exists many relations in between a and t. For our analysis, we consider power law (Sharif & Zubair (2012)) as

$$a = \alpha t^{\chi} .$$

Using equation (10) the rest energy density ρ is given by

$$\rho = \left[\frac{1}{(2\eta - 1)}\left(\frac{3\chi^2}{t^2} + \beta\right)\right]^{\frac{1}{\eta}} , \tag{12}$$

here the model is valid for $\eta > 0.5$.
Using equation (11), the tension density λ is given by

$$\lambda = \frac{\dfrac{2\alpha\chi(\chi - 1)t^2}{\alpha t^{\chi}} + \dfrac{1}{(2\eta - 1)}\left(\dfrac{3\chi^2}{t^2} + \beta\right) - \beta}{\eta\left[\dfrac{1}{(2\eta - 1)}\left(\dfrac{3\chi^2}{t^2} + \beta\right)\right]^{\frac{\eta-1}{\eta}}} . \tag{13}$$

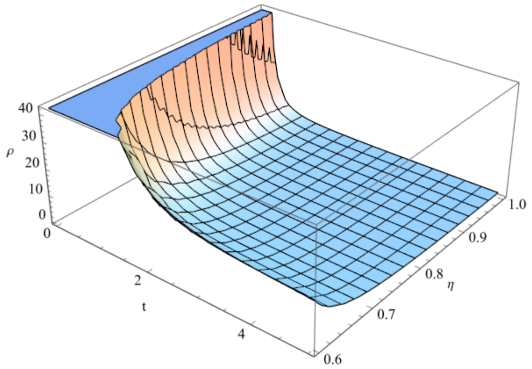

Figure 1. The rest density has been plotted by considering $\alpha = 0.2, \beta = 1, \chi = 0.8$

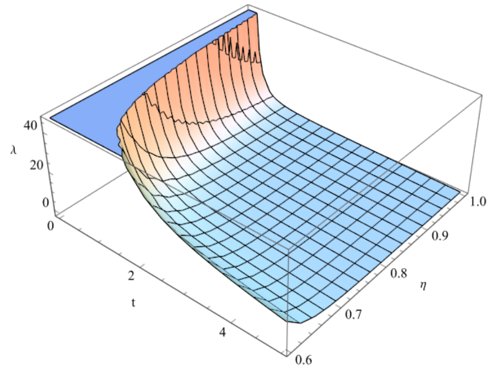

Figure 2. The tension density has been plotted by considering $\alpha = 0.2, \beta = 1, \chi = 0.8$

ii) Hybrid Expansion Law

The hybrid expansion law is given by (Agrawal and Nile 2024)

$$a = te^{mt},$$

where, $m > 0$ be any constant.

Using equation (10) the rest density ρ is given by

$$\rho = \left[\frac{1}{(2\eta - 1)}\left(\frac{3(mt+1)^2}{t^2} + \beta\right)\right]^{\frac{1}{\eta}}. \tag{14}$$

Using equation, the tension density λ is given by

$$\lambda = \frac{\frac{2(2m + m^2 t)}{t} + \left[\frac{1}{(2\eta - 1)}\left(\frac{3(mt+1)}{t^2} + \beta\right)\right] + \beta}{\eta\left[\frac{1}{(2\eta - 1)}\left(\frac{3(mt+1)}{t^2} + \beta\right)\right]^{\frac{\eta-1}{\eta}}}. \tag{15}$$

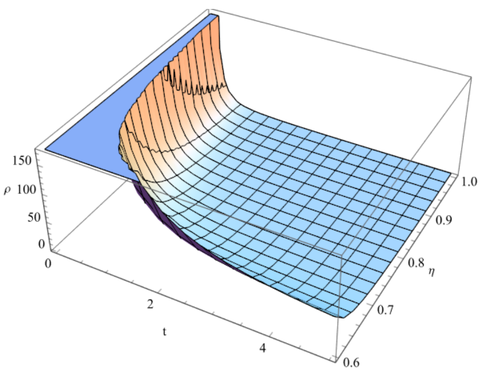

Figure 3. The rest density has been plotted by considering $\alpha = 0.2, \beta = 1, \chi = 0.8, m = 0.5$

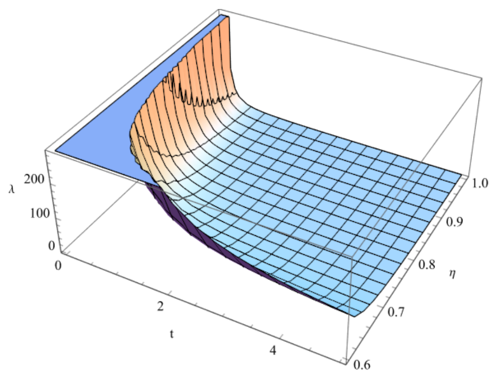

Figure 4. The tension density has been plotted by considering $\alpha = 0.2, \beta = 1, \chi = 0.8, m = 0.5$

Cosmological $f(R, L_m)$. Model-II

Here we consider the anther form of $f(R, L_m)$ model as (Harko *et al.* 2014),

$$f(R, L_m) = \Lambda e^{\frac{R}{2\Lambda} + \frac{L_m}{\Lambda}},$$

where $\Lambda > 0$ is arbitrary constant.

For this particular $f(R, L_m)$ model we take $L_m = \rho$ (Harko *et al.* 2014).

The Friedmann equation (8) and (9) becomes

$$\Lambda - \frac{3\ddot{a}}{a} + \frac{3\dot{a}}{2a\Lambda} = 2\rho\,, \tag{16}$$

$$\frac{\ddot{a}}{a} + \frac{3\dot{a}^2}{a^2} - \frac{1}{4\Lambda^2} - \frac{3\dot{a}}{2a\Lambda} - \Lambda = -\lambda\,. \tag{17}$$

The system comprises of two equations (16)-(17), with three unknown variables. To find the solution of this system of equations, it is important to identify a particular condition which guarantees a definite solution.

i) Power Law

By using the power law given by equation (12) and equations (16), we get the rest density as

$$\rho = \frac{1}{2}\left[\Lambda - \frac{3\chi(\chi-1)}{t^2} + \frac{3\chi}{2t\Lambda}\right]. \tag{18}$$

Using the equation (17) the tension density is given by

$$\lambda = \Lambda + \frac{1}{4\Lambda^2} - \frac{\chi(\chi-1)}{t^2} - \frac{3\chi^2}{t^2} + \frac{3\chi}{2t\Lambda}\,. \tag{19}$$

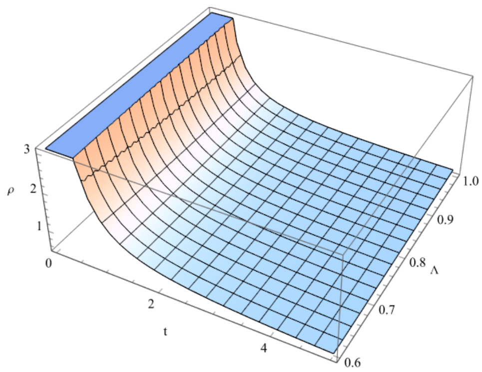

Figure 5. The rest density has been plotted by considering $\chi = 0.8, \Lambda = 0.1$

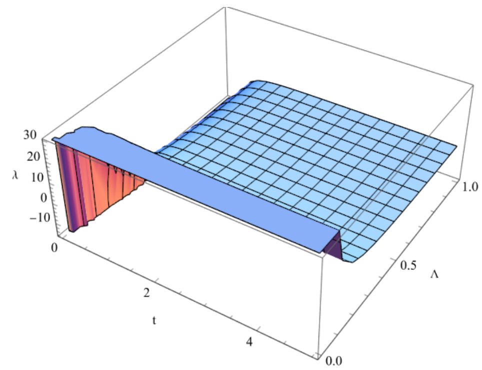

Figure 6. The tension density has been plotted by considering $\chi = 0.8, \Lambda = 0.1$

ii) Hybrid Expansion Law

By using the hybrid expansion law given by equation (14) and using equations (16), we get the rest density ρ as

$$\rho = \frac{1}{4\Lambda}\left[2\Lambda^2 - 6\left(m^2 + \frac{2m}{t}\right) + 3\left(m + \frac{1}{t}\right)\right]. \tag{20}$$

Using equation (17) the tension density λ given by

$$\lambda = \Lambda + \frac{1}{4\Lambda^2} + \frac{3(m + 1/t)}{2\Lambda} - \left(m^2 + \frac{2m}{t}\right) - 3\left(m + \frac{1}{t}\right)^2. \tag{21}$$

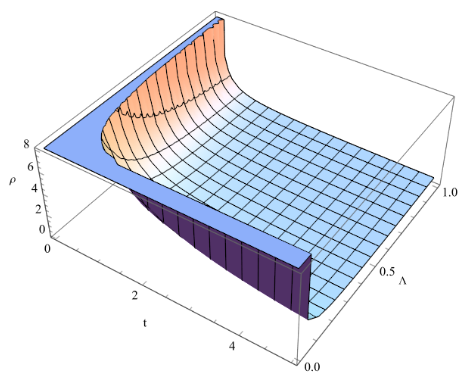

Figure 7. The rest density has been plotted by considering $\chi = 0.8, \Lambda = 0.1, m = 0.5$

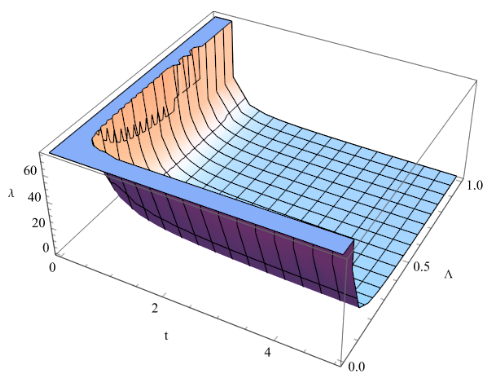

Figure 8. The tension density has been plotted by considering $\chi = 0.8, \Lambda = 0.1, m = 0.5$

Common Physical Parameters for Model-I and Model-II:

Power Law:

We obtained the Hubble parameter and the deceleration parameter in terms of t as

$$H = \frac{\dot{a}}{a} = \frac{\chi}{t}, \tag{22}$$

$$q = \frac{1}{\chi} - 1. \tag{23}$$

The spatial volume is given by

$$V = a^3 = (\alpha t)^3. \tag{24}$$

By using equation (29), we can obtain the Scalar expansion, mean anisotropic parameter and Shear scalar as

$$\theta = 3H = \frac{3\chi}{t}, \tag{25}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right) = 0, \tag{26}$$

$$\sigma^2 = \frac{3}{2}\Delta H = 0. \tag{27}$$

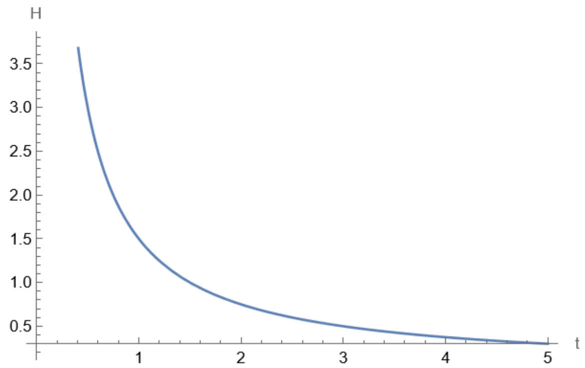

Figure 9. The mean Hubble Parameter has been plotted by considering $\chi = 0.8$

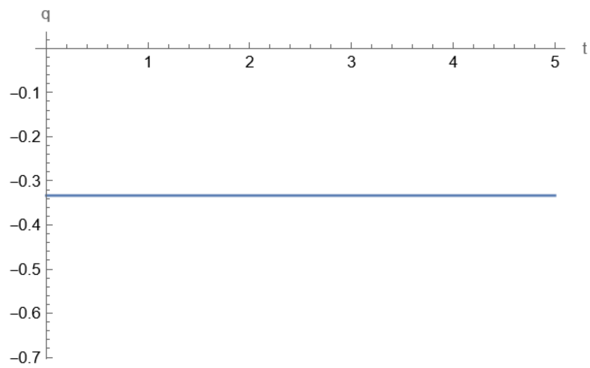

Figure 10. The deceleration parameter has been plotted by considering $\chi = 0.8$

Hybrid Expansion Law:

We obtained the Hubble parameter and the deceleration parameter in terms of t as

$$H = \frac{\dot{a}}{a} = m + \frac{1}{t}, \tag{28}$$

$$q = \frac{1}{(1+mt)^2} - 1. \tag{29}$$

The spatial volume is given by

$$V = a^3 = t^3 e^{3mt}. \tag{30}$$

By using equation (28), we can obtain the Scalar expansion, anisotropic parameter and Shear scalar as

$$\theta = 3H = 3\left(m + \frac{1}{t}\right), \tag{31}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right) = 0, \tag{32}$$

$$\sigma^2 = \frac{3}{2}\Delta H = 0. \tag{33}$$

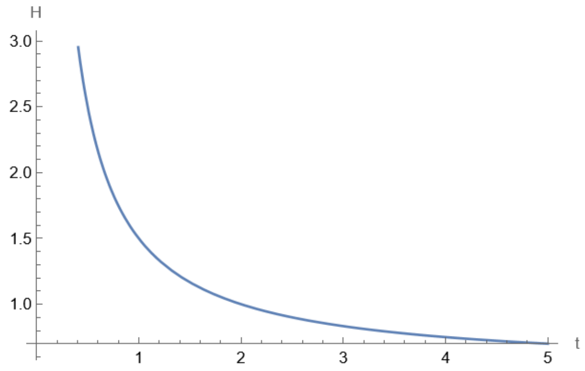

Figure 11. The mean Hubble parameter have been plotted by considering $m = 0.5$

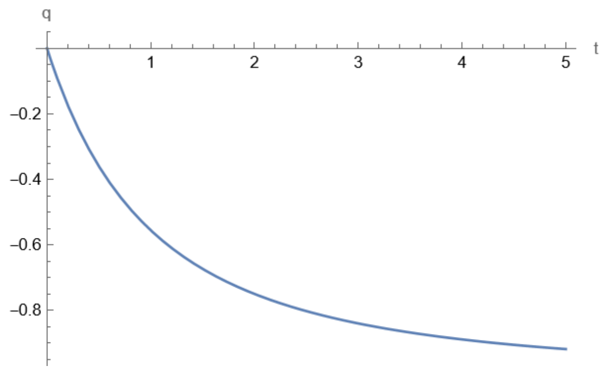

Figure 12. The deceleration parameter has been plotted by considering $m = 0.5$

RESULT AND DISCUSSIONS

In this paper, we have analyzed the FLRW model with cosmic string in the framework of $f(R, L_m)$ gravity considering two different models for two distinct forms of $f(R, L_m)$ gravity such as $f(R, L_m) = \frac{R}{2} + L_m^\eta + \beta$ and $f(R, L_m) = \Lambda e^{\frac{R}{2\Lambda} + \frac{L_m}{\Lambda}}$, where η, β and Λ are free model parameters. We obtained the physical and dynamical parameters such as mean Hubble parameter, deceleration parameter etc. specifically we obtained the rest density ρ and the tension density λ for the studied models.

In model-I, it is observed that the rest density for power law and hybrid expansion law is positively decreasing function of cosmic time t and approaches to 0 as depicted in figure-1 & figure-3. Also, the tension density λ for power law and hybrid expansion law is obtained and plotted with respective cosmic time t as shown in figure 2 & Figure-4.

In model-II, it is observed that the rest density for power law and hybrid expansion law is positively decreasing function of cosmic time t and approaches to 0 as $t \to \infty$ as depicted in figure-5 & figure-7. Also, the tension density λ for power law and hybrid expansion law is obtained and plotted against cosmic time t as shown in figure-6 & Figure 8.

This behavior aligns with widely accepted model of an expanding universe, where the energy density of matter, here string, decreases as the universe expands. As cosmic time progresses, the density of the matter, including cosmic strings, diminishes, which indicates the transitioning of universe from a matter-dominated phase to a dark energy dominated phase.

Also, in figure-9 and figure-11, the mean Hubble parameter for the both models shows the occurrence of the expansion of the universe, which is in consistent with the modern-day observations. Figure-10 the deceleration parameter shows the constant expansion. In the figure-12, the deceleration parameter shows the early deceleration to late acceleration of the universe. The shift from deceleration to acceleration is a key feature of modern cosmological models.

CONCLUSION

In conclusion, the analysis of FLRW model with cosmic string within the frame work of $f(R, L_m)$ gravity has provided valuable insights into the universe's expansion dynamics. The results for both Model – I and Model - II demonstrate that the rest and tension densities decrease with cosmic time, reflecting the universe's transition from a matter-dominated phase to a dark energy-dominated phase. The mean Hubble parameter confirms the ongoing expansion of the universe, consistent with current observations (Riess *et al*. (1998); Peebles and Ratra (2003)). Moreover, the behavior of the deceleration parameter, showing a shift from early deceleration to late-time acceleration, aligns with modern cosmological theories. These findings contribute to a deeper understanding of the role of cosmic strings and modified gravity in shaping the universe's evolution.

ORCID

S.D. Katore, https://orcid.org/0000-0003-0521-4334; **P.R. Agrawal,** https://orcid.org/0000-0002-8040-937X
H.G. Paralikar, https://orcid.org/0009-0003-1301-5340; **A.P. Nile,** https://orcid.org/0009-0003-1504-9085

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ДИНАМІКА СТРУННОЇ КОСМОЛОГІЧНОЇ МОДЕЛІ У $f(R, L_m)$ ТЕОРІЇ ГРАВІТАЦІЇ

С.Д. Каторе[a], П. Р. Агравал[b], Х. Г. Паралікар[a], А.П. Ніл[b]
[a]*Факультет математики, Університет Сант-Гадж Баба Амраваті, Амраваті 444602, Індія*
[b]*Факультет математики, Науковий коледж Бріджлала Біяні, Амраваті, 444602, Індія*

У цій статті розглядається модель FLRW з космічною струною в рамках гравітації $f(R,L_m)$, розглядаючи дві різні форми гравітації $f(R,L_m)$, такі як $f(R,L_m)=\frac{R}{2}+L_m^\eta+\beta$ і $f(R,L_m)=\Lambda e^{\frac{R}{2\Lambda}+\frac{L_m}{\Lambda}}$, де η,β і Λ є вільними параметрами моделі. Розв'язки моделей отримані з використанням як степеневого закону, так і гібридного закону розширення. Отримані фізичні та динамічні параметри моделей аналізуються та представлені у вигляді графічних зображень.

Ключові слова: *гравітація* $f(R,L_m)$ *; струнна космологічна модель; степеневий закон; закон гібридного розширення*