

## NONLINEAR ION-ACOUSTIC SOLITARY WAVES IN A WEAKLY RELATIVISTIC ELECTRON-POSITRON-ION PLASMA WITH RELATIVISTIC ELECTRON AND POSITRON BEAMS

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In this investigation, compressive and rarefactive solitons are demonstrated to exist in a plasma model that includes unmagnetized weak-relativistic positive ions, negative ions, electrons, electron beam and positron beam. For these weakly relativistic non-linear ion-acoustic waves in unmagnetized plasma with electron inertia and relativistic beam, the existence of compressive and rarefactive soliton is investigated by deriving the Korteweg-de Vries (KdV) equation. It has been observed that the amplitude and width of compressive and rarefactive solitons vary differently in response to pressure variation and the presence of electron inertia. The research determines the requirements that must be met for the existence of the nonlinear ion-acoustic solitons. The fluid equations of motion governing the one-dimensional plasma serve as the foundation for the analysis. Various relational forms of the strength parameter ( $\epsilon$ ) are chosen to stretch the space and time variables, leading to a variety of nonlinearities. The findings can have implications not only for astrophysical plasmas but also for inertial confinement fusion plasmas.

**Keywords:** *Relativistic plasma; Soliton; Ion acoustic wave; Positron Beam; Perturbation theory*

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### INTRODUCTION

Kalita, Das and Sarmah [1] have investigated the existence of relativistic compressive solitons of the fast ion acoustic mode in plasmas where the ion beam is drifting perpendicular to the direction of the magnetic field and  $Q$  ( $= m_b/m_i$ , the ratio of the mass of the ion beam to the mass of the ions) is greater than or equal to 1. In these considerations, however, the relativistic Lorentz factor  $\gamma$  is not taken into account by the Poisson's equation or the equation of continuity. Furthermore, some authors, like ElLabany and El-Taibany [2], have studied electronacoustic (EA) solitons without accounting for relativistic effects. Non-relativistic electron acoustic solitary waves were studied by El-Shewy and El-Shamy [3], taking non-thermal electrons into consideration. By using the pseudo potential approach, Alinejad [4] has investigated the characteristics of arbitrary amplitude dust ion acoustic solitary waves in a dusty plasma that contains warm adiabatic, electron following flat-trapped velocity distribution, and arbitrarily (positively or negatively) charged dust immobile dust. The formation of dust ion acoustic solitons in an unmagnetized plasma with the electrons drift velocity through the modified KdV equation has been studied by Das and Karmakar [5]. By resolving the time fractional modified KdV equation, Nazari-Golshan and Nourazar [6] have investigated the nonlinear propagation of small but finite amplitude dust ion-acoustic solitary waves in unmagnetized dusty plasma with trapped electrons and electron solitary waves have been investigated in [13-15] with trapped electrons. Kalita and Das [7] studied both compressive and rarefactive KdV solitons of interesting character in a plasma model consisting of ions and electrons with pressure variations in both the components in the presence of stationary dust. In multispecies plasma model, consisting of negative mobile dusts, non-thermal ions and Boltzmann electrons, dust-ion acoustic solitary waves are studied by Das [8] through reductive perturbative technique by deriving corresponding KdV equation. Different modes of dust ion acoustic waves have been studied theoretically and numerically by Hasnan, Biswas, Habib and Sultana [9] taking into account a four-component magnetised collisional k-nonthermal plasma containing non-inertial k-distributed super thermal electrons, stationary dust grains of opposite charges, and inertial ion fluid. Das [10] investigated the role of streaming speeds of ions and relativistic electrons together with the immobile dust charge to form dust-ion acoustic compressive and rarefactive relativistic solitons in a multispecies plasma model for immobile dusty plasma. Oblique propagation of the quantum electrostatic solitary waves in magnetized relativistic quantum plasma is investigated using the quantum hydrodynamic equations by Soltani, Mohsenpour and Sohbatzadeh [11]. Singh, Kakad, Kakad, Saini [12] studied the evolution of ion acoustic solitary waves (IASWs) in pulsar wind. The study of nonlinear phenomena in their various manifestations is an interesting field of study in many physical contexts [13-16]. Solitons represent a remarkable natural example of nonlinear structure in both magnetised and unmagnetized plasmas [13-16]. Electron solitary wave has been investigated in [16] with resonant electrons. The formation of nonlinear structures like electron acoustic solitons (EAS), ion acoustic solitons (IAS), and double layers [17-20] is being studied by a large number of researchers worldwide. Moreover, space missions such as Solar Anomalous and the Magnetospheric Particle Explorer have demonstrated that relativistic electrons are a threat to the International Space Stations. It may be tried to balance the nonlinear effect by dispersion that leads to solitons in the context of wave particle interactions, in order to avoid warning of the dangers. Space observations with energies greater

than 150 keV confirm the existence of highly relativistic electrons associated with ions in the "Outer zone of the radiation belt" that stretches to distance.

### EQUATIONS GOVERNING DYNAMICS OF PLASMA

The fluid equations of motion, governing the collision less dusty plasma in one dimension are: for positive ions,

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial \gamma_i u_i}{\partial t} + u_i \frac{\partial \gamma_i u_i}{\partial x} = - \frac{\partial \phi}{\partial x}, \quad (2)$$

for negative ions,

$$\frac{\partial n_j}{\partial t} + \frac{\partial(n_j u_j)}{\partial x} = 0, \quad (3)$$

$$\frac{\partial \gamma_j u_j}{\partial t} + u_j \frac{\partial \gamma_j u_j}{\partial x} = \frac{1}{Q'} \frac{\partial \phi}{\partial x}, \quad (4)$$

$$(Q' = \frac{\text{negative ion mass}}{\text{positive ion mass}} = m_j / m_i)$$

for electrons,

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e u_e)}{\partial x} = 0, \quad (5)$$

$$\frac{\partial \gamma_e u_e}{\partial t} + u_e \frac{\partial \gamma_e u_e}{\partial x} = \frac{1}{Q} \frac{\partial \phi}{\partial x} - \frac{1}{Q n_e} \frac{\partial p_e}{\partial x}, \quad (6)$$

$$\frac{\partial p_e}{\partial t} + u_e \frac{\partial p_e}{\partial x} + 3p_e \frac{\partial \gamma_e u_e}{\partial x} = 0, \quad (7)$$

$$(Q = \frac{\text{electron mass}}{\text{positive ion mass}} = m_e / m_i)$$

for electron beam,

$$\frac{\partial n_b}{\partial t} + \frac{\partial(n_b u_b)}{\partial x} = 0, \quad (8)$$

$$\frac{\partial \gamma_b u_b}{\partial t} + u_b \frac{\partial \gamma_b u_b}{\partial x} = \frac{\partial \phi}{\partial x} - 3\sigma n_b \frac{\partial n_b}{\partial x}, \quad (9)$$

where  $\sigma = T_b/T_e$  = electron beam temperature/electron temperature, for positron beam,

$$\frac{\partial n_s}{\partial t} + \frac{\partial(n_s u_s)}{\partial x} = 0, \quad (10)$$

$$\frac{\partial \gamma_s u_s}{\partial t} + u_s \frac{\partial \gamma_s u_s}{\partial x} + \frac{1}{\beta} \frac{\partial \phi}{\partial x} = 0. \quad (11)$$

$$(\beta = \frac{\text{positive ion-beam mass}}{\text{positive ion mass}} = m_s / m_i)$$

The basic governing equations are the continuity and the momentum equations of acoustic mode of the plasma. Equations (7) provides the adiabatic response which contributes additional sources of energy to the non-linearity in the usual ion-electron inertial dynamical system. Electron inertia, which is usually neglected, is considered. Again, these equations are to be supplemented by the following Poisson equation for the charge imbalances.

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + n_j - n_i + n_b - n_s, \quad (12)$$

where,  $\gamma_a = \{1 - (\frac{u_a}{c})^2\}^{-\frac{1}{2}} = 1 + \frac{u_a^2}{2c^2}$ ,  $a = i, j, e, b, s$  and  $c$  is the speed of light. Here, suffixes  $i, j, e, b$  and  $s$  stand for positive ion, negative ion, electron, electron beam and positron beam respectively. In this case, we normalize densities by the equilibrium plasma density  $n_0$ ; velocities ( including  $c$ ) by the acoustic speed  $c_s = (\frac{k_b T_e}{m_i})^{\frac{1}{2}}$ ; time  $t$  by the inverse of the characteristic ion plasma frequency  $\omega_{pi}^{-1} = (m_i/4\pi n_{e0} e^2)^{1/2}$ ; the distance  $x$  by the Debye length  $\lambda_{De} = (k_b T_e/4\pi n_{e0} e^2)^{1/2}$ ; the electron pressure  $p_e$  by the characteristic electron pressure  $p_{e0} = n_{e0} k_b T_e$ ; and, the electric potential  $\phi$  by  $(\frac{k_b T_e}{e})$ , where  $k_b$  is the Boltzmann constant and  $T_e$  is the characteristic electron temperature.

**KdV Equation and its Solution**

We use the stretched variables

$$\xi = \epsilon^2(x - Vt) \text{ and } \tau = \epsilon^2 t \tag{13}$$

(where  $V$  is the phase velocity) along with the expansions of the flow variables in terms of the smallness parameter  $\epsilon$  as  $n_k = n_{k0} + \epsilon n_{k1} + \epsilon^2 n_{k2} + \epsilon^3 n_{k3} + \dots$ ,  $u_k = u_{k0} + \epsilon u_{k1} + \epsilon^2 u_{k2} + \epsilon^3 u_{k3} + \dots$ ,  $p_e = 1 + \epsilon p_{e1} + \epsilon^2 p_{e2} + \epsilon^3 p_{e3} + \dots$ ,  $\phi = \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \dots$ , ( $k = i, j, e, b$  and  $s$ ), where  $n_{e0} = 1$ ,  $n_{b0} = 1$  and  $u_{e0} = 1$  to derive the KdV equation from the set of equations (1) to (11).

Using the transformation (13) and the expansions of  $n_i, n_j, n_e, n_b, u_i, u_j, u_e, u_b$  and  $p_e$  in equations (1) to (11) and equating the coefficient of the first lowest-order of  $\epsilon$  we get,

$$\begin{aligned} n_{i1} &= \frac{n_{i0}}{(V-u_{i0})^2 \beta_i} \phi_1, u_{i1} = \frac{1}{(V-u_{i0}) \beta_i} \phi_1, n_{j1} = \frac{-n_{j0}}{(V-u_{j0})^2 Q' \beta_j} \phi_1, u_{j1} = -\frac{1}{(V-u_{j0}) Q' \beta_j} \phi_1, \\ n_{e1} &= \frac{1}{\{3-(V-u_{e0})^2 Q\} \beta_e} \phi_1, u_{e1} = \frac{(V-u_{e0})}{\{3-(V-u_{e0})^2 Q\} \beta_e} \phi_1, p_{e1} = \frac{3}{\{3-(V-u_{e0})^2 Q\}} \phi_1, \\ n_{b1} &= -\frac{n_{b0} \phi_1}{(V-u_{b0})^2 \beta_b - 3\sigma n_{b0}}, u_{b1} = -\frac{(V-u_{b0})}{(V-u_{b0})^2 \beta_b - 3\sigma n_{b0}} \phi_1, \\ n_{s1} &= \frac{n_{s0} \phi_1}{(V-u_{s0})^2 \beta_s}, u_{s1} = \frac{1}{\beta(V-u_{s0}) \beta_s} \phi_1, \end{aligned}$$

$$\text{with } \beta_i = 1 + \frac{3u_{i0}^2}{2c^2}, \beta_j = 1 + \frac{3u_{j0}^2}{2c^2}, \beta_e = 1 + \frac{3u_{e0}^2}{2c^2}, \beta_b = 1 + \frac{3u_{b0}^2}{2c^2} \text{ and } \beta_s = 1 + \frac{3u_{s0}^2}{2c^2},$$

where  $u_{i0}, u_{j0}, u_{e0}, u_{b0}$  and  $u_{s0}$  are initial streaming velocities of relativistic positive ions, relativistic negative ions, relativistic electrons, relativistic electron beam and relativistic electron beam respectively.

Using the expansions of  $n_{i1}, n_{j1}, n_{e1}, n_{b1}$  and  $n_{s1}$  in  $n_{e1} + n_{j1} - n_{i1} + n_{b1} - n_{s1} = 0$ , the expression of phase velocity  $V$  is found as

$$\frac{1}{\{3-(V-u_{e0})^2 Q\} \beta_e} - \frac{n_{j0}}{(V-u_{j0})^2 Q' \beta_j} - \frac{n_{i0}}{(V-u_{i0})^2 \beta_i} - \frac{n_{b0} \phi_1}{(V-u_{b0})^2 \beta_b - 3\sigma n_{b0}} - \frac{n_{s0} \phi_1}{(V-u_{s0})^2 \beta_s} = 0.$$

Eliminating  $u_{i2}, u_{j2}, u_{e2}, u_{b2}, u_{s2}$  and  $p_{e2}$  from the equations obtained by equating the coefficient of second higher order terms of  $\epsilon$  we get the KdV equation as,

$$\frac{\partial \phi_1}{\partial \tau} + p \phi_1 \frac{\partial \phi_1}{\partial \xi} + q \frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \tag{14}$$

where

$$p = \frac{K_1}{K_2}, q = \frac{1}{-K_2},$$

$$K_1 = (n_{b0} L_5 D_4^2 \beta_i^3 D_1^4 Q'^2 \beta_j^3 D_2^4 \beta_e^3 L_1^2 - 2\beta_b n_{b0} D_4^2 \beta_i^3 D_1^4 Q'^2 \beta_j^3 D_2^4 \beta_e^3 L_1^2 - 3\sigma n_{b0}^3 \beta_i^3 D_1^4 Q'^2 \beta_j^3 D_2^4 \beta_e^3 L_1^2 - 3n_{i0} L_2 L_6^3 Q'^2 \beta_j^3 D_2^4 \beta_e^3 L_1^2 + 3n_{j0} L_3 L_6^3 \beta_i^3 D_1^4 \beta_e^3 L_1^2 + 3(L_4 - L_7) L_6^3 \beta_i^3 D_1^4 Q'^2 \beta_j^3 D_2^4) / (L_6^3 \beta_i^3 D_1^4 Q'^2 \beta_j^3 D_2^4 \beta_e^3 L_1^2)$$

$$K_2 = \frac{-2n_{i0} Q' \beta_j D_2^3 \beta_e L_1^2 L_6^2 - 2n_{j0} D_1^3 \beta_i \beta_e L_1^2 L_6^2 - 2Q D_3 D_1^3 \beta_i Q' \beta_j D_2^3 L_6^2 - 2n_{b0} \beta_b D_4 D_1^3 \beta_i Q' \beta_j D_2^3 \beta_e L_1^2}{D_1^3 \beta_i Q' \beta_j D_2^3 \beta_e L_1^2 L_6^2}$$

$$\text{and } D_1 = V - u_{i0}, D_2 = V - u_{j0}, D_3 = V - u_{e0}, D_4 = V - u_{b0},$$

$$L_1 = 3 - Q(V - u_{e0})^2, L_2 = \beta_i - \frac{u_{i0}(V-u_{i0})}{c^2}, L_3 = \beta_j - \frac{u_{j0}(V-u_{j0})}{c^2}, L_4 = \beta_e - \frac{u_{e0}(V-u_{e0})}{c^2},$$

$$L_5 = \frac{3u_{b0}(V-u_{b0})}{c^2} - \beta_b, L_6 = 3\sigma n_{b0} - (V - u_{b0})^2 \beta_b, L_7 = \frac{\beta_e(1+3\beta_e)}{L_1}$$

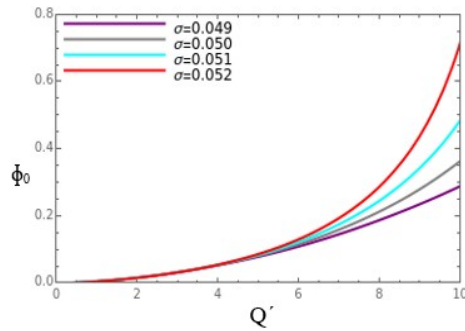
We introduce the variable  $\eta = \xi - U\tau$ , where  $U$  is the velocity of the wave in the linear  $\eta$  space, to find a stationary solution of the KdV equation (14). Equation (14) can be integrated using the boundary conditions  $\phi_1 = \frac{\partial \phi_1}{\partial \eta} = \frac{\partial^2 \phi_1}{\partial \eta^2} = 0$  as  $|\eta| \rightarrow \infty$ , to give

$$\phi_1 = \phi_0 \text{sech}^2(\eta/\Delta) \tag{15}$$

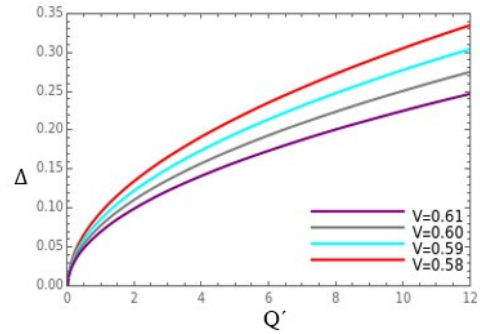
where  $\phi_0 = (3U/p)$  is the amplitude and  $\Delta = (4q/U)^{1/2}$  is the width of the soliton respectively.

**RESULTS AND DISCUSSION**

Weak relativistic effects are incorporated in pursuit of the formation of solitary waves in this plasma model. For nonlinear ion-acoustic solitary waves we have used the reductive perturbation theory to reduce the basic set of equations to KdV equation (14). We have investigated the effects of plasma parameters on the nature of the solitary waves in this model of plasma and displayed their variation graphically in figures 1 to 12. For numerical analysis, some appropriate values of plasma parameters are considered as:  $c = 300, Q = 0.00054$ .



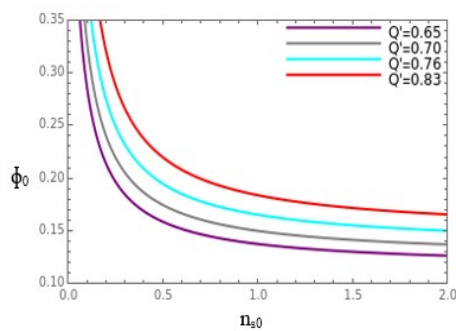
**Figure 1.** Variation of amplitude with respect to  $Q'$  for different  $\sigma$  and fixed  $U = 0.03, u_{i0} = 1.17, V = 0.74, u_{j0} = 0.95, n_{i0} = 1.06, n_{b0} = 1.06, u_{b0} = 0.95, n_{j0} = 1.06, u_{e0} = 1.22, n_{s0} = 1.27, u_{s0} = 1.01$



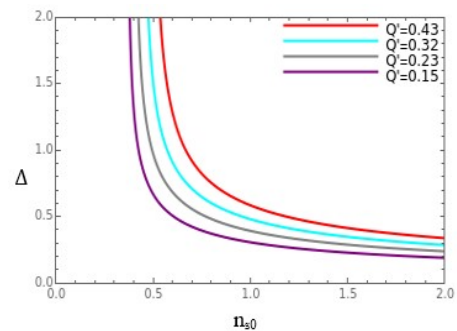
**Figure 2.** Variation of width with respect to  $Q'$  for different  $V$  and fixed  $U = 1.01, u_{i0} = 1.11, u_{j0} = 1.17, n_{i0} = 1.22, n_{b0} = 1.33, u_{b0} = 1.01, n_{j0} = 0.79, \sigma = 0.412, u_{e0} = 0.74, n_{s0} = 1.11, u_{s0} = 0.75$

Variation of amplitude with respect to mass ratio is shown in Figure 1. The figure shows that when we increase mass ratio while keeping other parameters mixed the amplitude increases positively. Therefore, we can say that our plasma model has compressive solitons whose amplitude increases as we increase the value of mass ratio. If we increase the value of  $\sigma$  from 0.049 to 0.052 we observe that the soliton amplitude gradually rises. Thus, we can say that we get compressive solitons of higher amplitude for greater values of  $\sigma$  in the comparison of amplitude with respect to mass ratio. Figure 2 shows variation of width with respect to mass ratio. While keeping other parameters fixed as mentioned in Figure 2 we observe that soliton width increases as mass ratio increases. We also observe that soliton width increases when phase velocity increases from 0.58 to 0.61 in the comparison of width with respect to mass ratio.

Figure 3 depicts the amplitude variation with respect to  $n_{s0}$ . The figure illustrates how the amplitude increases positively when we increase  $n_{s0}$  while maintaining fixed values of other parameters. Therefore, we can conclude that our plasma model has compressive solitons whose amplitude increases as we increase the value of  $n_{s0}$ . A gradual increase in the soliton amplitude is observed when the mass ratio is increased from 0.65 to 0.83. Therefore, in the comparison of amplitude with respect to  $n_{s0}$ , we can say that for larger values of  $\sigma$ , we obtain compressive solitons of higher amplitude. Figure 4 illustrates how width varies in relation to  $n_{s0}$ . Soliton width rises as  $n_{s0}$  increases, as shown in Figure 4, while other parameters remain fixed. In the comparison of width with respect to  $n_{s0}$ , we also find that soliton width increases when mass ratio increases from 0.15 to 0.43.

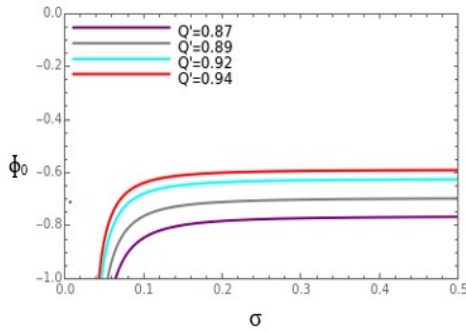


**Figure 3.** Variation of amplitude with respect to  $n_{s0}$  for different  $Q'$  and fixed  $U = 0.37, u_{i0} = 0.85, v = 0.47, u_{j0} = 0.9, n_{i0} = 0.85, n_{b0} = 0.9, u_{b0} = 0.42, n_{j0} = 0.74, \sigma = 0.047, u_{e0} = 0.95, u_{s0} = 0.23$

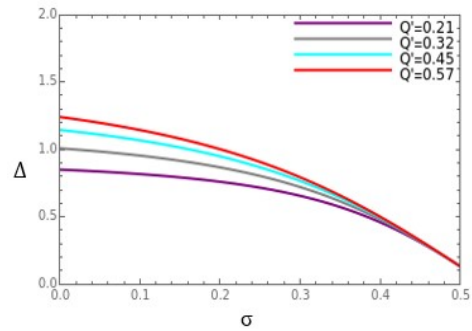


**Figure 4.** Variation of width with respect to  $n_{s0}$  for different  $Q'$  and fixed  $U = 1.83, u_{i0} = 1.82, V = 0.45, u_{j0} = 1.43, n_{i0} = 1.01, n_{b0} = 1.33, u_{b0} = 1.64, n_{j0} = 0.37, \sigma = 0.071, u_{e0} = 1.11, u_{s0} = 1.17$

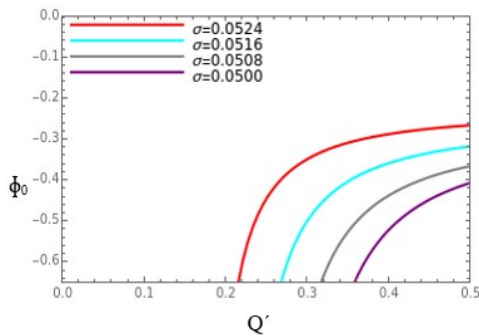
Figure 5 illustrates how the amplitude varies in relation to  $\sigma$ . As we hold other parameters constant, the figure illustrates how the amplitude varies negatively as we increase  $\sigma$ . We can therefore conclude the presence of rarefactive soliton in our plasma model which have increasing amplitude as we increase the value of  $\sigma$ . We find that the soliton amplitude gradually decreases as we increase the mass ratio from 0.87 to 0.94. Therefore, in the comparison of amplitude with respect to  $\sigma$ , we can say that for larger values of mass ratio, we obtain rarefactive solitons of lower amplitude. Figure 6 illustrates how width varies in relation to  $\sigma$ . It is observed that the soliton width increases as  $\sigma$  increases, while other parameters remain fixed as indicated in figure 6. In the comparison of width with respect to  $\sigma$ , we also find that soliton width increases when mass ratio increases from 0.21 to 0.57.



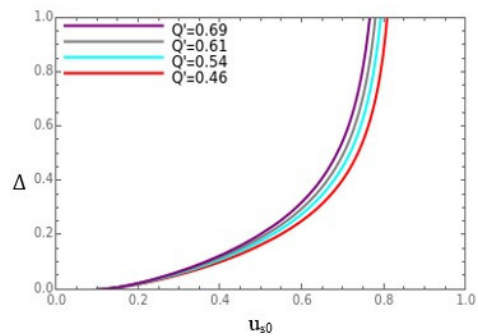
**Figure 5.** Variation of amplitude with respect to  $\sigma$  for different  $Q'$  and fixed  $U = 0.9$ ,  $u_{i0} = 1.54$ ,  $V = 1.3$ ,  $u_{j0} = 1.0$ ,  $n_{i0} = 1.06$ ,  $n_{b0} = 0.95$ ,  $u_{b0} = 1.43$ ,  $n_{j0} = 1.27$ ,  $u_{e0} = 1.06$ ,  $n_{s0} = 0.95$ ,  $u_{s0} = 0.85$



**Figure 6.** Variation of width with respect to  $\sigma$  for different  $Q'$  and fixed  $U = 0.85$ ,  $u_{i0} = 0.85$ ,  $v = 2.15$ ,  $u_{j0} = 1.01$ ,  $n_{i0} = 1.01$ ,  $n_{b0} = 0.9$ ,  $u_{b0} = 0.95$ ,  $n_{j0} = 1.01$ ,  $n_{s0} = 0.85$ ,  $u_{e0} = 1.11$ ,  $u_{s0} = 1.01$



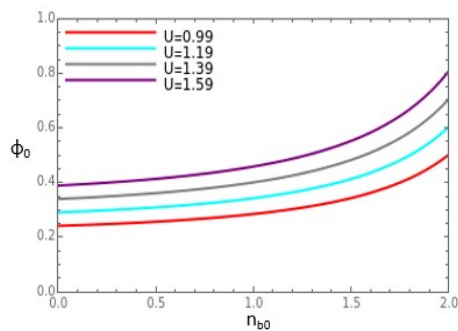
**Figure 7.** Variation of amplitude with respect to  $Q'$  for different  $\sigma$  and fixed  $U = 0.16$ ,  $u_{i0} = 0.9$ ,  $V = 0.7$ ,  $u_{j0} = 1.17$ ,  $n_{i0} = 0.79$ ,  $n_{b0} = 1.27$ ,  $u_{b0} = 1.17$ ,  $n_{j0} = 1.06$ ,  $\sigma = 0.0524$ ,  $u_{e0} = 1.11$ ,  $n_{s0} = 1.01$ ,  $u_{s0} = 1.11$



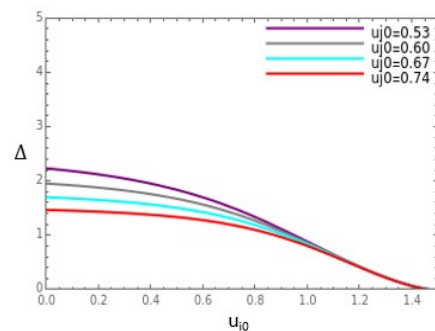
**Figure 8.** Variation of width with respect to  $u_{s0}$  for different  $Q'$  and fixed  $U = 1.8$ ,  $u_{i0} = 0.95$ ,  $V = 0.1$ ,  $u_{j0} = 1.49$ ,  $n_{i0} = 1.22$ ,  $n_{b0} = 1.27$ ,  $u_{b0} = 0.95$ ,  $n_{j0} = 1.11$ ,  $n_{s0} = 1.43$ ,  $u_{e0} = 1.17$ ,  $\sigma = 0.016$

Figure 7 displays the variation in amplitude in relation to the mass ratio. The graph indicates that the amplitude increases negatively as the mass ratio climbs while the other parameters remain unchanged. As such, we can state the presence of rarefactive solitons in our plasma model whose amplitude increases as we increase the value of mass ratio. The soliton amplitude gradually decreases as we increase the value of  $\sigma$  from 0.0500 to 0.0524. Therefore, when comparing amplitude to mass ratio, we obtain rarefactive solitons with lower amplitudes for higher values of  $\sigma$ . Figure 8 illustrates the variation in width in relation to  $u_{s0}$ . We note that soliton width increases as  $u_{s0}$  increases while maintaining other parameters fixed, as shown in figure 8. Furthermore, we note that in the comparison of width with respect to  $u_{s0}$ , soliton width increases when mass ratio increases from 0.46 to 0.69.

Variation of amplitude with respect to  $n_{b0}$  is shown in Figure 9. The figure shows that when we increase  $n_{b0}$  while keeping other parameters mixed the amplitude increases positively. Therefore, we can say that our plasma model has compressive solitons whose amplitude increases as we increase the value of  $n_{b0}$ . If we increase the value of wave speed from 0.99 to 1.59, we observe that the soliton amplitude gradually rises. Thus, we can say that we get compressive solitons of higher amplitude for greater values of  $\sigma$  in the comparison of amplitude with respect to  $n_{b0}$ . Figure 10 shows variation of width with respect to  $u_{i0}$ . While keeping other parameters fixed as mentioned in figure 10, we observe that soliton width increases as  $u_{i0}$  increases. We also observe that soliton width decreases when  $u_{j0}$  increases from 0.53 to 0.74 in the comparison of width with respect to  $u_{i0}$ .



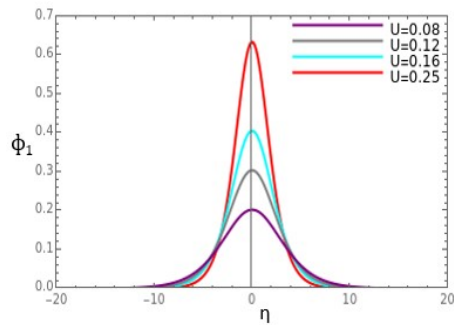
**Figure 9.** Variation of amplitude with respect to  $n_{b0}$  for different  $U$  and fixed  $u_{i0} = 0.62$ ,  $V = 0.21$ ,  $u_{j0} = 0.39$ ,  $n_{i0} = 0.69$ ,  $Q' = 0.85$ ,  $n_{s0} = 0.53$ ,  $u_{b0} = 0.74$ ,  $n_{j0} = 0.37$ ,  $\sigma = 0.026$ ,  $u_{e0} = 0.79$ ,  $u_{s0} = 0.53$



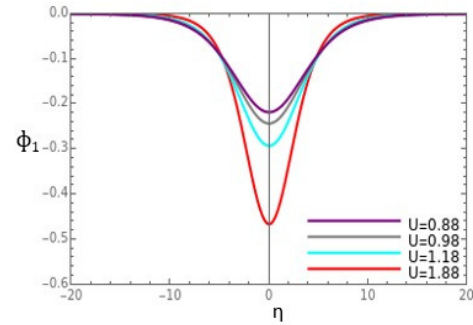
**Figure 10.** Variation of width with respect to  $u_{i0}$  for different  $u_{j0}$  and fixed  $U = 0.21$ ,  $Q' = 0.21$ ,  $V = 1.49$ ,  $n_{i0} = 0.26$ ,  $n_{b0} = 0.26$ ,  $u_{b0} = 0.26$ ,  $n_{j0} = 0.42$ ,  $\sigma = 0.018$ ,  $u_{e0} = 0.16$ ,  $n_{s0} = 0.26$ ,  $u_{s0} = 0.37$



Finally, we have observed variation of solitary wave potential  $\phi_1$  versus  $\eta$  for four different values of wave speed  $U$  as shown in Figure 11 and 12. We found that the wave potential of both compressive (Figure 11) and rarefactive (Figure 12) ion-acoustic soliton increases while the value of wave speed increases.



**Figure 11.** Variation of  $\phi_1$  with respect to  $\eta$  for different  $U$  and fixed  $u_{i0} = 4.2$ ,  $V = 5.4$ ,  $u_{j0} = 0.79$ ,  $n_{i0} = 1.17$ ,  $n_{b0} = 1.59$ ,  $u_{b0} = 1.49$ ,  $n_{j0} = 1.49$ ,  $n_{s0} = 1.43$ ,  $u_{e0} = 1.54$ ,  $Q' = 0.47$ ,  $u_{s0} = 0.47$ ,  $n_{s0} = 0.16$ ,  $\sigma = 0.282$



**Figure 12.** Variation of  $\phi_1$  with respect to  $\eta$  for different  $U$  and fixed  $u_{i0} = 0.58$ ,  $V = 3.56$ ,  $u_{j0} = 0.47$ ,  $n_{i0} = 0.53$ ,  $n_{b0} = 0.58$ ,  $u_{b0} = 0.32$ ,  $n_{j0} = 0.33$ ,  $n_{s0} = 1.43$ ,  $u_{e0} = 0.26$ ,  $Q' = 0.21$ ,  $n_{s0} = 0.42$ ,  $u_{s0} = 0.53$ ,  $\sigma = 0.045$

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### НЕЛІНІЙНІ ІОННО-АКУСТИЧНІ ОДИНОКІ ХВИЛІ В СЛАБОРЕЛЯТИВІСТСЬКІЙ ЕЛЕКТРОН-ПОЗИТРОН-ІОННІЙ ПЛАЗМІ З РЕЛЯТИВІСТСЬКИМИ ПУЧКАМИ ЕЛЕКТРОНІВ І ПОЗИТРОНІВ

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У цьому дослідженні було продемонстровано існування стискаючих і розріджених солітонів у моделі плазми, яка включає ненамагнічені слабкі релятивістські позитивні іони, негативні іони, електрони, електронний пучок і пучок позитронів. Для цих слабо релятивістських нелінійних іонно-акустичних хвиль у ненамагніченій плазмі з електронною інерцією та релятивістським пучком існування стисливого та розрідженого солітону досліджується шляхом виведення рівняння Кортевега-де Фріза (KdV). Було помічено, що амплітуда та ширина солітонів стиснення та розрідження змінюються по-різному у відповідь на зміну тиску та наявності інерції електронів. Дослідженнями визначено вимоги, які повинні бути виконані для існування нелінійних іонно-акустичних солітонів. Основою для аналізу є рівняння руху рідини, що керують одновимірною плазмою. Різні відносні форми параметра міцності ( $\epsilon$ ) вибираються для розширення просторових і часових змінних, що призводить до різноманітних нелінійностей. Отримані результати можуть мати наслідки не лише для астрофізичної плазми, але й для термоядерної плазми з інерційним утриманням.

**Ключові слова:** релятивістська плазма; солітон; іонна акустична хвиля; пучок позитронів; теорія збурень