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LRS BIANCHI COSMOLOGICAL MODEL IN SÁEZ-BALLESTER THEORY OF GRAVITY WITH TIME VARYING COSMOLOGICAL CONSTANT

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The present work deals with the study of a locally rotationally symmetric (LRS) Bianchi type-I cosmological model in the framework of a scalar-tensor theory of gravity formulated by Sáez and Ballester with time varying cosmological constant. To obtain the explicit solutions of the Sáez-Ballester field equations we assume the average scale factor to obey a power law expansion and the cosmological constant to be proportional to the energy density of the cosmic fluid. The dynamical behaviour of relevant cosmological parameters including the Hubble parameter, the deceleration parameter, the energy density, the pressure, the equation of state parameter, the cosmological constant, the shear scalar, the expansion scalar etc. are investigated graphically by examining their evolution versus the redshift parameter. The validation of the four energy conditions are also checked. We find the outcomes of the constructed model to be in good agreement with the recent observational data.

Keywords: *Cosmological constant; Deceleration parameter; Hubble parameter; LRS Bianchi type-I; Sáez-Ballester theory*

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1. INTRODUCTION

Throughout the last few decades, a number of observations in the field of cosmology and astrophysics have been indicating that our universe is currently passing through a phase of accelerated expansion. The list of the observations includes Supernova type Ia (SNIa), Large Scale Structure (LSS), Cosmic Microwave Background (CMB), Wilkinson Microwave Anisotropy Probe (WMAP) [1]- [10] etc. These observations have contradicted the earlier beliefs of the cosmologists that the expansion of the universe would be decelerating due to the gravitationally attractive nature of the matter in the universe. As a result of the contradiction in the belief, cosmologists become more inquisitive to know the root cause of the accelerated cosmic expansion. Within the framework of General Relativity, the leading cause behind the late time acceleration in the expansion of the universe is considered to be a mysterious form of energy with anti-gravity effect and tremendous negative pressure. This exotic form of energy is named dark energy which consists of nearly 68.3% of the total energy budget of the present universe. Another exotic component of the universe is the dark matter which takes approximately 26.8% of the total matter-energy content of the universe. The yet unknown nature of these two exotic components consisting of more than 95% of the universe raises some fundamental questions which can not be explained from the General Theory of Relativity although this theory is very successful in describing many gravitational phenomena up to cosmological scales. In order to ascertain the true nature of dark energy and the root cause of the observed cosmic acceleration, a variety of theoretical models are proposed in the literature which can be classified into two broad categories - the dark energy models and the modified gravity models. The dark energy models are constructed by modifying the matter part of the Einstein-Hilbert action. On the other hand, the modified gravity models are constructed by modifying the gravitational part of the Einstein-Hilbert action.

Among the several dark energy models, ΛCDM model is the simplest and the best fit model of the universe but it is plagued with some theoretical challenges such as the fine-tuning and cosmic coincidence problems. To overcome these problems, different dynamical scalar field models such as quintessence, k-essence, phantom, tachyons etc. [11], Chaplygin gas models [12], Holographic dark energy models [13]- [17] etc. are proposed in the literature. Several modified gravity models are also proposed in the literature such as the $f(R)$ gravity, $f(G)$ gravity, $f(Q)$ gravity, $f(R,T)$ gravity, $f(R,G)$ gravity, $f(Q,T)$ gravity etc., where R is the Ricci scalar curvature, G is the Gauss–Bonnet invariant, Q is the non-metricity scalar, T is the trace of the energy-momentum tensor and some scalar-tensor theories of gravity such as Brans-Dicke theory [18], Sáez-Ballester theory [19] etc. in order to unfold the mystery behind the late time acceleration in the cosmic expansion as well as to study various other aspects of the universe. The Sáez-Ballester theory of gravity was formulated by Sáez and Ballester in 1986. This theory is a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field ϕ in a simple manner. The coupling satisfactorily describes the weak fields and also provides a possible way of removing the missing matter problem in non-flat Friedmann-Lemaître-Robertson-Walker cosmologies. After the discovery of the acceleration in the rate of expansion of the universe, many researchers have constructed different cosmological models in Sáez-Ballester theory and investigated various aspects of the universe as it can be shown that there exists an antigravity regime in this theory. Rao *et al.* [20] presented exact string cosmological

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models for Bianchi type II, VIII and IX. Rao *et al*. [21] also discussed the exact Bianchi type II, VIII and IX perfect fluid cosmological models. Naidu *et al*. [22] investigated a Bianchi type-III universe in the presence of anisotropic dark energy. Mishra and Chand [23] studied the dynamical nature of Bianchi type-I model considering a bilinearly varying deceleration parameter. Mishra and Dua [24] investigated a Bianchi type-I model with cosmological constant, considering the deceleration parameter to be a linear function of the Hubble parameter. They have also studied the statefinder diagnostic and some cosmographic parameters graphically. Naidu *et al*. [25] investigated the dynamical behaviour of FRW type Kaluza-Klein (KK) cosmological model taking the Planck Collaboration data as a special reference and discussed three different models by using hybrid expansion law and varying deceleration parameters. Singh, *et al*. [26] examined a FRW model with bulk viscous fluid. Mishra and Dua [27] examined the behaviours of bulk viscous string cosmological models in the tilted Bianchi type-VI_0 universe. Wath and Nimkar [28] studied a Bianchi type VIII anisotropic dark matter fluid cosmological model. Dabgar and Bhabor [29] investigated a five-dimensional Bianchi type-III model with string cosmology considering both power law and exponential law models.

In the present work, we also consider the Sáez-Ballester theory of gravity and study the cosmological dynamics of a locally rotationally symmetric Bianchi type-I universe with a time varying cosmological constant Λ. The paper is organised as follows: In section 2, we derive the Sáez-Ballester field equations corresponding to a locally rotationally symmetric Bianchi type-I line-element. In section 3, we obtain cosmological solution of Sáez-Ballester field equations by considering the cosmological constant Λ to be proportional to the energy density ρ, and by using a power law expansion for the average scale factor. In section 4, we express the relevant cosmological parameters in terms of the redshift parameter and study their physical behaviour as the universe evolves. In section 5, the validity of the energy conditions are checked. The paper is concluded in section 6 with a brief summary of the main outcomes of our model.

2. BASIC EQUATIONS GOVERNING THE MODEL

The action for the Sáez-Ballester theory of gravity along with time-varying cosmological constant Λ can be expressed as

$$S = \int_{\Sigma} \left[(R - 2\Lambda) + 16\pi\mathcal{L} - W\phi^n \phi_{,i}\phi^{,i} \right] \sqrt{-g}\, dX^1 dX^2 dX^3 dX^4 \tag{1}$$

where, R is the Ricci scalar curvature, $\mathcal{L}$ is the matter Lagrangian, W and n are arbitrary dimensionless constants, ϕ is a dimensionless scalar field, $\phi_{,i}$ is the partial derivative of ϕ with respect to the coordinate X^i, $\phi^{,i}$ is the contraction $\phi_{,\alpha}g^{\alpha i}$ and $g = |g_{ij}|$.

By considering the scalar field ϕ to be vanishing at the boundary of the arbitrary region Σ of integration, the variation of the action (1) with respect to the tensor g^{ij} and the scalar field ϕ leads to the field equations

$$R_{ij} - \frac{R}{2}g_{ij} + \Lambda g_{ij} - W\phi^n \left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k} \right) = -8\pi T_{ij} \tag{2}$$

and

$$2\phi^n \phi^{,k}{}_{;k} + n\phi^n \phi_{,k}\phi^{,k} = 0 \tag{3}$$

where R_{ij} is the Ricci tensor, T_{ij} is the energy-momentum tensor and semicolon represents the covariant derivative.

Now, in order to construct a cosmological model, we consider a locally rotationally symmetric (LRS) Bianchi type-I space-time characterised by the metric

$$ds^2 = dt^2 - A^2 dx^2 - B^2 \left(dy^2 + dz^2 \right) \tag{4}$$

where A and B are the functions of the cosmic time t.

We assume the matter-energy distribution of the universe to be as isotropic perfect fluid of density ρ and pressure p so that the energy-momentum tensor T_{ij} can be taken as

$$T_{ij} = (\rho + p)\, u_i u_j - p g_{ij} \tag{5}$$

where u^i is the four velocity with $u_i u^i = 1$.

In a comoving coordinate system, the field equations (2) and (3) with equations (5) for the metric (4) lead to the following set of field equations:

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{W}{2}\phi^n \dot{\phi}^2 - \Lambda = -8\pi p \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{W}{2}\phi^n \dot{\phi}^2 - \Lambda = -8\pi p \tag{7}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{W}{2}\phi^n\dot{\phi}^2-\Lambda=8\pi\rho \tag{8}$$

$$\frac{\ddot{\phi}}{\dot{\phi}}+\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{n}{2}\frac{\dot{\phi}}{\phi}=0 \tag{9}$$

Integration of equation (9) yields

$$\phi=\left[E\left(\frac{n}{2}+1\right)\right]^{\frac{2}{n+2}},\ n\neq -2 \tag{10}$$

where $E(t)=\int\frac{k_1}{a^3}dt$, k_1 is a constant of integration.
From the equations (6) and (7), we have

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{B}}{B}\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)=0 \tag{11}$$

On integration, it gives

$$\frac{B}{A}=D(t) \tag{12}$$

where $D(t)=e^{\int\frac{k_2}{a^3}dt}$, k_2 is an integrating constant.
Therefore, the average scale factor, $a(t)$ can be expressed as

$$a=\left(AB^2\right)^{\frac{1}{3}}=AD^{\frac{2}{3}} \tag{13}$$

Equations (6)-(8) can be written in terms of the average scale factor a as

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{{k_2}^2}{3a^6}-\frac{W}{2}\frac{{k_1}^2}{a^6}-\Lambda=-8\pi p \tag{14}$$

$$3\frac{\dot{a}^2}{a^2}-\frac{{k_2}^2}{3a^6}+\frac{W}{2}\frac{{k_1}^2}{a^6}-\Lambda=8\pi\rho \tag{15}$$

3. ASSUMPTIONS AND SOLUTION OF THE FIELD EQUATIONS

We have three equations and five unknowns a, Λ, p, ρ and ϕ, which allows us to take two conditions in consideration in order to find the exact solutions of the field equations.
We assume the average scale factor a to obey a power law expansion as

$$a=a_0t^\alpha \tag{16}$$

where $\alpha>0$, a_0 is a constant and represents the present value of a.
In view of equation (15), we consider the cosmological constant $\Lambda(t)$ to be proportional to the energy density $\rho(t)$ with h as the constant of proportionality as

$$\Lambda=h\rho \tag{17}$$

Then, from the equations (10) and (14)-(17) the expressions for ϕ, Λ, p and ρ are obtained as

$$\phi(t)=\left[\frac{k_1}{{a_0}^3}\left(\frac{n}{2}+1\right)\left(\frac{t^{1-3\alpha}}{1-3\alpha}\right)\right]^{\frac{2}{n+2}},n\neq -2 \tag{18}$$

$$\Lambda(t)=\frac{h}{8\pi+h}\left[\frac{3\alpha^2}{t^2}-\frac{{k_2}^2}{3\left(a_0t^\alpha\right)^6}+\frac{W}{2}\frac{{k_1}^2}{\left(a_0t^\alpha\right)^6}\right] \tag{19}$$

$$p(t)=\frac{1}{8\pi}\left[\frac{2\alpha}{t^2}-\frac{2{k_2}^2}{3\left(a_0t^\alpha\right)^6}+W\frac{{k_1}^2}{\left(a_0t^\alpha\right)^6}\right]-\frac{1}{8\pi+h}\left[\frac{3\alpha^2}{t^2}-\frac{{k_2}^2}{3\left(a_0t^\alpha\right)^6}+\frac{W}{2}\frac{{k_1}^2}{\left(a_0t^\alpha\right)^6}\right] \tag{20}$$

$$\rho(t)=\frac{1}{8\pi+h}\left[\frac{3\alpha^2}{t^2}-\frac{{k_2}^2}{3\left(a_0t^\alpha\right)^6}+\frac{W}{2}\frac{{k_1}^2}{\left(a_0t^\alpha\right)^6}\right] \tag{21}$$

4. PROPERTIES OF THE MODEL

The Hubble parameter H measures the rate of expansion of the universe. It is related to the scale factor a by the relation $H=\frac{\dot{a}}{a}$ and therefore, $H>0$ infers the expanding universe. The deceleration parameter q reveals whether the expansion of the universe is uniform, accelerating or decelerating. It is defined by the relation $q=-\frac{a\ddot{a}}{\dot{a}^2}$ and therefore, q is

related to the Hubble parameter H through the relation $q = -1 - \frac{\dot{H}}{H^2}$.
For our model, these two parameters are obtained as

$$H(t) = \frac{\dot{a}}{a} = \frac{\alpha}{t} \tag{22}$$

$$q(t) = -1 + \frac{1}{\alpha} \tag{23}$$

The spatial volume (V), the expansion scalar (θ), the shear scalar (σ^2), the mean anisotropy parameter (A_m) and the equation of state (EoS) parameter (η) are obtained as

$$V(t) = a^3 = (a_0 t^\alpha)^3 \tag{24}$$

$$\theta(t) = 3H = 3\frac{\alpha}{t} \tag{25}$$

$$\sigma^2(t) = \frac{1}{3}\frac{{k_2}^2}{(a_0 t^\alpha)^6} \tag{26}$$

$$A_m(t) = \frac{2}{9}\left(\frac{t}{\alpha}\right)^2 \frac{{k_2}^2}{(a_0 t^\alpha)^6} \tag{27}$$

$$\eta(t) = \frac{p(t)}{\rho(t)} = \frac{(8\pi + h)\left[\frac{2\alpha}{t^2} - \frac{2{k_2}^2}{3(a_0 t^\alpha)^6} + W\frac{{k_1}^2}{(a_0 t^\alpha)^6}\right]}{8\pi\left[\frac{3\alpha^2}{t^2} - \frac{{k_2}^2}{3(a_0 t^\alpha)^6} + \frac{W}{2}\frac{{k_1}^2}{(a_0 t^\alpha)^6}\right]} - 1 \tag{28}$$

The scale factor redshift relation is given by

$$a = \frac{a_0}{1+z} \tag{29}$$

Using equation (16), we obtain

$$t = (1+z)^{-\frac{1}{\alpha}} \tag{30}$$

Therefore, the cosmic time t dependent cosmological parameters of our model can be expressed in terms of the redshift z as
Hubble parameter,

$$H(z) = \alpha(1+z)^{\frac{1}{\alpha}} \tag{31}$$

Deceleration parameter,

$$q(z) = -1 + \frac{1}{\alpha} \tag{32}$$

Spatial volume,

$$V(z) = a^3 = {a_0}^3(1+z)^{-3} \tag{33}$$

Expansion scalar,

$$\theta(z) = 3H = 3\alpha(1+z)^{\frac{1}{\alpha}} \tag{34}$$

Shear scalar,

$$\sigma^2(z) = \frac{1}{3}\frac{{k_2}^2}{{a_0}^6}(1+z)^6 \tag{35}$$

Mean anisotropy parameter,

$$A_m(z) = \frac{2}{9\alpha^2}\frac{{k_2}^2}{{a_0}^6}(1+z)^{6-\frac{2}{\alpha}} \tag{36}$$

Also, scalar field,

$$\phi(z) = \left[\frac{k_1}{{a_0}^3}\left(\frac{n}{2}+1\right)\left(\frac{1}{1-3\alpha}\right)(1+z)^{\frac{3\alpha-1}{\alpha}}\right]^{\frac{2}{n+2}}, \; n \neq -2 \tag{37}$$

Cosmological constant,

$$\Lambda(z) = \frac{h}{8\pi + h}\left[3\alpha^2(1+z)^{\frac{2}{\alpha}} - \frac{{k_2}^2}{3{a_0}^6}(1+z)^6 + \frac{W}{2}\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right] \tag{38}$$

Pressure,

$$p(z) = \frac{1}{8\pi}\left[2\alpha(1+z)^{\frac{2}{\alpha}} - \frac{2{k_2}^2}{3{a_0}^6}(1+z)^6 + W\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right]$$

$$-\frac{1}{8\pi+h}\left[3\alpha^2(1+z)^{\frac{2}{\alpha}}-\frac{{k_2}^2}{3{a_0}^6}(1+z)^6+\frac{W}{2}\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right] \tag{39}$$

Energy density,

$$\rho(z)=\frac{1}{8\pi+h}\left[3\alpha^2(1+z)^{\frac{2}{\alpha}}-\frac{{k_2}^2}{3{a_0}^6}(1+z)^6+\frac{W}{2}\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right] \tag{40}$$

The EoS (equation of state) parameter,

$$\eta(z)=\frac{8\pi+h\left[2\alpha(1+z)^{\frac{2}{\alpha}}-\frac{2{k_2}^2}{3{a_0}^6}(1+z)^6+W\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right]}{8\pi\left[3\alpha^2(1+z)^{\frac{2}{\alpha}}-\frac{{k_2}^2}{3{a_0}^6}(1+z)^6+\frac{W}{2}\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right]}-1 \tag{41}$$

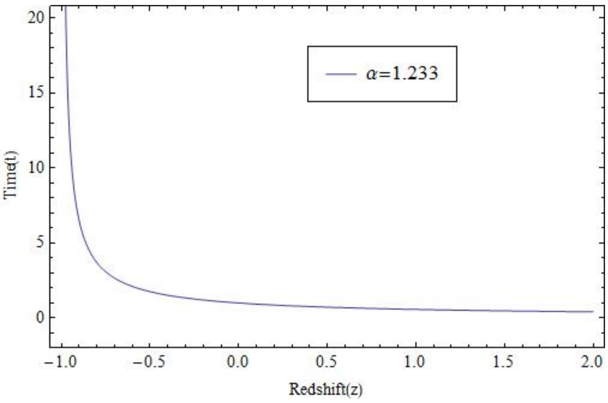

Figure 1. Plot of the cosmic time t v/s redshift z for $\alpha = 1.233$

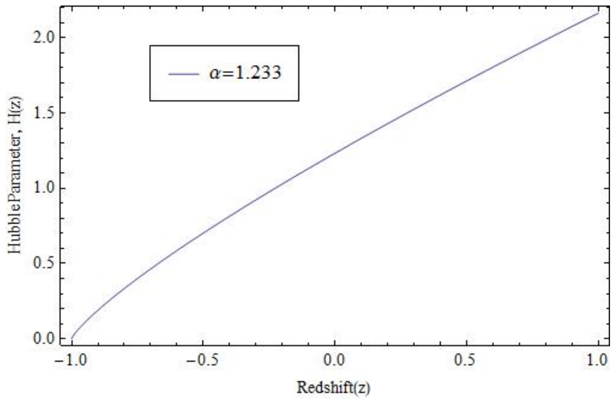

Figure 2. Evolution of the Hubble parameter H v/s redshift z for $\alpha = 1.233$

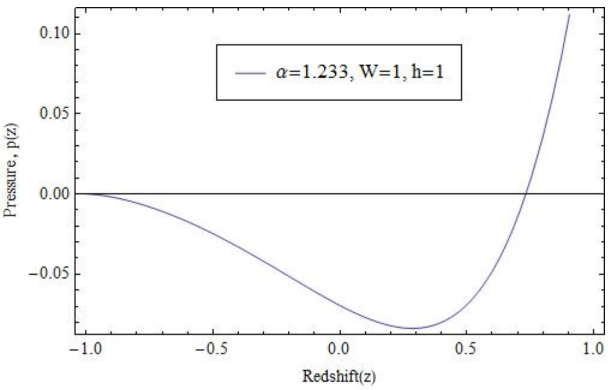

Figure 3. Evolution of the pressure p v/s redshift z for $\alpha = 1.233, h = 1, W = 1, a_0 = k_1 = k_2 = 1$

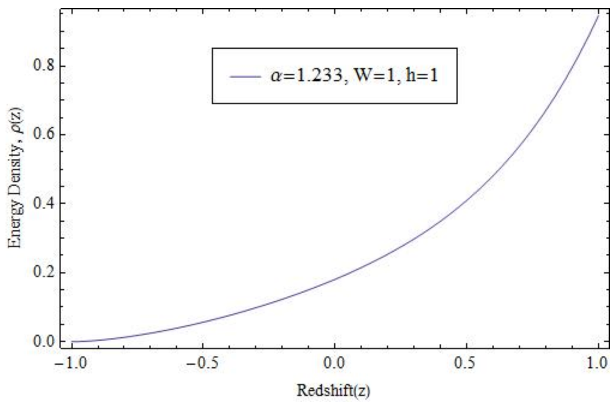

Figure 4. Evolution of the energy density ρ v/s redshift z for $\alpha = 1.233, h = 1, W = 1, a_0 = k_1 = k_2 = 1$

Figure 1 shows the graphical plot of cosmic time t v/s redshift z. From Figure 2 and Figure 8 we can see the decreasing and positive nature of the Hubble parameter H and the expansion scalar θ. In Figure 3, we observe that the pressure p of the cosmic fluid has a peculiar behaviour. It is positive in the early phases of the universe, subsequently becomes negative in the later phase and keeps increasing to attain the zero value at far future. Figure 4 depicts the behaviour of the energy density ρ. It decreases as the universe evolves, remains positive throughout the evolution of the universe and tends to zero at far future, thereby hinting about the expanding universe during the cosmic evolution. Figure 5 shows that the cosmological constant Λ is an increasing function of the redshift z, or equivalently it is a decreasing function of the cosmic time t. The Figure also depicts the positive nature of Λ in the evolving universe which fades away at far future. In Figure 6 we observe the decreasing nature of the EoS parameter η with the universe's evolution. The Figure indicates that the model starts in the radiation-dominated phase and subsequently it enters into the matter-dominated phase. At the late phase of universe's evolution, the model behaves as in the quintessence phase $\left(-1 < \eta < -\frac{1}{3}\right)$. Figure 7 depicts the increasing nature of the spatial volume V, with the evolution of the universe, which gives the indication of the acceleration

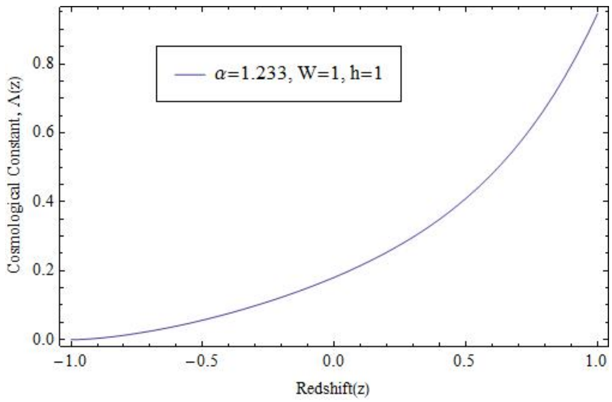

Figure 5. Evolution of the cosmological constant Λ v/s redshift z for $\alpha = 1.233, h = 1, W = 1, a_0 = k_1 = k_2 = 1$

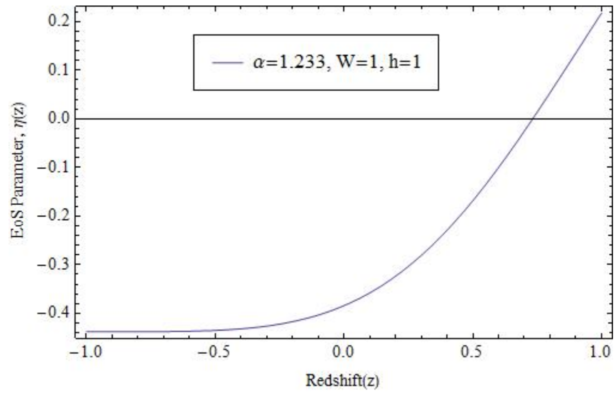

Figure 6. Evolution of the EoS parameter η v/s redshift z for $\alpha = 1.233, h = 1, W = 1, a_0 = k_1 = k_2 = 1$

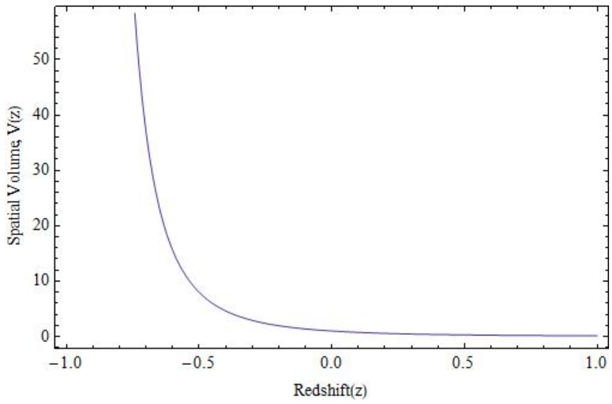

Figure 7. Evolution of the spatial volume V v/s redshift z for $a_0 = 1$

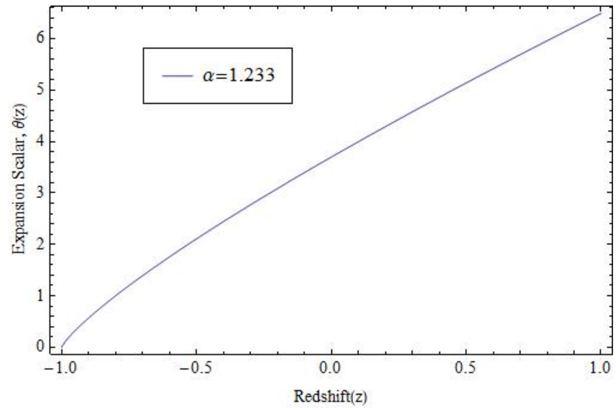

Figure 8. Evolution of the expansion scalar θ v/s redshift z for $\alpha = 1.233$

in the expansion rate of the universe at late times. Figure 9 and Figure 10 show the decreasing nature of the shear scalar σ^2 and the mean anisotropy parameter A_m which tends to zero at late times, thereby indicating the transition from early anisotropic phase to an isotropic phase at late time.

5. ENERGY CONDITIONS:

Energy conditions are simply some linear combinations of the energy density and the pressure with constraints. These conditions are helpful in studying the characteristics of the universe. A normal matter always satisfies all the energy conditions, for the reason that the energy density and the pressure of the normal matter are positive. Violation of the energy conditions hints about the presence of some unknown matter energy which is not normal in the universe. The four energy conditions are: Strong Energy Condition (SEC), Weak Energy Condition (WEC), Dominant Energy Condition (DEC) and Null Energy Condition (NEC).

The SEC suggests that the rate of expansion of the universe decelerates, independent of whether the universe is open, flat, or closed [30]. The WEC suggests that the energy density is always positive and non-increasing. The DEC provides an upper bound on the energy density and therefore an upper bound on the rate of expansion. The NEC implies a (very weak) upper bound on the Hubble parameter and indicates that the energy density of the universe goes down as its size increases.
The energy conditions are given as:
SEC: $\rho + 3p \geq 0$ and $\rho + p \geq 0$
WEC: $\rho + p \geq 0$ and $\rho \geq 0$
DEC: $\rho + p \geq 0$, $\rho - p \geq 0$ and $\rho \geq 0$
NEC: $\rho + p \geq 0$
For our model,

$$(\rho + 3p)(z) = \frac{3}{8\pi}\left[2\alpha(1+z)^{\frac{2}{\alpha}} - \frac{2{k_2}^2}{3{a_0}^6}(1+z)^6 + W\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right]$$

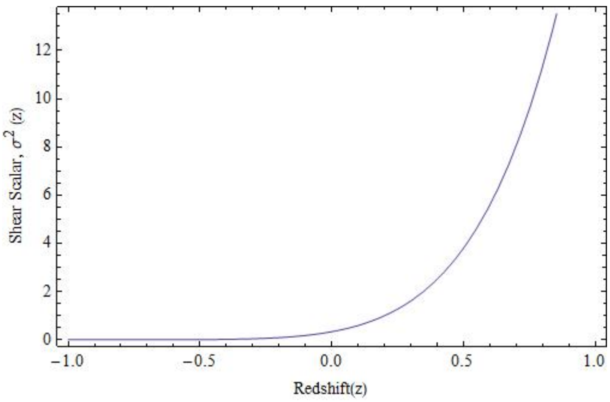

Figure 9. Evolution of the shear scalar σ^2 v/s redshift z for $a_0 = k_2 = 1$

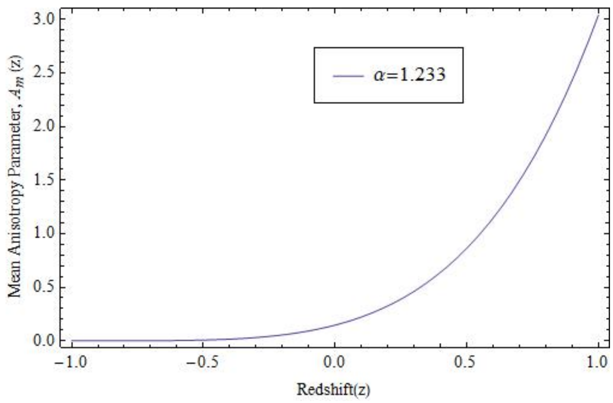

Figure 10. Evolution of the mean anisotropy parameter A_m v/s redshift z for $\alpha = 1.233, a_0 = k_2 = 1$

$$-\frac{2}{8\pi + h}\left[3\alpha^2 (1+z)^{\frac{2}{\alpha}} - \frac{{k_2}^2}{3{a_0}^6}(1+z)^6 + \frac{W}{2}\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right] \quad (42)$$

$$(\rho + p)(z) = \frac{1}{8\pi}\left[2\alpha (1+z)^{\frac{2}{\alpha}} - \frac{2{k_2}^2}{3{a_0}^6}(1+z)^6 + W\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right] \quad (43)$$

$$(\rho - p)(z) = -\frac{1}{8\pi}\left[2\alpha (1+z)^{\frac{2}{\alpha}} - \frac{2{k_2}^2}{3{a_0}^6}(1+z)^6 + W\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right] + \frac{2}{8\pi + h}\left[3\alpha^2 (1+z)^{\frac{2}{\alpha}} - \frac{{k_2}^2}{3{a_0}^6}(1+z)^6 + \frac{W}{2}\frac{{k_1}^2}{{a_0}^6}(1+z)^6\right] \quad (44)$$

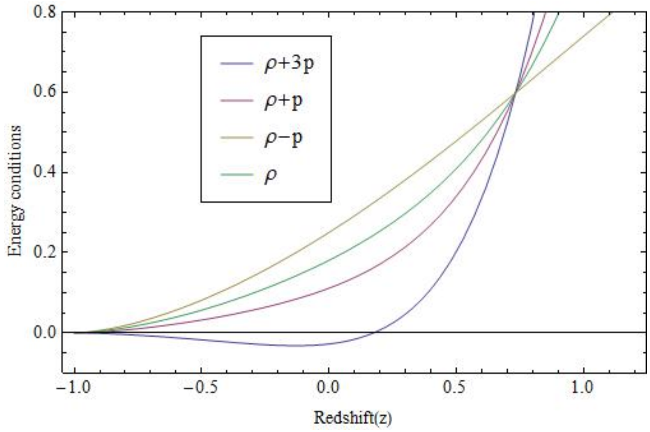

Figure 11. Plot of the energy conditions v/s redshift z for $\alpha = 1.233, h = 1, W = 1, a_0 = k_1 = k_2 = 1$

In Figure 11 we observe that at the very early stage of the universe, all the four energy conditions are satisfied and the three conditions other than the SEC are satisfied throughout the cosmic evolution. However, at a later stage the SEC is violated hinting about the accelerated rate of the universe's expansion, which is in agreement with recent obeservational data.

6. CONCLUDING REMARKS

In this paper, we explore LRS Bianchi type-I universe with a power law expansion in the framework of Sáez-Ballester scalar-tensor theory with a cosmological term Λ which is assumed to be directly proportional to the matter-energy density ρ. We study the evolution of some parameters of cosmological importance such as the Hubble parameter H, the deceleration parameter q, the equation of state (EoS) parameter η, spatial volume V, the expansion scalar θ, Shear scalar σ^2 and the mean anisotropy parameter A_m graphically by choosing the values of the parameters as $\alpha = 1.233, h = 1, W = 1, a_0 = k_1 = k_2 = 1$. We observe that

• The increasing nature of the scale factor a and the Spatial volume V of the universe throughout the cosmic evolution implies the acceleration in the rate of cosmic expansion.

• The decreasing nature of the Hubble parameter H and the expansion scalar θ gives the hint of accelerated expansion of

the universe.
• The deceleration parameter q is constant in nature which may be positive, negative or zero according as $0 < \alpha < 1$, $\alpha > 1$ or $\alpha = 1$.
• With the evolving universe, the cosmological constant Λ and the energy density ρ decrease and tend to zero at later phase of the universe's evolution.
• The decreasing nature of the EoS parameter η with the universe's evolution is seen in Figure 6, which indicates that the model starts in the radiation-dominated phase and subsequently it enters into the matter-dominated phase. At the late phase of universe's evolution, the model behaves as in the quintessence phase $\left(-1 < \eta < -\frac{1}{3}\right)$.
• The decreasing nature of the shear scalar σ^2 and the mean anisotropy parameter A_m which gradually fades away signifies the transitioning from early anisotropic phase to a later isotropic phase.
• Violation of the SEC is indicating the accelerated cosmic expansion agreeing with the observation.
Thus the results of our model, are found to be satisfactory with the current observational data.

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REFERENCES

[1] S. Perlmutter, *et al.*, "Measurements of the cosmological parameters Ω and Λ from the first seven supernovae at $z \geq 0.35$," The Astrophysical Journal, **483**, 565-581 (1997). https://doi.org/10.1086/304265

[2] S. Perlmutter, *et al.*, "Discovery of a supernova explosion at half the age of the Universe," Nature, **391**, 51–54 (1998). https://doi.org/10.1038/34124

[3] A.G. Riess, *et al.*, "Observational evidence from supernovae for an accelerating universe and a cosmological constant," The Astronomical Journal, **116**, 1009-1038 (1998). https://doi.org/10.1086/300499

[4] S. Perlmutter, *et al.*, "Measurements of Ω and Λ from 42 high- redshift supernovae," The Astrophysical Journal, **517**, 565-586 (1999). https://doi.org/10.1086/307221

[5] A.D. Miller, *et al.*, "A Measurement of The Angular Power Spectrum of The Cosmic Microwave Background Froml= 100 to 400," The Astrophysical Journal, **524**, L1–L4 (1999). https://doi.org/10.1086/312293

[6] D.N. Spergel, *et al.*, "First- Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters," The Astrophysical Journal Supplement Series, **148**, 175–194 (2003). https://doi.org/10.1086/377226

[7] C.L. Bennett, *et al.*, "First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results," The Astrophysical Journal Supplement Series, **148**, 1-27 (2003). https://doi.org/10.1086/377253

[8] K. Abazajian, *et al.*, "The Second Data Release of The Sloan Digital Sky Survey," The Astronomical Journal, **128**, 502–512 (2004). https://doi.org/10.1086/421365

[9] M. Tegmark, *et al.*, "The Three-Dimensional Power Spectrum of Galaxies from The Sloan Digital Sky Survey," The Astrophysical Journal, **606**, 702–740 (2004). https://doi.org/10.1086/382125

[10] D.N. Spergel, *et al.*, "Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology," The Astrophysical Journal Supplement Series, **170**, 377-408 (2007). https://doi.org/10.1086/513700

[11] E.J. Copeland, M. Sami, and S. Tsujikawa," Dynamics of Dark Energy," International Journal of Modern Physics D, **15**(11), 1753–1935 (2006). https://doi.org/10.1142/S021827180600942X

[12] A. Kamenshchik, U. Moschella, and V. Pasquier, "An alternative to quintessence," Physics Letters B, **511**, 265–268 (2001). https://doi.org/10.1016/S0370-2693(01)00571-8

[13] M. Li, "A model of holographic dark energy," Physics Letters B, **603**, 1–5 (2004). https://doi.org/10.1016/j.physletb.2004.10.014

[14] L.N. Granda, and A. Oliveros, "Infrared cut-off proposal for the holographic density," Physics Letters B, **669**, 275–277 (2008) https://doi.org/10.1016/j.physletb.2008.10.017

[15] M. Tavayef, "Tsallis holographic dark energy," Physics Letters B, **781**, 195-200 (2018). https://doi.org/10.1016/j.physletb.2018.04.001

[16] H. Moradpour, *et al.*, "Thermodynamic approach to holographic dark energy and the Rényi entropy," The European Physical Journal C, **78**, 829 (2018). https://doi.org/10.1140/epjc/s10052-018-6309-8

[17] E.N. Saridakis, "Barrow holographic dark energy," Physical Review D, **102**, 123525 (2020). https://doi.org/10.1103/PhysRevD.102.123525

[18] C. Brans, and R. H. Dicke, "Mach's Principle and a Relativistic Theory of Gravitation," Physical Review, **124**, 925 (1961). https://doi.org/10.1103/PhysRev.124.925

[19] D. Sáez, and V.J. Ballester, "A simple coupling with cosmological implications," Physics Letters, **113A**(9), (1986). https://doi.org/10.1016/0375-9601(86)90121-0

[20] V.U.M. Rao, M.V. Santhi, and T. Vinutha, "Exact Bianchi type II, VIII and IX string cosmological models in Saez-Ballester theory of gravitation," Astrophysics and Space Science, **314**, 73–77 (2008). https://doi.org/10.1007/s10509-008-9739-1

[21] V.U.M. Rao, M.V. Santhi, and T. Vinutha, "Exact Bianchi type-II, VIII and IX perfect fluid cosmological models in Saez-Ballester theory of gravitation," Astrophysics and Space Science, **317**, 27–30 (2008). https://doi.org/10.1007/s10509-008-9849-9

[22] R.L. Naidu, B. Satyanarayana, and D.R.K. Reddy, "Bianchi Type-III Dark Energy Model in a Saez-Ballester Scalar-Tensor Theory," International Journal of Theoretical Physics, **51**, 2857–2862 (2012). https://doi.org/10.1007/s10773-012-1161-3

[23] R.K. Mishra, and A. Chand, "Cosmological models in Sáez-Ballester theory with bilinear varying deceleration parameter," Astrophysics and Space Science, **365**, 76 (2020). https://doi.org/10.1007/s10509-020-03790-w

[24] R.K. Mishra, and H. Dua, "Bianchi type-I cosmological model in Sáez-Ballester theory with variable deceleration parameter," Astrophysics and Space Science, **366**, 47 (2021). https://doi.org/10.1007/s10509-021-03952-4

[25] R.L. Naidu, *et al*., "Kaluza-Klein FRW dark energy models in Saez-Ballester theory of gravitation," New Astronomy, **85**, 101564 (2021). https://doi.org/10.1016/j.newast.2020.101564

[26] S.S. Singh, *et al*., "Causal Viscous Universe in Sáez-Ballester Theory," Canadian Journal of Physics, **102**(1), (2023). https://doi.org/10.1139/cjp-2022-0342

[27] R.K. Mishra, and H. Dua, "Certain Investigations on Bulk Viscous String Models of the Universe with BVDP," Bulgarian Journal of Physics, **50**, 95–114 (2023). doi:https://doi.org/10.55318/bgjp.2023.50.2.095

[28] J.S. Wath, and A.S. Nimkar, "Cosmological Parameters and Stability of Bianchi Type-VIII in Sáez-Ballester Theory of Gravitation," Bulgarian Journal of Physics, **50**, 255–264 (2023). https://doi.org/10.55318/bgjp.2023.50.3.255

[29] R.K. Dabgar, and A.K. Bhabor, "Higher dimensional Bianchi type-III string cosmological models with dark energy in Saez–Ballester scalar-tensor theory of gravitation," Journal of Astrophysics and Astronomy, **44**, 78 (2023). https://doi.org/10.1007/s12036-023-09971-7

[30] M. Visser, "General Relativistic Energy Conditions: The Hubble expansion in the epoch of galaxy formation," Physical Review D, **56**(12), 7578 (1997). https://doi.org/10.1103/PhysRevD.56.7578

КОСМОЛОГІЧНА МОДЕЛЬ LRS BIANCHI В ТЕОРІЇ ГРАВІТАЦІЇ САЙЄЗ-БАЛЕСТЕРА ЗІ ЗМІННОЮ В ЧАСІ КОСМОЛОГІЧНОЮ КОНСТАНТОЮ

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Ця робота присвячена вивченню локально-обертально-симетричної (LRS) космологічної моделі Б'янкі типу I в рамках скалярно-тензорної теорії гравітації, сформульованої Сайєзом і Баллестером, зі змінною в часі космологічною сталою. Щоб отримати явні розв'язки рівнянь поля Сайєз-Балестера, ми припускаємо, що середній масштабний коефіцієнт підкоряється степеневому закону розширення, а космологічна стала пропорційна щільності енергії космічної рідини. Динамічну поведінку відповідних космологічних параметрів, включаючи параметр Хаббла, параметр уповільнення, щільність енергії, тиск, параметр рівняння стану, космологічну постійну, скаляр зсуву, скаляр розширення тощо, досліджується графічно шляхом вивчення їх еволюції проти параметр червоного зсуву. Також перевіряється перевірка чотирьох енергетичних умов. Ми вважаємо, що результати побудованої моделі добре узгоджуються з останніми даними спостережень.

Ключові слова: *космологічна стала; параметр уповільнення; параметр Хабла; LRS Бьянчі тип-I; теорія Сайєз-Балестера*