East European Journal of Physics. **3.** 83-92 (2024)
DOI:10.26565/2312-4334-2024-3-08 ISSN 2312-4334

A STUDY OF TIME EVOLUTION OF SOME COSMOLOGICAL PARAMETERS IN THE FRAMEWORK OF AN ANISOTROPIC KALUZA-KLEIN METRIC USING AN EMPIRICAL EXPONENTIAL SCALE FACTOR

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Received June 8, 2024; revised July 7, 2024; accepted July 15, 2024

The present study attempts to determine the time dependence of some cosmological parameters in flat space (i.e., a space of zero spatial curvature), in the framework of an anisotropic Kaluza-Klein metric. The field equations for this work have been derived from the metric by assuming a power-law relation between the normal scale factor and the scale factor corresponding to the extra (i.e., the fifth) dimension. An empirical scale factor, having the expression of $a = B\exp(\alpha t^{\beta})$, has been used here in order to derive the expressions for some cosmological parameters as functions of time. The reason for choosing this scale factor is that it generates an expression for the deceleration parameter which undergoes a change of sign, as time goes on, from positive to negative, indicating a transition of the universe from an initial state of decelerated expansion to that of an accelerated expansion (which is its present state), as has been inferred from astrophysical observations. We have graphically depicted the evolution of some cosmological parameters with respect to what one may call the *relative time*, expressed as t/t_0, where t_0 is the present age of the universe. The present study finds the dynamical cosmological constant (Λ) to be negative, and it becomes less negative with time, changing at a gradually decreasing rate. The dependence of pressure of the all-pervading cosmic fluid upon density, corresponding to the fifth dimension, has been described in terms of a skewness parameter (δ) which comes out to be decreasing with time. The anisotropy factor has been calculated in this study, whose numerical value has been found to be decreasing with time, indicating a journey of the universe towards phases of gradually smaller anisotropy.
Keywords: *Dark energy; Kaluza-Klein theory; Cosmological parameter (Λ); Anisotropy; Exponential scale factor*
PACS: 04.20.−q; 04.50.+h; 04.50.−h; 98.80.Es; 98.80.−k

1. INTRODUCTION

Based on cosmological observations throughout the world, it has been convincingly established that the universe undergoes a process of expansion with acceleration. Research is going on extensively to understand the nature of the agent causing an accelerated expansion. If gravitation had been the only interaction governing the motions of celestial bodies, the expansion of the universe would have continued with deceleration. On the basis of the observational findings from supernova 1a, it was concluded that there is a negative pressure generated by an exotic form of energy, referred to as dark energy (DE), which is considered to be responsible for the present phase of accelerated expansion of the universe [1, 2]. The functioning of this mysterious DE can only be determined by extensive investigations. A thorough analysis of supernova data has led to an inference that the universe has changed its phase from decelerated expansion to accelerated expansion, resulting in the change of sign of the deceleration parameter from positive to negative [3-5]. In the vast scientific literature regarding investigations to find the nature of cosmic acceleration, one generally finds approaches through mainly two ways. One of these ways is to construct mathematical models using modified theories of gravity (which are based on modifications of Einstein's theory of general relativity) and explore their characteristics. The other way is to investigate the cosmological observations by formulating dark energy models. A parameter, named cosmological constant (denoted by Λ), has been said to be representing DE in lots of models on cosmology. There are various dark energy models in scientific literature, namely quintessence, phantom, k-essence and quintom [6-9]. Although Λ was introduced in Einstein's theory as a constant parameter [10], but, due to some limitations connected with the coincidence problem and the cosmological constant problem, it is presently regarded as a time-dependent quantity [11]. By modifying Einstein's theory of gravity in various ways, researchers have formulated theories such as $f(R)$ and $f(R,T)$ [12-14] and scalar tensor models like Saez-Ballester (SB) and Brans-Dicke (BD) theories of gravity [15, 16]. Wide range of investigations have been carried out by constructing models on cosmological phenomena involving DE [17-20].

In order to unify electromagnetic force with gravitational force, two scientists, Kaluza and Klein, proposed a new theory in last century's first half, and it has always been referred to Kaluza-Klein (KK) theory since then [21, 22]. This theory talked about a new dimension (or the *fifth* dimension) which acted as a link between the two forces. A contraction of this new dimension with time was proved by Chodos and Detweiler through a five-dimensional model based on KK theory [23]. The current four-dimensional representation of the universe is theoretically demonstrated to be preceded by an era of a multidimensional state. The extra dimension shrinks along with the evolution of the universe and it cannot be detected now by the experimental techniques at our disposal. These phenomena have inspired many

Cite as: S. Roy, East Eur. J. Phys. 3, 83 (2024), https://doi.org/10.26565/2312-4334-2024-3-08

researchers to carry out studies by formulating models involving higher dimensions. The KK theory can be looked upon as a five-dimensional generalization of the general theory of relativity. Theoretical investigations, which are considered to be of great importance in this field, are those carried out by Chodos & Detweller [23], Witten [24], Appelquist et al. [25], Appelquist & Chodos [26] and Marchiano [27]. The motivation for the present work was obtained from a formulation of an anisotropic dark energy model based on KK theory by N. I. Jain [28].

An approximate solution to Kaluza-Klein's equations was shown in a study undertaken by J.A. Ferrari for a spherically symmetric charged system [29]. This study was carried out to find how a test particle behaves in a field of force produced by a charged particle. According to this study, Kaluza-Klein's theory allows us to determine the corrections to the Lorentz force. The five-dimensional relativity in KK framework is validated by these experimental observations. It is possible to verify experimentally the existence of an extra spacetime dimension, as obtained from a study by Kalligas et al. [30]. A set of equations was derived in this study which contains terms connected with the existence of an extra dimension. Using the data obtained from observations of the solar system it has been established that the terms representing the fifth dimension is extremely small in comparison to the usual dimensions of spacetime, in our region of space. It has been found that the parameters corresponding to the KK theory cannot be treated as universal constants, and, there can be place to place variation of these parameters depending upon the local characteristic of matter. Several non-asymptotically flat solutions of Kaluza–Klein space-time were found by Dzhunushaliev et al., which had both electric and magnetic charges [31]. It was proved that these solutions could be regarded to be acting as virtual quantum handles (wormholes) in the models on space-time foam. It was shown that, within an external magnetic and/or electric field, which is sufficiently large, these solutions might be *inflated* from a quantum state to a classical state. This finding leads to the expectation, in a multidimensional gravity, for a possible experimental signal for higher dimensions. An improvement of the theory based five-dimensional Kaluza-Klein metric is possible, according to a recent study by Jean Paul Mbelek [32], by incorporating an external scalar field (ψ). It came out of that formulation that the observational data (for the experiments in the laboratory and also in the context of astrophysics and cosmology) are consistent with the theoretical findings. It has been found in the study that, in consistency with predictions based on the theory, one measured a torque acting upon a torsion pendulum. Based on a novel experimental investigation, by Tajmar and Williams [33], a macroscopic interpretation of the fifth dimension of the Kaluza-Klein theory has been obtained. This experiment was carried out to verify an important aspect of theoretical findings which shows the fifth dimension to somehow correspond to the electric charge. Based on Kaluza-Klein theory, they arrived at an interpretation of the observations regarding the time dilation effect in an electrically charged clock. They explained it by saying that the five-dimensional metric should have a timelike signature for a classical explanation of the extra dimension.

The objective of the present study is to investigate the nature of time dependence of some cosmological parameters, based on an anisotropic Kaluza-Klein spacetime. This study involves a time-dependent cosmological term (Λ). A power-law type relation ($i.e., A = a^n$) has been assumed between the normal scale factor (a) and the scale factor representing the extra dimension (A), both of which belong to the Kaluza-Klein metric used here.

To obtain the solution of field equations, we have used an *ansatz* for the scale factor (i.e., $a = B\exp\left[\alpha t^{\beta}\right]$). The reason for choosing this function is that, the deceleration parameter ($q = -\ddot{a}a/\dot{a}^2$), obtained from this scale factor, undergoes a signature flip with time from positive to negative, which is consistent with the fact that the present phase of accelerated expansion of the universe was preceded by a phase of decelerated expansion [3-5]. Using this exponential scale factor, we have derived expressions for some cosmological quantities such as, Hubble parameter (H), deceleration parameter (q), energy density (ρ), cosmological constant (Λ), equation of state (EoS) parameter (ω), skewness parameter (δ) and the anisotropy factor (σ^2/θ). We have depicted their time variation by plotting them graphically as functions of the relative cosmic time (i.e., t/t_0) where t_0 denotes the age of the universe at the present time, which is nearly 13.7×10^9 years.

Based on our findings regarding both skewness parameter (δ) and the anisotropy factor (σ^2/θ) it can be said that we are heading towards phases of smaller anisotropy. The dynamical cosmological term (Λ), in our study, comes out to be negative (becoming less negative with time) and it changes very slowly in the present universe, indicating probably a slow rise in the dark energy content, which is considered to be causing the cosmic acceleration.

This article has six sections including the one for *introduction*. Sections 2 and 3 are respectively about the field equations and their solutions. Determination of cosmological quantities has been dealt with in the 4th section. Sections 5 and 6 are respectively about the findings of this theoretical investigation and its conclusions.

2. THE METRIC AND THE FIELD EQUATIONS

In order to obtain the cosmological field equations, we have used the Kaluza-Klein space-time of the following type [34].

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] + A^2(t)d\psi^2. \tag{1}$$

In equation (1), $a(t)$ and $A(t)$ are the fourth and fifth-dimension scale factors respectively. The symbol k is a measure of the spatial curvature, having the values -1, 0 and $+1$ respectively for the *open*, *flat* and *closed* universes. The energy-momentum tensor $\left(T_j^i\right)$, for the anisotropic space-time metric represented by equation (1), is given below [35].

$$T_j^i = diag(T_0^0, T_1^1, T_2^2, T_3^3, T_4^4) = diag(-\rho, p, p, p, p_\psi). \tag{2}$$

In equation (2), the symbols ρ and p denote respectively the energy density and pressure of the cosmic fluid (dark energy) pervading the universe. The symbol p_ψ denotes the pressure corresponding to the extra dimension. The barotropic equation of state (EoS) parameter for the normal dimensions is $\omega = p/\rho$. Based on some studies on anisotropy, in the framework of Kaluza-Klein theory, we have used the equation, $p_\psi = (\delta + \omega)\rho$, as the directional equation of state for the fifth dimension, where δ is the skewness parameter which represents the deviation from the normal equation-of-state parameter ω [28, 36-41]. The parameter δ serves as a measure of deviation from isotropy. Thus, the energy-momentum tensor of equation (2) can be rewritten as,

$$T_j^i = diag(-\rho, \omega\rho, \omega\rho, \omega\rho, (\omega + \delta)\rho) \tag{3}$$

The time dependence of ω and δ has been investigated in the present study.
Gravitational field equations are obtained from the following equation.

$$G_j^i = R_j^i - \frac{1}{2}R\delta_j^i = -8\pi G T_j^i + \Lambda\delta_j^i \tag{4}$$

To formulate the field equations, we have used an *ansatz* for the fifth-dimension scale factor (A), which is $A = a^n$ [42]. We have also used $8\pi G = c = 1$ and $k = 0$ (i.e., flat space). Combining equations (1), (3) and (4), one gets the following field equations.

$$(n + 2)\dot{H} + (n^2 + 2n + 3)H^2 = -\omega\rho + \Lambda \tag{5}$$

$$3\dot{H} + 6H^2 = -(\omega + \delta)\rho + \Lambda \tag{6}$$

$$3(n + 1)H^2 = \rho + \Lambda \tag{7}$$

Divergence of Einstein's tensor can be expressed as,

$$\left(R_j^i - \frac{1}{2}R\delta_j^i\right)_{;j} = \left(-T_j^i + \Lambda\delta_j^i\right)_{;j} = 0 \tag{8}$$

Based on equation (8), the equation representing energy conservation [35] is given by,

$$\dot{\rho} + 3(\rho + p)H + n(\rho + p_\psi)H + \dot{\Lambda} = 0 \tag{9}$$

Substituting the equations of state for the normal dimension and the extra dimension [i.e., $p = \omega\rho$ and $p_\psi = (\omega + \delta)\rho$ respectively] in equation (9), we get,

$$\dot{\rho} + (3 + n)(1 + \omega)\rho H + n\rho\delta H + \dot{\Lambda} = 0 \tag{10}$$

Equation (10) can be written as a sum of two equations which are equations (11) and (12), as given below.

$$\dot{\rho} + (3 + n)(1 + \omega)\rho H = Q \tag{11}$$

$$n\rho\delta H + \dot{\Lambda} = -Q \tag{12}$$

In equations (11) and (12), Q is an arbitrary parameter.
Subtracting equation (6) from equation (5), we get,

$$(n - 1)\dot{H} + (n^2 + 2n - 3)H^2 = \rho\delta \tag{13}$$

Substitution for $\rho\delta$ in equation (12), based on equation (13), leads to the following differential equation.

$$\dot{\Lambda} = -Q - nH(n - 1)\left[\dot{H} + (n + 3)H^2\right] \tag{14}$$

3. SOLUTION OF THE FIELD EQUATIONS USING AN EMPIRICAL SCALE FACTOR

To solve the field equations, we have used the following ansatz for the scale factor.

$$a = B\exp\left(\alpha t^\beta\right) \tag{15}$$

where the constant parameters $B, \alpha, \beta > 0$.

The reason for using this scale factor (expressed by eqn. 15) is that it leads to a deceleration parameter (given by equation no. 17) which (with suitable parameter values) undergoes a change of sign (as a function of time) from positive to negative, which is in agreement with the inferences drawn from recent astrophysical observations [ref. nos. 3-5] demonstrating a transition from decelerated expansion to accelerated expansion of the expanding universe. For the same purpose, one often uses a hybrid scale factor which is a combination of an *exponential* and a *power-law* function of time. It has been used in several recent cosmological studies [43-49]. There are some studies where hyperbolic functions of time have been used as empirical scale factors [50-55], having the same property (i.e., deceleration-to-

acceleration transition) of cosmic expansion. The parameter B in the expression for the scale factor (eqn. 15) does not appear in the equations representing the Hubble parameter and the deceleration parameter (eqns. 16 & 17 respectively), because of their expressions, which are, $H = \frac{\dot{a}}{a}$ and $q = -\frac{\ddot{a}\, a}{\dot{a}^2}$ respectively. All functions of H and q are therefore independent of B. The left-hand sides of equations (5), (6) and (7) are functions of the Hubble parameter (H) and its time derivative and they are thus independent of the parameter B. This is the reason why the parameter B is not found in any expression of the present article except that of the scale factor (eqn. 15).
Based on our empirical scale factor (eqn. 15), the Hubble parameter (H) is given by,

$$H = \frac{\dot{a}}{a} = \alpha\beta t^{\beta-1} \tag{16}$$

Based on our empirical scale factor (eqn. 15), the deceleration parameter (q) is given by,

$$q = -\frac{\ddot{a}\, a}{\dot{a}^2} = \frac{1-\beta}{\alpha\beta t^{\beta}} - 1 \tag{17}$$

In the present article, we have used the symbols, H_0 and q_0, which stand for the values of H and q respectively at the present time (i.e., $t = t_0$) where t_0 denotes the age of the universe ($t_0 = 13.7 \times 10^9$ years).
Putting $H = H_0$, $q = q_0$ and $t = t_0$ in equations (16) and (17), we get,

$$H_0 = \alpha\beta {t_0}^{\beta-1} \tag{18}$$

$$q_0 = \frac{1-\beta}{\alpha\beta {t_0}^{\beta}} - 1 \tag{19}$$

Combining equations (18) and (19) we get the following expressions for the constants α and β.

$$\alpha = \frac{H_0\, {t_0}^{(q_0+1)H_0 t_0}}{1-(q_0+1)H_0 t_0} \tag{20}$$

$$\beta = 1 - (q_0 + 1)H_0 t_0 \tag{21}$$

4. DETERMINATION OF COSMOLOGICAL PARAMETERS

Using equation (16) in equation (14) and solving the differential equation for Λ, we get,

$$\Lambda = C - Qt + \frac{n(1-n)\alpha^2\beta^2}{2} t^{2\beta-2} + \frac{n(1-n)(3+n)\alpha^3\beta^3}{3\beta-2} t^{3\beta-2} \tag{22}$$

where C is the integration constant. Using equations (16) and (22) in equation (7), the energy density (ρ) is obtained as,

$$\rho = -C + Qt + \frac{(n^2+5n+6)\alpha^2\beta^2}{2} t^{2\beta-2} - \frac{n(1-n)(3+n)\alpha^3\beta^3}{3\beta-2} t^{3\beta-2} \tag{23}$$

Using equation (5), we get equation (24) which represents the EoS parameter (ω).

$$\omega = \frac{\Lambda-(n+2)\dot{H}-(n^2+2n+3)H^2}{\rho} \tag{24}$$

Using equations (16), (22) and (23) in equation (24), we get,

$$\omega = \frac{C-Qt+\frac{n(1-n)(3+n)\alpha^3\beta^3}{3\beta-2}t^{3\beta-2}-(n+2)\alpha\beta(\beta-1)t^{\beta-2}-\frac{3\alpha^2\beta^2}{2}(n^2+n+2)t^{2\beta-2}}{-C+Qt+\frac{(n^2+5n+6)\alpha^2\beta^2}{2}t^{2\beta-2}-\frac{n(1-n)(3+n)\alpha^3\beta^3}{3\beta-2}t^{3\beta-2}} \tag{25}$$

Using equation (13) we get,

$$\delta = \frac{(n-1)\dot{H}+(n^2+2n-3)H^2}{\rho} \tag{26}$$

Using equations (16) and (23) in equation (26), we get,

$$\delta = \frac{(n-1)\alpha\beta(\beta-1)t^{\beta-2}+(n^2+2n-3)\alpha^2\beta^2t^{2\beta-2}}{-C+Qt+\frac{(n^2+5n+6)\alpha^2\beta^2}{2}t^{2\beta-2}-\frac{n(1-n)(3+n)\alpha^3\beta^3}{3\beta-2}t^{3\beta-2}} \tag{27}$$

Combining equation (16) with equation (22), Λ can be expressed as,

$$\Lambda = C - Qt + \frac{n(1-n)}{2} H^2 + \frac{n(1-n)(3+n)\alpha\beta}{3\beta-2} t^{\beta} H^2 \tag{28}$$

Using equation (28) in equation (7) we get,

$$\rho = -C + Qt + H^2\left[3(n+1) - \frac{n(1-n)}{2} - \frac{n(1-n)(3+n)\alpha\beta}{3\beta-2} t^{\beta}\right] \tag{29}$$

Putting $\rho = \rho_0$, $H = H_0$ and $t = t_0$ in equation (29), we get the following equation from which one can determine the value of C.

$$C = -\rho_0 + Qt_0 + {H_0}^2\left[3(n+1) - \frac{n(1-n)}{2} - \frac{n(1-n)(3+n)\alpha\beta}{3\beta-2}{t_0}^{\beta}\right] \quad (30)$$

Thus, among the three parameters n, Q and C (which are present in the expressions for Λ, ρ, ω, δ) , it is evident from equation (30) that the parameter C can be calculated using the values of the parameters n and Q. One may also express Q as a function of n and C in the following way.

$$Q = \frac{\rho_0 + C - {H_0}^2\left[3(n+1) - \frac{n(1-n)}{2} - \frac{n(1-n)(3+n)\alpha\beta}{3\beta-2}{t_0}^{\beta}\right]}{t_0} \quad (31)$$

In the present formulation, we have used n and Q as independent parameters, which determine the value of C, in accordance with equation (30).
The expansion scalar (θ) and the shear scalar (σ^2) are given by the following equations.

$$\theta = 3\frac{\dot{a}}{a} + \frac{\dot{A}}{A} = (n+3)H \quad (32)$$

$$\sigma^2 = \frac{3}{8}\left(\frac{\dot{a}}{a} - \frac{\dot{A}}{A}\right)^2 = \frac{3}{8}(1-n)^2H^2 \quad (33)$$

Using equations (32) and (33), the anisotropy factor (σ^2/θ) can be expressed as,

$$\frac{\sigma^2}{\theta} = \frac{3(1-n)^2}{8(n+3)}H = \frac{3(1-n)^2}{8(n+3)}\alpha\beta t^{\beta-1} \quad (34)$$

5. RESULTS AND DISCUSSION

In the present article, we have discussed the results of a theoretical investigation carried out to determine the time evolution of an anisotropic universe in terms of the time-variations of the directional equation-of-state (EoS) parameters for the normal and extra dimensions (ω & δ), defined respectively by the relations $p = \omega\rho$ and $p_\psi = (\omega + \delta)\rho$. Time-variations of different cosmological parameters, such as scale factor, Hubble parameter, deceleration parameter, energy density, cosmological constant, etc. have been shown graphically.

Using equations (20) and (21), respectively, we have obtained $\alpha = 4.925 \times 10^{-10}$ and $\beta = 0.543$. The time variations of Hubble parameter and deceleration parameter depend upon these parameters, according to equations (16) and (17) respectively. Apart from α and β, the scale factor (a) depends upon the parameter B, according to equation (15). Any change in B causes a proportionate change in the scale factor (a) and its time derivative (da/dt) without affecting the values of the Hubble parameter $\left(H = \frac{\dot{a}}{a}\right)$ and deceleration parameter $\left(q = -\frac{\ddot{a}\,a}{\dot{a}^2}\right)$ because they are independent of B. As per equation (29), the energy density (ρ) depends upon the parameters n, C and Q, where C itself depends upon Q & n according to equation (30). Based on equation (7) we have, $\Lambda = 3(n+1)H^2 - \rho$. Thus, Λ has a similar dependence upon Q, which is evident from equation (28). The larger the value of Q, the faster would be the change in Qt with time, which is present in both Λ and ρ. The parameter C was introduced in the expression for Λ (eqn. 22) as a constant of integration. Based on equation (30), we have $C = -8.043 \times 10^{-27}$ for $Q = 0$ and $n = -500$. For this study, we have used $Q = 0$, otherwise the energy density (ρ) comes out to be negative for a certain range of values of t. For $n > 0$, we have found the energy density (ρ) to be increasing with time which is not possible for an universe which is expanding with time. For $n = 0$, there is almost no change of ρ with time. For these reasons, we have depicted our findings graphically for $n < 0$ in this article.

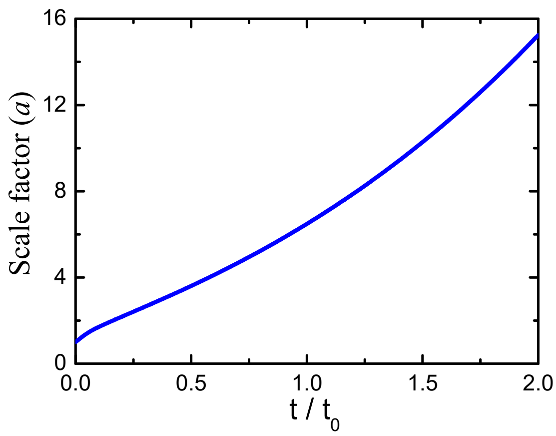

Figure 1. The scale factor versus time

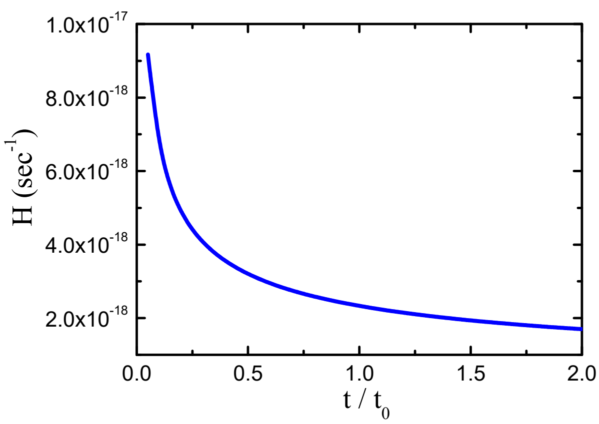

Figure 2. The Hubble parameter versus time

Figure 1 shows the time-variation of the scale factor (a). This cosmological quantity increases with time, which is in accordance with the property of an expanding universe. This figure shows that the rate of change of scale factor increases with time.

Figure 2 depicts the Hubble parameter (H) as a function of time. This plot shows this parameter to be decreasing with time. Its positive value is in accordance with the property of an expanding universe. Its decrease with time indicates that the scale factor (a) increases faster with time in comparison to the increase in its rate of change ($\dot{a}$). One can also say that, the time-rate of fractional change of the scale factor (a) decreases with time.

Figure 3 shows the time variation of the deceleration parameter (q). It shows a signature flip, from positive to negative, indicating clearly a transition of the universe from a phase of decelerated expansion to a phase of accelerated expansion, which is in agreement with the inferences drawn from astrophysical observations [3-5].

Figure 4 depicts the time dependence of the energy density (ρ) for three values of the parameter n, where we have $Q = 0$. The energy density decreases with time which is expected for a universe which is expanding with time. This figure shows that, more negative values of n cause a faster fall in ρ. We have found that, for $Q \neq 0$ and $n \geq 0$, ρ is often found to negative (which is not allowed by its definition) and increasing with time (which is not admissible for an expanding universe). For this reason, we have used same values, i.e., $Q = 0$ and $n < 0$ to depict the time dependence of other parameters (Λ, ω and δ).

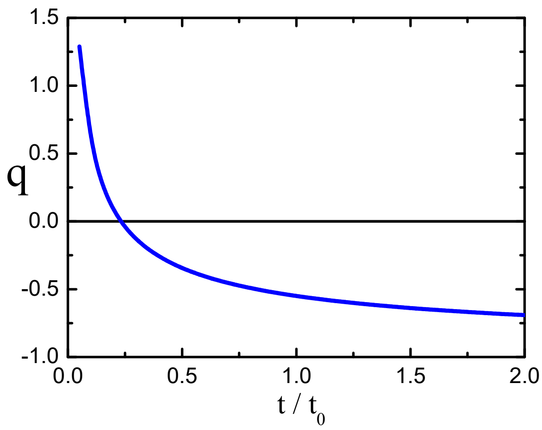

Figure 3. The deceleration parameter versus time

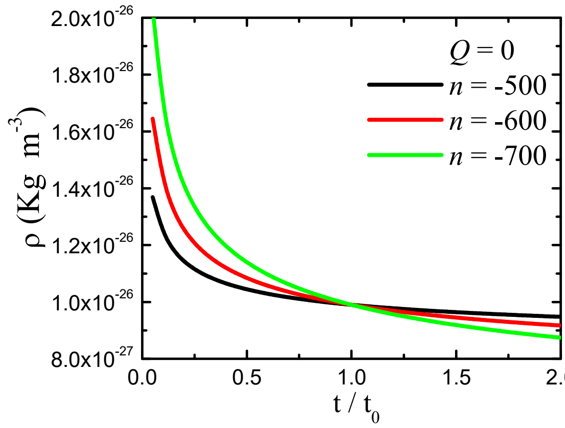

Figure 4. The energy density versus time for three values of the parameter n

Figure 5 shows the nature of dependence of the cosmological constant (Λ) upon time for three values of the parameter n, with $Q = 0$. The cosmological constant (Λ) is often used to represent the dark energy which is generally held responsible for the accelerated expansion of the universe. It is found to be negative in our study, becoming less negative with time at a gradually decreasing rate. This behaviour is found to be consistent with the observations of some recent studies [48, 56-58]. This figure shows the cosmological constant (Λ) to have a faster rise with time for more negative values of n.

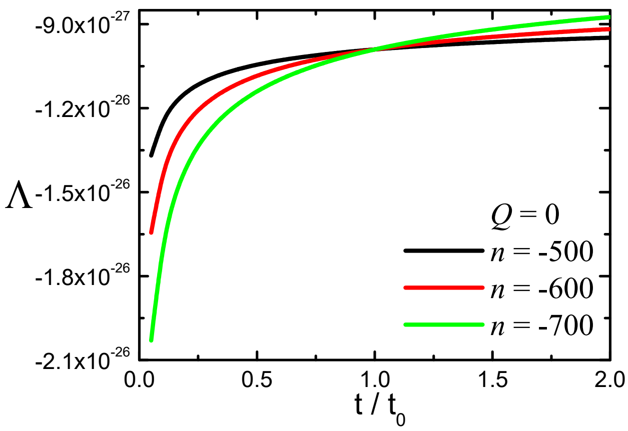

Figure 5. The dynamical cosmological parameter versus time for three values of the parameter n

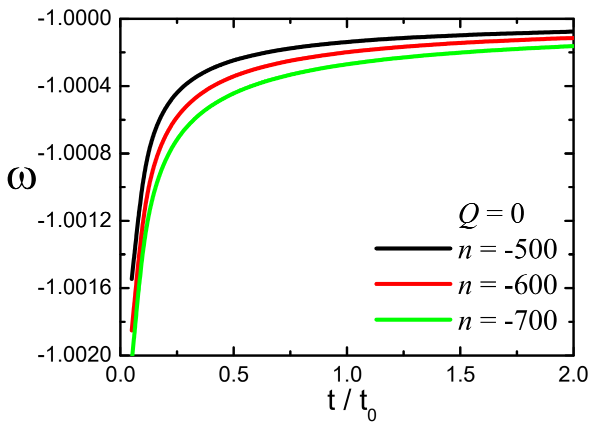

Figure 6. The EoS parameter versus time for three values of the parameter n

Figure 6 shows the nature of dependence of the EoS parameter (ω) upon time for three values of the parameter n, with $Q = 0$. It is negative and it becomes closer to -1 (minus one) with time at a rate which decreases gradually with time. Its values show that the universe is dominated by *phantom* dark energy (i.e., $\omega < -1$) at the early stage and also at the present time (i.e., $t = t_0$), but it is gradually making a transition to a state where it will be dominated by vacuum fluid (i.e., $\omega = -1$) in future, and thereafter to a state of *quintessence* dark energy (i.e., $\omega > -1$) later. These findings are in agreement with the results of some studies carried out in theoretical frameworks which are totally different from the Kaluza-Klein framework that we have used here [59-61]. As per SN Ia data we have $-1.67 < \omega < -0.6237$ while the range obtained by a combination of galaxy clustering statistics and SN Ia data (with CMB anisotropy) and is $-1.33 < \omega < -0.79$ [62, 63]. The values of ω at $t = t_0$, as obtained from equation (24), are consistent with these ranges obtained experimentally. It is found in this figure that less negative values of n makes ω closer to -1, causing a faster approach towards a vacuum fluid dominated universe.

Figure 7 shows the time dependence of the skewness parameter delta (δ) for three different values of the parameter n, with $Q = 0$. Its value is positive and it decreases with time at a gradually smaller rate. Its value is smaller for less negative values of n. Its present value is of the order of 10^{-4}, implying a very small anisotropy in the present universe, and this anisotropy is shown by this graph to become smaller with time. This finding is in sufficient agreement with some recent studies based on Kaluza-Klein anisotropic metric [28].

Figure 8 depicts the time-variation of the ratio σ^2/θ (anisotropy factor) for three different values of the parameter n, with $Q = 0$. Its values are negative and approaches smaller negative values with time at a gradually smaller rate. Its present value is nearly of the order of 10^{-16}, indicating a small anisotropy of the universe at the present time, and this anisotropy becomes smaller with time. It is observed that, the condition for isotropy, i.e., $\sigma^2/\theta \to 0$ as $t \to \infty$, is satisfied. This observation is consistent with the findings by Shamir *et al*, based on Bianchi type III space-time [64, 65]. Its absolute value, (i.e., $|\sigma^2/\theta|$), is closer to zero for less negative values of the parameter n.

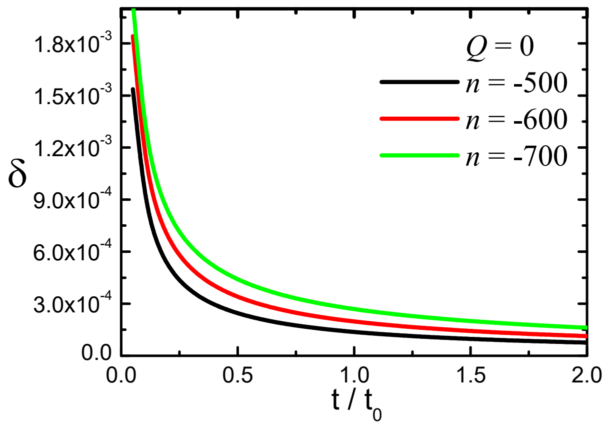

Figure 7. The skewness parameter (δ) versus time for three values of the parameter n

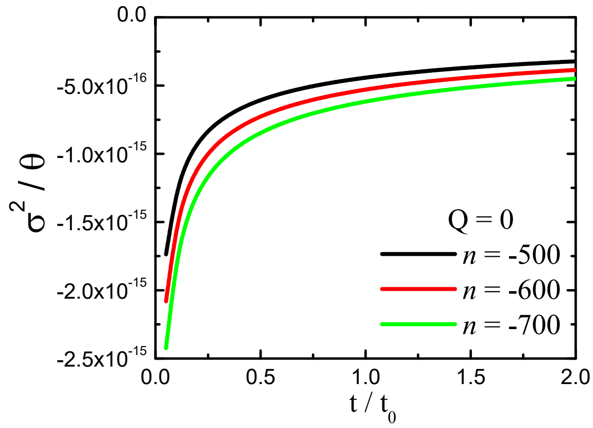

Figure 8. The anisotropy factor (σ^2/θ) versus time for three values of the parameter n

In the present article, we have used the following values of the measurable cosmological parameters, obtained from recent scientific literature [66-72]: $H_0 = 72.20\, km\, s^{-1}\, Mpc^{-1} = 2.34 \times 10^{-18} s^{-1}$, $q_0 = -0.55$, $\rho_0 = 9.83 \times 10^{-27}\, Kg\, m^{-3}$. The graph of cosmological parameter Λ versus time, in Fig. 5, is based on equations (28), where the value of C has been obtained from equation (30). The value of Λ at the present time (i.e., at $t = t_0$), obtained from these equations, is -9.9×10^{-27}. One of the values of the cosmological parameter (Λ), as determined from observational data, is $1.25 \times 10^{-52}\, m^{-2}$, according to recent scientific literature [73-76]. But, one finds a very long range of values, which is spread over several orders of magnitude, as its estimates from theoretical and observational investigations [76-78]. The objective of determining the cosmological constant (Λ) in the present study is to find the nature of its evolution with time in an anisotropic Kaluza-Klein space-time where an exponential function of time (Eqn. 15) has been used as an ansatz for the scale factor. Our findings regarding the time-variation of Λ are in qualitative agreement with some recent studies based on models quite different from ours [48, 56-58].

6. CONCLUDING REMARKS

The present study has been carried out in the framework of an anisotropic Kaluza-Klein space-time, having a time-dependent cosmological constant (Λ), to determine the time variation of various cosmological parameters. For this purpose, we have used an empirical scale factor of such a form that it leads to a deceleration parameter which changes sign with time from positive to negative, indicating clearly a transition from decelerated expansion to accelerated expansion, remaining consistent with inferences drawn from observations [3-5]. To enhance the theoretical validity and the ability of prediction of the model constructed here, we have determined the values of three constants (α, β, C) associated with this model by using the presently accepted values of H_0, q_0 and ρ_0. Due to lack of experimental evidence, we have not been able to fix the value of the parameter n, although it has been shown here that we must have $n < 0$ to get physically acceptable results. It is evident from Figure-4 that the slopes of the curves for ρ are different for different values of n at all values of t/t_0. The value of n could probably have been determined if we had any experimentally obtained estimate of the rate of change of energy density (i.e., $d\rho/dt$) at the present time (i.e., $t = t_0$). An important finding of this study is that the dynamical cosmological parameter (Λ) comes out to be negative and it becomes less negative with time, changing at a gradually decreasing rate. At the present time ($t = t_0$), its value is found to be -9.9×10^{-27}, irrespective of the value of n. This value is numerically equal to the value of ρ_0 (expressed in $Kg\ m^{-3}$). The time evolution of the EoS parameter (ω) shows that the universe has been in the *phantom* regime (i.e., $\omega < -1$) of dark energy since the earliest phase and we are gradually moving towards a vacuum fluid dominated stage (i.e., $\omega = -1$). This model shows that, as $t \to \infty$, $\sigma^2/\theta \to 0$, indicating a journey of the universe towards phases of gradually smaller anisotropy. In the present formulation, we have not been able to use equation (11), though it was obtained from equation (10) which is one of the field equations used here. Using $\omega = -1$ in equation (11), one can obtain an expression for ρ where the integration constant can be determined by using the fact that $\rho = \rho_0$ at $t = t_0$. Proceeding in this manner, the time dependence of other cosmological quantities can be determined. We have plans to use this method in a future project.

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ДОСЛІДЖЕННЯ ЧАСОВОЇ ЕВОЛЮЦІЇ ДЕЯКИХ КОСМОЛОГІЧНИХ ПАРАМЕТРІВ В РАМКАХ АНІЗОТРОПНОЇ МЕТРИКИ КАЛУЦИ-КЛЕЙНА З ВИКОРИСТАННЯМ ЕМПІРИЧНОГО ЕКСПОНЕНЦІАЛЬНОГО МАСШТАБНОГО ФАКТОРА

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У цьому дослідженні зроблено спробу визначити залежність від часу деяких космологічних параметрів у плоскому просторі (тобто просторі нульової просторової кривизни) в рамках анізотропної метрики Калуци-Клейна. Рівняння поля для цієї роботи були отримані з метрики шляхом припущення степеневого співвідношення між нормальним масштабним фактором і масштабним фактором, що відповідає додатковому (тобто п'ятому) виміру. Емпіричний масштабний коефіцієнт, що має вираз $a = B\exp(\alpha t^{\beta})$, був використаний тут для отримання виразів для деяких космологічних параметрів як функцій часу. Причина вибору цього масштабного коефіцієнта полягає в тому, що він створює вираз для параметра уповільнення, який зазнає зміни знака з плином часу від позитивного до негативного, що вказує на перехід Всесвіту від початкового стану уповільненого розширення до стану прискорене розширення (що є його поточним станом), як було зроблено з астрофізичних спостережень. Ми графічно зобразили еволюцію деяких космологічних параметрів відносно того, що можна назвати *відносним часом*, вираженим як t/t_0, де t_0 — поточний вік Всесвіту. У цьому дослідженні встановлено, що динамічна космологічна константа (Λ) є від'ємною, і з часом вона стає менш від'ємною, змінюючись зі швидкістю поступового зменшення. Залежність тиску всепроникної космічної рідини від щільності, що відповідає п'ятому виміру, була описана в термінах параметра асиметрії (δ), який, як виявилося, зменшується з часом. У цьому дослідженні було розраховано коефіцієнт анізотропії, чисельне значення якого, як виявилося, зменшується з часом, що вказує на рух Всесвіту до фаз поступово меншої анізотропії.

Ключові слова: *темна енергія; теорія Калуци-Клейна; космологічний параметр (Λ); анізотропія; експоненціальний масштабний коефіцієнт*