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NON-FLAT FRIEDMANN-LEMAÎTRE-ROBERTSON-WALKER UNIVERSE WITH BARROW HOLOGRAPHIC DARK ENERGY

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In this paper, we study a non-flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe filled with cold dark matter and Barrow holographic dark energy. We assume the Hubble horizon as IR cutoff and the scale factor to obey a hybrid expansion law to construct a cosmological model within the framework of General Relativity. The physical and geometrical properties of the model are discussed by studying the evolution of various parameters of cosmological importance. The behaviour of the dark energy equation of state parameter ω_{DE} is also studied for both interacting and non-interacting Barrow holographic dark energy. We observe that the Barrow exponent Δ significantly affects the dark energy equation of state parameter which in turn exhibits the behaviour of quintessence and phantom dark energy. The evolution of the jerk parameter is also studied.

Keywords: *Friedmann-Lemaître-Robertson-Walker universe; Hybrid expansion law; Barrow holographic dark energy; Cold dark matter; Equation of state parameter*

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1. INTRODUCTION

According to a number of recent cosmological and astrophysical observations, including Supernovae Type Ia (SN Ia) [1,2], Cosmic Microwave Background (CMB) [3,4], Large Scale Structure (LSS) [5,6], and others the present universe is thought to be dominated by a mysterious physical entity known as dark energy. This indicates that the universe has recently undergone the transition from the matter era with decelerated expansion to the current accelerated expansion phase. Many cosmological observations such as Wilkinson Microwave Anisotropy Probe (WMAP), Baryon Acoustic Oscillation (BAO), Sloan Digital Sky Survey(SDSS), Gravitational Lensing etc. provided strong support to the observed cosmic acceleration. According to Planck Collaboration results, 2018, dark energy contributes about 68.3% of the total energy of the present observable universe. Dark energy permeats all over the space and it has large negative pressure. Since the cosmological constant Λ, introduced by Einstein, is physically comparable to quantum vacuum energy with an equation of state parameter $\omega = -1$, it follows that Λ can be a good contender for dark energy. But the constant Λ is plagued with the fine tuning and cosmic coincidence problems although it fits the observations reasonably well. As an alternative to Λ, a wide range of scenarios with a number of dynamically evolving scaler field models have been considered in the literature to explain the late time cosmic acceleration. Currently, an attempt for probing the true nature of dark energy has been taken in the literature, called holographic dark energy (HDE) [7,8], that originates from the Holographic Principle [9–12]. The fundamental tenet of the Holographic Principle is that, as opposed to scaling with system volume, the number of degrees of freedom that are directly related to entropy of a physical system scales with the system's surrounding surface area. G.'t Hooft [9] first presented it with the goal to explain the thermodynamics of black hole physics. The gravitational entropy inside a closed surface shouldn't always be greater than the particle entropy that travels through the surface's past light-cone, according to a later extension of this idea to the cosmological context made by Fischler and Susskind [12]. In cosmology, a significant implications of the holographic principle is that the entropy of the universe's horizon is directly proportional to its surface area, similar to the Bekenstein-Hawking entropy of a black hole.

Utilizing the black hole entropy expression, one may apply the Holographic Principle to generate holographic dark energy. Consequently, by utilizing different entropies, one can derive multiple varieties of the theory. A new black hole entropy relation, recently suggested by Barrow, indicates that the black hole structure may acquire intricate fractal features due to quantum gravitational processes. Due to its intricate structure, the black-hole entropy equation is distorted and has a limited volume but an infinite (or finite) area [13]. The equation is given by

$$S_B = \left(\frac{A}{A_0}\right)^{1+\frac{\Delta}{2}} \tag{1}$$

where A_0 is the Planck area and A is the standard horizon area. When $\Delta = 0$, the Barrow exponent Δ corresponds to the standard Bekenstein-Hawking entropy and with $\Delta = 1$, it corresponds to the most complex design. Therefore, the Barrow holographic dark energy (BHDE) is formulated [14] when holographic dark energy is described in accordance with the

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general entropy formula given in (1).

Saridakis [15] offered an alternative cosmic arrangement to the Bekenstein-Hawking one by utilizing the Barrow entropy. Srivastava and Sharma [16] investigated the BHDE in a flat Friedmann-Lemaître-Robertson-Walker(FLRW) universe taking the Hubble horizon as the IR cutoff. Srivastava, Kumar and Srivastava [17] also investigated the BHDE model in the background of a flat FLRW universe. Recently many researchers examined the BHDE model in various cosmological and gravitational setups [18–21].

The goal of the current work is to study the evolution of a non-flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe filled with pressureless cold dark matter and Barrow holographic dark energy (BHDE) with Hubble horizon as IR cutoff. We present our work in the following sections of this paper. In section 2, we construct a cosmological model by assuming the scale factor to obey the hybrid expansion law proposed by Akarsu *et al.* [22]. In section 3, we study the properties of the constucted model by examining the geometrical and physical characteristics of a few relevant cosmological parameters. We also study the behaviour of the dark energy equation of state parameter ω_{DE} for both interacting and non-interacting Barrow holographic dark energy. In section 4, we wrap up our paper with a conclusion.

2. THE MODEL

In this section, we consider a non-flat FLRW line element in the form

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{2}$$

where $a(t)$ is the scale factor and $k = +1, 0, -1$ corresponds to closed, flat and open spatial curvature respectively and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$.

Normally, holographic dark energy density given in reference [14] can be obtained in the framework of the holographic model based on the Barrow entropy according to the expression (1)

$$\rho_{DE} = CL^{\Delta-2} \tag{3}$$

with C being a parameter with dimensions $[L]^{-2-\Delta}$ and L is a holographic horizon length. For $\Delta = 0$, equation (3) gives the standard holographic dark energy density $\rho_{DE} = CL^{-2}$, where $C = 3c^2 M_p^2$, c^2 is standard parameter of order one that is present in all holographic dark energy models and M_p, the Planck mass. Taking the IR cutoff ($L = H^{-1}$) as the Hubble horizon, the energy density of BHDE is obtained as

$$\rho_{DE} = CH^{2-\Delta} \tag{4}$$

We consider the universe to be filled with cold dark matter (CDM) and Barrow holographic dark energy (BHDE). Then in natural units, we can write Einstein's field equations as

$$R_{ij} - \frac{1}{2}g_{ij}R = T_{ij} + \bar{T}_{ij} \tag{5}$$

where R_{ij} is the Ricci tensor; R is the Ricci scalar; T_{ij} is the energy-momentum tensor for cold dark matter given by

$$T_{ij} = \rho_m u_i u_j \tag{6}$$

and $\bar{T}_{ij}$ is the energy-momentum tensor for Barrow holographic dark energy given by

$$\bar{T}_{ij} = (\rho_{DE} + p_{DE})u_i u_j + g_{ij}p_{DE} \tag{7}$$

Here, ρ_m is the energy density of cold dark matter, ρ_{DE} and p_{DE} are the energy density and the pressure of the Barrow holographic dark energy respectively.

The Friedmann equations in this case are written as

$$3H^2 + 3\frac{k}{a^2} = \rho_m + \rho_{DE} \tag{8}$$

$$3H^2 + 2\dot{H} + \frac{k}{a^2} = -p_{DE} \tag{9}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and an over dot denotes differentiation with respect to cosmic time t.

The continuity equation is given by

$$\dot{\rho}_m + \dot{\rho}_{DE} + 3H(\rho_m + \rho_{DE} + p_{DE}) = 0 \tag{10}$$

In view of equation (4), the field equations (8) and (9) constitute a system of non-linear differential equations in three unknowns a, ρ_m and p_{DE}. In order to construct a cosmological model we consider the cosmological scale factor a to obey the hybrid expansion law (HEL) [22]

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)} \tag{11}$$

where α and β are constants and a_0 and t_0 denote the scale factor and the age of the present universe respectively. The reason for using such scale factor is the fact that it will give the deceleration parameter as a function of cosmic time t. This is amongst the models that describe the transition of the universe from a phase of deceleration to the present phase of acceleration as suggested by the present cosmological observations. Several researchers applied HEL to study behaviour of a number of cosmological models in different gravitational and cosmological backgrounds.

3. PROPERTIES OF THE MODEL

The Hubble parameter $H(t)$, a cosmological parameter that shows the rate of expansion at the epoch t, corresponding to the HEL (11) is obtained as

$$H = \frac{\dot{a}}{a} = \frac{\alpha}{t} + \frac{\beta}{t_0} \tag{12}$$

The deceleration parameter, denoted by q, is a dimensionless measure of the acceleration in the expansion of the universe and is defined by $q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}$. Thus deceleration parameter q is obtained as

$$q = \frac{\alpha}{\left(\alpha + \frac{\beta t}{t_0}\right)^2} - 1 \tag{13}$$

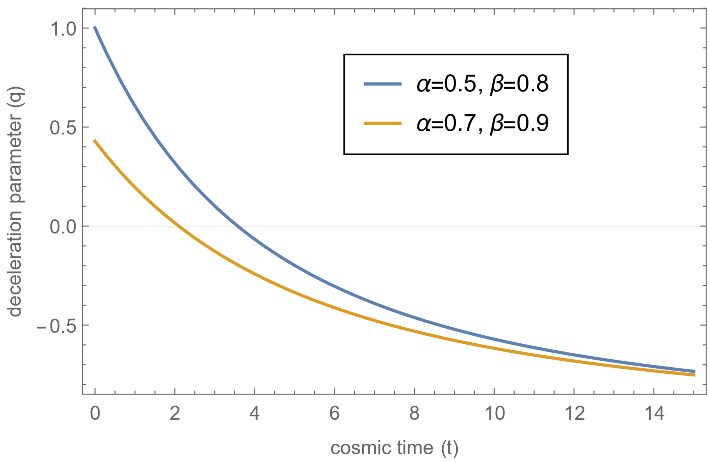

Figure 1. Evolution of deceleration parameter q vs cosmic time t

Figure 1 depicts the evolution of the deceleration parameter (q) for the constructed model against cosmic time (t). With reference to the displayed graph, the outcome reveals that the deceleration parameter (q) attributed a positive value at the initial stage followed by a sharp decrease and subsequently a slow convergence towards the value of negative one. This means that there is a transition from what is referred to as the decelerated phase of the universe which, in cosmological terms, seems to be the initial stage to the phase that is currently an accelerating phase of the universe. The parameters expressing rate of expansion in the earlier epoch have positive values means deceleration while moving toward negative in the later epochs means acceleration. These behaviors are in line with the acquired observational data concerning the validity of the late-time cosmic acceleration.

Using equations (12) in (4) we get the energy density of BHDE as

$$\rho_{DE} = C\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right)^{2-\Delta} \tag{14}$$

From equations (14),(12) and (8) we obtain the matter energy density as

$$\rho_m = 3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2 + 3k\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^{-2} - C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta} \tag{15}$$

For closed ($k = 1$) FLRW universe, equation (15) becomes

$$\rho_m = 3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2 + 3\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^{-2} - C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta} \tag{16}$$

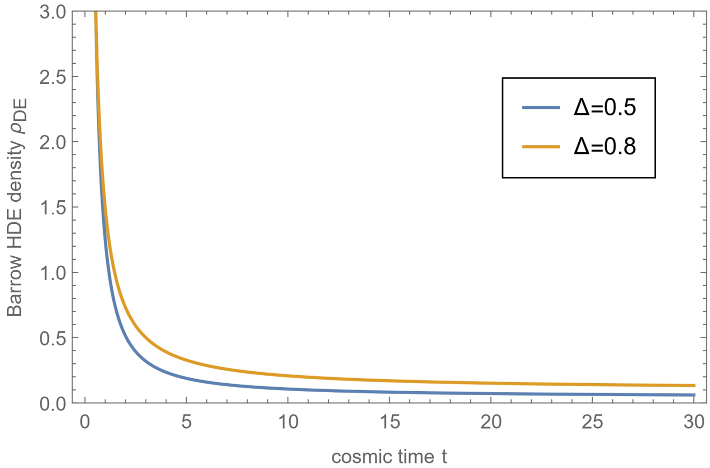

Figure 2. Evolution of the Barrow HDE density ρ_{DE} vs cosmic time t for $C = 3$, $t_0 = 13.8$, $\alpha = 0.5$ and $\beta = 0.8$

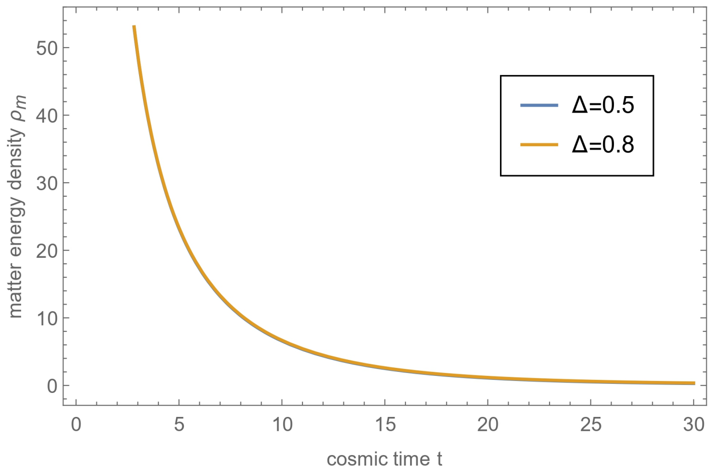

Figure 3. Evolution of the matter energy density ρ_m vs cosmic time t for $C = 3$, $\alpha = 0.5$, $\beta = 0.8$, $a_0 = 1$ and $t_0 = 13.8$

Figure 2 and Figure 3 clearly indicate that both ρ_{DE} and ρ_m are decreasing functions of the cosmic time t. The decrease in energy densities suggests that the volume of the universe is expanding.

The BHDE density parameter Ω_{DE} and the energy density parameter of matter Ω_m are obtained as

$$\Omega_{DE} = \frac{C}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{\Delta}} \tag{17}$$

$$\Omega_m = 1 + \frac{1}{\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2}\frac{1}{\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^2} - \frac{C}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{\Delta}} \tag{18}$$

Hence the total energy density parameter Ω is obtained as

$$\Omega = \Omega_m + \Omega_{DE} = 1 + \frac{1}{\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right)^2} \frac{1}{\left(a_0 \left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^2} \tag{19}$$

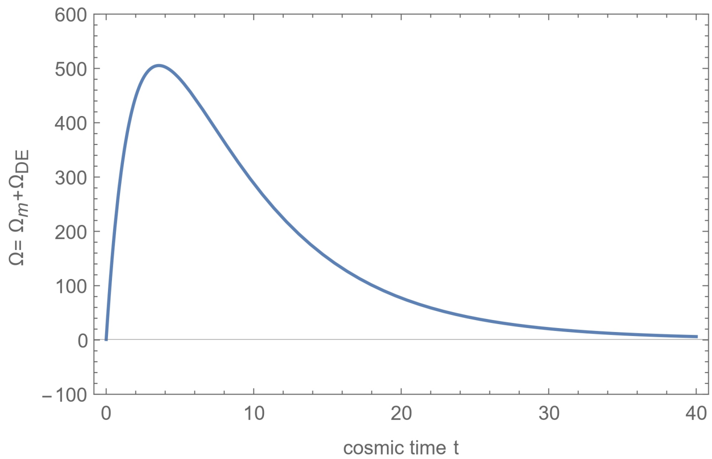

Figure 4. Evolution of the total energy density parameter Ω vs cosmic time t for $\alpha = 0.5$, $\beta = 0.8$, $a_0 = 1$ and $t_0 = 13.8$

Figure 4 represents the evolution of the total energy density parameter Ω as a function of cosmic time t. From the figure we observe that Ω tends to 1. This result is compatible with the observational results. Since our model predicts a flat universe for late times, and the present day universe is very close to flat, so the derived model is also compatible with the observational results.

Now for open ($k = -1$) FLRW universe, the energy density of cold dark matter ρ_m is obtained by using the equation (15) as

$$\rho_m = 3\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right)^2 - 3\left(a_0 \left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^{-2} - C\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right)^{2-\Delta} \tag{20}$$

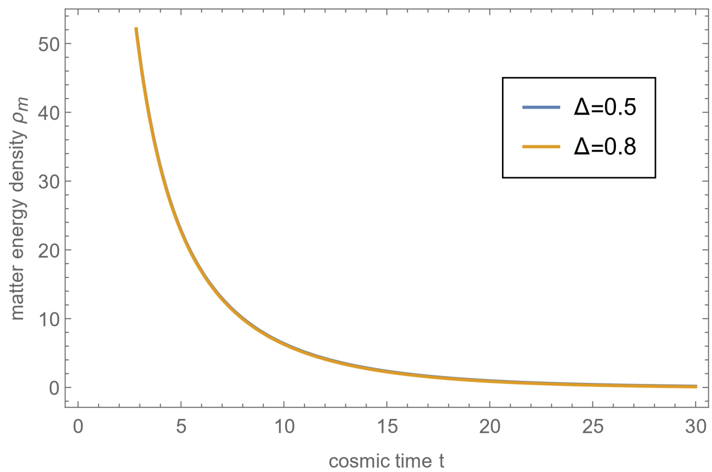

Figure 5. Evolution of the matter energy density ρ_m vs cosmic time t for $C = 3$, $\alpha = 0.5$, $\beta = 0.8$, $a_0 = 1$ and $t_0 = 13.8$

Figure 5 depicts the evolution of the matter energy density ρ_m vs cosmic time t. The graph shows that ρ_m is a decreasing function of cosmic time t, this indicates the expansion of the universe as it evolves.

The BHDE density parameter Ω_{DE} and the energy density parameter of matter Ω_m are obtained as

$$\Omega_{DE} = \frac{C}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{\Delta}} \tag{21}$$

$$\Omega_m = 1 - \frac{1}{\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2}\frac{1}{\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^2} - \frac{C}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{\Delta}} \tag{22}$$

Hence the total energy density parameter Ω is obtained as

$$\Omega = \Omega_m + \Omega_{DE} = 1 - \frac{1}{\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2}\frac{1}{\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^2} \tag{23}$$

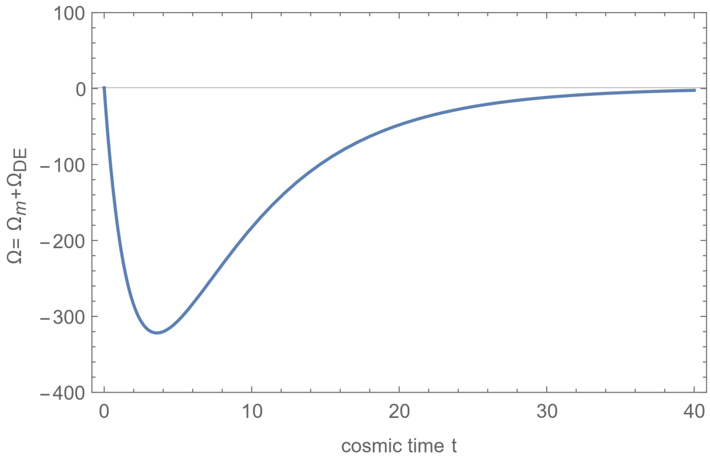

Figure 6. Evolution of the total energy density parameter Ω vs cosmic time t for $\alpha = 0.5$, $\beta = 0.8$, $a_0 = 1$ and $t_0 = 13.8$

From Figure 6, we see that Ω tends to 1. So our model approaches to flat FLRW universe at late times.

Case I: Interacting Barrow HDE

When there is an interaction between dark energy and dark matter, the energy densities do not conserve separately. So, from the continuity equation (10), we have

$$\dot{\rho}_m + 3H\rho_m = Q \tag{24}$$

and

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = -Q \tag{25}$$

where Q is the coupling parameter which describes the interaction between dark energy and dark matter. A common choice for the interaction term often takes the form $Q \propto H\rho_m$, $Q \propto H\rho_{DE}$ or a combination of these forms. We consider

$$Q = 3H(\gamma\rho_{DE} + \delta\rho_m) \tag{26}$$

where γ and δ are constants.

Using equations (26), (25), (14) and (12) we get

$$p_{DE} = -C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta} - \frac{C(2-\Delta)\left(-\frac{\alpha}{t^2}\right)}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{\Delta}} - \frac{\gamma C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta} + \delta\rho_m}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)} \tag{27}$$

where ρ_m is given by equation (15).

For $k = 1$, using equation (16) in (27), we get

$$p_{DE} = -C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta} - \frac{C(2-\Delta)\left(-\frac{\alpha}{t^2}\right)}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{\Delta}} - \frac{\gamma C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta} + 3\delta\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2} + 3\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^{-2} - C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta}}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)} \tag{28}$$

Using equations (28) and (14), we get the EoS parameter ω_{DE} as

$$\omega_{DE} = -1 + \frac{\alpha(2-\Delta)}{3\left(\alpha+\frac{\beta t}{t_0}\right)^2} - \left(\gamma + \frac{3\delta\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2} + 3\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^{-2} - C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta}}{C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta}}\right) \tag{29}$$

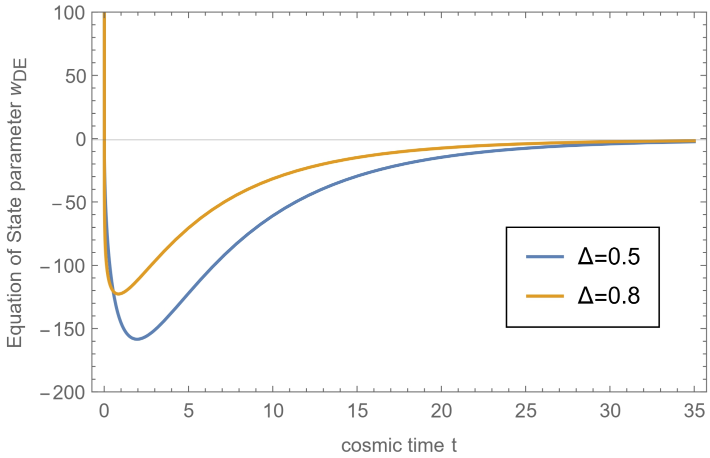

Figure 7. Evolution of EoS parameter ω_{DE} vs cosmic time t for $\alpha = 0.5$, $\beta = 0.8$, $t_0 = 13.8$, $\gamma = 1$ and $\delta = -1$.

From figure 7, we observe that ω_{DE} enters a high phantom region in the very early phases of the universe and as time passes it approaches to -1 showing thereby that BHDE behaves like the cosmological constant Λ at late times.

For $k = -1$, using (20) in (27), we get

$$p_{DE} = -C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta} - \frac{C(2-\Delta)\left(-\frac{\alpha}{t^2}\right)}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{\Delta}} - \frac{\gamma C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta} + 3\delta\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2} - 3\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^{-2} - C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta}}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)} \tag{30}$$

Using equations (30) and (14), the EoS parameter ω_{DE} is obtained as

$$\omega_{DE} = -1 + \frac{\alpha(2-\Delta)}{3\left(\alpha+\frac{\beta t}{t_0}\right)^2} - \left(\gamma + \frac{3\delta\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2} - 3\left(a_0\left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)}\right)^{-2} - C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta}}{C\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{2-\Delta}}\right) \tag{31}$$

From figure 8, we see that for different values of Δ, ω_{DE} gradually decreases and tends to -1 at late times.

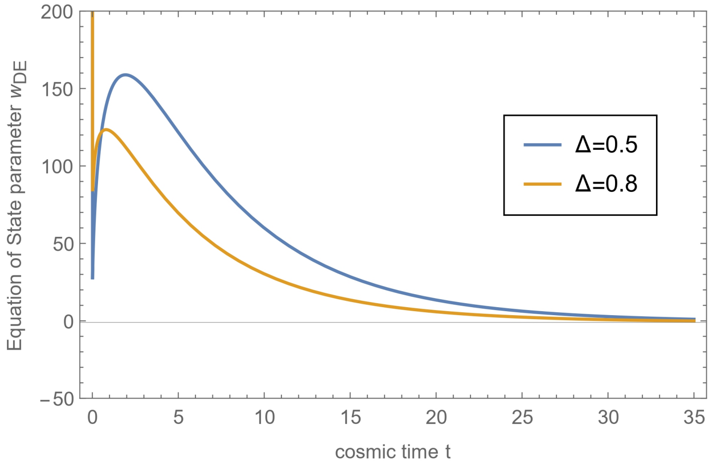

Figure 8. Evolution of EoS parameter ω_{DE} vs cosmic time t for $\alpha = 0.5$, $\beta = 0.8$, $t_0 = 13.8$, $\gamma = 1$ and $\delta = -1$.

Case II: Non-interacting Barrow HDE

If there is no interaction between dark energy and dark matter, the energy densities conserved separately and therefore from equation (10), we have

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{32}$$

and

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0 \tag{33}$$

From equations (33), (14) and (12), we get

$$p_{DE} = -C\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right)^{2-\Delta} - \frac{C(2-\Delta)\left(-\frac{\alpha}{t^2}\right)}{3\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right)^{\Delta}} \tag{34}$$

Using equations (34) and (14), the EoS parameter ω_{DE} is obtained as

$$\omega_{DE} = -1 + \frac{\alpha(2-\Delta)}{3\left(\alpha + \frac{\beta t}{t_0}\right)^2} \tag{35}$$

Since it is independent of the curvature parameter k, therefore, for both closed and open universe, the model represents ΛCDM model when $\omega_{DE} = -1$.

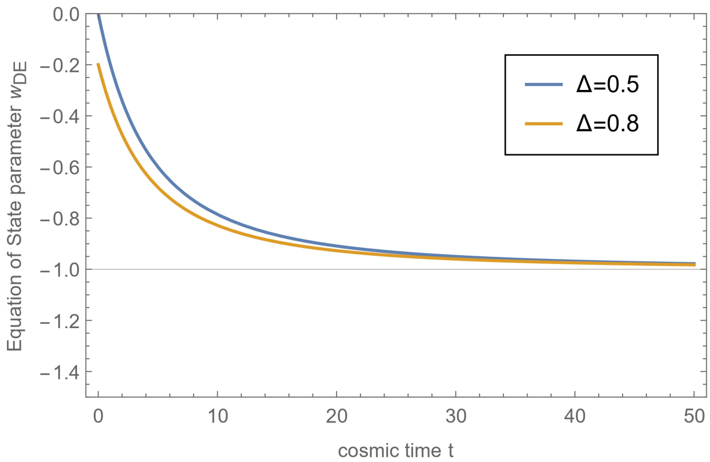

Figure 9. Evolution of EoS parameter ω_{DE} vs cosmic time t for $\alpha = 0.5$, $\beta = 0.8$ and $t_0 = 13.8$

From the figure 9, we observe the quintessence behaviour of BHDE for different values of Δ.

Jerk parameter: The jerk parameter j is a crucial tool for identifying deviations of cosmological model from the ΛCDM model and describing the universe's dynamical evolution. It characterizes models close to ΛCDM through a dimensionless third derivative of the scale factor relative to the cosmic time t. In cosmology it is defined as $j(t) = \frac{1}{aH^3}\frac{d^3a}{dt^3}$ and for our model it is obtained as

$$j = 1 + \frac{(2t_0 - 3\beta t - 3\alpha t_0)\alpha t_0^2}{(\beta t + \alpha t_0)^3} \tag{36}$$

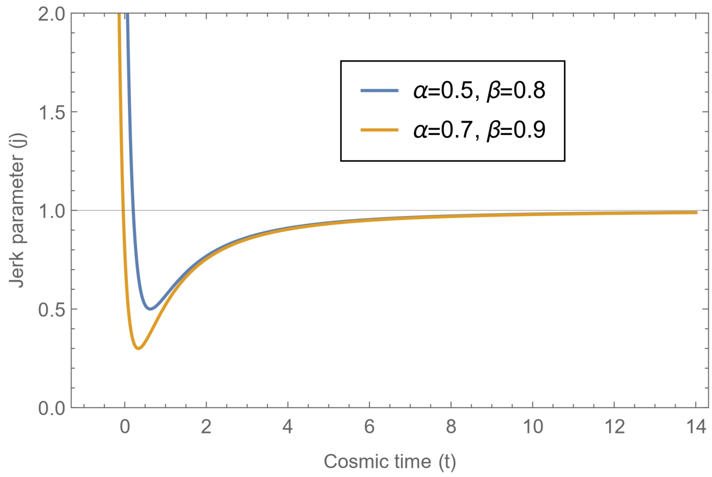

Figure 10. Evolution of jerk parameter j vs cosmic time t for $t_0 = 13.8$

Figure 10 illustrates that the cosmic jerk parameter remains positive throughout the universe's evolution, approaching the value 1 in the later stages.

4. CONCLUSION

In this paper, a non-flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe filled with cold dark matter and Barrow holographic dark energy (BHDE) is studied within the framework of General Relativity. Exact solution of the Einstein field equations are obtained by considering a hybrid expansion law and the physical and geometrical properties of the derived model are studied graphically. We also study the behaviour of the EoS parameter ω_{DE} in two cases: when the BHDE is interacting with CDM and when the BHDE does not interact with CDM. The evolution of the jerk parameter is also studied. We observe that

- The evolution of the deceleration parameter in our model illustrates the universe's transition from its earlier deceleration phase to the current acceleration phase.
- Barrow holographic dark energy density ρ_{DE} and cold dark matter energy density ρ_m decrease with the increase of cosmic time t in both closed ($k = 1$) and open ($k = -1$) universes. This indicates that the universe is expanding.
- The EoS parameter ω_{DE} tends to -1 at late times for both interacting and non-interacting BHDE in a closed ($k = 1$) and open ($k = -1$) universe. However in case of interacting BHDE, ω_{DE} enters into high phantom region before the BHDE behaves like the cosmological constant.
- The Barrow holographic dark energy density parameter Ω_{DE} takes the same value in both closed and open universe. However the energy density parameter of matter Ω_m is different for closed and open universe.
- The total energy density parameter Ω tends to 1 as the universe evolves. This suggests that the universe is approaching towards a flat universe at late times.
- The cosmic jerk parameter remains positive throughout the universe's evolution and approaches 1 in late times. It is consistent with the current observational data.

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НЕПЛОСКИЙ ВСЕСВІТ ФРІДМАНА-ЛЕМЕТРА-РОБЕРТСОНА-УОКЕРА З ГОЛОГРАФІЧНОЮ ТЕМНОЮ ЕНЕРГІЄЮ БАРРОУ

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У цій статті ми вивчаємо неплоский Всесвіт Фрідмана-Леметра-Робертсона-Уокера (FLRW), наповнений холодною темною матерією та голографічною темною енергією Барроу. Ми припускаємо, що горизонт Хаббла є ІЧ-відсіканням, а масштабний коефіцієнт підкоряється гібридному закону розширення для побудови космологічної моделі в рамках загальної теорії відносності. Фізичні та геометричні властивості моделі обговорюються шляхом вивчення еволюції різних параметрів космологічного значення. Поведінка рівняння темної енергії параметра стану ω_{DE} також вивчається як для взаємодіючої, так і для невзаємодіючої голографічної темної енергії Барроу. Ми спостерігаємо, що експонента Барроу Δ суттєво впливає на рівняння темної енергії параметра стану, яке, у свою чергу, демонструє поведінку квінтесенції та фантомної темної енергії. Також вивчається еволюція параметра ривка.

Ключові слова: *Всесвіт Фрідмана-Леметра-Робертсона-Уокера, гібридний закон розширення, голографічна темна енергія Барроу, холодна темна матерія, рівняння параметра стану*