East European Journal of Physics. 3. 71–82 (2024)
DOI: 10.26565/2312-4334-2024-3-07 ISSN 2312-4334

EXAMINING THE VISCOUS RICCI DARK ENERGY COSMOLOGICAL MODEL IN GENERAL THEORY OF GRAVITATION

T. Chinnappalanaidu[a], M. Vijaya Santhi[b], N. Sri Lakshmi Sudha Rani[b]
[a]*Department of Mathematics, Vignan's Institute of Information Technology(Autonomous), Visakhapatnam 530049, India*
[b]*Department of Applied Mathematics, College of Science and Technology, Andhra University, Visakhapatnam 530003, India*
Corresponding Author e-mail: chinnappalanaidu.tadi@gmail.com
Received June 30, 2024; revised July 24, 2024; accepted August 15, 2024

This study focuses on dynamically exploring Marder-type spacetime containing viscous Ricci dark energy within the framework of general relativity theory. To find a solution of the field equations, we use the relation between metric potentials and the average scale factor $a(t) = (\sinh\beta_1 t)^{\frac{1}{\beta_2}}$, this leads to a seamless transition of the Universe from its initial decelerating phase to the current accelerating phase. Here, we have obtained the cosmological parameters and $\omega_{de} - \omega'_{de}$ plane for the derived model. Also, dynamical features of the derived cosmological model are analyzed through diagrams.

Keywords: *Dark energy; Viscous Ricci dark energy; Marder type metric; General relativity*

PACS: $04.20.-q$; $98.80.-k$; $95.36.+x$

1. INTRODUCTION

Dark Energy(DE), which has yet to be identified, is thought to be the cause of the Universe's recent aggressive expansion. One of the pillars of contemporary cosmology is DE. Many measurements, like the cosmic microwave background radiation(CMB) [1], the distant modulus of the type Ia supernova (SnIa) [2–4] and more recently by large-scale structure studies [5], show that DE is a peculiar phenomena that is developing into an Einstein-de Sitter structure. Just $\sim 4\%$ of the entire Universe's energy density is represented by baryonic matter, $\sim 24\%$ by non-baryonic matter, and almost $\sim 72\%$ by an unidentified component that has negative pressure, which is the most unexpected and counter intuitive outcome of these studies. A cosmic acceleration could be produced by the cosmological constant, which represents quantum vacuum energy density. In general relativity (GR), this simple DE model is plagued by coincidence. The accelerating universe is explained by different dynamical DE models. There are several notable models in this group, including $K-$essence and Quintessence [6, 7]. You might want to also note that there are other modified matter models based on perfect fluids, such as pilgrim DE models [8–10], generalized Chaplygin gas models [11], and holographic DE models [12, 13]. GR is modified by another class of DE models. Models of DE correspond to modified theories of gravity [14, 15] and scalar-tensor theories of gravity [16, 17].

In 1916, Einstein [18] presented his GR, which gives a geometrical account of gravity. Even today, GR is an elegant scientific as well as geometrical framework used to accurately characterizes gravity fields. As well as being useful for discussing cosmological models of the Universe. Even so, Einstein admitted that some desirable characteristics were not accounted for in his theory. By way of instance, such an approach fails to take into account Mach's principle. In the simplest instance, gravity is maintained by Einstein's equations of GR. The Einstein-Maxwell equations are a set of differential second order equations with partial derivatives that are extremely nonlinear. On the one hand, these equations provide formulas for the elements of energy-momentum tensor. Because metric potentials are formed from Ricci tensors, they enter in a more complicated manner.

Holographic dark energy (HDE) has recently received significant attention as a possible candidate for DE. The holographic principle inspired this type of model [19, 20], a model that was further developed into string theory [20]. Using this principle for cosmology, Li [21] suggested that HDE energy density can be calculated as $\rho_d = 3b^2 M_p^2 L^{-2}$, where L denotes the infrared (IR) cut-off, $b^2 =$ constant and $M_p^{-2} = 8\pi G$. Using Hubble's horizon as a cutoff for the IR distance from HDE, it was found that there is no evidence that the Universe accelerates. The infrared cut-off of the Universe was later considered by him to be a prospective event horizon for the Universe that will emerge in the future. The HDE model can give insight into our current observations indicating an accelerated Universe expansion. Meanwhile, many works have been published [22–27] on HDE models to illustrate the rapid behavior of the cosmos. Based on the Ricci dark energy (RDE) model proposed by Gao et al. [28], DE density appears to be oppositely related to Ricci scalar curvature. In RDE, the future event horizon is replaced by the inverse of the Ricci scalar curvature. In the case of RDE, the energy density can be expressed as follows:

$$\rho_d = 3\alpha\left(\dot{H} + 2H^2\right), \tag{1}$$

Cite as: T. Chinnappalanaidu, M. Vijaya Santhi, N. Sri Lakshmi Sudha Rani, East Eur. J. Phys. 3, 71 (2024), https://doi.org/10.26565/2312-4334-2024-3-07

where H is the Hubble parameter, a is the scale factor and there is no dimension to α, but it describes how the energy density runs. Over dot refers to a derivative of cosmic time t.

Generally, it is recognized as a fact the Cosmos' content in the form of a perfect fluid is aggressive because dissipation is not involved, dissipation being something that exists extensively and supposedly plays a vital role in the cosmos's development. Most practically, the cosmos is thought to consist of a series of mediums which are made up of scattering media. The development of the Universe is characterized by a series of scattering operations. These mechanisms include shear viscosity, heat transmission and bulk viscosity. Numerous writers [29–32] have examined their involvement with the explanation of the Universe's initial inflation as well as its subsequent cosmic development. Over the past few decades, the concept of viscous DE models is being developed to better comprehend the development of the Universe. Based on the writings [33–44], many studies have been conducted to investigate viscosity of the bulk fluid in light of possibility of DE, without the cosmological constant, cosmological expansion may be explained by even the correct form of viscosity of the bulk fluid [45]. The main concept of viscous cosmology for the ancient and modern Universes was given by Brevik et al. [46]. Norman and Brevik [47,48] have investigated the characteristics of two different viscous cosmology models and established general mathematics formulations for bulk viscous fluids. Several recent literature [49,50] have examined the HDE model under the impact of bulk viscosity on the model. By assuming a linear barotropic fluid and an RDE with bulk viscosity, researchers Feng and Li [51] have developed a viscous RDE model. In their paper [52], Singh and Kumar propose a diagnostic method for viscous HDE cosmology using a statefinder.

Furthermore, the Universe's homogeneity and isotropic characteristics have been ascertained from various investigations. Inflationary Universes were homogeneous and isotropic at the end of the inflationary period [53], and FLRW models were essential in depicting cosmos that were both spatially homogeneous and isotropic. Yet the theoretical justification and anomalies found in the CMB facilitate the identification of a phase of anisotropy that is later referred to as an isotropic phase. Immediately following the Plank probe data was released [54], it was discovered that the early cosmos was not entirely uniform. As a result, cosmological models constructed with anisotropic backgrounds have gained popularity due to the inhomogeneous and anisotropic nature of the Universe. Accordingly, the attributes of the Universe change based on the direction in which they're observed, which indicates that we live in an anisotropic Universe. Based on the data provided by WMAP, the current Universe can be described as anisotropic [55]. Marder space time possesses certain traits that help elucidate the genesis of galaxies in the early phases of universal evolution [56,57]. Given that the Marder line element represents an anisotropic and homogeneous space time, it aids in comprehending the Universe's inception and its shift from anisotropy to isotropy. This encourages us to contemplate such space time configurations. Moreover, employing the transformation $t \to \int A(t)dt$, allows for the simplification of Marder spacetime to the Bianchi type-I model, subsequently converging to the FRW Universe. Thus, we categorize the line element accordingly based on whether anisotropy is prevalent in later times or during the early stages of the Universe [58]. Therefore, Marder spacetime not only enables us to study an anisotropic universe but also an isotropic one. We opted Marder's space-time in Einstein's GR and scalar-tensor theories, which is an anisotropic and homogeneous space-time that facilitates an anisotropic to isotropic transition. Many authors have discussed Marder space-time for different matter content. Aygün et al., [59], Aygün [60,61], Kömürcü and Aktas [62] analyzed the Marder's type Universe in the $f(R,T)$ theory of gravity in different contexts. Aygün et al., [56,63], Kabak and Aygun [64], Ali et al. [65] provide a couple of references to some authors who have examined Marder's space-time in various theories with a variety of tensors of energy and momentum. Singh et al., [66], Prakash [67] have used Marder's metric in the development of plane-symmetric models of the universe. Roy and Chatterjee [68], Mukherjee [69] study the Sen-Dunn theory of gravitation and obtain exact solutions to Marder's metric. Pawar and Solanke [70], Pawar and Panpatte [71] studied Marder's space-time in the Saez–Ballester framework. Pawar et al., [72] developed an anisotropic homogeneous Marder's space-time model of wet dark fluids. Santhi et al., [73,74] have examined a cosmological model based on a bulk viscous string in a modified theory of gravity, as well as a viscous HDE cosmological model with Marder space-time in GR respectively. It was just studied by Santhi and Naidu [75] that Marder space-time with Tsallis HDE in GR.

With the help of the GR, we look at Marder-type metric with a viscous RDE (VRDE). The paper looks like this: Section 2 contains the metric and field equations. We derive solutions for Marder-type cosmological models in section 3. Our physical discussion of the cosmological parameters is in section 4. The last section summarizes the results.

2. METRIC AND FIELD EQUATIONS

A space time of Marder type that is anisotropic and spatially homogeneous looks like this:

$$-b_1^2(t)dt^2 + b_1^2(t)dx^2 + b_2^2(t)dy^2 + b_3^2(t)dz^2 = ds^2. \tag{2}$$

- The average scale factor $(a(t))$ and volume (V) of the Marder type metric as follows:

$$V = [a(t)]^3 = b_1^2 b_2 b_3. \tag{3}$$

- The anisotropic parameter $\mathcal{A}_h$ is given by

$$\mathcal{A}_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{4}$$

where $H_1 = \frac{\dot{b_1}}{b_1}$, $H_2 = \frac{\dot{b_2}}{b_2}$, $H_3 = \frac{\dot{b_3}}{b_3}$ are directional Hubble's parameters and $H = \frac{1}{3}\left(2\frac{\dot{b_1}}{b_1} + \frac{\dot{b_2}}{b_2} + \frac{\dot{b_3}}{b_3}\right)$ is mean Hubble's parameter. Cosmological time t differentiation is indicated by an overhead dot.

- The expansion scalar (θ) is given by

$$\theta = \frac{1}{b_1}\left(\frac{\dot{b_2}}{b_2} + 2\frac{\dot{b_1}}{b_1} + \frac{\dot{b_3}}{b_3}\right). \tag{5}$$

- The shear scalar (σ^2) is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{3}\right). \tag{6}$$

In Einstein's theory of GR, the equations of fields are defined as follows:

$$G_{ij} = -T_{ij}, \tag{7}$$

where T_{ij} is the energy momentum tensor and $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor. As well as that, the equation for conservation is as follows:

$$T^{ij}{}_{;j} = 0. \tag{8}$$

With viscosity of the bulk fluid and a deviation from thermodynamic equilibrium of the first order, the stress-energy-momentum tensor takes the form (Wilson et al. [76]).

$$T_{ij} = (\rho_m + \rho_d)\,u_i u_j + \bar{p}_d\left(g_{ij} + u_i u_j\right), \tag{9}$$

where ρ_d and ρ_m are the energy densities of RDE and DM, respectively and $\bar{p}_d = p_d - 3\zeta H$, where $\bar{p}_d$ is the effective pressure of RDE, H is Hubble parameter and ζ is the bulk viscous coefficient which is given as $\zeta = \zeta_0 + \zeta_1 h + \zeta_2 h'$, where $h = \frac{H}{H_0}$ and ζ_0, ζ_1 are positive constants and $u_i u^i = -1$.
By using Eq. (9), the field equations (7) can be written as follows:

$$\frac{1}{b_1^2}\left(\frac{\ddot{b_2}}{b_2} + \frac{\ddot{b_3}}{b_3} + \frac{\dot{b_2}\dot{b_3}}{b_2 b_3} - \frac{\dot{b_1}\dot{b_2}}{b_1 b_2} - \frac{\dot{b_1}\dot{b_3}}{b_1 b_3}\right) = -\bar{p}_d, \tag{10}$$

$$\frac{1}{b_1^2}\left(\frac{\ddot{b_1}}{b_1} + \frac{\ddot{b_3}}{b_3} - \frac{\dot{b_1}^2}{b_1^2}\right) = -\bar{p}_d, \tag{11}$$

$$\frac{1}{b_1^2}\left(\frac{\ddot{b_1}}{b_1} + \frac{\ddot{b_2}}{b_2} - \frac{\dot{b_1}^2}{b_1^2}\right) = -\bar{p}_d, \tag{12}$$

$$\& \quad \frac{1}{b_1^2}\left(\frac{\dot{b_1}\dot{b_2}}{b_1 b_2} + \frac{\dot{b_2}\dot{b_3}}{b_2 b_3} + \frac{\dot{b_3}\dot{b_1}}{b_3 b_1}\right) = \rho_m + \rho_d. \tag{13}$$

Furthermore, the energy conservation equation leads to

$$(\dot{\rho}_m + \dot{\rho}_d) + \left(2\frac{\dot{b_1}}{b_1} + \frac{\dot{b_2}}{b_2} + \frac{\dot{b_3}}{b_3}\right)(\dot{\rho}_m + \dot{\rho}_d + \bar{p}_d) = 0. \tag{14}$$

3. SOLUTION OF THE FIELD EQUATIONS

There are four independent equations in (10)-(13) which have six independent components : $\bar{p}_d$, ρ_m, ρ_d, b_1, b_2, and b_3. In order to resolve the equations mentioned above, the following conditions are required:

- There is a relationship between the metric potentials when the shear scalar (σ) is proportional to the expansion scalar (θ)(Collins et al. [77]). That is

$$b_1 = (b_2 b_3)^n, \tag{15}$$

as long as $n \neq 1$ is constant, space-time's anisotropy is preserved.

- Mishra et al. [78] proposed a varying deceleration parameter as

$$q(t) = -\frac{a\ddot{a}}{\dot{a}^2} = b(t), \tag{16}$$

where $b(t)$ is an arbitrary function of time, where $a(t)$ is the average scale factor of the Universe. We can get the average scale factor by solving the above Eq. (16) using some suitable assumptions (Mishra et al. [78]).

$$a(t) = (\sinh \beta_1 t)^{\frac{1}{\beta_2}}, \tag{17}$$

where β_1 and β_2 represent arbitrary positive constants. Using such an average scale factor, we obtain a deceleration parameter(DP) (q) that changes from an early deceleration phase to a current acceleration phase. Santhi et al. [79] and Rao and Prasanthi [80], Reddy et al. [81] used this average scale factor to evaluate various cosmological models.

From Eqs.(11) and (12), we have

$$b_2 = \gamma_2^2 b_3, \tag{18}$$

here, $\gamma_2^2 \neq 1 \,\&\, \gamma_2^2 > 0$.
The metric potentials are derived from equations (3), (15), (17), and (18) as depicted below:

$$b_1 = (\sinh(\beta_1 t))^{\frac{3n}{\beta_2(n+1)}}, \tag{19}$$

$$b_2 = \gamma_2 (\sinh(\beta_1 t))^{\frac{3}{\beta_2(2n+2)}}, \tag{20}$$

$$b_3 = \frac{(\sinh(\beta_1 t))^{\frac{3}{\beta_2(2n+2)}}}{\gamma_2}. \tag{21}$$

The energy density of VRDE is expressed as follows:

$$\rho_d = \frac{6\alpha(\beta_2 - 2)\beta_1^3 \cosh(\beta_1 t)}{\beta_2^2 \sinh(\beta_1 t)^3}. \tag{22}$$

Matter's energy density is given by

$$\rho_m = \frac{3\beta_1^2 \left(3\left(\cosh(\beta_1 t)^2 - 1\right)\cosh(\beta_1 t)^2 \left(n + \frac{1}{4}\right)\sinh(\beta_1 t)^{\frac{2\beta_2(n+1)+(-4n-4)\beta_2-6n}{\beta_2(n+1)}} + \alpha(n+1)^2\left(\beta_2 - 2\cosh(\beta_1 t)^2\right)\right)}{\beta_2^2 (n+1)^2 \sinh(\beta_1 t)^2}. \tag{23}$$

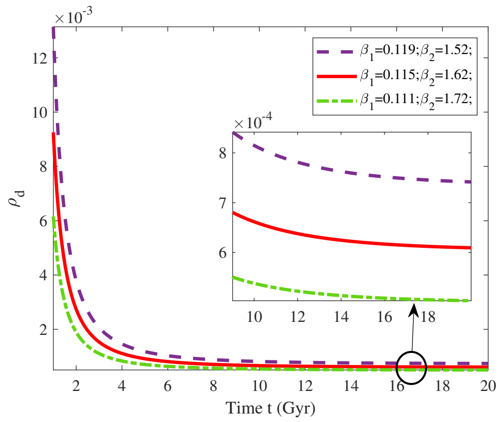

Figure 1. VRDE energy density (ρ_d) v/s time (t) (Gyr)

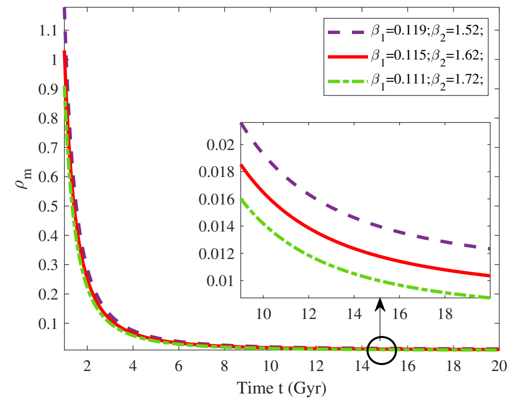

Figure 2. Matter energy density(ρ_m) v/s time (t) (Gyr)

The pressure of VRDE is given by

$$\bar{p}_d = -\frac{2\beta_1^2 (\sinh(\beta_1 t))^{\frac{(-2n-2)\beta_2 - 6n}{\beta_2(n+1)}} \left(\left(\frac{3n}{2} - \frac{15}{8}\right)\cosh(\beta_1 t)^2 + \beta_2(n+1)^2\right)}{\beta_2^2 (n+1)^2}. \tag{24}$$

The bulk viscosity is given by

$$\zeta = \frac{\zeta_1 \beta_1 \cosh(\beta_1 t)\sinh(\beta_1 t) H_0 + H_0 \beta_2 \cosh(\beta_1 t)^2 \zeta_0 - \zeta_0 \beta_2 H_0 - \zeta_2 \beta_1^2}{\beta_2 H_0 \sinh(\beta_1 t)^2}. \tag{25}$$

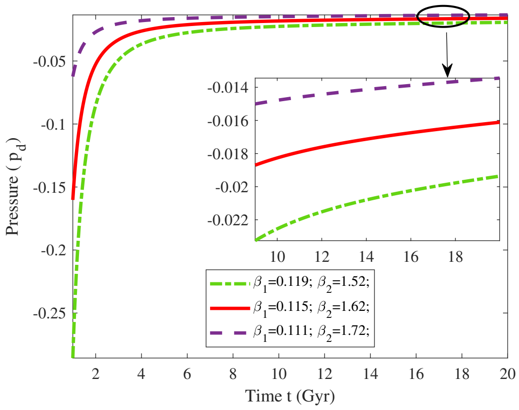

Figure 3. VRDE pressure ($\bar{p}_d$) v/s time (t)(Gyr)

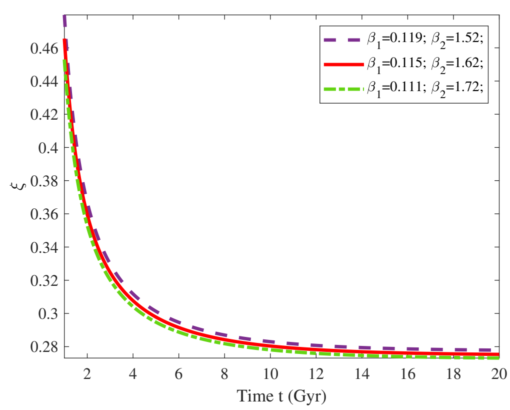

Figure 4. Bulk viscosity (ξ) v/s time (t)(Gyr)

We used the following constants as a constraint in our analysis as well as diagrammatic illustrations of the Marder VRDE model of the universe in which our analysis discussed the corresponding parameters: $\beta_1 = 0.199, 0.115, 0.111;\beta_2 = 1.52, 1.62, 1.72;$ $n = 0.019;$ $\alpha = 0.02;$ $\zeta_0 = 0.25;\zeta_1 = 0.35;\zeta_2 = 0.15;$ $H_0 = 65$. It is shown in Fig.(1) and (2) that the characteristics of VRDE energy density (ρ_d) and matter energy density (ρ_m) is plotted against cosmic time t. Observing the trajectories of VRDE energy density (ρ_d) and matter energy density (ρ_m) for various values of β_1 and β_2, as we can see a variation towards the positive region, through time, which indicates that the Universe is expanding. The total pressure (p) for different values of β_1 and β_2 can also be observed by examining Fig. (3). With the passing of time (t), the total pressure (p) becomes negative. According to Fig. (4), the trajectory of bulk viscosity decreases with time t, while it varies in the positive region.
It is now possible to write the metric (2) as

$$ds^2 = (\sinh(\beta_1 t))^{\frac{6n}{\beta_2(n+1)}}\, dx^2 + \left(\gamma_2^2\,(\sinh(\beta_1 t))^{\frac{6}{\beta_2(2n+2)}}\right) dy^2 + \left(\frac{(\sinh(\beta_1 t))^{\frac{6}{\beta_2(2n+2)}}}{\gamma_2^2}\right) dz^2 - (\sinh(\beta_1 t))^{\frac{6n}{\beta_2(n+1)}}\, dt^2. \tag{26}$$

Thereby, in the general theory of gravitation, Eq. (26) corresponds to a Marder-type space time that is spatially homogeneous and anisotropic with VRDE.

- The average scale factor ($a(t)$) and volume (V) of the Marder type space time are given by

$$V = (\sinh(\beta_1 t))^{\frac{3}{\beta_2}} \quad \text{and} \quad a(t) = [V]^3 = (\sinh(\beta_1 t))^{\frac{1}{\beta_2}}\,. \tag{27}$$

- The Hubble parameter (H) is given by the following equation:

$$H = \frac{\beta_1 \coth(\beta_1 t)}{\beta_2}. \tag{28}$$

- The expansion scalar (θ) is given by

$$\theta = 3\left(\frac{\beta_1 \coth(\beta_1 t)}{\beta_2}\right)\left(\sinh(\beta_1 t)^{\frac{(-n-1)\beta_2 - 3n}{\beta_2(n+1)}}\right). \tag{29}$$

- The shear scalar (σ^2) is given by

$$\sigma^2 = \left(\frac{-3}{2}\right)\frac{\beta_1^2 \cosh(\beta_1 t)^2\left((n+1)^2 \sinh(\beta_1 t)^{\frac{2\beta_2(n+1)+(-2n-2)\beta_2-6n}{\beta_2(n+1)}} - 3n^2 - \frac{3}{2}\right)}{\beta_2^2 (n+1)^2 \sinh(\beta_1 t)^2}. \tag{30}$$

- You can find the anisotropic parameter by

$$\mathcal{A}_h = \frac{(2n-1)^2}{2(n+1)^2}. \tag{31}$$

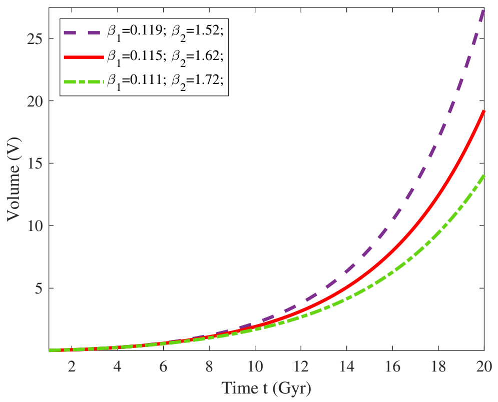

Figure 5. V v/s time t (Gyr)

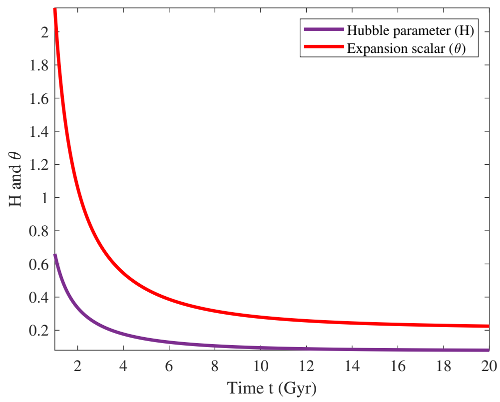

Figure 6. Hubble parameter (H), expansion scalar (θ) v/s time t (Gyr)

We observe from Eq.(31) that $\mathcal{A}_h \neq 0$ represents the likelihood that the Marder type cosmic model will always be anisotropic(except $n = \frac{1}{2}$). For different values of β_1 and β_2, the volume has monotonously increased against time (t) in Fig. (5). This describes the spatial volume (V) is growing a significant increase and presents the Universe's growth in an exponential manner. As can also be observed from the equations (27)-(30) within the scope of the earliest phase $i.e.$, at $t = 0$, the average scale factor $a(t)$ and the volume (V) decrease as time passes, indicating an expanding Universe. Hubble parameter H, expansion scalar θ, and shear scalar σ diverge at the initial epoch but attain constant values later. The graphical representation of the θ & H can be found in figure (6). As time progresses, both functions decrease and become constant. Based upon these observations, it can be seen that with respect to time, the model begins its intensification as a volume-zero model, and as it expands further, it reaches an infinite-volume model.

4. COSMIC PARAMETERS

We will investigate the Universe expands in terms of well recognized parameters relevant to the study of the cosmos, namely the EoS parameter (ω_{de}), squared sound speed (v_s^2), jerk parameter (j), DP (q), density parameters $i.e.$, Ω_m, Ω_d & Ω, energy conditions and $\omega_{de} - \omega'_{de}$ plane, statefinder $(r - s)$ for constructing an anisotropic VRDE model.

- **Deceleration Parameter:** A further parameter that we have to consider as a result of to examine the transition phase of the Universe is referred to as the DP. This measure quantifies the Universe's expansion rate in a dimensionless form. q represents this value and it can be described in the following manner:

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \beta_2 \sec h(\beta_1 t)^2 - 1. \tag{32}$$

Depending on its sign, it signifies either deceleration (if positive) or acceleration (if negative), whereas de-Sitter expansion is observed at $q = -1$ and marginal inflation happens when $q = 0$. The model indicates a super exponential expansion for $q < -1$ and an accelerated expansion for $-1 \leq q < 0$. According to Fig. (7), we can see that the DP q is plotted against time t, and by looking at the trajectory of the DP we observe that it shows a nice line with three different values of β_1 and β_2 corresponding to the shift from the initial slowing down to the current speeding up.

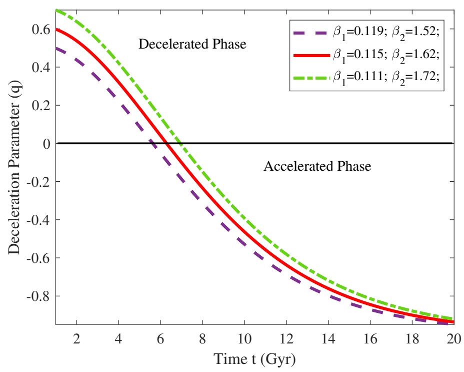

Figure 7. VRDE deceleration parameter(q) v/s time t (Gyr)

- **Jerk Parameter:** It is widely accepted that the jerk parameter(JP) in cosmology is the third derivative of the scale factor with respect to time, expressed without dimensions, and it is defined as follows:

$$j = \frac{\dddot{a}}{aH^3} = \frac{2\beta_2^2 + \cosh(\beta_1 t)^2 - 3\beta_2}{\cosh(\beta_1 t)^2}. \tag{33}$$

Nowadays it is generally accepted that in modern cosmology, the JP can be used to describe how the Universe transitions from a decelerating phase into an accelerating phase as a result of its jerk. It seems that this transition of the Universe occurs in various models if the JP is set to a positive value and the DP is set to a negative value (Chiba and Nakamura [82]; Visser [83]). Based on three different values of β_1 and β_2, we have plotted a graphical representation of the JP against time (t) in Fig. (8). Throughout the evolution, the JP varies in positive regions and achieves a value of one at the end of the process. Thus, we can conclude from our model that recent observations are consistent with it. Additionally, we observe that β_1 and β_2 completely affect the JP.

- **Statefinder diagnostics:** For the purpose of distinguishing between multiple candidates for dark energy, Sahni et al. [84] has developed an analytical tool that can be used to identify and quantify the statefinder pair of terms $\{r, s\}$, for which the term r able to derived as a result of $a(t)$ and in terms of cosmic time (t), it is a third-order derivative and s can be derived as a function of r and the DP q. The parameters for Statefinder are as follows:

$$r = \frac{\dddot{a}}{aH^3} = \frac{2\beta_2^2 + \cosh(\beta_1 t)^2 - 3\beta_2}{\cosh(\beta_1 t)^2}, \quad s = \frac{r-1}{3(q-\frac{1}{2})} = \frac{4\beta_2^2 - 6\beta_2}{6\beta_2 - 9\cosh(\beta_1 t)^2}. \tag{34}$$

"A few useful regions can be described by these parameters: $(r, s) = (1, 0)$ indicates ΛCDM, $(r, s) = (1, 1)$ shows CDM limit, Chaplygin gas region ($r > 1, s < 0$), quintessence and phantom regions ($r < 1, s > 0$). Fig. (9) shows the behavior of $r - s$ plane of our model. A Λ CDM model is observed within our model of the Universe."

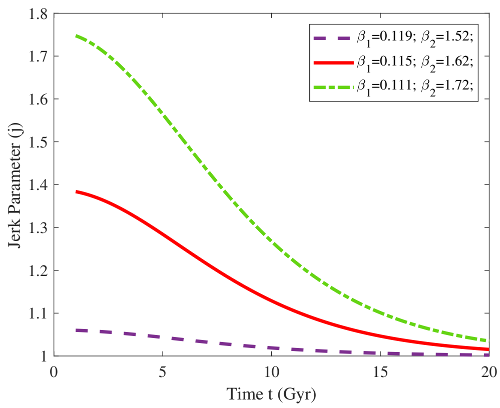

Figure 8. VRDE JP (j) v/s time t (Gyr)

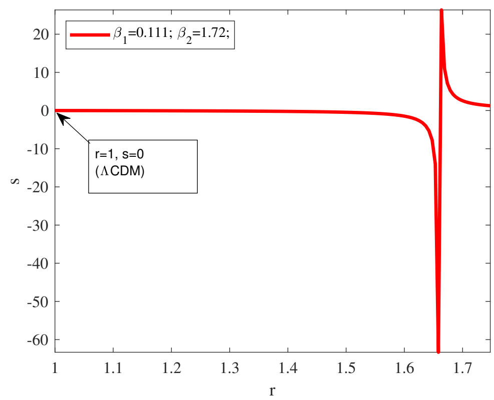

Figure 9. $r - s$ plane

- **EoS Parameter:** The following equation will be used to obtain EoS parameter

$$\omega_{de} = \frac{\bar{p}_d}{\rho_d}. \tag{35}$$

Here, ρ_d and $\bar{p}_d$ are used to represent the DE density and the pressure of the VRDE, respectively. It is estimated that the EoS parameter can be used to define the Universe's progression into stages of deceleration and acceleration. It is possible to divide the DE dominated phase into the following eras:

(i) $\omega_{de} = 0$ corresponds to non-relativistic matter.
(ii) $-1 < \omega_{de} < \frac{-1}{3}$ quintessence.
(iii) $\omega_{de} = -1$ cosmological constant.
(iv) $\omega_{de} < -1$ phantom.

In the model, the EoS parameter is represented as the following:

$$\omega_{de} = \frac{2\beta_1^2 \sinh(\beta_1 t)^{\frac{\beta_2(-2n-2)-6n}{\beta_2(n+1)}} \left(\left(\frac{3n}{2} - \frac{15}{8} \right) \cosh(\beta_1 t)^2 + \beta_2 (n+1)^2 \right)}{(n+1)^2 6\alpha (\beta_2 - 2) \beta_1^3 \coth(\beta_1 t) cosech(\beta_1 t)^2}. \tag{36}$$

VRDE's EoS parameter is given in Eq. (26). For the model (26), we present in figure (10) the development of the EoS parameter ω_{de} in terms of time (t), which has various numbers for β_1 and β_2. It is clear form the figures that the EoS parameter for model (26) starts from phantom region and vary in high phantom region.

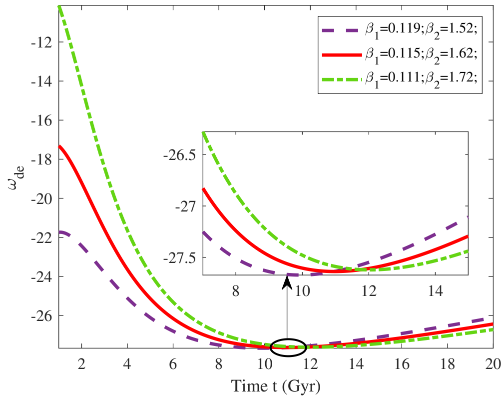

Figure 10. VRDE EoS parameter(ω_{de}) v/s time t (Gyr)

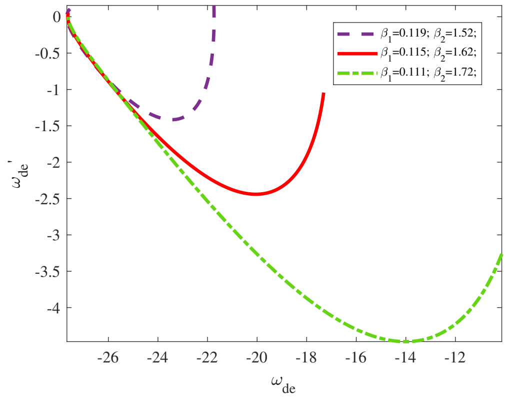

Figure 11. $\omega_{de} - \omega'_{de}$ plane

- $\omega_{de} - \omega'_{de}$ **plane:** Caldwell and Linder [85] propose the $\omega_{de} - \omega'_{de}$ plane analysis, which is a useful tool for distinguishing DE models based on the trajectory of their planes. Using this approach, we can create two types of planes in the essence model, *i.e.*, the region $\omega_{de} < 0$, $\omega'_{de} < 0$ implies the freezing region and the region $\omega_{de} < 0$, $\omega'_{de} > 0$ a region of thawing. The expression for ω'_{de} can be obtained by taking the derivative of Eq. (36) with respect to $\ln a$

$$\left.\begin{aligned}\omega'_{de} = & \frac{(-1)\left(4\left(\frac{9n^2}{2}-\frac{45n}{8}\right)\cosh(\beta_1 t)^2 \sinh(\beta_1 t)^{\frac{3\beta_2(n+1)+\beta_2(-3n-3)-6n}{\beta_2(n+1)}}\right)}{\beta_2(n+1)^3 6\alpha\,(\beta_2-2)} \\ & + \frac{(-4)\left(\beta_2(n+1)^2+3n^2+\frac{9n}{2}-\frac{15}{8}\right)\sinh(\beta_1 t)^{\frac{3\beta_2(n+1)+\beta_2(-3n-3)-6n}{\beta_2(n+1)}}}{(n+1)^2 6\alpha\,(\beta_2-2)} \\ & + \frac{(-1)\sinh(\beta_1 t)^{\frac{3\beta_2(n+1)+\beta_2(-2n-2)-6n}{\beta_2(n+1)}}\left(\left(\frac{3n}{2}-\frac{15}{8}\right)\cosh(\beta_1 t)^2+\beta_2(n+1)^2\right)}{(n+1)^2 6\alpha\,(\beta_2-2)\,\beta_1^3\cosh(\beta_1 t)}.\end{aligned}\right\} \quad (37)$$

As shown in Fig. (11), all values of β_1 and β_2 place the model in the freezing region. According to observations, the Universe is expanding relatively fast in the freezing region. For the obtained model (26), $\omega_{de} - \omega'_{de}$ plane analysis gives the Universe is expanding faster than ever.

- **Energy conditions:** Astrophysical and cosmological energy conditions (ECs) are derived from Raychaudhuri equations [86]. In general, the energy momentum tensor(EMT) is an important factor to consider when studying energy conditions, so, for this discussion, we are going to use the term EMT in relation to pressure $\bar{p}_d$ and energy density ρ_d, therefore, all four of the energy conditions can be written in the following ways: null energy condition (NEC), dominant energy condition (DEC), strong energy condition (SEC), and weak energy condition (WEC). As part of this paper, we have examined how energy conditions have evolved over time. In general, these energy conditions serve as a measure of the expansion of the Universe. It is imperative to recognize that these conditions impose additional limitations on the cosmological model's viability. These conditions include the following:

  WEC: $\rho_d \geq 0$

  NEC: $rho_d + \bar{p}_d \geq 0$

  DEC: $\rho_d - \bar{p}_d \geq 0$

  SEC:$\rho_d + 3\bar{p}_d \geq 0$

  It's important to mention that SEC represents a strong energy condition, DEC represents a dominant energy condition,and NEC represents a null energy condition, and WEC represents a weak energy condition.

  Fig. (12) shows the behavior of these energy conditions in the constructed model. In cosmic evolution, WEC and DEC are well satisfied while NEC and SEC are violated at late times, which corresponds to accelerated expansion.

Our model also clearly shows that DEC dominates NEC and SEC in accordance with our observation. I think this is an interesting observation that should be taken into account.

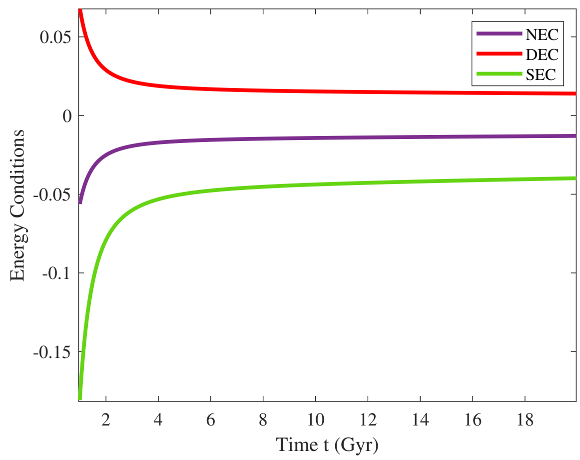

Figure 12. VRDE ECs v/s time t (Gyr)

- **Squared speed of sound (v_s^2):** Our next step will be to consider and study one of the most important quantities in cosmology, that is the squared speed of sound(v_s^2), which is an important quantity to take into account when checking every DE model's stability. Depending on the sign of this parameter, we can examine stability of DE models. Models with $v_s^2 < 0$ are unstable whereas models with $v_s^2 > 0$ are stable. Here is a definition of the squared speed of sound:

$$v_s^2 = \frac{\dot{p}_d}{\dot{\rho}_d} = \left.\frac{-2\left(\left(\frac{9n^2}{2}-\frac{45n}{8}\right)\cosh(\beta_1 t)^2+(n+1)\left(\beta_2(n+1)^2+3n^2+\frac{92}{2}-\frac{15}{8}\right)\beta_2\right)\sinh(\beta_1 t)^{\frac{-6n}{\beta_2(n+1)}}}{3\alpha\,(\beta_2-2)\,\beta_2(n+1)^3}\right\}. \qquad (38)$$

Fig. (13) illustrates v_s^2 versus time t for various values of β_1 and β_2. All values of β_1 and β_2 show trajectories in the negative region, which indicates unstable cosmos behavior.

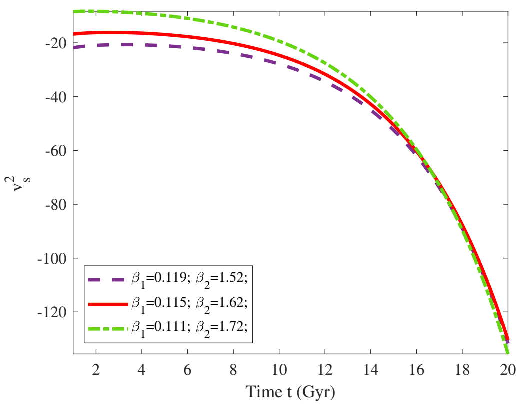

Figure 13. VRDE squared speed of sound(v_s^2) v/s time t (Gyr)

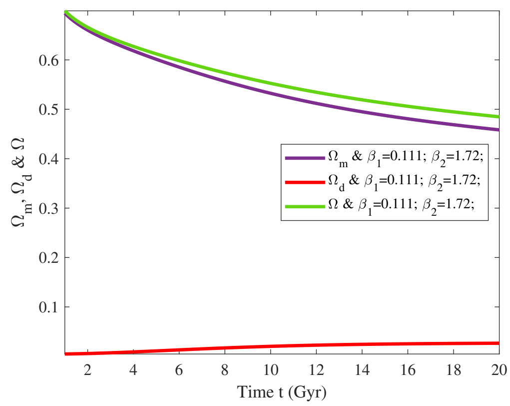

Figure 14. VRDE density parameters (Ω, Ω_d Ω_m) v/s time t (Gyr)

- **Density parameters:** "It has been suggested by most authors that the $\Omega \approx 1$. The ultimate destiny of the Universe can be revealed by knowing whether Ω is greater than 1, less than 1, or exactly equal to 1. Eventually, the Universe will stop expanding and collapse if $\Omega > 1$. In the case where $\Omega < 1$, then the Universe is open and will continue to expand forever, whereas if $\Omega = 1$, then the Universe is flat and has enough material to stop expansion, but not enough to collapse. As a result of this definition, a dimensionless density parameter expression can be found as follows:"

$$\Omega_d = \frac{\rho_d}{3H^2}, \qquad (39)$$

$$\Omega_m = \frac{\rho_m}{3H^2}, \tag{40}$$

$$\Omega = \Omega_d + \Omega_m. \tag{41}$$

For model (26), the density parameters of VRDE, matter, and total density are given by

$$\Omega_d = \frac{-\alpha\left(\beta_2 - 2\cosh(\beta_1 t)^2\right)}{\cosh(\beta_1 t)^2}, \tag{42}$$

$$\Omega_m = \frac{12\left(\cosh(\beta_1 t)^2 - 1\right)\cosh(\beta_1 t)^2\left(n + \frac{1}{4}\right)\sinh(\beta_1 t)^{\frac{\beta_2(-2n-2)-6n}{\beta_2(n+1)}}}{4\cosh(\beta_1 t)^2(n+1)^2} + \frac{4\alpha(n+1)^2\left(\beta_2 - 2\cosh(\beta_1 t)^2\right)}{4\cosh(\beta_1 t)^2(n+1)^2}, \tag{43}$$

$$\& \quad \Omega = \frac{3(4n+1)\sinh(\beta_1 t)^{\frac{-6n}{\beta_2(n+1)}}}{4(n+1)^2}. \tag{44}$$

According to Fig. (14), we analyze the behavior of the density parameters of VRDE (Ω_d), matter (Ω_m) as well as the total density (Ω) as a function of cosmic time t for model (26) with β_1 and β_2. Based on the trajectory of density parameters, we observe that Ω_d, Ω_m & Ω decrease with cosmic time and approach a number less than one at later times, subsequently the total density (Ω) dominates the VRDE and matter parameters.

5. CONCLUSION

Using Einstein GR, we have constructed a Marder type cosmological model with viscous Ricci dark energy. A variable declaration parameter has been applied to study the dynamics of the Universe and to determine solutions to the field equations. A transition from a decelerating to accelerating phase was observed in DP. This model has the following main outcomes:

Our obtained model at present accelerated and deceleration at past, based on the assumed form of DP. Observations agree well with these aspects of the model. At first V, a are dies whereas H, θ, & σ tends to ∞ but as time passes V, a are tends to ∞ and H, θ, & σ are become constant. It can be seen that our constructed model is anisotropic throughout the evolution of the Universe since (A_h) is constant and does not vanish ($A_h \neq 0$). Throughout cosmic expansion, VRDE has a positive energy density. Furthermore, cosmic pressure increases over time and is negative. Also, the changes of EoS parameter $\omega_{de} = \frac{\bar{p}_d}{\rho_d}$ has been graphically observed. According to this analysis, ω_{de} evolves from the phantom region and varies in the high phantom region. Our constructed model's evolution of $\omega_{de} - \omega'_{de}$ varies in the freezing phase. In the freezing region of the Universe, according to the observations, the growth of the Universe is relatively accelerating at the present time and also we examine that ECs for obtained model and hence notice that SEC, NEC are breach whereas DEC is fulfilled. We observe that Ω_d, Ω_m & Ω decrease with cosmic time and approach a number less than one at later times Throughout the evolution, the JP is positive and reaches one. In general, (r,s) trajectories start from the chaplygin gas region and finally reach SCDM, where $s = 0$, $r = 1$. Consequently, in the future, the constructed model of the Universe will behave in a manner similar to the ΛCDM model.

Compliance with Ethical Standards

Funding: not applicable.
Conflict of interest: The Authors declare that they have no conflicts of interest.
Author contribution: The study was carried out in collaboration of all authors. All authors read and approved the final manuscript.
Ethical Conduct.
Data Availability: The data used to support the findings of this study are included within the article and are cited at relevant places within the text as references.
Acknowledgments MVS acknowledges Department of Science and Technology (DST), Govt of India, New Delhi for financial support to carry out the Research Project [No. EEQ/2021/000737, Dt. 07/03/2022].

ORCID

T. Chinnappalanaidu, https://orcid.org/0000-0001-6902-2820; **M. Vijaya Santhi**, https://orcid.org/0000-0002-0050-3033; **N. Sri Lakshmi Sudha Rani**, https://orcid.org/0009-0009-9593-954X

REFERENCES

[1] N. Aghanim, *et al.*, (Planck Collaboration), (2018). https://doi.org/10.48550/arXiv.1807.06209

[2] A.G. Riess, *et al.*, Astron. J. **116**, 1009 (1998). https://doi.org/10.1086/300499

[3] S.Perlmutter, *et al.*, Astrophys. J. **517**, 565 (1999). https://doi.org/10.1086/307221
[4] D.M. Scolnic, *et al.*, Astrophys. J. **859**, 101 (2018). arXiv:1710.00845 [astro-ph.CO] https://doi.org/10.3847/1538-4357/aab9bb
[5] S.Nadathur, *et al.*, Phys.Rev.Lett. **124**, 221301 (2020). https://doi.org/10.1103/PhysRevLett.124.221301
[6] B. Ratra, and P. Peebles, Am. Phys. Soc. **37**(12), 3406 (1988). https://doi.org/10.1103/PhysRevD.37.3406
[7] T. Chiba, *et al.*, Phys. Rev. D, **62**, 023511 (2000). https://doi.org/10.1103/PhysRevD.62.023511
[8] H. Wei, and R.G. Cai, Phys. Lett. B, **660**(3), 113 (2008). https://doi.org/10.1016/j.physletb.2007.12.030
[9] M.V. Santhi, *et al.*, Int. J. Theor. Phys. **56**, 362 (2017). https://doi.org/10.1007/s10773-016-3175-8
[10] H. Wei, and R.G. Cai, Phys. Lett. B, **663**, 1 (2008). https://doi.org/10.1016/j.physletb.2008.03.048
[11] A. Kamenshchik, *et al.*, Phys. Lett. B, **511**, 265 (2001). https://doi.org/10.1016/S0370-2693(01)00571-8
[12] A. Cohen, *et al.*, Phys. Rev. Lett. **82**, 4971 (1999). https://doi.org/10.1103/PhysRevLett.82.4971
[13] P. Horava, and D. Minic, Phys. Rev. Lett. **85**, 1610 (2000). https://doi.org/10.1103/PhysRevLett.85.1610
[14] V. Husain, and O. Winkler, Phys. Rev. D, **75**, 024014 (2007). https://doi.org/10.1103/PhysRevD.75.024014
[15] T. Harko, *et al.*, Phys. Rev. D, **84**, 024020 (2011). https://doi.org/10.1103/PhysRevD.84.024020
[16] C.H. Brans, and R.H. Dicke, Phys. Rev. **124**, 925 (1961). https://doi.org/10.1103/PhysRev.124.925
[17] D. Saez, and V.J. Ballester, Phys. Lett. **113**, 467 (1986). https://doi.org/10.1016/0375-9601(86)90121-0
[18] A. Einstein, Ann. Phys. **354**, 769 (1916). https://doi.org/10.1002/andp.19163540702
[19] G t'Hooft, arXiv:gr-qc/9310026 (1993). https://arxiv.org/abs/gr-qc/9310026
[20] L. Susskind, J. Math. Phys. **36**, 6377 (1995). https://doi.org/10.1063/1.531249
[21] M. Li, Phys. Lett. B, **603**, 1 (2004). https://doi.org/10.1016/j.physletb.2004.10.014
[22] W. Fischler, and L. Susskind, arXiv:hep-th/9806039 (1998). https://arxiv.org/abs/hep-th/9806039
[23] Y. Gong, Phys. Rev. D, **70**, 064029 (2004). https://doi.org/10.1103/PhysRevD.70.064029
[24] B.Wang, *et al.*, Phys. Lett. B, **624**, 141 (2005). https://doi.org/10.1016/j.physletb.2005.08.008
[25] X. Zhang, Int. J. Mod. Phys. D, **14**, 1597 (2005). https://doi.org/10.1142/S0218271805007243
[26] D. Pavon, and W. Zimdahl, arXiv:hep-th/0511053 (2005). https://arxiv.org/abs/hep-th/0511053
[27] A.G. Cohen, *et al.*, Phys. Rev. Lett. **82**, 4971 (1999). https://doi.org/10.1103/PhysRevLett.82.4971
[28] C. Gao, *et al.*, Phys. Rev. D, **79**, 043511 (2009). https://doi.org/10.1103/PhysRevD.79.043511
[29] J.D. Barrow, Phys. Lett. B, **180**, 335 (1986). https://doi.org/10.1016/0370-2693(86)91198-6
[30] I. Brevik, and S.D. Odintsov, Phys. Rev. D, **65**, 067302 (2002). https://doi.org/10.1103/PhysRevD.65.067302
[31] I. Brevik, and O. Gorbunova, Gen. Relativ. Gravit. **37**, 2039 (2005). https://doi.org/10.1007/s10714-005-0178-9
[32] D.J. Liu, and X.Z. Li, Phys. Lett. B, **611**, 8 (2005). https://doi.org/10.1016/j.physletb.2005.02.048
[33] S. Nojiri, and S.D. Odintsov, Phys. Rev. D, **72**, 023003 (2005). https://doi.org/10.1103/PhysRevD.72.023003
[34] S. Capozziello, *et al.*, Phys. Rev. D, **73**, 043512 (2006). https://doi.org/10.1103/PhysRevD.73.043512
[35] J. Ren, and X.H. Meng, Phys. Lett. B, **633**, 1 (2006). https://doi.org/10.1016/j.physletb.2005.11.055
[36] Hu M.G., Meng X.H., Phys. Lett. B, **635**, 186 (2006). https://doi.org/10.1016/j.physletb.2006.02.059
[37] N. Cruz, S. Lepe, and F. Pena, Phys. Lett. B, **646**, 177 (2007). http://dx.doi.org/10.1016/j.physletb.2017.01.035
[38] X.H. Meng, J. Ren, and M.G. Hu, Commun. Theor. Phys. **47**, 379 (2007). https://doi.org/10.1088/0253-6102/47/2/036
[39] C.P. Singh, S. Kumar, and A. Pradhan, Class. Quantum Gravit. **24**, 455 (2007). https://doi.org/10.1088/0264-9381/24/2/011
[40] C.P. Singh, Pramana J. Phys. **71**, 33 (2008). https://doi.org/10.1007/s12043-008-0139-4
[41] C.J. Feng, and X.Z. Li, Phys. Lett. B, **680**, 355 (2009). https://doi.org/10.1016/j.physletb.2009.09.013
[42] Y.D.Xu, Z.G. Huang, and X.H. Zhai, Astrophys. Space Sci. **337**, 493 (2012). https://doi.org/10.1007/s10509-011-0850-3
[43] C.P. Singh, P. Kumar, Eur. Phys. J. C, **74**, 3070 (2014). https://doi.org/10.1140/epjc/s10052-014-3070-5
[44] P. Kumar, and C.P. Singh, Astrophys. Space Sci. **357**, 120 (2015). https://doi.org/10.1007/s10509-015-2348-x
[45] J.S. Gagnon, and J. Lesgourgues, J Cosmol. Astropart. Phys. **09**, 026 (2011). https://doi.org/10.1088/1475-7516/2011/09/026
[46] I. Brevik, *et al.*, Int. J. Mod. Phys. D, **26**, 1730024 (2017). https://doi.org/10.1142/S0218271817300245
[47] B.D. Normann, and I. Brevik, Mod. Phys. Lett. A, **32**, 1750026 (2017). https://doi.org/10.1142/S0217732317500262
[48] B.D. Normann, and I. Brevik, Entropy, **18**, 215 (2016). https://doi.org/10.3390/e18060215
[49] M. Jamil, and M.U. Farooq, Int. J. Theor. Phys. **49**, 42 (2010). https://doi.org/10.1007/s10773-009-0176-x
[50] S. Chattopadhyay, Int. J. Mod. Phys. D, **26**, 1750042 (2017) https://doi.org/10.1142/S0218271817500420
[51] C.J. and Feng, X.Z. Li, Phys. Lett. B, **680**, 355 (2009). https://doi.org/10.1016/j.physletb.2009.09.013

[52] C.P. Singh, and P. Kumar, Astrophys. Space Sci. **361**, 157 (2016). https://doi.org/10.1007/s10509-016-2740-1

[53] A.D. Linde, Inflationary cosmology, Lect. Notes Phys. **738**, 1 (2008). https://doi.org/10.1007/978-3-540-74353-8_1

[54] P.A.R. Ade, *et al*., Planck Collab., Planck 2013 results. XVI. Cosmological parameters, Astron. Astrophys. **571**, A16 (2014). https://doi.org/10.1051/0004-6361/201321591

[55] E. Komatsu, *et al*., Astrophys. J. Suppl. **180**, 330 (2008). https://doi.org/10.1088/0067-0049/180/2/330

[56] S. Aygün, *et al*., Journal of Geometry and Physics, **62**, 100 (2012). https://doi.org/10.1016/j.geomphys.2011.09.011

[57] C.B. Kilinc, Astrophys. Space Sci. **222**, 171 (1994). https://doi.org/10.1007/BF00627091

[58] M. Sharif, and H.R. Kausar, Phys. Lett. B, **697**, 1 (2011). https://doi.org/10.1016/j.physletb.2011.01.027

[59] S. Aygün, *et al*., Astrophys. Space Sci. **361**, 380 (2016). https://doi.org/10.1007/s10509-016-2956-0

[60] S. Aygün, Turk. J.Phys. **41**, 436 (2017). https://doi.org/10.3906/fiz-1704-14

[61] S. Aygün, *et al*., Indian J.Phys. **93**, 407 (2019). https://doi.org/10.1007/s12648-018-1309-y

[62] C. Kömürcü, and C. Aktas, Mod. Phys. Lett. A, **2050263**, 1 (2020). https://doi.org/10.1142/S0217732320502636

[63] Aygün, S., *et al*., Pramana J. Phys. **68**, 21 (2007). https://doi.org/10.1007/s12043-007-0002-z

[64] A. Kabak, and S. Aygün, Int. J. Mod. Phys. **9**, 50 (2019).

[65] A. Ali, *et al*., Int. J. innov. sci. math. **7**, 2347 (2019). https://www.ijism.org/administrator/components/com_jresearch/files/publications/IJISM_798_FINAL.pdf

[66] K.P. Singh, *et al*., Journal of Physics A: Mathematical, Nuclear and General, **6** 1090, (1973). https://doi.org/10.1088/0305-4470/6/7/029

[67] S. Prakash, Astrophys. space sci. **111**, 383, (1985). https://doi.org/10.1007/BF00649977

[68] A.R. Roy, and B. Chatterjee, Acta Phys. Acad. Sci. Hung. **48**, 383, (1980). https://doi.org/10.1007/BF03155547

[69] B. Mukherjee, Heavy Ion Phys. **18**, 115 (2003).

[70] D.D. Pawar, and Y.S. Solanke, arXiv:1602.05222, (2016). https://arxiv.org/abs/1602.05222

[71] D.D. Pawar, and M.K. Panpatte, Prespacetime Journal, **7**, 1187 (2016). https://prespacetime.com/index.php/pst/issue/view/83

[72] D.D. Pawar, *et al*., New Astrnomy, **87**, 101599 (2021). https://doi.org/10.1016/j.newast.2021.101599

[73] M.V. Santhi, *et al*., MSEA, **71** 1056 (2022).

[74] M.V. Santhi, *et al*., Indian J. Phys. **97**, 1641 (2022). https://doi.org/10.1007/s12648-022-02515-9

[75] M.V. Santhi, and T. Chinnappalanaidu, Int. J. Geom. Methods Mod. **19** 2250211 (2022). https://doi.org/10.1142/S0219887822502115

[76] J.R. Wilson, G.J. Mathews, and G.M. Fuller, Phys. Rev. D, **75** 043521 (2007). https://doi.org/10.1103/PhysRevD.75.043521

[77] C.B. Collins, *et al*., Gen. Relativ. Gravit. **12**, 805 (1980). https://doi.org/10.1007/BF00763057

[78] R.K. Mishra, *et al*., Int. J. Theor. Phys. **55**, 1241 (2016). https://doi.org/10.1007/s10773-015-2766-0

[79] M.V. Santhi, *et al*. Can. J. Phys. **95**, 381 (2017). https://doi.org/10.1139/cjp-2016-0781

[80] V.U.M. Rao, and U.Y. Divya Prasanthi, Eur. Phys. J. Plus, **132**, 64 (2017). https://doi.org/10.1140/epjp/i2017-11328-9

[81] D.R.K. Reddy, *et al*., arXiv:1601.02648 (2016). https://arxiv.org/abs/1601.02648

[82] T. Chiba, and T. Nakamura, Prog. Theor. Phys. **100**, 1077 (1998). https://doi.org/10.1143/PTP.100.1077

[83] M. Visser, Class. Quantum Gravity, **21**, 2603 (2004). 10.1088/0264-9381/21/11/006

[84] V. Sahni, *et al*., J. Exp. and Theor. Phys. Lett. **77**, 201 (2003). https://doi.org/10.1134/1.1574831

[85] R. Caldwell, and E.V. Linder, Phys. Rev. Lett. **95**, 141301 (2005). https://doi.org/10.1103/PhysRevLett.95.141301

[86] A. Raychaudhuri, Phys. Rev. **98**, 1123 (1955). https://doi.org/10.1103/PhysRev.98.1123

АНАЛІЗ КОСМОЛОГІЧНОЇ МОДЕЛІ В'ЯЗКОЇ ТЕМНОЇ ЕНЕРГІЇ РІЧЧІ В ЗАГАЛЬНІЙ ТЕОРІЇ ГРАВІТАЦІЇ

Т. Чіннаппаланайду[a], М. Віджая Санті[b], Н. Шрі Лакшмі Судха Рані[b]

[a] *Департамент математики, Інститут інформаційних технологій Віньяна (автономний), Вісакхапатнам 530049, Індія*

[b] *Департамент прикладної математики, Коледж науки і технологій, Університет Андхра, Вісакхапатнам 530003, Індія*

Це дослідження зосереджено на динамічному дослідженні простору-часу типу Мардера, що містить в'язку темну енергію Річчі, в рамках загальної теорії відносності. Щоб знайти розв'язок рівнянь поля, ми використовуємо співвідношення між метричними потенціалами та середнім масштабним коефіцієнтом $a(t) = (\sinh\beta_1 t)^{\frac{1}{\beta_2}}$, це призводить до плавного переходу Всесвіту від початкової фази уповільнення до поточної фази прискорення. Тут ми отримали космологічні параметри та площину $\omega_{de} - \omega'_{de}$ для похідної моделі. Також за допомогою діаграм аналізуються динамічні особливості отриманої космологічної моделі.

Ключові слова: *темна енергія; в'язка темна енергія Річчі; метрика типу Мардера; загальна теорія відносності*