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ANISOTROPIC COSMOLOGICAL MODEL WITH SQM IN $f(R, L_m)$ GRAVITY

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A locally rotationally symmetric Bianchi-I model filled with strange quark matter (SQM) is explored in $f(R, L_m)$ gravity as a non-linear functional of the form $f(R, L_m)=\frac{R}{2}+L_m^{\alpha}$, where α is the free model parameter. We considered the special law of variation of Hubble's parameter proposed by Berman (1983) and also used the power law relation between the scale factors to obtain the exact solution of the field equation, which matches the model of the universe. We also analyze the physical and geometrical aspects of the universe's kinematic and dynamic behavior. Additionally, we employ equation-of-state (EoS) parameters and statefinder parameters as analytical tools to gain insights into the evolution of the universe. We use the ΛCDM model as a benchmark to validate the results. By placing the deviations of the universe from λCDM model and yet making important contributions to the study of the anisotropic nature of $f(R, Lm)$ gravity within the framework of cosmological dynamics, the paper increases our comprehension of our cosmic evolution.

Keywords: *LRS Bianchi type I cosmological model; $f(R, L_m)$ gravity; Strange quark matter; Cosmic time*

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1. INTRODUCTION

Over the last twenty years, plenty of cosmological investigations have come out to suggest that we are living in an accelerated growth phase of the universe. Strong evidence from Type Ia supernovae [1, 2, 3], which are crucial probes of cosmic distances and expansion rates, supports this fast expansion. Moreover, studies on Baryon Acoustic Oscillations (BAO) [4, 5], Wilkinson Microwave Anisotropy Probe [6], the large-scale structure of the universe [7, 8], assessments of galaxy redshifts [9], and examinations of the cosmic microwave background radiation (CMBR) [10, 11] all provide convincing empirical proof for this phenomenon. Collectively, these several lines of evidence indicate the impressive fact that two mysterious substances, referred to as dark matter (DM) and dark energy (DE) with negative pressure, constitute 95% of the total universe [12]. Dark energy is thought to be the driving force causing the expanding universe's noticeable accelerated expansion, while dark matter, a substance that is gravitationally effective but unable to produce light, interacts mostly through gravitational forces. The enormous significance of these mysterious components for determining the evolution and fate of our universe is brought into focus by the convergence of universe facts.

Numerous theoretical descriptions of this acceleration have been suggested in the literature. The concept of dark energy is basically associated with the rapid acceleration of the universe. It can be understood in two different ways. The first one argues that the universe is currently expanding not due to the gravitation force but because of the existence of an unknown force with a negative pressure higher than that of gravitation, and this force is called "dark energy" (DE). The literature proposes time-varying dark energy models such as quintessence [13], k-essence [14, 15], and even the perfect fluid models, especially the Chaplygin gas model [16, 17] as a solution to this problem. Interpreting spacetime's geometry is the second tactic for explaining the universe's acceleration. The left-hand side of the Einstein equation can be changed for this purpose. Modified theories of gravity are the alteration of the Einstein-Hilbert action of general relativity to reach the acceleration of the universe. These theories are geometric extensions of relativity by Einstein. Among the recent developments, cosmologists have been detecting dark energy through the modified gravity theories as an explanation. It is argued that dark energy would be the product of introducing a modification to the force of gravity. Various scientific evidence shows that modified versions of gravity theories are likely to be the reasons for the acceleration of the universe at the early and late stages, forming a consistent picture of the universe. Hence, there are many reasons to search for theories that extend beyond general relativity, and the theories of gravity need to be revised. In the literature, there are several modified theories that have been proposed. A few of the modified theories consist of $f(R)$ gravity, the modification of general relativity by introducing an arbitrary function of the Ricci scalar (R) into the gravitational action [18],the $f(R,T)$ theory, an extension of $f(R)$ gravity coupled with the trace of energy-momentum tensor T [19], $f(G)$ theory where G is the Gauss-Bonnet invariant [20, 21, 22], $f(R,G)$ theory [23, 24], $f(T)$ gravity [25, 26, 27], $f(Q,T)$ theory [28] and $f(R, L_m)$ gravity [29].

The $f(R, L_m)$ gravity [29, 30] is a theory that is based on general relativity, attaching additional terms to the action that are dependent on matter density Lagrangian (L_m) and the Ricci scalar (R), respectively. It is an attempt to overcome

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the arising issues of general relativity and observation, such as the need for dark matter and dark energy to explain universe occurrences. This function $f(R, L_m)$ is likely to be required for several theoretical reasons, such as resolving the cosmological constant puzzle, broadcasting the universe's accelerated expansion, or offering an alternative explanation for gravitational incidents observed at the universe scales. Researchers [31] derived the energy condition and Dolgov-Kawasaki (DK) instability criterion [32] in $f(R, Lm)$ gravity and provided the highly versatile energy requirements that can reduce commonly accepted energy conditions found in f(R) theories of gravity and general relativity with any connection between matter and geometry, non-minimal connection, and non-coupling. Geometry-matter couplings in the presence of scalar fields were discussed in [33]. Kasner-type static, cylindrically symmetric interior string solutions in the $f(R, L_m)$ theory of modified gravity are studied [34]. Some of the researchers discussed various cosmological models [35, 36] and phenomenon of gravitational baryogenesis [37] in $f(R, Lm)$ gravity. Kavya et al. [38] have discussed the anisotropic cosmological model in $f(R, L_m)$ gravity. The universe's accelerating scenarios [39] and warmhole solution [40] have all been investigated recently.

The LRS Bianchi-type I cosmological model is a homogeneous and anisotropic cosmological solution to Einstein's field equations. It specifies a spatially homogeneous universe that allows anisotropic expansion since it experiences different rates of expansion along distinct spatial directions. This model has been extensively studied in the context of both general relativity (GR) and modified gravity theories to understand its implications and test the viability of such theories against observations. Yadav et al. [41] have studied the LRS Bianchi I bulk viscous cosmological model in $f(R,T)$ gravity. Interacting two fluid dark energy radiating cosmological models [42] and power-exponential law models [43] have been investigated in $f(R)$ gravity. Later on, several researchers [44, 45, 46] discussed the various cosmological aspects of the LRS Bianchi type I cosmological model in $f(R,T)$ gravity. Recently, Solanke et al.[47] investigated the LRS Bianchi type-I cosmological model in the $f(Q,T)$ theory of gravity with observational constraints.

This research paper emphasizes the exploration of an exact solution for the LRS Bianchi Type I space-time within the framework of $f(R, L_m)$ gravity, Hubble's law, and incorporating the presence of strange quark matter (SQM). The study aims to advance understanding regarding the universe's dynamics and properties within this gravitational framework. The article is organized as follows: The basic field equation and detailed review of $f(R, L_m)$ modified gravity, including the metric and energy momentum tensor, are given in Sections 2 and 3. Moving to Section 4, efforts are directed to find the exact solution of the $f(R, L_m)$ cosmological model. The subsequent Sections, 5 and 6, covered the details about the strange quark model and some physical parameters respectively, within the framework of the discussed modified gravity theory. The important analytical tool statefinder parameters are discussed in Section 7. The figures and conclusion are summarized in sections 8 and 9.

2. BASIC FIELD EQUATIONS IN $f(R, L_m)$ GRAVITY

The action integral for the framework of $f(R, L_m)$ interpreted with the matter Lagrangian density L_m and the Ricci scalar R is given as,

$$S = \int f(R, L_m)\sqrt{-g}dx^4, \tag{1}$$

where $f(R, L_m)$ is arbitrary function of Ricci scalar R and matter Lagrangian L_m.
By contracting the Ricci tensor R_{mn}, one may get the Ricci scalar R,

$$R = g^{ij}R_{ij} \tag{2}$$

where, the Ricci tensor is defined by,

$$R_{ij} = -\delta_\lambda \Gamma^\lambda_{ij} + \delta_j \Gamma^\lambda_{i\lambda} - \Gamma^\lambda_{\lambda\sigma}\Gamma^\sigma_{ij} + \Gamma^\lambda_{j\sigma}\Gamma^\sigma_{i\lambda} \tag{3}$$

Here $\Gamma^\alpha_{\beta\gamma}$ represents the components of well-known Levi-Civita connection defined by

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}g^{\alpha\lambda}\left(\frac{\delta g_{\gamma\lambda}}{\delta x^\beta} + \frac{\delta g_{\lambda\beta}}{\delta x^\gamma} - \frac{\delta g_{\beta\gamma}}{\delta x^\lambda}\right) \tag{4}$$

The corresponding field equations of $f(R, L_m)$ gravity are obtained by varying the action (1) for metric g_{ij} is given by,

$$f_R(R, L_m)R_{ij} + \left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right)f_R(R, L_m) - \frac{1}{2}\left[f(R, L_m) - f_{L_m}(R, L_m)L_m\right]g_{ij} = \frac{1}{2}f_{L_m}(R, L_m)T_{ij} \tag{5}$$

Where, $f_R(R, L_m) = \frac{\delta f(R,L_m)}{\delta R}$, $f_{L_m}(R, L_m) = \frac{\delta f(R,L_m)}{\delta R_m}$ Here covariant derivative is represented by ∇_i and the energy momentum tensor T_{ij} can be expressed as,

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\,L_m\right)}{\delta g^{ij}} = g_{ij}L_m - 2\frac{\delta L_m}{\delta g^{ij}} \tag{6}$$

Now, from the explicit form of the field equation (5), the covariant divergence of Energy momentum tensor T_{ij} can be obtained as,

$$\nabla^i T_{ij} = 2\nabla^i ln\left[f_{L_m}(R, L_m)\right]\frac{\delta L_m}{\delta g^{ij}} \tag{7}$$

The relation between the trace of energy momentum-tensor T, Ricci scalar R, and the Lagrangian density of the matter L_m obtained by contracting the field equation (5)

$$f_R(R, L_m)R + 3\nabla_i\nabla^i f_R(R, L_m) - 2\left[f(R, L_m) - f_{L_m}(R, L_m)L_m\right] = \frac{1}{2}f_{L_m}(R, L_m)T \tag{8}$$

The relation between the trace of the energy momentum tensor $T = T^i_i$, L_m, and R can be established by taking account of the previously mentioned equation.

3. METRIC AND FIELD EQUATION IN $f(R, L_m)$ GRAVITY

The spatially homogeneous and anisotropic LRS Bianchi type I spacetime can be written in the form of,

$$ds^2 = -dt^2 + L^2dx^2 + M^2\left(dy^2 + dz^2\right) \tag{9}$$

Where L and M are the metric potential that are the functions of cosmic time t only.
The Ricci scalar for LRS Bianchi - I spacetime can be expressed as

$$R = -2\left[\frac{\ddot{L}}{L} + 2\frac{\ddot{M}}{M} + 2\frac{\dot{L}\dot{M}}{LM} + \frac{\dot{M}^2}{M^2}\right] \tag{10}$$

The overhead dot (.) denotes the derivative with respect to time t. The spatial volume V of the universe is defined as

$$V = LM^2 \tag{11}$$

The generalized mean Hubble parameter (H), which describes the space-time expansion rate, can be stated as

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{12}$$

where H_1, H_2, H_3 are the directional Hubble's parameters in the direction of the x-, y-, and z-axes, respectively. In order to figure out whether the models approach isotropy or not, we define the expansion's anisotropy parameter as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{13}$$

The expansion scalar and shear scalar are defined as follows:

$$\theta = u^i_{;i} = \frac{\dot{L}}{L} + 2\frac{\dot{M}}{M} \tag{14}$$

$$\sigma^2 = \frac{3}{2}H^2A_m \tag{15}$$

Let us take the matter that contains the energy momentum tensor for quark matter, which is of the form

$$T_i^{j(quark)} = (p+\rho)u^ju_i + pg_i^j = diag\,(-\rho, p, p, p) \tag{16}$$

where $\rho = \rho_q + B_c$ is a quark matter total energy density, $p = p_q - B_c$ is the quark matter total pressure, and u_i is the four-velocity vector such that $u_iu^i = -1$.
The EoS parameter for quark matter is defined as,

$$p_q = \omega\rho_q, \quad 0 \le \omega \le 1 \tag{17}$$

The linear equation of state for strange quark matter is provided by

$$p = \omega(\rho - \rho_0) \tag{18}$$

where ω is constant and ρ_0 is the energy density at zero pressure. When $\omega = \frac{1}{3}$ and $\rho_0 = 4B_c$, in the bag model, strange quark matter changes the above linear equation of state to the one that follows EoS.

$$p = \frac{(\rho - 4B_c)}{3} \tag{19}$$

where B_c is the bag constant.
By using the help of equation (16), the field equation (5) can be translated into the action of metric (10) in the co-moving coordinate system as,

$$-\left(\frac{\ddot{L}}{L} + 2\frac{\dot{L}\dot{M}}{LM}\right) f_R - \frac{1}{2}\left(f - f_{L_m} L_m\right) - 2\frac{\dot{M}}{M}\dot{f}_R - \ddot{f}_R = \frac{1}{2} f_{L_m}\left(p_q - B_c\right) \tag{20}$$

$$-\left(\frac{\ddot{M}}{M} + \frac{\dot{M}^2}{M^2} + \frac{\dot{L}\dot{M}}{LM}\right) f_R - \frac{1}{2}\left(f - f_{L_m} L_m\right) - \left(\frac{\dot{L}}{L} + \frac{\dot{M}}{M}\right)\dot{f}_R - \ddot{f}_R = \frac{1}{2} f_{L_m}\left(p_q - B_c\right) \tag{21}$$

$$-\left(\frac{\ddot{L}}{L} + 2\frac{\ddot{M}}{M}\right) f_R - \frac{1}{2}\left(f - f_{L_m} L_m\right) - \left(\frac{\dot{L}}{L} + 2\frac{\dot{M}}{M}\right)\dot{f}_R - \ddot{f}_R = -\frac{1}{2} f_{L_m}\left(\rho_q + B_c\right) \tag{22}$$

4. COSMOLOGICAL $f(R, L_m)$ MODEL

In the present study,to examine the dynamics of the cosmological model in $f(R, L_m)$ gravity, we use the relation between R and L_m [38]

$$f(R, L_m) = \frac{R}{2} + L_m^{\alpha} \tag{23}$$

where $\alpha \neq 0$ is a parameter and one can retain GR for $\alpha = 1$.
For this particular $f(R, L_m)$ model, we have to consider $L_m = \rho$ [48]
Using the above particular choice of L_m, the field equations (20),(21) and (22) becomes,

$$2\frac{\ddot{M}}{M} + \frac{\dot{M}^2}{M^2} - (1-\alpha)\left(\rho_q + B_c\right)^{\alpha} = \alpha\left(\rho_q + B_c\right)^{\alpha-1}\left(p_q - B_c\right) \tag{24}$$

$$\frac{\ddot{L}}{L} + \frac{\ddot{M}}{M} + \frac{\dot{L}\dot{M}}{LM} - (1-\alpha)\left(\rho_q + B_c\right)^{\alpha} = \alpha\alpha\left(\rho_q + B_c\right)^{\alpha-1}\left(p_q - B_c\right) \tag{25}$$

$$\frac{\dot{M}^2}{M^2} + 2\frac{\dot{L}\dot{M}}{LM} = (1-2\alpha)\left(\rho_q + B_c\right)^{\alpha} \tag{26}$$

The field equations (24), (25) and (26) are three independent differential equations with four unknowns: L, M, ρ_q, and p_q. Hence, to determine solutions, we have to use physically plausible conditions.

Berman [49] indicate that there exists a connection between the deceleration parameter as well as the average scale factor given as,

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{27}$$

Here, a is the average scale factor with $a = \left(LM^2\right)^{\frac{1}{3}}$ and q is the deceleration parameter. If we use Hubble's law and relate Hubble's parameter H to the average scale factor a then we get a constant value of the deceleration parameter q.
Hence the Hubble's law gives,

$$H = ba^{-m} \tag{28}$$

where b and m are constants.Also, Hubble's parameter (12) can be written as

$$H = \frac{1}{3}\left(\frac{\dot{L}}{L} + 2\frac{\dot{M}}{M}\right) = \frac{\dot{a}}{a} \tag{29}$$

Using equation (29), we can re-write equation (28) as

$$\dot{a} = ba^{-m+1} \tag{30}$$

Using the equations (28), (29), and (30) in (27), we get

$$q = -m+1 \tag{31}$$

This equation demonstrates that the deceleration parameter is going to stay constant, whatever its significance, and regardless of whether the value is positive or negative. The standard deceleration model is indicated by positive values of the deceleration parameters. Negative numbers, on the other hand, lead the model to accelerate or lead to inflation. on solving the equation (28) with the help of equation (29), we get

$$a = (ct+d)^{\frac{1}{q+1}}, q \neq -1 \tag{32}$$

considering that d is the integration constant and $c \neq 0$.Using equation (**??**) and $LM^2 = a^3$, we can obtain,

$$LM^2 = (ct+d)^{\frac{3}{q+1}}, q \neq -1 \tag{33}$$

In order to obtain a favorable solution to the field equations, we have to consider the constraining equation. Here we presume the anisotropic relation can be written in terms of expansion scalar (θ) and shear scalar (σ)as,

$$\sigma \propto \theta$$

With reference to the scale factors L and M, the above assumption leads to the following anisotropic relation:

$$L = M^k \tag{34}$$

where $k \neq 1$ is an arbitrary constant. The model becomes isotropic if $k = 1$, indicating that the distribution of matter in the universe is homogeneous; otherwise, it turns out to be anisotropic.
Using the equation (34), equation (33) implies that

$$L = (ct+d)^{\frac{3k}{(q+1)(k+2)}} \tag{35}$$

$$M = (ct+d)^{\frac{3}{(q+1)(k+2)}} \tag{36}$$

Equations (35) and (36) indicate that the model's metric potentials L and M are time-dependent functions that rise with time at $q > -1,\ k \neq -2$ and fall with time at $q < -1, k \neq -2$; they also do not exist at $q = -1$ or $k = -2$. Moreover, it is important to note that for $q > -1, k \neq -2$, these parameters begin at a constant value, but at the point $t = -\frac{d}{c}$, they start at zero, indicating that the model exhibits point-type singularity at that point.
Thus, the metric (9) with the help of equations (35) and (36) can be written as,

$$ds^2 = -dt^2 + (ct+d)^{\frac{6k}{(q+1)(k+2)}} dx^2 + (ct+d)^{\frac{6}{(q+1)(k+2)}} \left(dy^2 + dz^2\right) \tag{37}$$

Equation (37) represents the homogeneous anisotropic plane symmetric cosmological model with quark and strange quark matter in the framework of $f(R, L_m)$ gravity. The model increases with time for the constants $q < -1, k \neq -2$ and has a singularity at the point $t = -\frac{d}{c}$

5. STRANGE QUARK MATTER FOR COSMOLOGICAL MODEL

From the equations (25) and (26), with the help of metric potential, the energy density and pressure of strange quark matter are given as,

$$\rho = \left[\frac{9(1+2k)c^2}{(1-2\alpha)(q+1)^2(k+2)^2(ct+d)^2}\right]^{\frac{1}{\alpha}} \tag{38}$$

$$p = D\left[\frac{9c^2(1+2k)^{1-\alpha}}{(1-2\alpha)(q+1)^2(k+2)^2(ct+d)^2}\right]^{\frac{1}{\alpha}} \tag{39}$$

where, $D = \frac{[(2\alpha-1)q+(2-4\alpha)]k^2+[3q(2\alpha-1)+6(\alpha-1)]k+[2(2\alpha-1)q+(\alpha-2)]}{3\alpha}$

Using the above equations, the pressure and energy density of the quark matter as follows:

$$\rho_q = \left[\frac{9(1+2k)c^2}{(1-2\alpha)(q+1)^2(k+2)^2(ct+d)^2}\right]^{\frac{1}{\alpha}} - B_c \tag{40}$$

$$p_q = D\left[\frac{9c^2(1+2k)^{1-\alpha}}{(1-2\alpha)(q+1)^2(k+2)^2(ct+d)^2}\right]^{\frac{1}{\alpha}} + B_c \tag{41}$$

Using the equations (38) and (39), the equation of state (EoS) for strange quark matter and quark matter are given as

$$\omega = \frac{D}{(1+2k)} \tag{42}$$

$$\omega_q = \frac{D\left[\frac{9c^2(1+2k)^{1-\alpha}}{(1-2\alpha)(q+1)^2(k+2)^2(ct+d)^2}\right]^{\frac{1}{\alpha}} + B_c}{\left[\frac{9(1+2k)c^2}{(1-2\alpha)(q+1)^2(k+2)^2(ct+d)^2}\right]^{\frac{1}{\alpha}} - B_c} \tag{43}$$

6. SOME PHYSICAL PARAMETERS

The spatial volume V of the universe is given as

$$V = (ct+d)^{\frac{3}{q+1}} \tag{44}$$

The spatial volume of the universe increases with increasing cosmic time, starting with a constant value at $t = 0$ and with a big bang at $t = -\frac{d}{c}$. As a result of this approach, inflation. This illustrates that the universe begins to evolve at zero volume and grows over cosmic time. The mean generalized Hubble's parameter (29) of the model is given by

$$H = \frac{c}{(q+1)(ct+d)} \tag{45}$$

The expansion scalar of the model turns out to be

$$\theta = \frac{3c}{(q+1)(ct+d)} \tag{46}$$

At the initial stage, both the Hubble's parameter and the expansion scalar are constant and approach zero steadily at $t \to \infty$, but at $t = -\frac{d}{c}$ both are infinitely large. The mean anisotropic parameter of the model is given as

$$A_m = \frac{2k^2 - 4k + 2}{k^2 + 4k + 4} \tag{47}$$

The shear scalar of the model is represented as

$$\sigma^2 = \frac{3c^2}{(q+1)^2}\frac{k^2 - 2k + 1}{k^2 + 4k + 4}\frac{1}{(ct+d)^2} \tag{48}$$

The shear scalar, the scalar expansion, and the Hubble parameter are all the functions of time that are rapidly decreasing with the increase of cosmic time and getting closer to zero in the later stages. This fact reveals that in the earliest stages of the universe, the rate of expansion was very high for a while, but gradually it became slower. This shows that the evolution of the universe starts at an infinite rate, but with expansion, it declines.

7. STATEFINDER PARAMETERS

The so-called cosmic acceleration may arise from a quite wide range of dark energy models, many of which are distinguishable by the utilization of the statefinder diagnostic tool. It is a model-free way of quantifying the dark energy intrinsic properties of higher derivatives to the scale factor. Through employing the cosmic statefinder diagnostic fiction pair $\{r, s\}$, the technique permits research to investigate dark energy properties, free of any particular models. The statefinder parameters are defined as [50, 51].

$$r = \frac{1}{aH^3}\frac{d^3}{dt^3}(a), \quad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{49}$$

Identifying between different cosmological domains is mostly dependent on the paths in the $\{r, s\}$ plane. For example, in the $\{r, s\}$ plane, the λCDM model is characterized by the point $(r = 1, s = 0)$, Standard Cold Dark Matter is for $(r = 1, s = 1)$, and the holographic DE model is represented by $(r = 1, s = \frac{2}{3})$. The phantom region is associated with $(r > 1, s < 0)$, and the quintessence region is identified by $(r < 1, s > 0)$. with the help of equations (32), (44) and (45), the equation (49) becomes

$$\{r, s\} = \{2q^2 + q, \frac{2}{3}(q+1)\} \tag{50}$$

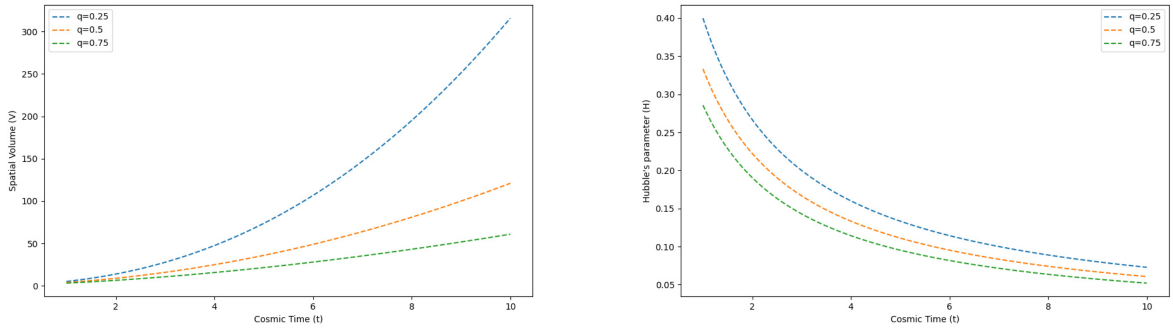
We can see that for a given model, q, r, s are constant. For different values of q, we have different expansion factors, which can be analyzed in the following Table 1.

Table 1. Description of Models

q	r	s	Type of Model
0.5	1	1	SCDM
-0.5	0	$\frac{1}{3}$	quintessence
-1	1	0	λ CDM
-2	6	$-\frac{2}{3}$	Phantom

8. FIGURES

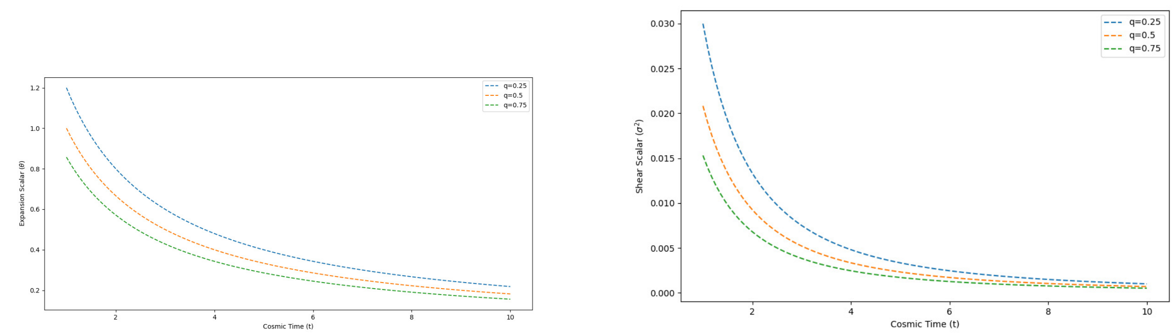
In this section, in order to gain a deeper insight into our cosmological model, let us plot different physical and dynamic parameters against cosmic time.



(a) **(b)**

Figure 1. The Variation of Spatial volume (V) *(left)* and Hubble's parameter (H) *(right)* as a function of cosmic time (t) are shown. V,H and t are in arbitrary units. To derive above plot we have used $c = d = 1$.

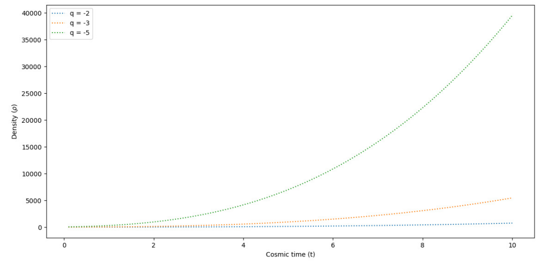
- The graph of volume against cosmic time is increasing in nature (Fig. 1(a)), and that of the Hubble parameter H is a decreasing function of cosmic time (t) in the positive region (Fig. 1(b)). From it, we collect important insights about the expansion of the universe. This observation serves as a necessary foundation for our understanding of the universe's dynamics.



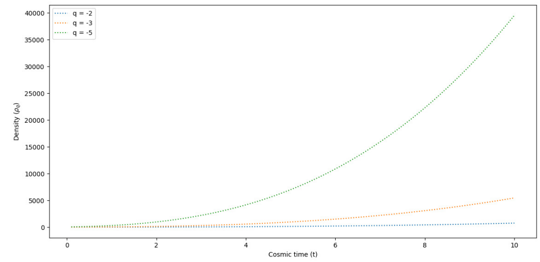
(a) **(b)**

Figure 2. Variation of Expansion scalar (θ) *(left)* and Shear scalar (σ^2) *(right)* as a function of cosmic time (t) are shown. θ ,σ^2 and t are in arbitrary units. To derive above plot we have used $c = d = 1$.

- We found that the expansion scalar (Fig. 2(a)) and shear scalar (Fig. 2(b)) are the diminishing functions of cosmic time (t).



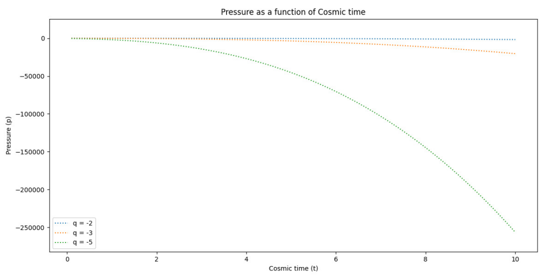
(a)



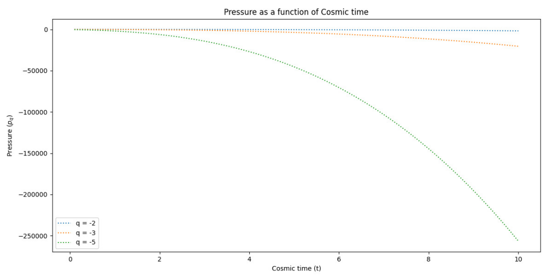
(b)

Figure 3. Variations of energy density of strange quark matter(ρ) *(left)* and quark matter(ρ_q) *(right)* as a function of cosmic time (t) for $q < -1$. All quantities are in arbitrary units. These plots are derived using $c = d = B_c = 1, k = 2$.

- The energy density of strange quark matter (ρ) and quark matter (ρ_q) both appear constant at the initial stage and becomes infinite as t tends to infinity, as shown in Fig. 3.



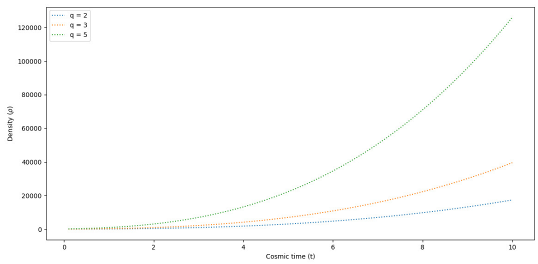
(a)



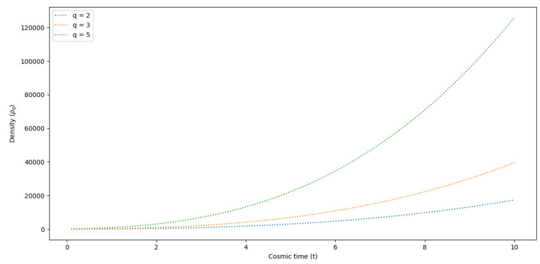
(b)

Figure 4. Variations of pressure of strange quark matter(p) and quark matter(p_q) as a function of cosmic time (t) for $q < -1$. All quantities are in arbitrary units. These plots are derived using $c = d = B_c = 1, k = 2$.

- The pressure of strange quark matter (p) and quark matter (p_q), both decreasing functions of time, remains negative throughout the evolution, as shown in Fig. 4.



(a)



(b)

Figure 5. Variations of energy density of strange quark matter (ρ) and quark matter (ρ_q) as a function of cosmic time (t) for $q > -1$. All quantities are in arbitrary units.These plots are derived using $c = d = B_c = 1, k = 2$.

9. CONCLUSIONS

In this article, we explore an accelerating model of the universe in the context of the $f(R, L_m)$ theory of gravity as a non-linear functional of the form $f(R, L_m)=\frac{R}{2} + L_m^{\alpha}$, where α is the free model parameter. Utilizing a unique formulation of the deceleration parameter, we derive cosmological solutions that closely resemble the characteristics of the dark energy-driven λCDM model. We assumed a power law relation between the scale factors. We also considered the special law of variation of Hubble's parameter proposed by Berman (1983), which yields the constant deceleration parameter.The findings of the research are very interesting and ultimately result in the following conclusions:

Around $t = 0$, the metric potentials L and M are constant, and then both vanish. This brings one to the conclusion that the model exhibits an initial singularity at $t = -\frac{d}{c}$. As a result, similar to the standard Big Bang theory, the values of L and M increase steadily over time. At a singular point, the model is similar to the work of [52]. We discovered that the spatial volume (V), expansion scalar (θ), shear scalar (σ^2), and mean Hubble's parameter (H) are all functions of cosmic time (t). These parameters tend to zero as t tends to infinity $(t \to \infty)$, but they diverge with the exception of spatial volume when cosmic time approaches $t = -\frac{c}{d}$, as shown in Fig.(1) and Fig.(2) The spatial volume (V) of the model is zero when cosmic time is $t = -\frac{d}{c}$. Depending on the value of q we have the following two cases:

case(i)*when* $q < -1$: The proposed model starts expanding with the Big Bang singularity at $t = -\frac{d}{c}$. At $t = 0$, both the pressure p and energy density ρ of strange quark matter are constant, and at $t \to \infty$, both p and ρ become infinite, as shown in Fig. 3(a) and Fig. 4(a). The pressure of strange quark matter (p) and quark matter (p_q), both decreasing functions of time, remains negative throughout the evolution. Negative pressure (p) and (p_q) corresponds to the accelerating expansion of the universe. Also, the pressure p_q and energy density ρ_q for quark matter behave the same as for strange quark matter. The shift in the ρ values than that of p_q is due to the additional term of bag constant in equation (40). In this study, we chose the bag constant as unity, as shown in Fig. 3(b) and Fig. 4(b).

case (ii) *when* $q > -1$: At cosmic time $t = 0$, the proposed model has constant volume, which increases with an increase in time and becomes infinite at $t \to \infty$. At $t = 0$, energy density ρ of strange quark matter are constant, and at $t \to \infty$, it become infinite, as shown in Fig. 5(a) and Fig. 5(b).The energy and pressure profiles for quark and strange quark matter are the same except from the extra bag constant. The bag constant is subtracted for energy density and added for the pressure of quarks.

For $\alpha < 0$, equations (38) to (41) give the real value of pressure and energy density for quark matter and strange quark matter (SQM), and values turn out to be complex for $\alpha > 0$. The anisotropic parameter A_m is nonzero for $k \neq 1$ provided $k \neq -2$, and in such a case, the model does not approach isotropy, but for $k = 1$ provided $k \neq -2$, the mean anisotropic parameter is zero, and the model becomes isotropic. Moreover, the mean anisotropic parameter (A_m) remains constant throughout the evolution of the universe as it is independent of the cosmic time (t). Regarding the current statefinder parameters, the value $\{r, s\}$=$\{1, 0\}$ generated by our investigation is in the same line with the λCDM model, which is very close to the recent data [53, 54]

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АНІЗОТРОПНА КОСМОЛОГІЧНА МОДЕЛЬ ІЗ SQM У $f(R, L_m)$ ГРАВІТАЦІЇ

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Локально обертально-симетрична модель Bianchi-I, заповнена дивною кварковою матерією (SQM), досліджується в $f(R, L_m)$ гравітації як нелінійний функціонал у формі $f(R, L_m)=\frac{R}{2} + L_m^{\alpha}$, де α — вільний параметр моделі. Ми розглянули спеціальний закон зміни параметра Хаббла, запропонований Берманом (1983), а також використали степеневий зв'язок між масштабними факторами, щоб отримати точний розв'язок рівняння поля, який відповідає моделі Всесвіту. Ми також аналізуємо фізичні та геометричні аспекти кінематичної та динамічної поведінки Всесвіту. Крім того, ми використовуємо параметри рівняння стану (EoS) і параметри визначення стану як аналітичні інструменти, щоб отримати уявлення про еволюцію Всесвіту. Ми використовуємо модель ΛCDM як еталон для перевірки результатів. Розміщуючи відхилення Всесвіту від моделі λCDM і водночас роблячи важливий внесок у дослідження анізотропної природи $f(R, Lm)$ гравітації в рамках космологічної динаміки, стаття покращує наше розуміння нашої космічної еволюції.

Ключові слова: *космологічна модель LRS типу Bianchi-I; $f(R, L_m)$ гравітація; дивна кваркова матерія; космічний час*